2021 Prelim P2

1 A piece of wire, 28 cm in length is cut into two parts. One part is used to make a rectangle and the other a square. The length of the rectangle is three times its width. The width of the rectangle is x cm.

(a) (i) Write down an expression, in terms of x, for the perimeter of the rectangle.

$P = 2(L+B)$

$= 2(3x + x)$

$= 8x$

Answer $8x$ .... [1]

(ii) Find, and simplify, an expression, in terms of x, for the length of a side of the square.

$P = 4L$

$28 - 8x = 4L$

$L = \frac{28-8x}{4}$

$= 7 - 2x$

Answer $7 - 2x$ .... [1]

(b) It is given that the areas of the rectangle and the square are equal.

(i) Form an equation in x and show that it reduces to $x^2 - 28x + 49 = 0$. [3]

Answer

$(7-2x)^2 = 3x \times x$

$49 - 28x + 4x^2 = 3x^2$

$x^2 - 28x + 49 = 0$

(ii) Solve the equation $x^2 - 28x + 49 = 0$, giving your answers correct to three decimal places.

$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(49)}}{2(1)}$

$= 26.1243, \; 1.8756$

$= 26.124, \; 1.876$

Answer .... , ... [3]

(iii) Calculate the area of the square.

area $= L^2$

$= (7-2x)^2$

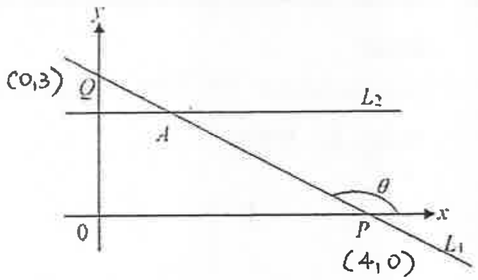
$= [7 - 2(1.876)]^2$

$= 10.549$

$= 10.5$

Answer 10.5 .... cm^2 [1]

2 In the diagram, line L_1 cuts the x-axis at P (4, 0) and the y-axis at Q (0, 3). On the same axes, line L_2, which is parallel to the x-axis, meets line L_1 at A.



(a) Write down, as a fraction in its lowest terms, the exact value of $\cos\theta$.

$\cos\theta = -\frac{4}{5}$

Answer $-\frac{4}{5}$ [1]

(b) Find the equation of line L_1.

grad $= \frac{3-0}{0-4}$

$= -\frac{3}{4}$

$\therefore y = -\frac{3}{4}x + 3$

Answer $y = -\frac{3}{4}x + 3$ [2]

[Turn Over

(c) M lies on the line PQ. The shortest distances, d units, from M to each of the axes are the same. Find the value of d.

Let $M(d, d)$

$y = -\frac{3}{4}x + 3$

$d = -\frac{3}{4}d + 3$

$\frac{7}{4}d = 3$

$d = \frac{12}{7}$

Answer $\frac{12}{7}$ units [1]

(d) R and S are points on the diagram. R lies on the x-axis and $QRPS$ is a parallelogram. Given that the area of $QRPS$ is 6 units², find the coordinates of R.

area = 6

$b \times h = 6$

$b \times 3 = 6$

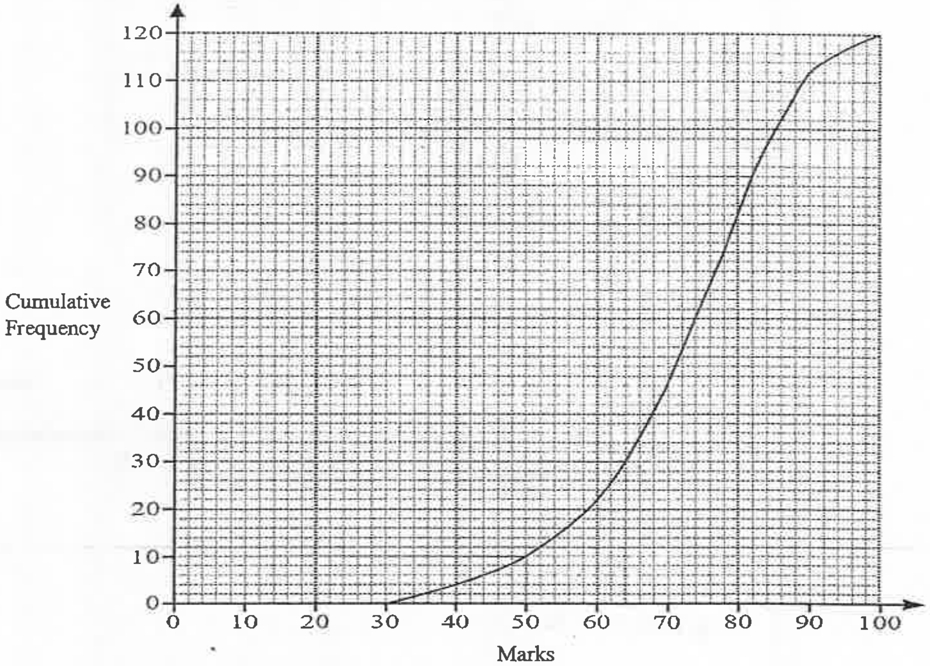
$b = 2$

$\therefore R(4-2, 0)$

$= R(2, 0)$

Answer (2, 0) [2]

3 The cumulative frequency curve shows the marks obtained by students in an examination in May. The maximum mark was 100.



(a) Find

(i) the median mark,

Answer 74 [1]

(ii) the interquartile range.

IQR = 82 - 64

= 18

Answer 18 [2]

(b) If the pass mark was 50, how many students passed the examination?

No passed = 120 - 10

= 110

Answer 110 [1]

[Turn Over

(c) The same group of students took another examination in August. The maximum mark was 100. The median mark was 70. The interquartile range was 5 marks.

(i) In which examination were the marks better? Give your reason. [1]

Answer

Examination in May because the median in May is higher

(ii) Which examination had a greater spread of the marks? Give your reason. [1]

Answer

Examination in May because the interquartile range in May is higher

(iii) If the marks of the examination in August were revised by the formula $y = \frac{6}{5}x$, where x was the original marks and y was the corresponding new marks, find the revised interquartile range.

Answer 6 [1]

(d) The marks for another examination in May is represented in the following frequency table.

Marks, x	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$
Number of students	12	72	24	12

Calculate

(i) the estimated mean of the marks,

$$\text{mean} = \frac{12\times15+72\times25+24\times35+12\times45}{120}$$

$$= \frac{3360}{120}$$

$$= 28$$

Answer 28 [2]

(ii) the estimated standard deviation of the marks.

$$SD = \sqrt{\frac{12\times15^2+72\times25^2+24\times35^2+12\times45^2}{120} - 28^2}$$

$$= \sqrt{61}$$

$$= 7.8102$$

$$= 7.81$$

Answer 7.81 [2]

[Turn Over

4 (a) $\mathscr{E} = \{a : a \text{ is a natural number}, a < 9\}$.
$P = \{a : a \text{ is divisible by } 4\}$ and $Q = \{a : a \text{ is a perfect cube}\}$.

(i) List the element(s) of

(a) $P \cap Q$,

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$P = \{4, 8\}$

$Q = \{8\}$

$P \cap Q = \{8\}$

Answer $\{8\}$ [1]

(b) $(P \cup Q)'$.

$P \cup Q = \{1, 4, 8\}$

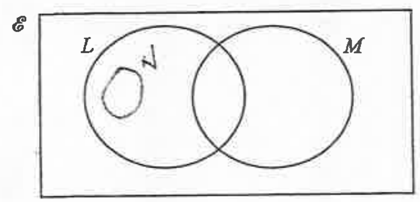
$(P \cup Q)' = \{2, 3, 5, 6, 7\}$

Answer $\{2, 3, 5, 6, 7\}$ [1]

(ii) Write down all the subsets of P.

Answer ϕ, $\{4\}$, $\{8\}$, $\{4, 8\}$ [1]

(b) On the Venn diagram, add a set N such that $N \subset L$ and $N \cap M = \phi$. [1]



(c) The table shows the result obtained by 80 students in a test.

	Grade A	Grade B	Grade C	Total
Boys	12	28	8	48
Girls	4	8	20	32
Total	16	36	28	80

(i) A student is randomly selected from the 80 students. Find the probability that

(a) the selected student is a boy who has obtained grade A,

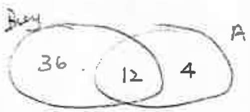
$P(\text{Boy and A}) = \frac{12}{80}$

$= \frac{3}{20}$

Answer $\frac{3}{20}$ [1]

(b) the selected student is a boy or the selected student result is grade A.

$P(\text{Boy or A student}) = \frac{48}{80} + \frac{16}{80} - \frac{12}{80}$



$= \frac{13}{20}$

Answer $\frac{13}{20}$. [1]

(ii) Two students are randomly selected from the 80 students. Find the probability that the selected students both obtained the same grade.

$P(\text{same grade})$

$= \frac{16}{80} \times \frac{15}{79} + \frac{36}{80} \times \frac{35}{79} + \frac{28}{80} \times \frac{27}{79}$

$= \frac{141}{395}$

Answer $\frac{141}{395}$ [2]

[Turn Over

(d) John throws a fair six-sided die until he obtains a '6'. Find, as simply as possible and in terms of n, the probability that

(i) John fails to obtain a '6' in his first n throws,

$P(\text{not } 6) : \left(\frac{5}{6}\right)^n$

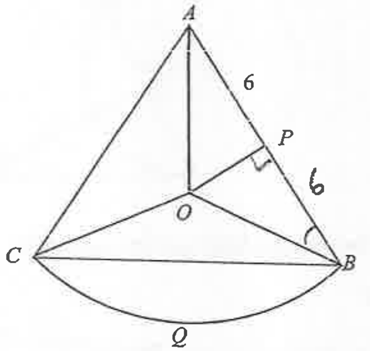
Answer $\left(\frac{5}{6}\right)^n$ [1]

(ii) John obtains a '6' in one of his first n throws.

$P(6) = 1 - \left(\frac{5}{6}\right)^n$

Answer $1 - \left(\frac{5}{6}\right)^n$ [1]

5 In the diagram, triangles OAC, OAB and OCB are congruent and $OA = OB = OC$. O is the centre of the sector $OBQC$. P is a point on AB such that OP is perpendicular to AB and $AP = 6$ cm.



(a) Show that angle $ABO = \frac{\pi}{6}$ radians. State your reasons clearly. [2]

Answer

$\angle AOB = \frac{2\pi}{3}$ ($\angle$s at a pt)

$\angle ABO = \dfrac{\pi - \frac{2\pi}{3}}{2}$ (isos Δ)

$= \frac{\pi}{6}$.

(b) Find the radius of the sector $OBQC$.

$\cos\frac{\pi}{6} = \frac{6}{OB}$.

$OB = \dfrac{6}{\cos\frac{\pi}{6}}$

$= 6.9282$

$= 6.93$

Answer 6.93 cm [2]

[Turn Over

(c) Find the area of the sector $OBQC$.

area $= \frac{1}{2}(6.9282)^2\left(\frac{2\pi}{3}\right)$

$= 50.265$

$= 50.3$

Answer 50.3 cm^2 [2]

(d) Find the perimeter of the figure $ABQC$.

$P = 6.9282\left(\frac{2\pi}{3}\right) + 2(12)$

$= 38.51$

$= 38.5$

Answer 38.5 cm [2]

6 In **Diagram I**, $ABCD$ is a square. P and Q are the midpoints of AD and AB respectively. QC and PB intersect at M.

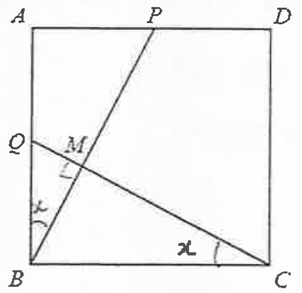


Diagram I

(a) (i) Show that triangles APB and BQC are congruent. [3]

Answer

$AB = BC$ (sides of square)

$\angle PAB = \angle QBC$ (int $\angle$s of square)

$AP = BQ$ (P and Q are midpts of AD and AB)

$\therefore \triangle APB \equiv \triangle BQC$ (SAS)

(ii) Show that angle $BMQ = 90°$. State your reasons clearly. [2]

Answer

let $\angle QCB = x$

$\angle QCB = \angle PBA$ ($\triangle APB \equiv \triangle BQC$)

$\angle BQC = 180° - 90° - x$ ($\angle$ sum of $\triangle$)

$= 90° - x$

$\angle BMQ = 180° - (90° - x) - x$ ($\angle$ sum of $\triangle$)

$= 90°$

[Turn Over

(b) **Diagram II** shows a circle being drawn on **Diagram I** with Q as the centre. The diameter of the circle is AB and the circle intersects BP at N.

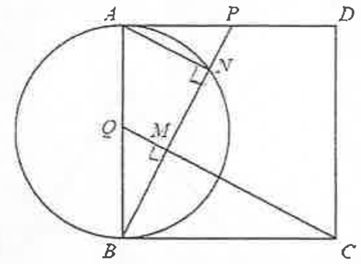


Diagram II

(i) Show that triangles BMQ and BNA are similar. [2]

Answer

$\angle BNA = 90°$ ($\angle$ in semicircle)

$= \angle BMQ$. (given in aii)

$\angle ABN = \angle QBM$ (common)

$\therefore \triangle BMQ$ and $\triangle BNA$ are similar

(ii) Find, as a fraction in its lowest terms, the value of

(a) $\dfrac{\text{area of triangle } BMQ}{\text{area of quadrilateral } MQAN}$,

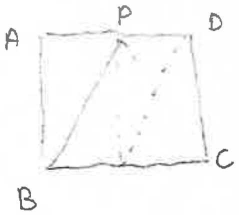
$\dfrac{\text{area } \triangle BMQ}{\text{area } \triangle BNA} = \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

$\dfrac{\text{area } \triangle BMQ}{\text{area } MQAN} = \frac{1}{3}$

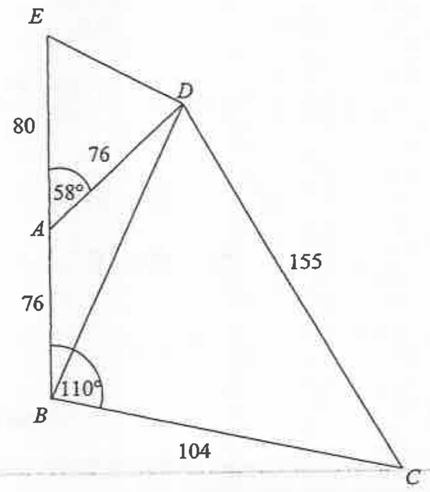
Answer $\frac{1}{3}$ [2]

(b) $\dfrac{\text{area of triangle } APB}{\text{area of quadrilateral } BCDP}$.



Answer $\frac{1}{3}$ [1]

7 In the diagram, A, B, C, D and E are five points on a horizontal field.
Paths run along AD and BD as well as the edges BAE, DE, CD and BC of the field.
$AD = AB = 76$ m, $AE = 80$ m, $BC = 104$ m and $DC = 155$ m.
Angle $DAE = 58°$ and angle $CBE = 110°$.



(a) Given that A and B are both due south of E, find the bearing of D from B.

Bearing of D from B $= \frac{58°}{2}$ (ext < of Δ = sum of 2 int opp <s)
$= 029°$

Answer 029° [1]

(b) Calculate

(i) DE,

$DE^2 = 80^2 + 76^2 - 2(80)(76)\cos 58°$

$DE = \sqrt{5732.181747}$

$= 75.71$

$= 75.7$

Answer 75.7 m [2]

(ii) angle *BDC*.

$$\frac{\sin \angle BDC}{104} = \frac{\sin 81^\circ}{155}.$$

$$\angle BDC = \sin^{-1}\left[\frac{\sin 81^\circ}{155} \times 104\right]$$

$$= 41.506^\circ$$

$$= 41.5^\circ$$

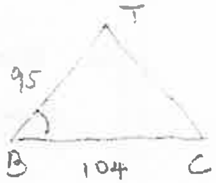
Answer ....41.5°.......... [2]

(c) *T* is a point inside triangle *BCD* such that the area of triangle *BCT* is 4477 m^2 and *BT* = 95 m. Find angle *TBC*.

Area $\triangle BCT = 4477$

$\frac{1}{2} \times 95 \times 104 \times \sin \angle TBC = 4477$

$\angle TBC = \sin^{-1}\left(\frac{4477 \times 2}{104 \times 95}\right)$



$= 64.9955^\circ$

$= 65.0^\circ$

Answer ....65.0°.......... [2]

(d) A vertical mast of height 3 m, is erected at *C*. Find the greatest angle of elevation of the top of the mast from a point on the path *BD*.

$\sin 81^\circ = \frac{\text{short dist}}{104}$

short dist $= 104 \sin 81^\circ$

$= 102.719$

$\tan\theta = \frac{3}{102.719}$

$\theta = \tan^{-1}\frac{3}{102.719}$

$= 1.672^\circ$

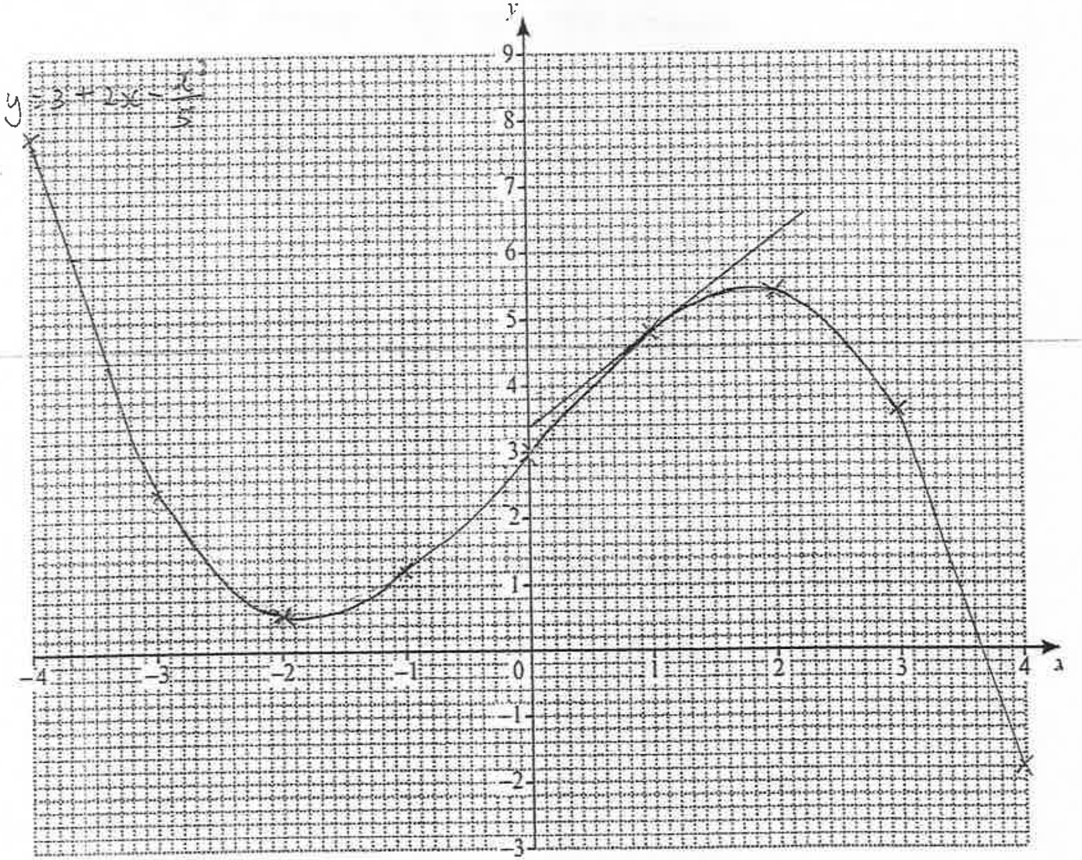
$= 1.7^\circ$

Answer ....1.7°.......... [3]

8 The table below is for $y = 3 + 2x - \frac{x^3}{5}$.

x	−4	−3	−2	−1	0	1	2	3	4
y	7.8	2.4	0.6	1.2	3	4.8	5.4	3.6	−1.8

(a) On the grid, draw the graph of $y = 3 + 2x - \frac{x^3}{5}$, for $-4 \le x \le 4$. [3]



[Turn Over

(b) Use your graph to

(i) estimate the gradient of the curve when $x = 1$,

$(1, 4.8)$, $(0, 3.4)$

$\text{grad} = \frac{4.8 - 3.4}{1 - 0}$

$= 1.4$

Answer .... 1.4 .... [2]

(ii) solve the equation $2x - \frac{x^3}{5} = 3$,

$3 + 2x - \frac{x^3}{5} = 3 + 3$

$y = 6$

$x = -3.7$

Answer .... −3.7 .... [2]

(iii) find the maximum value of $2x - \frac{x^3}{5}$ for $0 \le x \le 4$.

$y = 3 + 2x - \frac{x^3}{5}$

max is approx $= 5.4$

$2x - \frac{x^3}{5} = 5.4 - 3$

$= 2.4$

Answer .... 2.4 .... [1]

(c) The line $2y + x = 8$ and the curve $y = 3 + 2x - \frac{x^3}{5}$ can be used to solve the equation $2x^3 = ax + b$. Find the values of a and of b.

$y = 3 + 2x - \frac{x^3}{5}$

$2y = 6 + 4x - \frac{2x^3}{5}$

$8 - x = 6 + 4x - \frac{2x^3}{5}$

$2x^3 = 5(5x - 2)$

$2x^3 = 25x - 10$

$\therefore a = 25,$

$b = -10$

Answer a = ... 25 ....

b = ... −10 .... [2]

9 (a) Construct triangle ABC in which $AB = 12$ cm, angle $ABC = 37°$ and $BC = 14$ cm. [2]

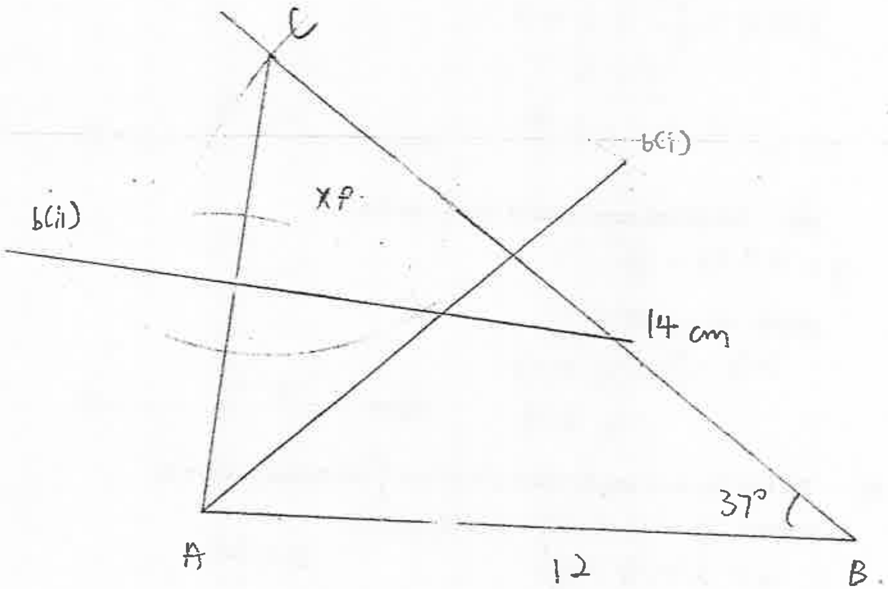
(b) On your diagram construct and label,

(i) the perpendicular bisector of AC, [1]

(ii) the bisector of angle BAC. [1]

(c) The point P, inside triangle ABC, is nearer to C than to A and nearer to AC than to AB. Indicate clearly a possible position of the point P on your diagram. [1]

Answer



(d) A circle centre O is drawn such that it touches AC at its midpoint and also touches AB. The midpoint of AC is X and the circle touches AB at Y.

Give a brief reason why

(i) $AX = AY$, [1]

Answer

tangents from external point are equal

(ii) angle OYA is a right angle. [1]

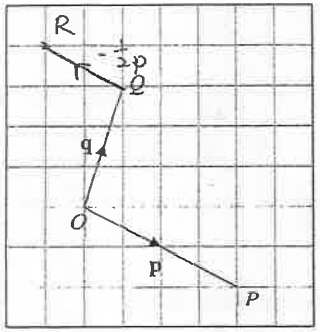
Answer

tangent to perpendicular to radius

10 (a) In the diagram, O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

$\overrightarrow{OR} = \mathbf{q} - \frac{1}{2}\mathbf{p}$. Mark point R on the diagram. [1]

Answer



(b) The column vectors of C and D with reference to the origin O are $\mathbf{c} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$, $\mathbf{d} = \begin{pmatrix} 0 \\ p \end{pmatrix}$ respectively.

(i) Find the magnitude of $\overrightarrow{OC}$.

$|\overrightarrow{OC}| = \sqrt{6^2+1^2}$

$= \sqrt{37}$

$= 6.08$

Answer 6.08 units [1]

(ii) Given that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{AB}$ is parallel to $\overrightarrow{DC}$, determine

(a) the value of p,

$\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$

$= \begin{pmatrix} 0 \\ -p \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 1-p \end{pmatrix}$

$\frac{6}{1-p} = \frac{2}{1}$

$3 = 1-p$

$p = -2$ A1

Answer -2 [2]

(b) the ratio, in its simplest form, $AB : DC$.

$\overrightarrow{DC} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

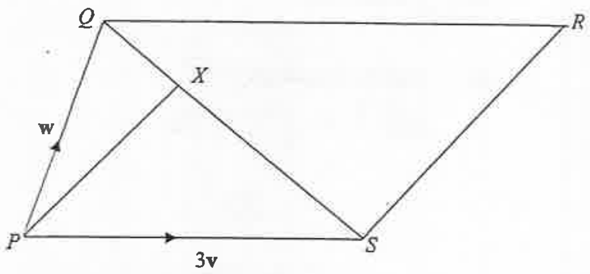
$= 3\overrightarrow{AB}$

$\therefore AB : DC$

$1 : 3$

Answer 1 : 3 [1]

(c) In the diagram, $PQRS$ is a quadrilateral.
The lines PS and QR are parallel and $QS = 4QX$.
$\overrightarrow{PQ} = \mathbf{w}$ and $\overrightarrow{PS} = 3\mathbf{v}$.



(i) Express, in terms of **w** and **v**,

(a) $\overrightarrow{QS}$,

$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$

$= -\underset{\sim}{w} + 3\underset{\sim}{v}$

Answer $-\underset{\sim}{w} + 3\underset{\sim}{v}$ [1]

(b) $\overrightarrow{QX}$,

$\overrightarrow{QX} = \frac{1}{4} QS$

$= \frac{1}{4}(-\underline{w} + 3\underline{v})$

Answer $\frac{1}{4}(-\underset{\sim}{w} + 3\underline{v})$ [1]

(c) $\overrightarrow{PX}$.

$\overrightarrow{PX} = \overrightarrow{PQ} + \overrightarrow{QX}$

$= \underline{w} + \frac{1}{4}(-\underline{w} + 3\underline{v})$

$= \frac{3}{4}\underline{w} + \frac{3}{4}\underline{v}$

Answer $\frac{3}{4}\underline{w} + \frac{3}{4}\underline{v}$ [1]

[Turn Over

(ii) Given that the point Y lies on the line QR and $\overrightarrow{XY} = \frac{1}{4}(\mathbf{w} + \mathbf{v})$, prove, by vector method, that P, X and Y lie on a straight line. [1]

Answer

$\overrightarrow{XY} = \frac{1}{4}(\underset{\sim}{w} + \underline{v})$

$\overrightarrow{PX} = \frac{3}{4}(\underset{\sim}{w} + \underline{v})$

$= 3\left[\frac{1}{4}(\underset{\sim}{w} + \underset{\sim}{v})\right]$

$= 3\overrightarrow{XY}$

$\therefore$ P, X and Y lie on a straight line

(iii) Find the numerical value of

(a) $\frac{PX}{PY}$,

Answer $\frac{3}{4}$ [1]

(b) $\frac{\text{area of triangle } PQS}{\text{area of triangle } PQX}$.

$\frac{\text{area } \triangle PQS}{\text{area } \triangle PQX} = \frac{\frac{1}{2} \times b_1 \times h}{\frac{1}{2} \times b_2 \times h}$

$= \frac{b_1}{b_2}$

$= \frac{QS}{QX} = 4$

Answer 4 [1]

11 A tea shop sell three types of tea in two different sizes. The price for each type of tea in their respective size is shown in the table below.

	Small Cup (750 ml)	Big Cup (1000 ml)
Tea *A*	$3	$4.5
Tea *B*	$3.5	$5
Tea *C*	$5	$6.5

(a) A customer bought five small cups and ten big cups for each type of teas.

(i) Evaluate $\begin{pmatrix} 3 & 4.5 \\ 3.5 & 5 \\ 5 & 6.5 \end{pmatrix}\begin{pmatrix} 5 \\ 10 \end{pmatrix}$.

$$\begin{pmatrix} 3 & 4.5 \\ 3.5 & 5 \\ 5 & 6.5 \end{pmatrix}\begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 3\times5 + 4.5\times10 \\ 3.5\times5 + 5\times10 \\ 5\times5 + 6.5\times10 \end{pmatrix}$$

$$= \begin{pmatrix} 60 \\ 67.5 \\ 90 \end{pmatrix}$$

Answer $\begin{pmatrix} 60 \\ 67.5 \\ 90 \end{pmatrix}$ [1]

(ii) Explain what the elements of the matrix in (a)(i) represent. [1]

Answer

Total price of 5 small cups and 10 big cups of tea A, B and C respectively

[Turn Over

(b) During a promotion, customers who brought their own containers, the tea shop would give a 20 cents discount for a small cup of tea purchased and 50 cents discount for a big cup of tea purchased. When the prices of the small cup and big cup of the same type of tea were compared, it was discovered that some of the items were not of better value for money after the discount.

The tea shop wants the big cup to be always better value for money for all the types of tea after the discount. It also wants to earn as much profit as possible. It still gives a 20 cents discount for a small cup of tea for those who brings their container. How much discount can be given to a big cup of tea? Justify the recommended discount and show your calculation clearly. [6]

Answer

Tea A (after discount)
Small – $2.80
Big – $4.

Price of small cup at 1000ml
$= \frac{2.80}{750} \times 1000$
$= \$3.733 < \$4$
∴ not value for money for big cup
M1
min Possible Discount $= 4.5 - 3.733$
$= \$0.767$

Tea B (after discount)
Small – $3.30
Big – $4.50

Price of small cup at 1000ml
$= \frac{3.30}{750} \times 1000$
$= \$4.40 < \$4.50$
∴ not value for money for big cup
M1
min Possible discount $= 5 - 4.4$
$= \$0.6.$

Tea C (after discount)
Small – $4.80
Big – $6

Price of small cup 1000 ml
$= \frac{4.80}{750} \times 1000$
$= \$6.40 > \$6$
∴ value for money for big cup
M

The shop can give a discount of $0.80 to a big cup of tea to ensure better value of money. $0.80 is chosen because it is closer to the minimum discount of $0.767 hence ensure the shop earning as much profit as possible (or $0.80 is easier to provide small change when the customers do not pay the exact amount