NAME: ________________________________ (        )                    CLASS: 4 (      )

**ANGLICAN HIGH SCHOOL**
**SECONDARY FOUR**
**PRELIMINARY EXAMINATIONS 2022**

**S4**

# ADDITIONAL MATHEMATICS

**4049 / 01**

**Paper 1**

**29 August 2022**

**2 hours 15 mins**

Candidates answer on the Question Paper.
No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your name, index number and class in the space at the top of this page.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

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**For Examiners' Use**

| Question | Marks | Question | Marks | Table of Penalties | |
|---|---|---|---|---|---|
| 1 | | 8 | | | |
| 2 | | 9 | | Clarity/Logic | |
| 3 | | 10 | | Accuracy/Precision | |
| 4 | | 11 | | Units | |
| 5 | | 12 | | Total: | |
| 6 | | 13 | | | |
| 7 | | | | | |
| Parent's Name & Signature: <br> Date: | | | | 90 | |

This document consists of **18** printed pages.

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

**1** A polynomial, $\mathrm{P}(x)$, is $2x^3 + ax^2 + 6x + 27$, where $a$ is a constant.

The quadratic expression $x^2 + bx + 9$ is a factor of $\mathrm{P}(x)$, where $b$ is a constant.

**(a)** Find the remaining factor of $\mathrm{P}(x)$. [2]

**(b)** Show that the equation $\mathrm{P}(x) = 0$ has only one real root. [3]

**2** Find the remainder when the polynomial $\mathrm{f}(x) = x^2 - \frac{12}{121}\sqrt{5}$ is divided by $\left(x - \frac{1}{3 - 2\sqrt{5}}\right)$. [4]

**3** Solve the equation $\log_4(x-3) = \frac{\log_7 x}{\log_7 16} + \frac{1}{2}$. [5]

**4** Carbon–14 has a half–life of 5,730 years, which means that it takes 5,730 years for the original mass of Carbon–14 to be reduced by 50%.
To determine the age of a plant or animal fossil, scientists determine the amount of Carbon–14 in the specimen as the Carbon–14 undergoes radioactive decay.
The amount of Carbon–14 in a piece of fossilised bone is given by $M = M_0 \, \mathrm{e}^{-kt}$, where $k$ is a constant, $M$ is the mass of Carbon–14 in the specimen, $M_0$ is the initial mass of Carbon–14 and $t$ is measured in years.

**(i)** Determine the value of $k$. [2]

**(ii)** Determine the percentage of Carbon–14 that has decayed in the specimen if the fossil is estimated to be about 900 years old. [2]

**(iii)** Express the rate of change of percentage of Carbon–14 in terms of $t$. [2]

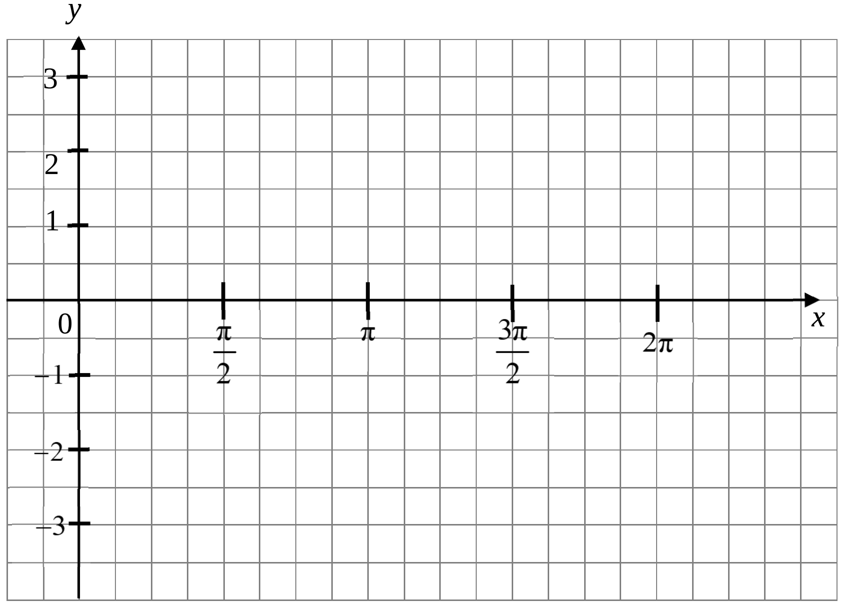
**5** Express $\dfrac{x^2-10}{\left(x^2+3\right)(2x-1)}$ in partial fractions. [5]

**6** The function f is defined as $f(x) = \cos 2x + 1$ for $0 \le x \le 2\pi$.

The function g is defined as $g(x) = 2\tan x$ for $0 \le x \le 2\pi$.

**(i)** State the period of f. [1]

**(ii)** On the same axes, sketch, for $0 \le x \le 2\pi$, the graphs of f and of g. [3]




**(iii)** Hence, state the number of solutions for which $\cos 2x - 2\tan x + 1 = 0$ [1]

**7** Given that $\sin(A+B)=\frac{4}{5}$ and $\cos(A-B)=\frac{12}{13}$ where $(A+B)$ and $(A-B)$ are acute angles,

**(i)** what can be deduced about the size of $A$ relevant to $B$? [1]

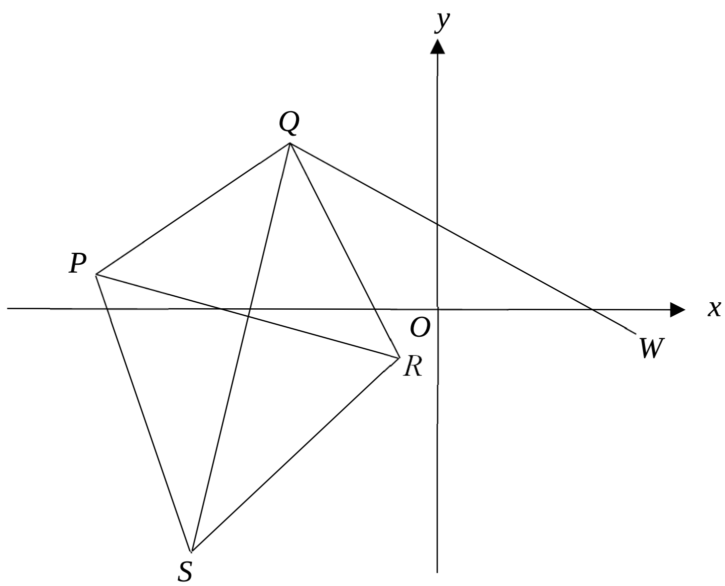
**(ii)** find, without using the additional formulae, the value of

**(a)** $\tan(A+B)$, [2]

**(b)** $\tan(A-B)$, [2]

**(iii)** Hence, using the answers in **(ii)**, prove that $\tan 2A=\frac{63}{16}$. [3]

**8** The diagram shows a quadrilateral *PQRS* where the coordinates of *P*, *Q* and *R* are $(-6,1)$, $(-2,2)$ and $(-1,k)$ respectively.
The line *QS* is the perpendicular bisector of *PR*, and $PQ = QR$.
The equation of the line *PS* is $y+4x=-23$.
The line *QW* makes an acute angle $\alpha$ with the *x*–axis and $\tan\alpha = \frac{3}{5}$.




**(a)** Find

**(i)** the value of *k*, [3]

**(ii)** the coordinates of $S$. [4]

**(b)** Explain why the gradient of $QW$ is $-\frac{3}{5}$. [1]

**(c)** Given that $\frac{PR}{QW} = \frac{2}{3}$, find the coordinates of $W$. [2]

**(d)** Determine if a circle can be drawn to pass through the vertices of the quadrilateral $PQRS$. [2]

**9** A student conducts a Physics experiment where he heats a metal cube to a certain temperature and allows it to cool.
Assume that the metal cube retains its shape throughout the experiment.
He measures the length of the cube, $x$ mm, and the corresponding time, $t$ minutes.

The student plots $x$ against $t$ and models the curve using the equation $x = \frac{5}{3t+1} + 10$, where $t \geq 0$.
After $p$ minutes, $x$ decreases at a rate of 0.03 mm/minute.

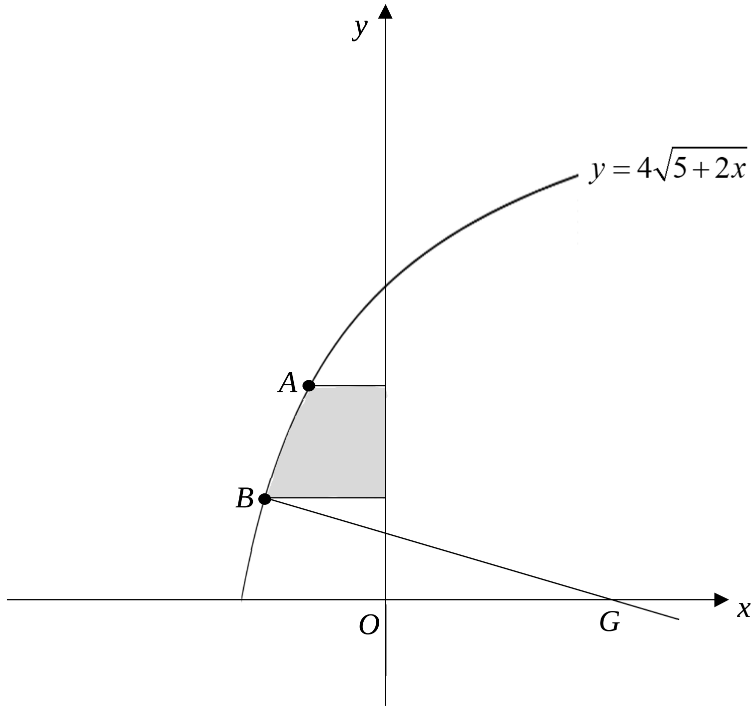
**(a)** Find the value of $p$ and the corresponding value of $x$. [4]

**(b)** The student records the surface area of the cube, $A$ mm$^2$.
After $k$ minutes, the surface area is decreasing at 121 times the rate of decrease of the side.
Find the value of $k$. [3]

**10** **(i)** Differentiate $\left(x^2+1\right)\ln\left(x^2+1\right)$ with respect to $x$. [3]

**(ii)** Hence find $\int x\ln\left(x^2+1\right)\,\mathrm{d}x$. [3]

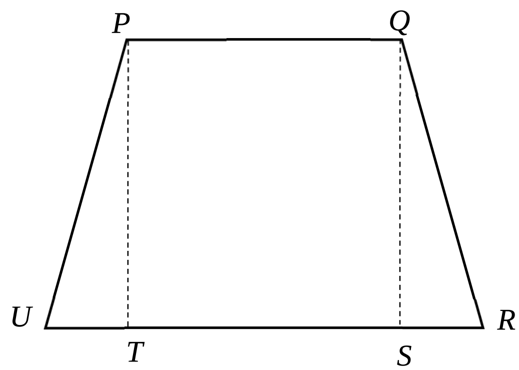
**11** The diagram shows part of the curve $y = 4\sqrt{5+2x}$.
The normal to the curve at point $B$ meets the $x$-axis at the point $G\ (14, 0)$ respectively.
The $x$-coordinate of point $A$ is $-0.5$.




**(a)** Show that the coordinate of $B$ is $(-2, 4)$. [5]

**(b)** Find the area of the shaded region. [6]

**12** The diagram shows the cross section of a solid gold pendant which is in the shape of a trapezium $PQRSTU$.

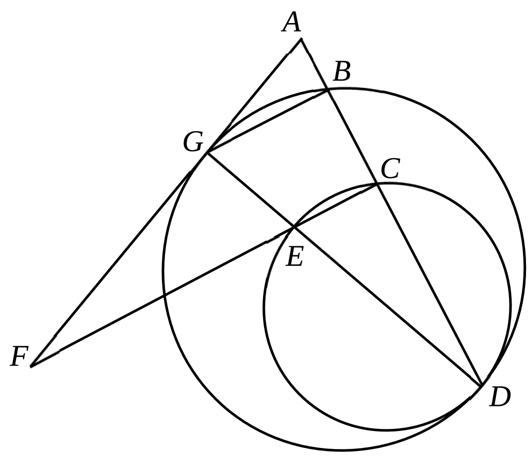



$PQST$ is a rectangle. $PQ = 3$ cm, $PU = QR = 4$ cm and angle $UPT$ = angle $RQS = \theta$ radians.
The pendant has a uniform thickness of 0.2 cm.
The density of gold is 19 g/cm$^3$.

**(a)** Show that the mass, $M$ g, of the pendant is $M = 30.4\sin 2\theta + 45.6\cos\theta$. [3]

**(b)** Find the stationary value of $M$ and determine its nature. [6]

**13** In the diagram, $GED$ is the diameter of circle $GBD$, and $ED$ is the diameter of circle $ECD$.
$AGF$ is a tangent to circle $GBD$ at $G$.
$ABCD$ and $CEF$ are straight lines and $GE = EC$ .




**(a)** Prove that triangle $DBG$ is similar to triangle $DCE$. [2]

**(b)** Prove that $GF = CD$ . [3]

**(c)** Prove that triangle $ABG$ is similar to triangle $EGF$. [2]

**END OF PAPER**