



CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE
NAME

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CLASS

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INDEX
NUMBER

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CENTRE/
INDEX NO

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ADDITIONAL MATHEMATICS

Paper 1

4049/01

13 September 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 84.

For Examiner's Use

84

This document consists of **21** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \dots (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Answer **all** the questions.

- 1 P and Q are the points of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$, ($a > 0, b > 0$), with the x and y axes respectively. The distance PQ is 10 and the gradient of PQ is -2 . Find the exact value of a and of b .

[5]

- 2 The polynomial $P(x) = ax^3 + bx^2 - 19x + 4$, where a and b are constants, has a factor $x + 4$ and is such that $2P(1) = 5P(0)$.

(a) Find the value of a and of b . [4]

(b) Hence, factorise $P(x)$ completely.

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- 3 A function is defined by the equation $y = \frac{2x^2}{x-1}$, $x \neq 1$.

(a) Show that y is increasing for $x > 2$. [3]

(b) If y is increasing at the rate of 2 units per second, find the rate of change of x when $x = 3$. [2]

- 4 Consider the expansion $\left(\frac{a^2}{\sqrt{x}} - \frac{\sqrt{x}}{a}\right)^8$ where a is a positive constant.

Find, in terms of a ,

- (a) the term independent of x .

[2]

- (b) the coefficient of x^2 in the expansion of $\left(\frac{3x^4 - 4x^2}{x^2}\right)\left(\frac{a^2}{\sqrt{x}} - \frac{\sqrt{x}}{a}\right)^8$.

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[4]
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- 5 There are approximately 5 times as many animals of Species A as animals of Species B in a nature reserve.
The population of animals of Species A decreases at a rate of 5% per year and the population of animals of Species B increases at a rate of 10% per year.

It is given that N is the number of animals of Species B in the nature reserve.

(a) Write down, in terms of N ,

- (i) an expression, P_A , to model the population of animals of Species A after t years, [1]

- (ii) write down an expression, P_B , to model the population of animals of Species B after t years. [1]

- (b) Using your models, find the number of years that will elapse before the population of animals from Species A is equal to the population of animals from Species B . [3]

- 6 (a) Solve $\tan 3x = 0$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ giving your answers in terms of π . [2]

- (b) Use your answers to part (a) to sketch the graph of $y = 4 \tan 3x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ on the axes below. Show the coordinates of the points where the curve meets the axes.

y

[3]
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- (c) On the same diagram given in part (b), sketch the graph of $y = \sin x + 1$ for the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. Hence state the number of solutions of the equation

$$4 \tan 3x - \sin x = 1 \text{ in the interval } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}. \quad [3]$$

- 7 (a) Show that $2 \sin(A + 45^\circ) \cos(A + 45^\circ) = \cos 2A$. [3]

- (b) Hence, solve the equation $\sin(A + 45^\circ) = \frac{1}{8} \sec(A + 45^\circ)$, for $0^\circ \leq A \leq 360^\circ$. [4]

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- 8 A curve is such that $\frac{d^2y}{dx^2} = 5 \cos 2x$. This curve has a gradient of $\frac{3}{4}$ at the point $\left(-\frac{\pi}{12}, \frac{5\pi}{4}\right)$.

Show that the equation of the curve is $y = -\frac{5}{4} \cos 2x + 2x + \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}$. [6]

9 (a) Factorise $x^3 + y^3$.

[1]

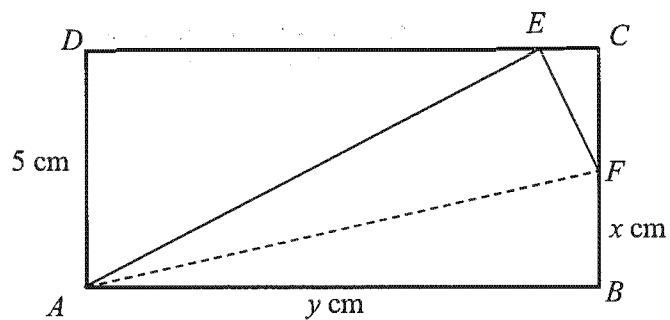
(b) Hence, prove the identity $\frac{\sin^3 \theta + \cos^3 \theta}{2 \sin^2 \theta - 1} = \frac{\sec \theta - \sin \theta}{\tan \theta - 1}$.

[4]

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Continuation of working space for Question (9b).

10



The diagram shows a rectangular piece of paper $ABCD$ such that $AD = 5$ cm, $BF = x$ cm and $AB = y$ cm. The paper is folded along AF such that B meets E on DC .

- (a) Express EC and AE in terms of x .

[4]

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- (b) Hence show that the area, A cm², of triangle AEF is given by

$$A = \frac{5x^2}{2\sqrt{10x-25}}.$$

[2]

- (c) Show that $x = \frac{10}{3}$ for which $\frac{dA}{dx} = 0$. [3]

- (d) By considering the change in the sign of $\frac{dA}{dx}$, show that A is a minimum at $x = \frac{10}{3}$. Hence find the minimum value of A . [3]

- 11 The height, in metres, of a ball above the ground t seconds after it is thrown from a tower is given by the function $h(t) = -5t^2 + 10t + c$, where c is a constant.

(a) (i) Given that $h(0) = 6$, find the value of c . [1]

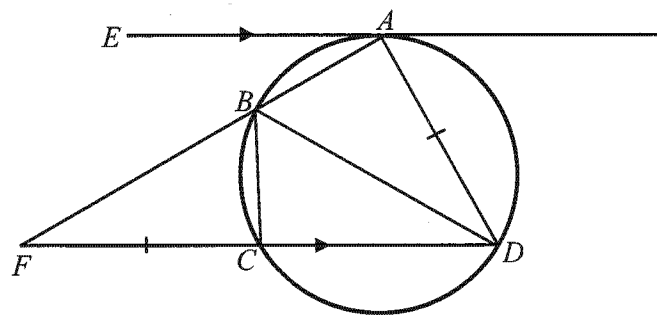
(ii) Interpret the meaning of the answer in (a) (i). [1]

(b) **Without the use of differentiation** and using the value of c from (a) (i), determine the maximum height reached by the ball and the corresponding value of t . [4]

Continuation of working space for Question (11b).

- (c) Find the time it takes for the ball to drop to the ground. [2]

12



In the diagram above, EA is the tangent to the circle at A .
 FBA and FCD are straight lines.
 EA is parallel to FD and $FC = AD$.

(a) Show that $\angle BAD = \angle BCF$.

[2]

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- (b) Hence, prove that triangle ABD is congruent to triangle CBF . [4]

- 13** A particle starts from rest at a fixed point O and moves in a straight line with its acceleration, $a \text{ m/s}^2$, is given by $a = 9 - kt$, where t seconds is the time since leaving O , and k is a constant. When $t = 2$, its velocity is 15 m/s .

(a) Find the value of k . [3]

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(b) The particle changes its direction of motion when $t = T$.
Find the value of T . [2]

- (c) Show that the particle returns to O when $t = 18$.

[3]

- (d) Hence find the distance travelled by the particle between $t = 0$ and $t = 18$. [3]

END OF PAPER

