Name: Worked Solutions	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2022
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **23 August 2022**

Candidates answer on the Question Paper. **2 hours 15 min**
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your name, register number and class in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the answer spaces provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Marks													

Table of Penalties		Question Number		
Presentation	–1			
Accuracy / Units	–1		Parent's/Guardian's Signature	90

This document consists of **21** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 A trapezium of area $\left(12\sqrt{7}+1\right)\text{cm}^2$ has parallel sides of length $\left(2\sqrt{7}-1\right)$cm and $\left(\sqrt{7}+6\right)$cm. Without using a calculator, obtain an expression for the height, *h*, of the trapezium in the form $a\sqrt{7}+b$, where *a* and *b* are integers. [4]

Solution:

$$\frac{1}{2}\left(2\sqrt{7}-1+\sqrt{7}+6\right)h=12\sqrt{7}+1$$ [M1]

$$\left(3\sqrt{7}+5\right)h=24\sqrt{7}+2$$

$$h=\frac{24\sqrt{7}+2}{3\sqrt{7}+5}$$

$$=\frac{24\sqrt{7}+2}{3\sqrt{7}+5}\times\frac{3\sqrt{7}-5}{3\sqrt{7}-5}$$ [M1] – rationalize denoiminator

$$=\frac{504-120\sqrt{7}+6\sqrt{7}-10}{63-25}$$ [M1] – correct expansion

$$=\frac{494-114\sqrt{7}}{38}$$

$$=13-3\sqrt{7}$$ [A1]

2 Integrate $\sin^2 x+\frac{1}{x}$ with respect to *x*. [3]

Solution:

$$\int \sin^2 x+\frac{1}{x}\ \mathrm{d}x=\int \frac{1-\cos 2x}{2}+\frac{1}{x}\ \mathrm{d}x$$ [M1]

$$=\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+\ln x+c$$ [A1] – Integrate $\frac{1-\cos 2x}{2}$

$$=\frac{1}{2}x-\frac{\sin 2x}{4}+\ln x+c$$ [A1] – Integrate $\frac{1}{x}$

–1 mark if *c* is not written

3 Solve the simultaneous equations.

$$2^{x+1}+5^{y}=89$$
$$2^{x-1}-5^{y-1}=11$$ [4]

Solution:

$$5^{y}=89-2^{x+1} \text{ --- (1)}$$
$$2^{x-1}-5^{y-1}=11 \text{ --- (2)}$$

Sub (1) into (2):

$$2^{x-1}-5^{-1}\left(89-2^{x+1}\right)=11$$ [M1]

$$\frac{2^{x}}{2}-\frac{89}{5}+\frac{2\times 2^{x}}{5}=11$$

$$\frac{9}{10}\times 2^{x}=\frac{144}{5}$$ [M1]

$$2^{x}=32$$

$$x=5$$ [A1]

Sub $x=5$ into (1),

$$5^{y}=89-2^{6}$$
$$=25$$
$$y=2$$ [A1]

4 Express $\dfrac{7x^2-23x+6}{(x-2)(x^2-4)}$ in partial fractions. [5]

Solution:

$$\frac{7x^2-23x+6}{(x-2)(x^2-4)}=\frac{7x^2-23x+6}{(x-2)^2(x+2)}$$

[B1] – Factorise denominator completely

Let $\dfrac{7x^2-23x+6}{(x-2)^2(x+2)}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+2}$

[B1]

$$7x^2-23x+6=A(x-2)(x+2)+B(x+2)+C(x-2)^2$$

Let $x=2$, $-12=4B$

$B=-3$

Let $x=-2$, $80=16C$

$C=5$

Let $x=0$, $6=-4A-6+20$

$A=2$

[M2] – solve for A, B, C using substitution or comparing coefficients. 2 marks if all correct, 1 mark if 2 correct.

$$\frac{7x^2-23x+6}{(x-2)^2(x+2)}=\frac{2}{x-2}-\frac{3}{(x-2)^2}+\frac{5}{x+2}$$

[A1]

5 The gradient function of a curve is $\dfrac{4}{(1-2x)^2}+p$. It is given that the gradient of the curve at the point (1, –3) is 7.

(a) Find the value of p. [2]

Solution:

$$\frac{4}{(1-2)^2}+p=7 \qquad \text{[M1]}$$

$$p=3 \qquad \text{[A1]}$$

(b) Find the coordinates of the point where the curve meets the y-axis. [4]

Solution:

$$y=\int \frac{4}{(1-2x)^2}+3\ \mathrm{d}x$$

$$=\frac{4(1-2x)^{-1}}{(-1)(-2)}+3x+c \qquad \text{[M1]}$$

$$=\frac{2}{1-2x}+3x+c$$

Sub (1, –3) into y,

$$-3=\frac{2}{-1}+3+c \qquad \text{[M1]}$$

$$c=-4$$

$$y=\frac{2}{1-2x}+3x-4 \qquad \text{[M1]}$$

Since the curve meets the y-axis, $x = 0$.

$y = –2$.

The coordinates are (0, –2). [A1]

6 **(i)** Express $\log_2 \sqrt{\dfrac{4^{3n-2}}{16^n}}$ in the form $an+b$, where a and b are integers. [3]

Solution:

$$\log_2 \sqrt{\frac{4^{3n-2}}{16^n}} = \frac{1}{2}\log_2\left(\frac{2^{6n-4}}{2^{4n}}\right) \quad \text{[M1]}$$

$$= \frac{1}{2}\log_2\left(2^{2n-4}\right) \quad \text{[M1]}$$

$$= n-2 \quad \text{[A1]}$$

(ii) Solve the equation $\log_2(x+4) = \log_4(7x+4) + \dfrac{1}{2}$. [4]

Solution:

$$\log_2(x+4) = \log_4(7x+4) + \frac{1}{2}$$

$$\log_2(x+4) = \frac{\log_2(7x+4)}{\log_2 4} + \frac{1}{2}$$ [M1] – change of base

$$\log_2(x+4) = \frac{\log_2(7x+4)}{2} + \frac{1}{2}$$

$$2\log_2(x+4) = \log_2(7x+4) + 1$$

$$\log_2(x+4)^2 - \log_2(7x+4) = 1$$

$$\log_2\left(\frac{(x+4)^2}{7x+4}\right) = 1$$ [M1] – application of quotient law

$$\frac{(x+4)^2}{7x+4} = 2$$ [M1] – change to logarithmic form

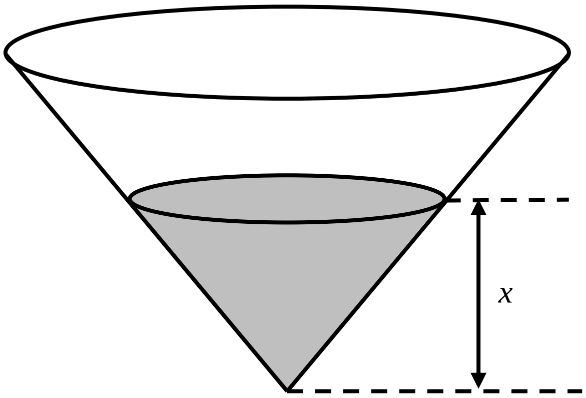
$$x^2 + 8x + 16 = 14x + 8$$

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$$x = 2 \quad \text{or} \quad 4$$ [A1] – both answers are correct

7 A container is in the shape of an inverted right circular cone where its base radius is equal to its height. Alcohol in the container evaporates at a constant rate of 10 cm^3/s. The volume of alcohol, V, in the container is $\frac{1}{3}\pi x^3$ when the height of the alcohol is x cm.



(i) Calculate the rate of change of the height of alcohol when the height of alcohol is 5 cm, leaving your answer in exact form. [3]

Solution:

$$V = \frac{1}{3}\pi x^3$$

$$\frac{dV}{dx} = \pi x^2$$ [M1]

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$$

$$-10 = \pi x^2 \times \frac{dx}{dt}$$ [M1]

When x = 5 cm,

$$-10 = \pi (5)^2 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = -\frac{2}{5\pi} \text{ cm/s}$$

The rate of change of the height of alcohol is $-\frac{2}{5\pi}$ cm/s. [A1]

(ii) Calculate the rate of change of the exposed surface area of the alcohol when the depth of alcohol is 5 cm. [3]

Solution:

$A = \pi x^2$

$\frac{\mathrm{d}A}{\mathrm{d}x} = 2\pi x$ [M1]

$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$

$= 2\pi x \times \left(-\frac{2}{5\pi}\right)$ [M1]

$= -\frac{4}{5}x$

When x = 5 cm,

$\frac{\mathrm{d}A}{\mathrm{d}t} = -\frac{4}{5}(5)$

$= -4 \text{ cm}^2/\text{s}$

The rate of change of exposed surface area of the alcohol is $-4 \text{ cm}^2/\text{s}$. [A1]

8 The vertical displacement of the waves, y m, at a port can be modelled by the equation $y = -3\cos kt + 8.5$, where k is a constant and t is the number of hours after midnight.

The time between two consecutive high tides is 6 hours.

(i) Show that the value of k is $\dfrac{\pi}{3}$. [1]

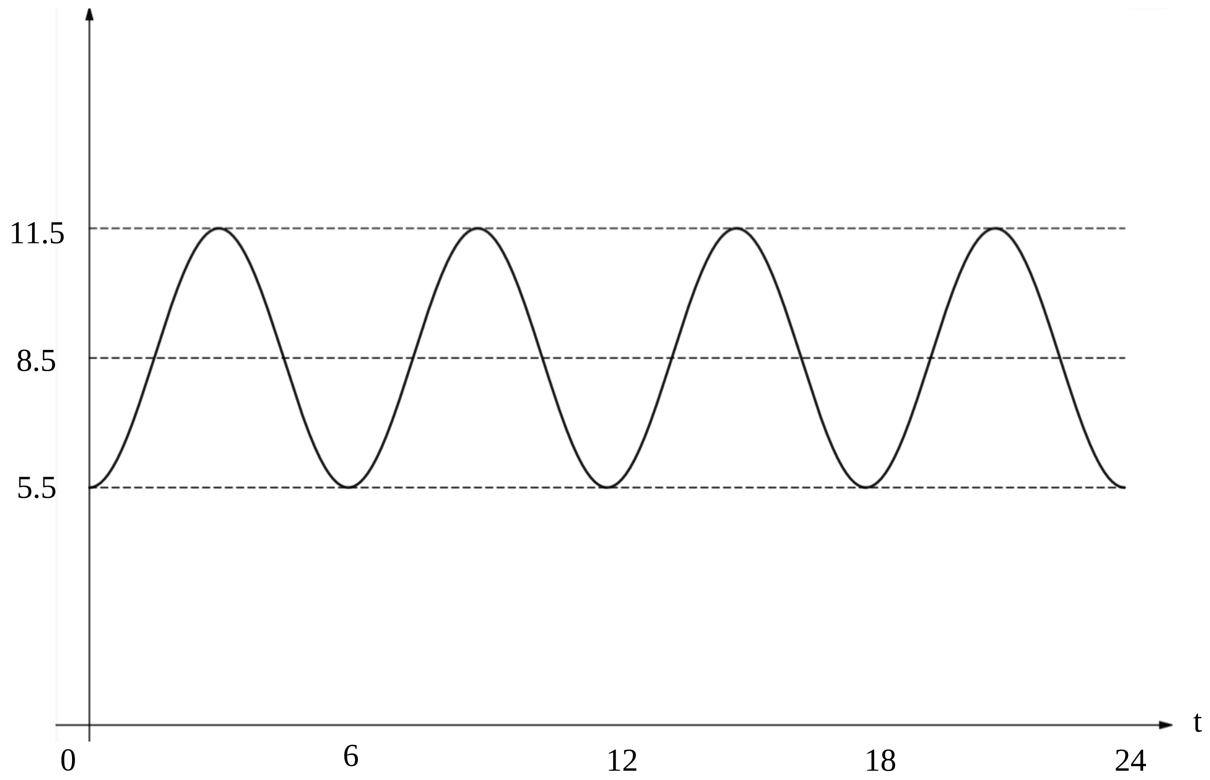
Solution:

$$\frac{2\pi}{k} = 6 \qquad \text{[B1]}$$

$$k = \frac{\pi}{3}$$

(ii) Sketch the graph of $y = -3\cos\dfrac{\pi}{3}t + 8.5$ for $0 \le t \le 24$. [3]

Solution:



[B1] – Shape
[B1] – Correct period and 4 cycles
[B1] – Correct max/min points
* axis of graph must be indicated

(iii) If a ship can only enter the port safely when the vertical displacement of the waves is at least 7 m, find the total length of time in hours in a day when the ship can enter the port. [4]

Solution:

$$-3\cos\frac{\pi}{3}t+8.5=7$$

$$\cos\frac{\pi}{3}t=0.5$$ [M1]

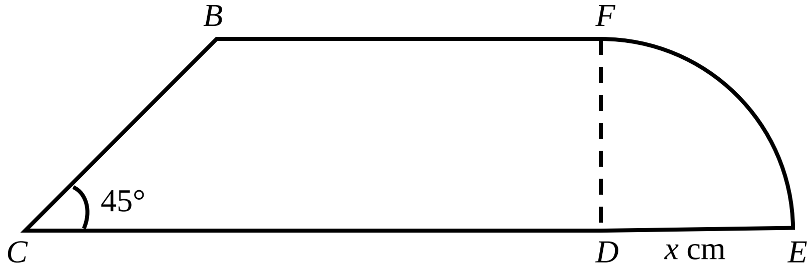
$$\alpha=\frac{\pi}{3}$$ [M1]

$$\frac{\pi}{3}t=\frac{\pi}{3},\ \frac{5\pi}{3}$$

$$t=1,\ 5$$ [M1]

Total length of time = 4(5 – 1)
= 16 hours [A1]

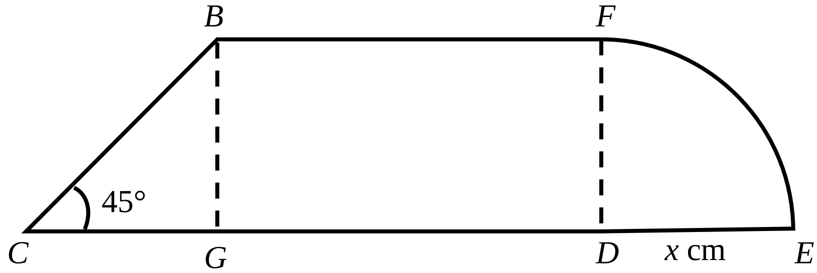
9 A piece of wire of length 460 cm is bent to form a figure consisting of a trapezium *BCDF* and a quadrant *DEF* where angle $BCD = 45°$ and $DE = x$ cm.



(i) Show that the area of the figure is given by $A = 230x - \frac{\sqrt{2}+1}{2}x^2$. [5]

Solution:

Let the length of *BF* be *y* cm.



$CG = BG = DF = DE = x$ cm [B1]

$$\sin 45° = \frac{x}{BC}$$
$$BC = \sqrt{2}x$$ [B1]

$$\sqrt{2}x + y + \frac{1}{4}(2\pi x) + x + y + x = 460$$
$$2y + \sqrt{2}x + \frac{\pi}{2}x + 2x = 460$$
$$y = 230 - \frac{\sqrt{2}}{2}x - \frac{\pi}{4}x - x$$ [M1]

$$A = \frac{1}{2}x^2 + \left(230 - \frac{\sqrt{2}}{2}x - \frac{\pi}{4}x - x\right)x + \frac{1}{4}\pi x^2$$ [M1]
$$= \frac{1}{2}x^2 + 230x - \frac{\sqrt{2}}{2}x^2 - x^2$$
$$= 230x - \frac{1}{2}x^2 - \frac{\sqrt{2}}{2}x^2$$
$$= 230x - \frac{\sqrt{2}+1}{2}x^2 \text{ (shown)}$$ [A1]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

Solution:

$$A = 230x - \frac{\sqrt{2}+1}{2}x^2$$

$$\frac{dA}{dx} = 230 - \left(\sqrt{2}+1\right)x$$ [M1]

For stationary value of A, $\frac{dA}{dx} = 0$.

$$230 - \left(\sqrt{2}+1\right)x = 0$$

$$x = \frac{230}{\sqrt{2}+1}$$ [M1]

$$A = 230\left(\frac{230}{\sqrt{2}+1}\right) - \frac{\sqrt{2}+1}{2}\left(\frac{230}{\sqrt{2}+1}\right)^2$$

$$= 11000 \ (3sf)$$ [A1]

$$\frac{d^2A}{dx^2} = -\left(\sqrt{2}+1\right) < 0$$

Since $\frac{d^2A}{dx^2} < 0$, A is a maximum value. [A1]

10 **(a)** A dolphin jumps out of the water during a dolphin show. The height, y m, of the dolphin is given by $y=-4.5t^2+9t$, where t is the time in seconds after the dolphin jumps out of the water.

(i) Express $y=-4.5t^2+9t$ in the form $y=a(t-h)^2+k$. [2]

Solution:

$y=-4.5t^2+9t$

$=-4.5(t^2+2t)$

$=-4.5(t^2+2t+1-1)$ [M1]

$=-4.5(t-1)^2+4.5$ [A1]

(ii) How long did it take the dolphin to reach the maximum height and what is the maximum height? [2]

Solution:

Time taken to reach maximum height = 1 seconds [B1]
Maximum height = 4.5 m [B1]

(iii) Find the range of values of t where the height of the dolphin is at least 3.375 m. [2]

Solution:

$-4.5t^2+9t\geq 3.375$

$4.5t^2-9t+3.375\leq 0$

$4t^2-8t+3\leq 0$

$(2t-3)(2t-1)\leq 0$ [M1]

$\frac{1}{2}\leq t\leq 1\frac{1}{2}$ [A1]

(b) The function *f* is defined by $f(x)=2x^3+ax-3$, where *a* is a constant. $f(x)$ has a remainder of $-2p$ when divided by $x+1$ and a remainder of $3p+7$ when divided by $x-2$. Find the value of *a* and of *p*. [4]

Solution:

$f(-1)=-2p$

$2(-1)^3+a(-1)-3=-2p$ [B1]

$a=2p-5$ --- (1)

$f(2)=3p+7$

$2(2)^3+a(2)-3=3p+7$ [B1]

$2a=3p-6$ --- (2)

Sub (1) into (2): [M1]

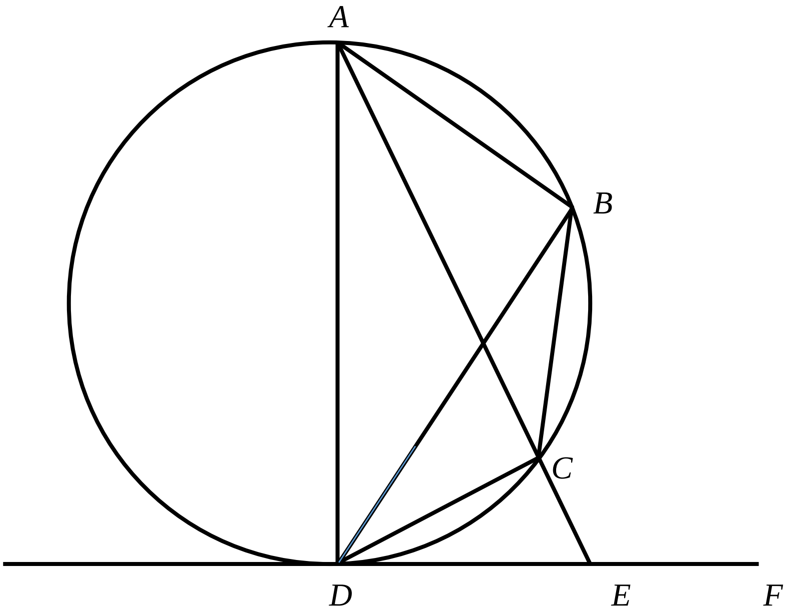
$2(2p-5)=3p-6$

$p=4$

[A1]

Sub $p=4$ into (1), $a=3$.

11



In the figure above, DEF is a tangent to the circle at D. The line EA bisects angle DAB and intersects the circle at C.

(i) Prove that angle CDB = angle CDE. [3]

Solution:

$\angle CDB = \angle CAB$ (angles in the same segment) [B1]

$= \angle DAC$ (FA bisects $\angle DAB$) [B1]

$= \angle CDE$ (alt. segment theorem) [B1]

(ii) Prove that $AD \times DE = AE \times CD$. [3]

Solution:

$\angle DEC = \angle AED$ (common angle) [B1]

$\angle CDE = \angle DAE$ (alt segment theorem) [B1]

ΔDCE is similar to ΔADE.

$$\frac{AD}{CD} = \frac{AE}{DE}$$ [M1]

$AD \times DE = AE \times CD$ (proven)

(iii) Prove that $BC = CD$. [2]

Solution:

$\angle CDB = \angle CDE$ (from (i))

$= \angle CBD$ (alt segment theorem) [B1]

ΔBCD is isosceles.

$BC = CD$ (proven) [B1]

12 **(a)** Prove that $\sin^3 x\sec^2 x+\sin x=\tan x\sec x$. [4]

Solution:

$$\sin^3 x\sec^2 x+\sin x=\sin x(\sin^2 x\sec^2 x+1) \quad \text{[M1]}$$
$$=\sin x(\tan^2 x+1) \quad \text{[M1]}$$
$$=\sin x(\sec^2 x) \quad \text{[M1]}$$
$$=\sin x\left(\frac{1}{\cos x}\right)(\sec x)$$
$$=\tan x\sec x \text{ (proven)} \quad \text{[A1]}$$

(b) Find all the values of x between 0 and 5 for which $2\cos^2 x-6\sin x\cos x=1$. [5]

Solution:

$$2\cos^2 x-6\sin x\cos x=1$$
$$\cos 2x+1-3\sin 2x=1$$
$$\cos 2x=3\sin 2x$$

[M1] – application of double angle (either one)

$$\tan 2x=\frac{1}{3}$$

[M1]

$$\alpha=0.32175$$

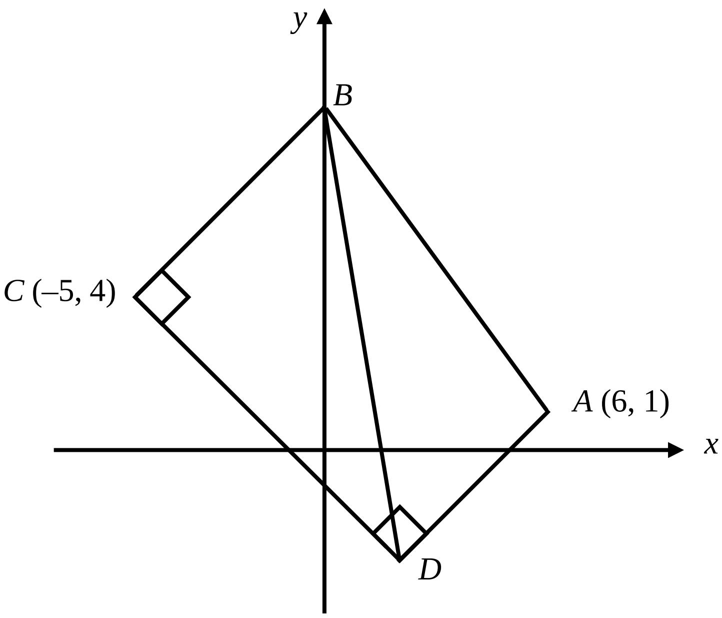
[M1]

Since tan $2x$ is positive, $2x$ lies in the 1st or 3rd quadrant.

$$2x=0.32175,\ \pi+0.32175,\ 2\pi+0.32175,\ 3\pi+0.32175$$
$$=0.32175,\ 3.4633,\ 6.6049,\ 9.7465$$
$$x=0.161,\ 1.73,\ 3.30,\ 4.87$$

[A2]
– 1m for 3 correct answers
– 2m for 4 correct answers

13 *ABCD* is a trapezium such that the coordinates of *A* and *C* are (6, 1) and (–5, 4) respectively. Angle *ADC* and angle *BCD* are right angles and the gradient of *AB* is $-\frac{4}{3}$.



(i) Show that the coordinates of *B* are (0, 9). [2]

Solution:

$\frac{y-1}{0-6}=-\frac{4}{3}$ [M1]

$y=9$

B (0, 9) [A1]

(ii) Show that the coordinates of D are (2, –3). [4]

Solution:

$$m_{CB} = \frac{9-4}{0-(-5)}$$
$$= 1$$
$$m_{CD} = -1$$
[M1]

Equation of CD:

$y - 4 = -1(x+5)$

$y = -x - 1$ --- (1) [M1]

$m_{AD} = 1$

Equation of AD:

$y - 1 = 1(x-6)$

$y = x - 5$ --- (2)

(1) = (2)

$-x - 1 = x - 5$ [M1]

$x = 2$

Sub $x = 2$ into (1),

$y = -3$

D (2, –3) [A1]

(iii) Find the area of triangle ABD. [2]

Solution:

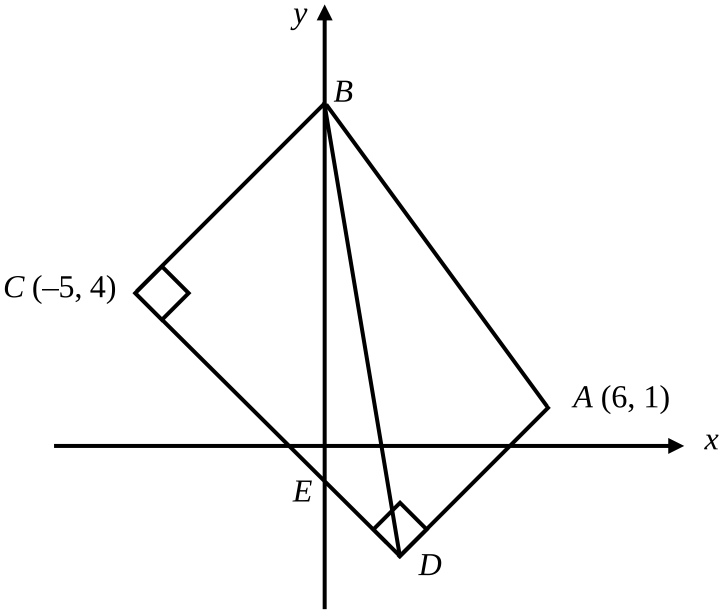
$$\text{area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 2 & 6 & 0 \\ 9 & -3 & 1 & 9 \end{vmatrix}$$ [M1]

$$= \frac{1}{2}\left[(2+54)-(18-18)\right]$$

$$= 28 \text{ units}^2$$ [A1]

(iv) Find angle ABC. [3]

Solution:



$\angle EBA = 90° - \tan^{-1}\left(\frac{4}{3}\right)$ [M1]

$= 36.870°$

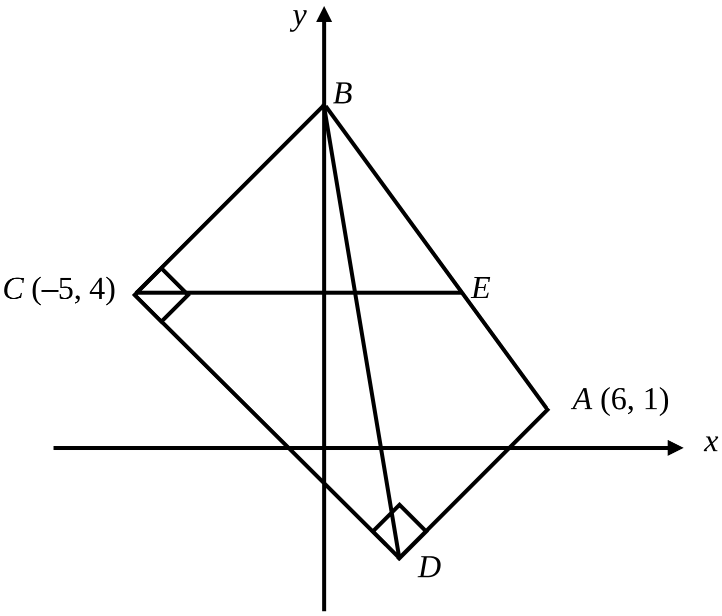
$\angle CBE = 90° - \tan^{-1}(1)$ [M1]

$= 45°$

$\angle ABC = 45° + 36.870°$

$= 81.9°$ (1dp) [A1]

Alternative Solution:



$\angle BCE = \tan^{-1}(1)$ [M1]

$= 45°$

$\angle BEC = \tan^{-1}\left(\frac{4}{3}\right)$ [M1]

$= 53.130°$

$\angle ABC = 180° - 45° - 53.130°$ (sum of angles in a triangle) [A1]

$= 81.9°$ (1dp)

END OF PAPER