

NAME: _____ ()

CLASS: 4 ()



**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2022**

S4

ADDITIONAL MATHEMATICS

Paper 2

Candidates answer on the Question Paper.

One sheet of graph paper is required.

4049 / 02

30 August 2022

2 hours 15 mins

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the space at the top of this page.

Write in dark blue or black pen.

You may use a 2B pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use

Use of Examiners					
Question	Marks	Question	Marks	Table of Penalties	
1		8			
2		9		Clarity/Logic	
3		10		Accuracy/Precision	
4		11		Units	
5		12		Total:	
6		13			
7					
Parent's Name & Signature:				90	
Date:					

This document consists of **16** printed pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle = \frac{1}{2} ab \sin C$$

- 1 Determine the range of values of k such that the curve $y = 3x^2 + k^2 - k - 3$ intersects the line $y = 3kx$ at two distinct points. [4]

- 2 Find the range of values of x where the curve $y = \frac{6 - x^2}{4x^2 - 12x + 9}$ is below the x -axis. [4]

- 3 The curve $y = 3x^3 + 16x^2 - 6$ intersects the line $y = 10 - 12x$.
Find the x -coordinates of the points of intersection. [5]

- 4 Solve the equation $\left(2^{2x-1}\right)^2 + 2\left(2^{2x-1}\right) = 15$. [5]

- 5 **(a)** By considering the general term in the binomial expansion of $\left(\frac{1}{5x^3} - x\right)^{10}$, explain why there are only three terms of non-negative powers of x in this expansion. [4]

- (b)** In the expansion of $(-5 + 2x)(1 - 4x)^n$, in ascending powers of x , the sum of the coefficients of the term in x and the term in x^2 is -8218 . Find the value of n . [4]

- 6 A formula for calculating the number of living cells, y , of a host body per hour, t , is given by $e^{y-a} = (et)^b$, where a and b are constants.
- When y is plotted against $\ln t$, a straight line graph is obtained and it passes through the points $(0, 3)$ and $(2, 7)$.
- Find the value of a and of b . [5]

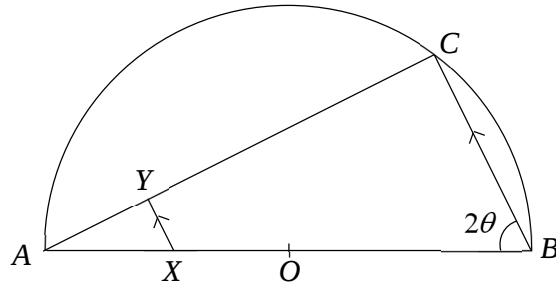
- 7 Prove the following identities.

(a)
$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = \frac{2}{\sin^2\theta}.$$
 [2]

(b) $\frac{\cos \theta}{\sin 2\theta} - \frac{\sin \theta}{\cos 2\theta - 1} = \operatorname{cosec} \theta.$ [2]

(c) By using (a) and (b), show that there is no solution to the equation $\frac{1}{1+\cos \theta} + \frac{1}{1-\cos \theta} - \frac{\cos \theta}{\sin 2\theta} + \frac{\sin \theta}{\cos 2\theta - 1} = 0$ for $0 < \theta < 2\pi$. [2]

- 8 The diagram shows a triangle ABC in a semi-circle, centre O and radius 4 cm. It is given that $AX = 2$ cm, $\angle ABC = 2\theta$ and XY is parallel to BC .

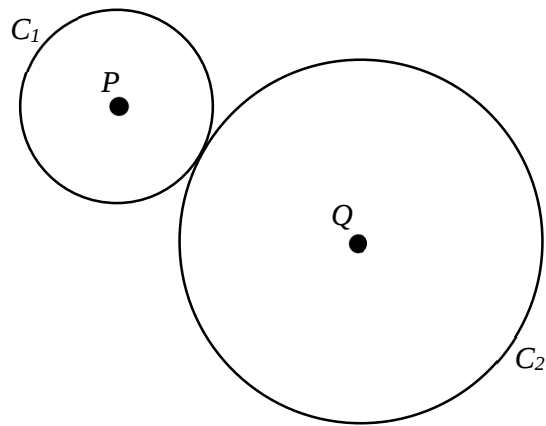


- (a) Show that the perimeter of the figure $XBCY$, P cm, is given by the equation $P = 10 \cos 2\theta + 6 \sin 2\theta + 6$. [3]

- (b) Express P in the form $R \cos(2\theta - \alpha) + 6$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

- (c) Find the maximum value of P and the corresponding value of θ . [3]

- 9 A circle C_1 is given by the equation $x^2 + y^2 + 10x - 6y + 9 = 0$.
Another circle C_2 with centre $Q(7, -2)$ touches the circle C_1 externally as shown in the diagram below.



- (i) Find the coordinates of the centre P and the radius of circle C_1 . [3]

- (ii) Show that the equation of circle C_2 is $(x-7)^2 + (y+2)^2 = 64$. [2]

It is given that the equation of the tangent to the circle C_1 at the point R is $3y + 4x = k$.

- (iii) Given that point R lies to the left of centre P , find the value of the constant k . [6]

10 The equation of a curve is $y = \ln(e^{2x} \tan^2 4x)$.

- (a) Express the first derivative in the form $A + B(\tan 4x + \cot 4x)$ where A and B are real constants. [5]

- (b) Find the equation of the tangent to the curve when $x = \frac{\pi}{16}$, leaving your answer in terms of π . [3]

- 11 Given that $y = \frac{\cos x}{\cos x - \sin x}$, express $\frac{dy}{dx}$ in the form of $\frac{k}{(\cos x - \sin x)^2}$, where k is a constant.

Hence evaluate $\int_0^{\frac{3\pi}{4}} \frac{2\cos^2 x - \cos 2x}{(\cos x - \sin x)^2} dx$. [8]

- 12 A particle moves along a straight line, and its initial displacement from a fixed point O is -6 cm.
At time, t seconds, its acceleration, a cm²/s is modelled by $a = 6t - 20$.
When $t = 1$, the velocity of the particle is 8 cm/s.
- (i) Find the values of t at which the particle is instantaneously at rest. [3]

- (ii) Find the displacement of the particle, s cm, in terms of t . [2]

- (iii) Find the distance travelled by the particle in the fourth second. [2]

- (iv) Explain clearly why the particle will cross O exactly three times. [2]

13 Answer the whole question on a piece of graph paper.

The table shows the experimental values of two variables x and y .

x	1.08	1.20	1.35	1.68	2.18
y	1.05	0.87	0.67	0.43	0.26

- (a) On the graph paper provided, plot $\ln y$ against $\ln x$ and draw a straight-line graph to represent the data. [3]
- (b) Find the gradient and y -intercept and hence express y in the form of $\frac{a}{x^n}$ where a and n are constants. [5]

End of Paper