Class	Index Number	Name

ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR EXPRESS / FIVE NORMAL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

Monday **12 September 2022** **2 hour 15 minutes**

Answer on the question paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For Examiner's Use
90

This document consists of **22** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that $\cos A = p$ where $\pi < A < \frac{3\pi}{2}$, express the following in terms of *p*.

(i) $\sin A$, [2]

(ii) $\tan(A - \pi)$. [2]

2 Find the range of values of h given that the graph of $y = -x^2 + (h-6)x - 3$ lies entirely below the line $y = -2$ for all values of x. [4]

3 A particle moves along the curve $y = 8 + \frac{4}{x^2}$ such that the *y*-coordinate of the particle is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the *x*-coordinate when $x = 2$. [4]

4 The value $\$V$, in millions, of an apartment in a certain district after t years can be represented by the model $V = 1.2 + 0.015e^{kt}$, where k is a constant.

(i) Find the initial value of the apartment. [1]

(ii) The value of the apartment is 1.3 million dollars after 2 years. Find the value of k. [2]

(iii) Find the number of years for the value of the apartment to be twice its initial value. [2]

5 Jim kicks a soccer ball from the ground. The height, h metres, of the ball t seconds after it has been kicked can be modelled by the quadratic equation $h = -4t^2 + 16t$.

(i) Express $-4t^2 + 16t$ in the form $a(t-p)^2 + q$, where a, p and q are constants. [2]

(ii) State the maximum height of the ball and the time at which it occurs. [2]

(iii) A goalkeeper claims he can catch the ball when it is dropping towards the ground and it is at most 2.5 m above the ground. Find the range of values of t for him to be able to catch the ball. [3]

6 The equation of a curve is $y = \dfrac{x^2 + 6}{2x + 1}$ for $x \geq 0$.

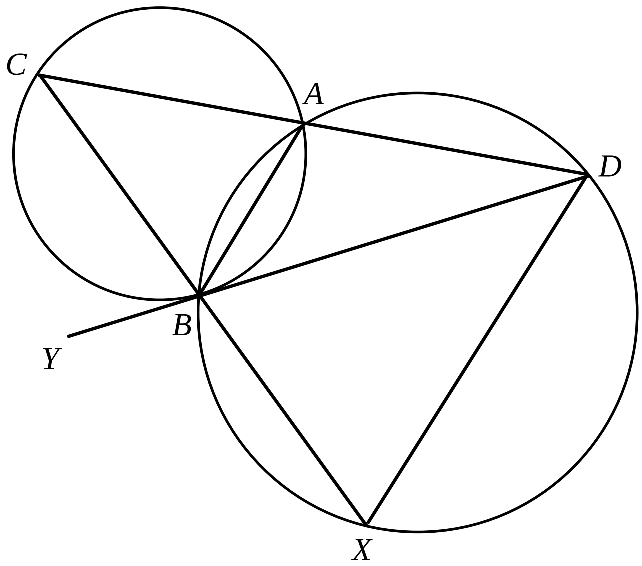
(i) Find the coordinates of the turning point of the curve. [5]

(ii) Find the equation of the normal to the curve $y = \dfrac{x^2 + 6}{2x + 1}$ at the point where the curve intersects the y-axis. [3]

7 **(a)** In the expression of $\left(4+\frac{x}{2}\right)^6+(1-mx)^8$, where *m* is a positive integer, the coefficient of x^2 is 1212. Find the value of *m*. [4]

(b) Given that the first 3 terms in the expansion of $(1+kx)^n$ are $1-27x+324x^2$, find the value of k and of n. [5]

8 In the diagram, two circles intersect at *A* and *B*. *YD* is tangent to the smaller circle at *B*. C*AD* and *CBX* are straight lines.

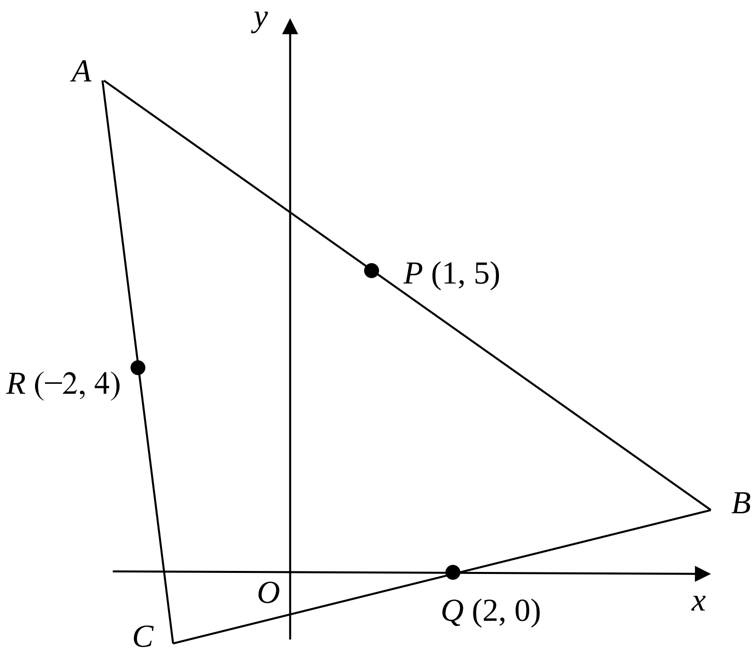


(i) Prove that triangle *ABC* and triangle *XDC* are similar. [3]

(ii) Prove that $DB = DX$. [3]

9 **Solutions to this question by accurate drawing will not be accepted**

The diagram shows a triangle *ABC*. The mid-points of the sides of the triangle are $P\ (1, 5)$, $Q\ (2, 0)$ and $R\ (-2, 4)$. The equation of *AC* is $5x + y = -6$.



(i) Explain why *RQ* is parallel to *AB*. [1]

(ii) Find the equation of *AB*. [2]

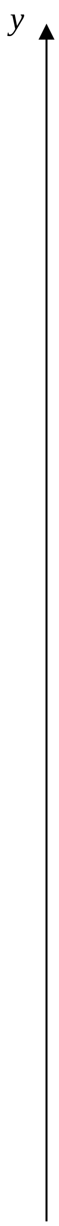
(iii) Find the coordinates of point *A*. [2]

(iv) Find the area of quadrilateral *APQR*. [2]

10 It is given that $y_1 = 1 - 2\cos x$ and $y_2 = \sin\frac{1}{2}x$. For the interval $0 \le x \le 2\pi$,

(i) state the amplitude and period of y_1, [2]

(ii) sketch, on the same diagram, the graphs of y_1 and y_2, [4]



(iii) find the x-coordinates of the points of intersection of the two graphs drawn in **(ii)**, [4]

(iv) hence state the range of values of x for which $y_1 \geq y_2$. [1]

11 A particle P moves in a straight line so that t seconds after leaving a point B, its velocity v m/s is given by $v = 9t - 3t^2$. When t = 0, its displacement from fixed point O is 3 m. Calculate

(i) the acceleration when t = 1, [2]

(ii) the maximum velocity attained by the particle, [3]

(iii) the distance of P from O when the particle comes to instantaneous rest. [5]

12 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9}{(2x-1)^2}$, where $x \neq \frac{1}{2}$. The point (2, 12) lies on the curve.

(i) Find the equation of the curve. [3]

(ii) Find the value of $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ when $x = 0$. [2]

(iii) A student concludes from **(ii)** that since the value of $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} > 0$ at $x = 0$, hence there is a minimum point at $x = 0$. State with reason and mathematical working whether the above conclusion is correct. [2]

13 **(a)** A pyramid has a square base of side $\left(2\sqrt{3}+3\right)$ cm and volume of $\left(15+\sqrt{48}\right)$ cm^3. Find, without using a calculator, the **exact** value of the perpendicular height of this square pyramid in the form $\left(a+b\sqrt{3}\right)$ cm, where a and b are integers. [4]

(b) Simplify $\dfrac{5^{x-3}\left(125^{x+2}\right)}{625^{x+1}-10\left(25^{2x}\right)}$. [4]

END OF PAPER