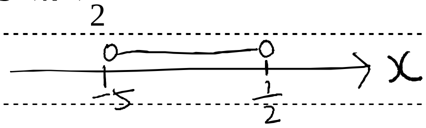
Anglo-Chinese School
(Barker Road)

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 1
Preliminary Examination 2022

1	(a)	$C = 180^\circ - (A+B)$			
		$\tan C = \tan\left(180^\circ - (A+B)\right)$	M1		
		$= \dfrac{\tan 180^\circ - \tan(A+B)}{1 + \tan 180^\circ \tan(A+B)}$	M1	Use of addition formula	
		$= \dfrac{-\tan(A+B)}{1}$			
		$= -\tan(A+B)$	AG		
	(b)	$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$			
		$= \dfrac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\left(\frac{5}{12}\right)}$	M2	Subst. using addition formula With correct $\tan A = \frac{4}{3}$ and $\tan B = \frac{5}{12}$	
		$= \dfrac{63}{16}$	A1		
		$\tan C = -\dfrac{63}{16}$	A1		
					[6]
2	(a)	$4x^2 + 6x - 4 < 6 - 12x$	M1	Line is above curve $6 - 12x > 4x^2 + 6x - 4$	
		$4x^2 + 18x - 10 < 0$			
		$2x^2 + 9x - 5 < 0$			
		$(x+5)(2x-1) < 0$	M1		
		$-5 < x < \dfrac{1}{2}$	A1		
			B1	correct number line	
	(b)	$\left(\sqrt{45} - \sqrt{3}\right) \times h = \sqrt{45} + \sqrt{3}$			
		$\left(3\sqrt{5} - \sqrt{3}\right) \times h = 3\sqrt{5} + \sqrt{3}$			
		$h = \dfrac{3\sqrt{5} + \sqrt{3}}{3\sqrt{5} - \sqrt{3}}$			

		$=\dfrac{3\sqrt{5}+\sqrt{3}}{3\sqrt{5}-\sqrt{3}}\times\dfrac{3\sqrt{5}+\sqrt{3}}{3\sqrt{5}+\sqrt{3}}$	M1	$h=\dfrac{\sqrt{45}+\sqrt{3}}{\sqrt{45}-\sqrt{3}}\times\dfrac{\sqrt{45}+\sqrt{3}}{\sqrt{45}+\sqrt{3}}$	
		$=\dfrac{45+6\sqrt{15}+3}{45-3}$	M1		
		$=\dfrac{48+6\sqrt{15}}{42}$			
		$=\left(\dfrac{8}{7}+\dfrac{1}{7}\sqrt{15}\right)\text{cm}$	A1		
					[7]
3	(a)	$a=40$	B1		
	(b)	$0=40\sin b(0.25)$	M1		
		$\sin b(0.25)=0$			
		$0.25b=0,\ \pi$	M1		
		$b=\dfrac{\pi}{0.25}$			
		$b=4\pi$ (shown)	AG		
	(c)	$-20=40\sin 4\pi t$	M1		
		$\sin 4\pi t=-\dfrac{1}{2}$			
		$\alpha=\sin^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{6}$	M1		
		$4\pi t=\pi+\dfrac{\pi}{6}$			
		$t=\dfrac{7}{24}$ s	A1		
					[6]
4	(a)	angle *BDC* = angle *BAD* (tangent chord)	B1		
		= angle *ADE* (alternate angles, *CA* and *DE* are parallel)	B1		
	(b)	From (a) angle *BDC* = angle *ADE*	B1		
		angle *ADE* $=180^\circ-$ angle *DBC*	B1	either one	
		angle *DAE* = angle *DBC*			
		Hence triangles *BDC* and *EDA* are similar			
	(c)	*BD* = *BC*			
		BDC and *EDA* are similar isosceles triangles			
		angle DAE = angle ADE	B1		

		= angle BAD (alternate angles ,*CA* and *DE* are parallel)	B1		
					[6]
5	(a)	$\dfrac{1+\sin A-\cos 2A}{\cos A+\sin 2A}=\tan A$			
		$=\dfrac{1+\sin A-\left(1-2\sin^2 A\right)}{\cos A+2\sin A\cos A}$	M1 M1	$1-2\sin^2 A$ $2\sin A\cos A$	
		$=\dfrac{\sin A+2\sin^2 A}{\cos A+2\sin A\cos A}$			
		$=\dfrac{\sin A(1+2\sin A)}{\cos A(1+2\sin A)}$	M1		
		$=\tan A$ =RHS (Shown)			
	(b)	$\dfrac{\cos 2B+\sin 4B}{1+\sin 2B-\cos 4B}=2$			
		$\dfrac{1}{\tan 2B}=2$	M1		
		$\tan 2B=\dfrac{1}{2}$			
		$2B=26.565^{\circ},206.565^{\circ},386.565^{\circ},566.565$	M1 M1	answers for 1st and 3rd quadrant answers for extended quadrants	
		$B=13.3^{\circ},103.283^{\circ},193.3^{\circ},283.3^{\circ}$	A1	All answers	
					[7]
6	(a)	$\dfrac{d}{dx}\left(\dfrac{\ln x^2}{5x}\right)=\dfrac{5x\left[\dfrac{2}{x}\right]-\ln x^2[5]}{(5x)^2}$	M2		
		$=\dfrac{5x\left[\dfrac{2}{x}\right]-\ln x^2[5]}{(5x)^2}$			
		$=\dfrac{10-5\ln x^2}{25x^2}$	M1		
		$=\dfrac{5\left(2-\ln x^2\right)}{25x^2}$			
		$=\dfrac{2-\ln x^2}{5x^2}$ (shown)	AG		

	(b)	$\int_1^e \frac{2-\ln x^2}{5x^2}\ \mathrm{d}x = \left[\frac{\ln x^2}{5x}\right]_1^e$			
		$\int_1^e \frac{2}{5x^2}\mathrm{d}x - \int_1^e \frac{\ln x^2}{5x^2}\ \mathrm{d}x = \left[\frac{\ln x^2}{5x}\right]_1^e$	M1	split	
		$-\frac{1}{5}\int_1^e \frac{\ln x^2}{x^2}\ \mathrm{d}x = \left[\frac{\ln e^2}{5e} - \frac{\ln 1^2}{5(1)}\right] - \int_1^e \frac{2}{5x^2}\ \mathrm{d}x$			
		$-\frac{1}{5}\int_1^e \frac{\ln x^2}{x^2}\ \mathrm{d}x = \left[\frac{2}{5e} - 0\right] - \int_1^e \frac{2}{5}x^{-2}\ \mathrm{d}x$	M1	Evaluate $\left[\frac{\ln x^2}{5x}\right]_1^e$	
		$-\frac{1}{5}\int_1^e \frac{\ln x^2}{x^2}\ \mathrm{d}x = \frac{2}{5e} - \left[-\frac{2}{5}x^{-1}\right]_1^e$	M1	Integrate $\int_1^e \frac{2}{5}x^{-2}$ correctly	
		$-\frac{1}{5}\int_1^e \frac{\ln x^2}{x^2}\ \mathrm{d}x = \frac{2}{5e} + \left[\frac{2}{5x}\right]_1^e$			
		$-\frac{1}{5}\int_1^e \frac{\ln x^2}{x^2}\ \mathrm{d}x = \frac{2}{5e} + \left[\frac{2}{5(e)} - \frac{2}{5(1)}\right]$			
		$-\frac{1}{5}\int_1^e \frac{\ln x^2}{x^2}\ \mathrm{d}x = \frac{4}{5e} - \frac{2}{5}$	M1	Simplify	
		$\int_1^e \frac{\ln x^2}{x^2}\ \mathrm{d}x = -5\left(\frac{4}{5e} - \frac{2}{5}\right)$			
		$= -\frac{4}{e} + 2$	A1		
					[8]
7	(a)	$x^4 - 1 = (x^2+1)(x^2-1)$			
		$= (x^2+1)(x+1)(x-1)$	B1		
	(b)	$\frac{x^4}{x^4-1} = 1 + \frac{1}{x^4-1}$	M1		
		$= 1 + \frac{1}{(x-1)(x+1)(x^2+1)}$			
		$\frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$	M1		
		$1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$			
		Subst. $x=1$, $A = \frac{1}{4}$	A1		
		Subst. $x=-1$, $B = -\frac{1}{4}$	A1		

		Comparing coefficient of x^2:			
		$0 = A + B + C$			
		$0 = \frac{1}{4} - \frac{1}{4} + C$			
		$C = 0$	A1		
		Comparing constant:			
		$1 = A - B - D$			
		$1 = \frac{1}{4} + \frac{1}{4} - D$			
		$D = -\frac{1}{2}$	A1		
		$\frac{x^4}{x^4 - 1} = 1 + \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x^2+1)}$	A1		
					[8]
8	(a)	$V = \frac{4}{3}\pi(3)^3$			
		$= 36\pi \text{ cm}^3$	B1		
		$\frac{\frac{4}{3}\pi r^3 - 36\pi}{117} = 8\pi$	M1		
		$\frac{4}{3}\pi r^3 - 36\pi = 8\pi(117)$			
		$\frac{4}{3}\pi r^3 = 8\pi(117) + 36\pi$			
		$r^3 = \frac{972\pi}{\frac{4}{3}\pi}$			
		$r^3 = 729$	M1		
		$r = 9$	AG		
	(b)	$\frac{dV}{dr} = 4\pi r^2$ $\qquad$ $\frac{dA}{dr} = 8\pi r$	B2	seen	
		$\frac{dA}{dr} = \frac{dA}{dr} \times \frac{dr}{dt}$	M1	Chain rule	
		$= \frac{dA}{dr} \times \left(\frac{dV}{dt} \div \frac{dV}{dr}\right)$	M1	$\frac{dV}{dr} = \frac{dV}{dt} \times \frac{dt}{dr}$	
		$= 8\pi r \times (8\pi \div 4\pi r^2)$			

		$=\dfrac{16\pi}{r}$			
		when $r=9$			
		$\dfrac{dA}{dr}=\dfrac{16\pi}{9}=5.59\text{ cm}^2/\text{s}$	A1		
					[8]
9	(a)	Let D be (a, b) and B be $(c, 0)$			
		Midpoint AC = $\left(\dfrac{0+5}{2},\dfrac{-1+4}{2}\right)$			
		$=\left(\dfrac{5}{2},\dfrac{3}{2}\right)$			
		Midpoint BD = $\left(\dfrac{a+c}{2},\dfrac{b+0}{2}\right)$			
		$=\left(\dfrac{a+c}{2},\dfrac{b}{2}\right)$			
		Midpoint AC = Midpoint BD			
		$\dfrac{5}{2}=\dfrac{a+c}{2}$ $\qquad$ $\dfrac{3}{2}=\dfrac{b}{2}$	M1	Midpoint concept	
		$5=a+c$ ---(1) $\qquad$ $b=3$			
		Gradient BD = $\dfrac{b-0}{a-c}$			
		$\dfrac{b-0}{a-c}=-1$	M1	Gradient	
		$\dfrac{3}{a-c}=-1$			
		$3=-a+c$ ----(2)			
		(1) +(2)			
		$8=2c$			
		$c=4$			
		$a=1$			
		$B(4, 0)$	A1		
		$D(1, 3)$	A1		
	(b)	Length AB = $\sqrt{(4-0)^2+(0-(-1))^2}$			
		$=\sqrt{17}$			
		Length BC = $\sqrt{(5-4)^2+(4-0)^2}$			
		$=\sqrt{17}$			
		$AB = BC$	M1		
		Since $ABCD$ is a **parallelogram**, $\boldsymbol{AB = BC = DC = AD}$	R1		

		Hence *ABCD* is a rhombus.			
	(c)	Area *ABCD* = $\frac{1}{2}\begin{vmatrix} 0 & 4 & 5 & 1 & 0 \\ -1 & 0 & 4 & 3 & -1 \end{vmatrix}$			
		$=\frac{1}{2}\left[(0+16+15-1)-(-4+0+4+0)\right]$	M1		
		$=15\text{ units}^2$	A1		
					[8]
10	(a)	$\frac{a}{y}=\frac{b}{x}+1$			
		$\frac{1}{y}=\frac{b}{a}\left(\frac{1}{x}\right)+\frac{1}{a}$	M1		
		Gradient = $\frac{0.625-0.4}{0.5-0.2}=\frac{3}{4}$	M1	$\frac{b}{a}=\frac{3}{4}$ Incorrect conclude $a=4,\quad b=3$ at this stage	
		$\frac{1}{y}=\frac{3}{4}\left(\frac{1}{x}\right)+\frac{1}{a}$			
		$0.4=\frac{3}{4}(0.2)+\frac{1}{a}$	M1		
		$\frac{1}{a}=\frac{1}{4}$			
		$\frac{b}{4}=\frac{3}{4}$			
		$a=4,\quad b=3$	A2		
	(b)	$\frac{1}{y}=\frac{b}{a}\left(\frac{1}{x}\right)+\frac{1}{a}$			
		$\frac{x}{y}=\frac{b}{a}+\frac{x}{a}$			
		$\frac{x}{y}$ (vertical axis)	B1		
	(c)	gradient $=\frac{1}{a}$			
		$a=\frac{1}{\text{gradient}}$	B1		

		vertical-intercept $= \frac{b}{a}$			
		b = vertical-intercept $\times$ a			
		$= \dfrac{\text{vertical-intercept}}{\text{gradient}}$	B1		
					[8]
11	(a)	$\frac{dy}{dx} = 3\sec^2(3x+1)$	B1		
		$\frac{dy}{dx} = \frac{3}{\cos^2(3x+1)}$			
		Since $\cos^2(3x+1) \neq 0$,	B1	$\cos^2(3x+1) \neq 0$	
		$\frac{dy}{dx} \neq 0$, y has no stationary points.	R1	$\frac{dy}{dx} \neq 0$	
	(b)	$y = \int k\left(\cos x - \frac{3}{4}\sin 2x\right) dx$			
		$y = k\sin x + \frac{3k}{8}\cos 2x + c$	M1		
		At $(0,4)$			
		$y = k\sin x + \frac{3k}{8}\cos 2x + c$			
		$4 = k\sin 0 + \frac{3k}{8}\cos 0 + c$			
		$4 = \frac{3k}{8} + c$			
		$32 = 3k + 8c$ --- (1)	M1	o.e	
		At $\left(\frac{\pi}{2}, 0\right)$			
		$0 = k\sin\frac{\pi}{2} + \frac{3k}{8}\cos\frac{\pi}{2} + c$			
		$0 = k - \frac{3k}{8} + c$			
		$0 = 8k - 3k + 8c$			
		$8c = -5k$ --- (2)	M1		
		Subst. (2) into (1)			
		$32 = 3k - 5k$			
		$k = -16$	A1		
		$c = 10$	A1		
		$y = -16\sin x - 6\cos 2x + 10$	A1		
					[9]

12	(a)	Surface area of lid			
		$= 2x^2 + 2(6x) + 2(3x)$			
		$= 2x^2 + 18x$			
		Surface area of open box	M1	Attempt to find surface area of lid/box	
		$= 2x^2 + 2xy + 2(2xy)$			
		$= 2x^2 + 6xy$			
		Total surface area =3000			
		$3000 = 4x^2 + 6xy + 18x$	M1		
		$6xy = 3000 - 4x^2 - 18x$			
		$y = \dfrac{3000 - 4x^2 - 18x}{6x}$			
		$y = \dfrac{500}{x} - \dfrac{2x}{3} - 3$	A1		
		$V = 2x^2 y$			
		$= 2x^2\left(\dfrac{500}{x} - \dfrac{2x}{3} - 3\right)$	M1	Susbt.	
		$= 1000x - \dfrac{4x^3}{3} - 6x^2$ (shown)	AG		
	(b)	$\dfrac{dV}{dx} = 1000 - 4x^2 - 12x$	B1		
		$1000 - 4x^2 - 12x = 0$	M1	Stationary Point	
		$x^2 + 3x - 250 = 0$			
		$x = 14.382$ or $x = -17.382$ (NA)	M1		
		Subst. $x = 14.382$			
		$V = 1000(14.382) - \dfrac{4(14.382)^3}{3} - 6(14.382)^2$			
		$V = 9170\,\text{cm}^3$ (3sf)	A1		
	(c)	$\dfrac{d^2V}{dx^2} = -8x^2 - 12$			
		when x =14.382			
		$\dfrac{d^2V}{dx^2} = -8(14.382)^2 - 12 = -1666.74$			
		Since $\dfrac{d^2V}{dx^2} < 0$, this value of x gives the largest volume of the box	R1	Showing $\dfrac{d^2V}{dx^2} =$ -1666.74 and conclude	

					[9]