HOUGANG SECONDARY SCHOOL
PRELIMINARY EXAMINATION / 2022
SECONDARY FOUR (EXPRESS)

CANDIDATE NAME:		CLASS:	

CENTRE NUMBER:	S					INDEX NUMBER:				

ADDITIONAL MATHEMATICS 4049/01

Paper 1

Friday 19 August 2022

2 hours 15mins

Candidates answer on the Question Paper

Instructions to students:

- Write your name, index number and class clearly in the spaces at the top of this page.
- Write in dark blue or black pen on spaces provided.
- You may use an HB pencil for any diagrams or graphs.
- Do not use staples, paper clips, glue or correction fluid.
- Answer **all** the questions in this paper.
- The use of an approved scientific calculator is expected, where appropriate.
- Give non-exact numerical answers correct to 3 significant figures, or one decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
- You are reminded of the need for clear presentation in your answers.

Information for pupils

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total mark for this paper is 90.

Calculator Model: ______________________

	90

Mathematical Formulae

1. **ALGEBRA**

Quadratic Equation :

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem :

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n$$

where n is appositive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)..........(n-r+1)}{r!}$

2. **TRIGONOMETRY**

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta ABC = \frac{1}{2}ab\sin C$$

1 The line $2y = 3x - 7$ meets the curve $y = x^2 + 3x - 8$ at two points A and B.
Find the distance between A and B. [4]

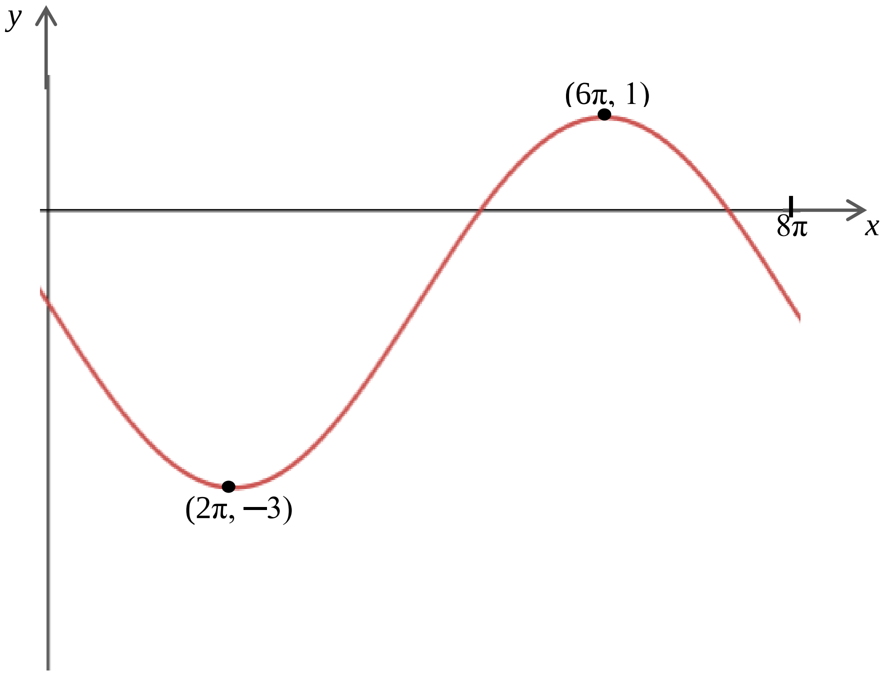
2 Express $\dfrac{7x+8}{(2x+1)(x-1)^2}$ in partial fractions. [5]

3 **Without using the calculator**, find the values of the integer a and b such that

$$\frac{4-\sqrt{3}}{a+b\sqrt{3}}=\frac{2+\sqrt{3}}{4-\sqrt{3}}.$$ [3]

4 The line of symmetry of a quadratic curve is $x=-2$ and the curve lies above the x-axis for all x. Given that the point $(-1,4)$ lies on the curve, find a possible equation of the curve in the form $y=a(x-h)^2+k$ where a, h, and k are integers. [4]

5



The diagram shows the curve $y = p\sin\dfrac{x}{q} + r$ for $0 \le x \le 8\pi$ radians. The curve has a minimum point at $(2\pi, -3)$ and a maximum point $(6\pi, 1)$.

(a) Show that $r = -1$. [1]

(b) Find the values of p and q. [2]

(c) Hence write down the equation of the curve. [1]

6 The function f is defined for all real values of x and is such that $\mathrm{f}''(x) = 6x + 2$.

The gradient to the curve $y = \mathrm{f}(x)$ at the point $(-1, 10)$ is 11.

(a) Find an expression for $\mathrm{f}'(x)$. [3]

(b) Hence find the equation of the curve $y = \mathrm{f}(x)$. [2]

(c) Determine whether the curve $y = \mathrm{f}(x)$ have stationary point(s).
Explain with clear working. [3]

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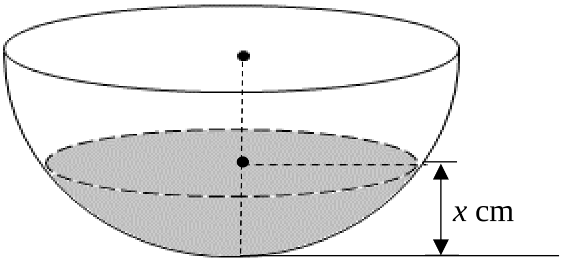

When the hemispherical bowl above contains water to a depth of x cm, the volume, V cm^3, of the water is given by $V = \frac{1}{3}\pi x^2 (18 - x)$. The bowl is initially empty. After water has been poured into the bowl at a constant rate for 9 seconds, the depth of water is 4.5 cm.

(a) Find the constant rate of change of volume in terms of π. [3]

(b) Find the rate at which the water level is rising when the depth is 4.5cm**.** [4]

8 **(a)** Factorise $\sin^3 x + \cos^3 x$ completely. [1]

(b) Show that $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \dfrac{1}{2}\sin 2x$. [3]

(c) Hence solve the equation $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = \dfrac{5}{4}$ for $0^\circ \le x \le 360^\circ$. [4]

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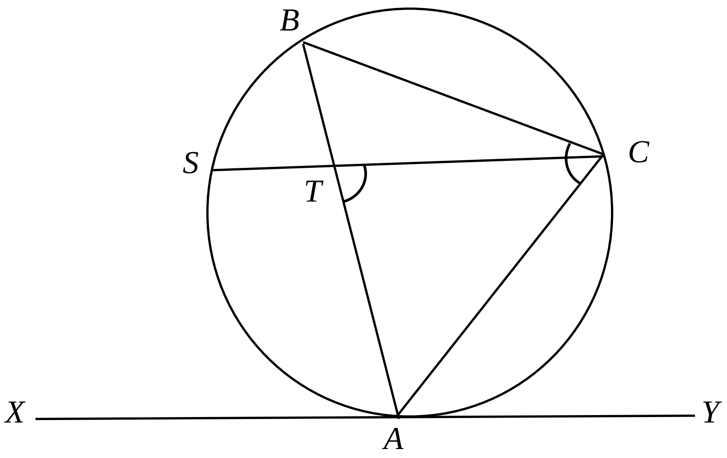

The diagram shows a point A on the circle and XAY is a tangent to the circle. Points S, B and C lie on the circle. The chords AB and SC intersect at T and angle ACB = angle ATC.

(a) Prove that triangles ABC and ACT are similar. [2]

(b) Show that $AC^2 - AT^2 = AT \times TB$. [3]

(c) Determine, with working, whether the lines SC and XY are parallel. [3]

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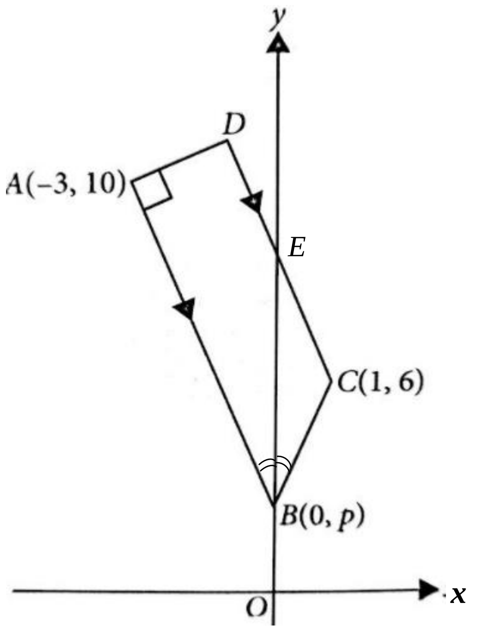

The diagram shows a trapezium with vertices $A(-3,10)$, $B(0,p)$, $C(1,6)$ and D. The sides AB and DC are parallel and the angle BAD is a right angle. Angle ABE is equal to angle CBE.

(a) Express the gradients of lines AB and CB in terms of p and hence, or otherwise, show that $p = 4$. [3]

(b) Show that the equation of line *AD* is $y = \frac{1}{2}x + \frac{23}{2}$.

Hence find the coordinates of the point *D*. [5]

(c) Find the area of the trapezium *ABCD*. [2]

11 (a) The polynomial $P(x) = 5x^3 + ax^2 - x + b$, where a and b are constants is exactly divisible by $x^2 + 4x + 3$. Show that the value of a = 16 and find the value of b. [4]

(b) Hence solve the equation $P(x)=0$. [3]

(c) Using a suitable substitution and your answers in **(b)**, solve the equation
$5x^6+ax^4-x^2+b=0$. [2]

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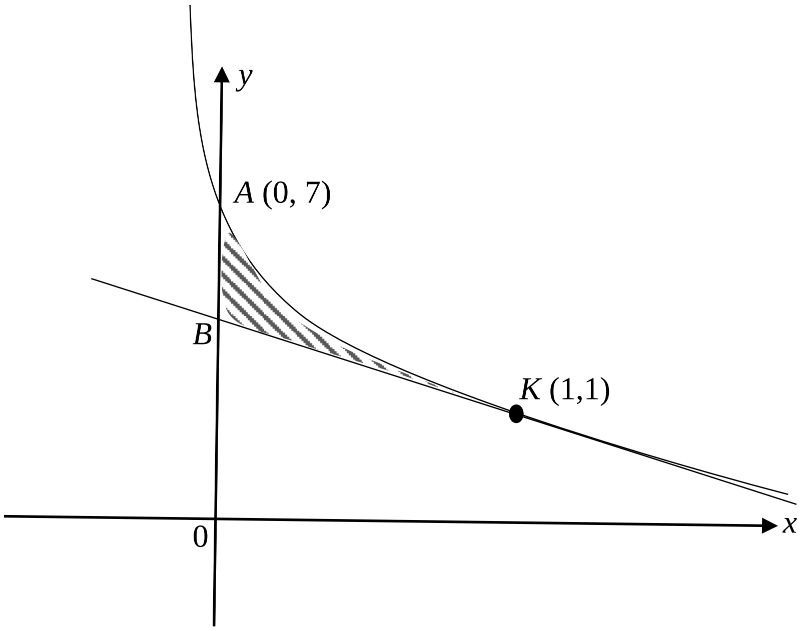


The diagram shows part of the curve $y = \dfrac{7}{6x+1}$ intersecting the *y*-axis at $A(0,7)$

The tangent to the curve at the point $K(1\,,1)$ intersects the *y*-axis at B.

(a) Find the coordinates of *B*. [5]

(b) Find the area of the shaded region bounded by the curve, the tangent *KB* and the y-axis. [5]

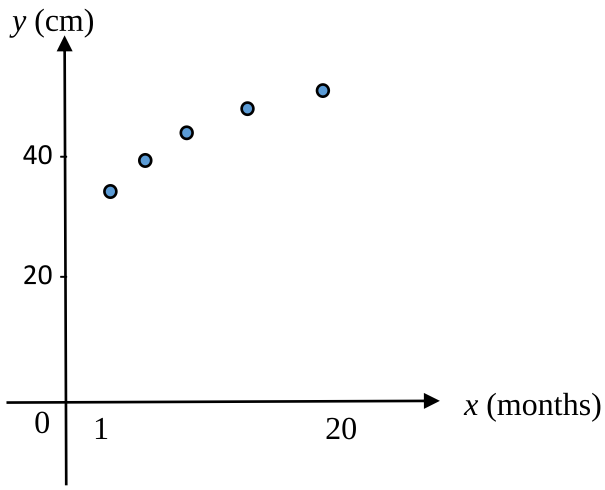
13 **(a)** Solve the equation $\left(\ln x\right)^2 + \dfrac{2}{\log_x e} = 3$. [4]

(b) It is given that $\lg p - \lg 2q = \lg(p + 2q)$.

(i) Express p in terms of q [2]

(ii) State the range of values of p and explain clearly why $0 < q < \frac{1}{2}$. [2]

(c) Mrs Tan decides to track the relationship between the age, x (in months) of her newborn baby girl and the circumference of her baby girl's head, y (in centimetres).
After plotting the data collected for 1st, 6th, 12th, 18th and 20th month, the following graph was obtained.



Determine, with a reason, which of the 2 equations below is suitable to model the data plotted in the above diagram.

(A) $y = ae^{bx}$ (Exponential function)
(B) $y = a\ln x + b$ (Logarithmic function) [2]

End of paper