Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 2
Preliminary Examination 2022

1	(a)	$-2x^2+3x+2$			
		$=-2\left[x^2-\frac{3}{2}x-1\right]$			
		$=-2\left[\left(x-\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-1\right]$	M1	Use of completing the square/ comparing	
		$=-2\left[\left(x-\frac{3}{4}\right)^2-\frac{25}{16}\right]$			
		$=-2\left(x-\frac{3}{4}\right)^2+\frac{25}{8}$	A1		
	(b)	$y=-2\left(x-\frac{3}{4}\right)^2+\frac{25}{8}$			
		Max value of $y=\frac{25}{8}=3.125$	B1	Max value	
		$(x+3)^2+(y-8)^2=20$			
		Centre $(-3,8)$			
		Radius $\sqrt{20}$			
		Lowest point of the circle $8-\sqrt{20}$	B1	Lowest point	
		$= 3.528$			
		Since the max value of $y=-2x^2+3x+2<3.528$,	R1		
		the curves will not intersect.			
					[5]
2		$\left(1-\frac{x}{2}\right)^n=1^n+\binom{n}{1}(1)^{n-1}\left(-\frac{x}{2}\right)+\binom{n}{2}(1)^{n-2}\left(-\frac{x}{2}\right)^2+..$			
		$=1-\frac{n}{2}x+\frac{n(n-1)}{8}x^2+..$	M1	Attempt to use binomial theorem seen.	
		$(1+ax)\left(1-\frac{x}{2}\right)^n$			
		$=(1+ax)\left(1-\frac{n}{2}x+\frac{n(n-1)}{8}x^2+..\right)$			
		$=1+ax-\frac{n}{2}x-\frac{an}{2}x^2+\frac{n(n-1)}{8}x^2+...$	M1	Expand	
		$=1+\left(a-\frac{n}{2}\right)x+\left(\frac{n(n-1)}{8}-\frac{an}{2}\right)x^2+...$			

		Comparing coefficient of x			
		$a-\frac{n}{2}=0$	M1		
		$a=\frac{n}{2}$			
		Comparing coefficient of x^2			
		$\frac{n(n-1)}{8}-\frac{an}{2}=-\frac{15}{4}$	M1		
		$n(n-1)-4an=-30$			
		$n(n-1)-4\left(\frac{n}{2}\right)n=-30$			
		$n^2+n-30=0$			
		$(n+6)(n-5)=0$			
		$n=-6$ (NA) or $n=5$	A1		
		$a=\frac{5}{2}$	A1	$a=2\frac{1}{2}$	
					[6]
3	(a)	Gradient AB = $\frac{5-12}{-4-3}$			
		$=1$			
		Gradient of perpendicular bisector = -1	M1		
		Midpoint AB = $\left(\frac{3-4}{2},\frac{12+5}{2}\right)$			
		$=\left(-\frac{1}{2},\frac{17}{2}\right)$	M1		
		$y-\frac{17}{2}=-\left(x+\frac{1}{2}\right)$			
		$y=-x+8$ ---- (1)			
		$y=3x-4$ ----(2)			
		$-x+8=3x-4$	M1		
		$-4x=-12$			
		$x=3$			
		Subst. $x=3$ into (1)			
		$y=-3+8=5$	M1		
		Center C $(3,5)$ shown	AG		
	(b)	Radius $\sqrt{(3-3)^2+(5-12)^2}$			

		$= 7$ units	B1	Use $A(3,12)$ and $C(3,5)$ o.e	
		$(x-3)^2+(y-5)^2=7^2$	M1	or use general form equation of circle.	
		$x^2-6x+9+y^2-10y+25=49$			
		$x^2+y^2-6x-10y-15=0$	A1		
					[7]
4		$y=3x^3-6x^2$			
		$\frac{dy}{dx}=9x^2-12x$	M1	First derivative	
		$\frac{d^2y}{dx^2}=18x-12$			
		$18x-12=0$			
		$x=\frac{2}{3}$	A1		
		Subst. $x=\frac{2}{3}$ $y=3\left(\frac{2}{3}\right)^3-6\left(\frac{2}{3}\right)^2=-\frac{16}{9}$			
		Coordinates of $P\left(\frac{2}{3},-\frac{16}{9}\right)$			
		Subst. $y=0$ $3x^3-6x^2=0$			
		$3x^2(x-2)=0$			
		$x=0$ or $x=2$	A1	$x=2$	
		$Q(2,0)$			
		Shaded area $=-\int_{\frac{2}{3}}^{2}3x^3-6x^2\,dx-\frac{1}{2}\times\left(2-\frac{2}{3}\right)\left(\frac{16}{9}\right)$	M1 M1	Area under curve Area of triangle or finding the equation of line *PQ*	
		$=-\left[\frac{3}{4}x^4-2x^3\right]_{\frac{2}{3}}^{2}-\frac{32}{27}$			
		$=-\left[\frac{3}{4}(16)-16-\frac{3}{4}\left(\frac{2}{3}\right)^4+2\left(\frac{2}{3}\right)^3\right]-\frac{32}{27}$	M1	Subst.	
		$=-\left[\frac{3}{4}(16)-16-\frac{3}{4}\left(\frac{2}{3}\right)^4+2\left(\frac{2}{3}\right)^3\right]-\frac{32}{27}$			

or $\int_{\frac{2}{3}}^{2}\frac{4}{3}x-\frac{8}{3}-(3x^3-6x^2)\,dx$

$=\left[\frac{2}{3}x^2-\frac{8}{3}x-\frac{3}{4}x^4+2x^3\right]_{2/3}^{2}$

Line PQ: $\frac{y-0}{x-2}=\frac{0-(-\frac{16}{9})}{2-(\frac{2}{3})}$

$y=\frac{4}{3}x-\frac{8}{3}$

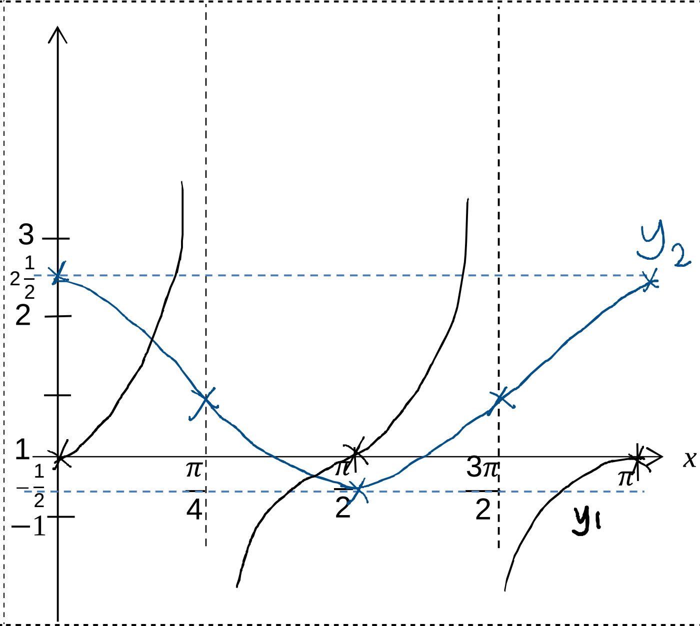
$= \frac{4}{3} - \left(-\frac{28}{27}\right) = \frac{64}{27}$

		$=\frac{64}{27}\text{ units}^2$	A1	Must be exact	
					[7]
5	(a)	$\lg\left(\frac{x+y}{2}\right)=\frac{1}{2}\lg x+\frac{1}{2}\lg y$			
		$\lg\left(\frac{x+y}{2}\right)=\frac{1}{2}(\lg x+\lg y)$			
		$\lg\left(\frac{x+y}{2}\right)=\lg(xy)^{\frac{1}{2}}$	M1	Use of product law	
		$\frac{x+y}{2}=\sqrt{xy}$			
		$x+y=2\sqrt{xy}$			
		$(x+y)^2=4xy$			
		$x^2+2xy+y^2=4xy$	M1		
		$x^2-2xy+y^2=0$			
		$(x-y)^2=0$	A1		
		$x-y=0$			
		$x=y$ (shown)	AG		
	(b)	$\log_2(3-x)-\log_4(x^2+15)=-1$			
		$\log_2(3-x)-\frac{\log_2(x^2+15)}{\log_2 2^2}=-1$	M1	Change of base	
		$2\log_2(3-x)-\log_2(x^2+15)=-2$			
		$\log_2\frac{(3-x)^2}{(x^2+15)}=-2$	M1	Log laws	
		$2^{-2}=\frac{(3-x)^2}{(x^2+15)}$	M1	Definition of log	
		$\frac{1}{4}(x^2+15)=(3-x)^2$			
		$\frac{1}{4}(x^2+15)=9-6x+x^2$			
		$3x^2-24x+21=0$			
		$x^2-8x+7=0$	M1		
		$(x-7)(x-1)=0$			
		$x=7$ (NA) or $x=1$			

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 2
Preliminary Examination 2022

Anglo-Chinese School (Barker Road)

		$x=1$	A1	Reject $x=7$	
					[8]
6	(a)	$f'(x)=(3-x)(-2e^{-2x})+e^{-2x}(-1)$	M1		
		$=e^{-2x}\left[-2(3-x)-1\right]$			
		$=e^{-2x}(2x-7)$	A1		
	(b)	$y=(3-x)e^{-2x}$			
		$y=0$			
		$(3-x)e^{-2x}=0$			
		$x=3 \qquad e^{-2x}>0$	M1		
		$(3,0)$			
		$\frac{\mathrm{d}y}{\mathrm{d}x}=e^{-2x}(2x-7)$			
		$x=3$			
		Gradient = $e^{-2(3)}(2(3)-7)$	M1		
		$=-e^{-6}$			
		Gradient of normal = e^{6}	M1		
		Equation of normal where the curve crosses x-axis			
		Subst. (3,0)	M1		
		$y=e^{6}(x-3)$	A1	o.e	
	(c)	$f'(x)=e^{-2x}(2x-7)$			
		For $x<3\frac{1}{2}$, $2x-7<0$	B1		
		Since $2x-7<0$ and $e^{-2x}>0$, $f'(x)<0$.	B1	$f'(x)<0$ with conclusion	
		f is a decreasing function.			
					[9]
7	(a)	$A=\frac{1}{2}(4)(3)\sin(90^\circ-\theta)+\frac{1}{2}(4)(4)\sin\theta+20$	M1	Area of triangle	
		$=6\cos\theta+8\sin\theta+20$	M1	$\cos\theta=\sin(90^\circ-\theta)$	
		$=8\sin\theta+6\cos\theta+20$ (shown)	AG		
	(b)	$6\cos\theta+8\sin\theta=R\cos(\theta-\alpha)$			
		$6\cos\theta+8\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$			
		$6=R\cos\alpha$ and $8=R\sin\alpha$			
		$R^2=6^2+8^2$	M1		

		$R=\sqrt{100}=10$			
		$\tan\alpha=\frac{8}{6}=\frac{4}{3}$	M1		
		$\alpha=53.1301°$			
		$\therefore A=20+10\cos(\theta-53.1°)$	A1		
	(c)	Max A = 20 +10 =30	A1		
		When $\cos(\theta-53.1301°)=1$			
		$\theta=53.1°$	A1		
	(d)	$20+10\cos(\theta-53.1301°)=25$			
		$10\cos(\theta-53.1301°)=5$			
		$\cos(\theta-53.1301°)=\frac{1}{2}$	M1		
		$\alpha=\cos^{-1}\left(\frac{1}{2}\right)=60°$			
		$\theta-53.1301°=60°$			
		$\theta=113.1°$			
		Since $\theta=113.1°$, A cannot be equal to 25 m².	R1	seen $\theta=113.1°$	
					[9]
8	(a)	Period of $y_1=\frac{\pi}{2}$	B1		
		Amplitude of $y_2=\frac{3}{2}$	B1		
			B2 B2	y_1: 1m for shape 1m for values as plotted (reasonably placed) y_2: 1m for shape 1m for values as plotted (reasonably placed)	

	(c)	$\tan 2x = \frac{3}{2}\cos 2x + 1$			
		$2\tan 2x = 3\cos 2x + 2$			
		$2\tan 2x - 3\cos 2x - 2 = 0$	M1		
		The intersections of the two graphs will be the x solutions of $2\tan 2x - 3\cos 2x - 2 = 0$	B1	With explanation	
					[8]
9	(a)	Let $\mathrm{f}(x) = ax^3 - (ax)^2 + x^2 - 23x + 12$			
		$\mathrm{f}(2) = a(1-4a^2)$			
		$a(2)^3 - (2a)^2 + (2)^2 - 23(2) + 12 = a - 4a^3$	M1	Subst. $x = 2$	
		$4a^3 - 4a^2 + 7a - 30 = 0$	M1	Reduce to cubic equation	
		$(a-2)(4a^2 + 4a + 15) = 0$	M1	Quadratic factor using long division o.e	
		For $4a^2 + 4a + 15 = 0$			
		discriminant $= 4^2 - 4(4)(15)$ $= -224$	M1	Use of discriminant to show $b^2 - 4ac < 0$ Or use of quadratic formula to show no solution for $4a^2 + 4a + 15 = 0$	
		Since discriminant < 0, there is no real roots for $4a^2 + 4a + 15 = 0$			
		$a = 2$ is the only real root.	A1		
	(b)	$2x - (2k-1) = \frac{1}{4}x^2 - kx + 4$			
		$\frac{1}{4}x^2 - kx - 2x + 4 + 2k - 1 = 0$			
		$\frac{1}{4}x^2 - (k+2)x + 2k + 3 = 0$	M1	Reduce to quadratic equation	
		discriminant $= [-(k+2)]^2 - 4\left(\frac{1}{4}\right)(2k+3)$	M1	Subst.	
		$= k^2 + 4k + 4 - 2k - 3$			
		$= k^2 + 2k + 1$			
		$= (k+1)^2$	M1		

		Since $(k+1)^2 \geq 0$, the line will meet the curve for all values of k.	R1	$(k+1)^2 \geq 0$ seen	
	(c)	$k=-1$	B1	$\surd$	
					[10]
10	(a)	$25^{x+2}=36\times 9^{1-x}$			
		$\left(5^2\right)^{x+2}=36\times\left(3^2\right)^{1-x}$			
		$5^{2x}\times 5^4=36\times 3^2\times 3^{-2x}$	M1	Simplify using law of indices	
		$5^{2x}\times 3^{2x}=\dfrac{36\times 3^2}{5^4}$			
		$15^{2x}=\dfrac{324}{625}$	M1		
		$15^x=\dfrac{18}{25}$ $\quad 15^x>0$	A1	or 0.72	
	(b)	$\lg 15^x=\lg\left(\dfrac{18}{25}\right)$	M1		
		$x=\dfrac{\lg\left(\frac{18}{25}\right)}{\lg 15}$			
		$x=-0.12\,(2\text{d.p})$	A1		
	(c)	$P=Ae^{kt}$			
		$t=0, \quad P=1000$			
		$1000=Ae^0$			
		$A=1000$	B1		
		$t=6.5, \quad P=2A$			
		$2A=Ae^{6.5k}$	M1	o.e	
		$2=e^{6.5k}$			
		$\ln 2=\ln e^{6.5k}$			
		$6.5k=\ln 2$			
		$k=\dfrac{\ln 2}{6.5}$	A1	$k=\dfrac{2\ln 2}{13}$ or 0.107	
	(d)	$t=36, \quad P=1000e^{\left(\frac{\ln 2}{6.5}\right)(36)}$		$\surd$	
		$=46480\neq 35000$	A1		
		The scientist prediction is not accurate.	B1		
					[10]

11	(a)	$v_P = 1 - \dfrac{9}{(3t+1)^2}$			
		$a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$			
		$a = \dfrac{54}{(3t+1)^2}$	M1		
		Intial acceleration = 54 m/s^2	M1		
	(b)	At instanteous rest , $v = 0$			
		$1 - \dfrac{9}{(3t+1)^2} = 0$	M1		
		$(3t+1)^2 = 9$			
		$(3t+1) = \pm 3$			
		$3t = 3 - 1$ or $3t = -3 - 1$	M1		
		$t = \dfrac{2}{3}$ or $t = -\dfrac{4}{3}$ (NA)			
		$t = \dfrac{2}{3}$ (shown)	AG		
	(c)	$s_P = \int 1 - \dfrac{9}{(3t+1)^2}\ \mathrm{d}x$			
		$s_P = t + \dfrac{3}{3t+1} + c$	M1		
		When $t = 0$, $s_P = 0$			
		$0 = 3 + c$			
		$c = -3$			
		$s_P = t + \dfrac{3}{3t+1} - 3$	A1		
		When $t = 0$, $s_P = 0$ m			
		When $t = \dfrac{2}{3}$, $s_P = -1\dfrac{1}{3}$ m			
		When $t = \dfrac{2}{3}$, $s_P = \dfrac{3}{10}$ m			
		Total distance = $1\dfrac{1}{3} \times 2 + \dfrac{3}{10} = \dfrac{89}{30}$ m			

		Average speed = $\dfrac{1\frac{1}{3}\times 2+\frac{3}{10}}{3}$	M1		
		$= 0.989$ m/s	A1	$\dfrac{89}{90}$	
	(d)	$s_P = s_Q$			
		$t+\dfrac{3}{3t+1}-3=t-1$	M1		
		$\dfrac{3}{3t+1}=2$			
		$\dfrac{3}{3t+1}=\dfrac{3}{2}$			
		$t=\dfrac{1}{6}$			
		When $t=\dfrac{1}{6}$, $s_Q=\dfrac{1}{6}-1$			
		$s_Q=-\dfrac{5}{6}$ m			
		Distance from the fixed point O is $\dfrac{5}{6}$ m	A1		
					[11]