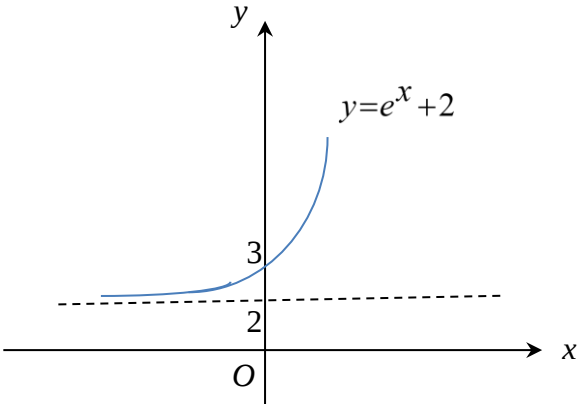


Marking Scheme for A Maths Paper 2				
Qn	Working	Marks	Remarks	
1(a)	 The graph shows the function $y = e^x + 2$. The curve is blue and passes through the point $(0, 3)$ on the y-axis. A horizontal dashed line at $y = 2$ represents the horizontal asymptote. The x and y axes are labeled, and the origin is marked O.	B1 B1	Shape of the curve y-intercept and horizontal asymptote	

(b)	$3 - e^{-x} = 2e^x$ $3 - \frac{1}{e^x} = 2e^x$ $3e^x - 1 = 2e^{2x}$ $2e^{2x} - 3e^x + 1 = 0$ Let $y = e^x$ $2y^2 - 3y + 1 = 0$ $(2y - 1)(y - 1) = 0$ $y = \frac{1}{2} \quad \text{or} \quad y = 1$ $e^x = \frac{1}{2} \quad \text{or} \quad e^x = 1$ $x = \ln \frac{1}{2} \quad \text{or} \quad x = \ln 1$ $x = -0.693 \quad \text{or} \quad x = 0$	M1		
		M1		
		M1		
		A1		

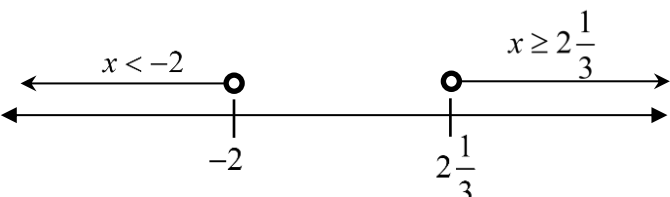
2(i)	Since roots of $f(x) = 0$ are 1, k and k^2 $f(x) = -(x-1)(x-k)(x-k^2)$ $f(2) = -7$ $-(2-1)(2-k)(2-k^2) = -7$ $(2-k)(2-k^2) = 7$ $4 - 2k^2 - 2k + k^3 = 7$ $k^3 + 2k^2 + 2k - 3 = 0 \text{ (shown)}$	M1 M1 A1		
(ii)	Let $g(k) = k^3 - 2k^2 - 2k - 3$ $g(3) = 3^3 - 2(3)^2 - 2(3) - 3$ $= 0$ Since $g(3) = 0$, $k - 3$ is a factor of $g(k)$. $k^3 - 2k^2 - 2k - 3 = 0$ $(k-3)(k^2 + k + 1) = 0$ $k = 3$ or $k^2 + k + 1 = 0$ $b^2 - 4ac = 1^2 - 4(1)(1)$ $= -3 < 0 \text{ (no real roots)}$ Therefore $k^3 - 2k^2 - 2k - 3 = 0$ has only 1 real root.	M1 M1 M1, M1 M1 B1		

3(i)	$y = (x+2)\sqrt{x-1}$ $\frac{dy}{dx} = (x+2)\left[\frac{1}{2}(x-1)^{-\frac{1}{2}}\right] + \sqrt{x-1}(1)$ $= (x-1)^{-\frac{1}{2}}\left[\frac{1}{2}x+1+x-1\right]$ $= (x-1)^{-\frac{1}{2}}\left(\frac{3}{2}x\right)$ $= \frac{3x}{2\sqrt{x-1}}$	M1		
(ii)	By Chain Rule, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $2 = \frac{3x}{2\sqrt{x-1}} \times \frac{dx}{dt}$ $\frac{dx}{dt} = 2 \div \frac{3(2)}{2\sqrt{2-1}}$ $= \frac{2}{3} \text{ units/s}$	M1		

(iii)	$\int_2^5 \frac{x}{\sqrt{x-1}} \, dx = \frac{2}{3} \int_2^5 \frac{3x}{2\sqrt{x-1}} \, dx$ $= \frac{2}{3} [(x+2)\sqrt{x-1}]_2^5$ $= \frac{2}{3} [7\sqrt{4} - 4\sqrt{1}]$ $= \frac{2}{3} (10)$ $= 6\frac{2}{3}$	M1		
		M1		
		A1		

4(i)	Let E be the point on AB such that BE is perpendicular to CE. F is the point on CE such that CF is perpendicular to FD. $\left. \begin{array}{l} \cos \theta = \frac{BE}{4} \\ BE = 4 \cos \theta \end{array} \right\}$ $\left. \begin{array}{l} \sin \theta = \frac{FD}{1} \\ FD = \sin \theta \end{array} \right\}$ $AB = BE + FD$ $= 4 \cos \theta + \sin \theta \quad (\text{shown})$	M1 for either one correct A1		
(ii)	$R = \sqrt{4^2 + 1^2}$ $= \sqrt{17}$ $\tan \alpha = \frac{1}{4}$ $\alpha = \tan^{-1} \frac{1}{4}$ $= 14.036^\circ$ $\therefore AB = \sqrt{17} \cos(\theta - 14.0^\circ)$	M1 for finding R, M1 for finding α A1		
(iii)	Max $AB = \sqrt{17}$ m	B1	Accept 4.12 m	

	For AB to be maximum, the value of $\cos(\theta - 14.036^\circ)$ must be 1. $\cos(\theta - 14.036^\circ) = 1$ $\theta - 14.036^\circ = 0^\circ$ $\theta = 14.036^\circ$ $= 14.0^\circ \text{ (1 d.p.)}$	B1		
(iv)	$3 = \sqrt{17} \cos(\theta - 14.036^\circ)$ $\cos(\theta - 14.036^\circ) = \frac{3}{\sqrt{17}}$ $\theta - 14.036^\circ = 43.313^\circ$ $\theta = 43.313^\circ + 14.036^\circ$ $= 57.349^\circ$ $= 57.3^\circ \text{ (1 d.p.)}$			

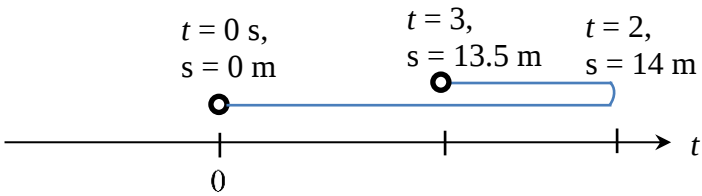
5(a)	$y = 2x^2 - 6x + c$ ----- (1) $y + 2x = 8$ ----- (2) Equating (1) & (2): $2x^2 - 6x + c = 8 - 2x$ $2x^2 - 4x + c - 8 = 0$ The line is a tangent to the curve, $b^2 - 4ac = 0$ $(-4)^2 - 4(2)(c - 8) = 0$ $16 - 8c + 64 = 0$ $c = 10$	M1 M1 A1		
(b)	$3(x^2 - 5) > x - 1$ $3x^2 - 15 - x + 1 > 0$ $3x^2 - x - 14 > 0$ $(3x - 7)(x + 2) > 0$ $x < -2$ or $x > 2\frac{1}{3}$ 	M1 A1 A1		

(c)	$-2x^2 + x - p$ $a = -2, b = 1, c = -p$ For real roots, $b^2 - 4ac \geq 0$ $1 - 4(-2)(-p) \geq 0$ $1 - 8p \geq 0$ $p \leq \frac{1}{8}$ Greatest value of integer $p = 0$	M1 M1 A1		
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6(i)	$\left(\frac{1}{2} - 2x\right)^5$ $= \left(\frac{1}{2}\right)^5 + \binom{5}{1}\left(\frac{1}{2}\right)^4(-2x)^1 + \binom{5}{2}\left(\frac{1}{2}\right)^3(-2x)^2 + \binom{5}{3}\left(\frac{1}{2}\right)^2(-2x)^3 + \dots$ $= \frac{1}{32} - \frac{5x}{8} + 5x^2 - 20x^3 + \dots$	M1 A1		
6(ii)	$\left(1 + ax + 3x^2\right)\left(\frac{1}{2} - 2x\right)^5$ $= \left(1 + ax + 3x^2\right)\left(\frac{1}{32} - \frac{5x}{8} + 5x^2 - 20x^3 + \dots\right)$ Considering x^2, $5x^2 - \frac{5}{8}ax^2 + \frac{3}{32}x^2 = \frac{13}{2}x^2$ $5 - \frac{5}{8}a + \frac{3}{32} = \frac{13}{2}$ $a = -2\frac{1}{4}$	M1 M1 M1 A1		
6(iii)	$(0.47)^5 = \left(\frac{1}{2} - 2x\right)^5$ $\frac{1}{2} - 2x = 0.47$ $x = 0.015$ $(0.47)^5 = \frac{1}{32} - \frac{5(0.015)}{8} + 5(0.015)^2 - 20(0.015)^3 + \dots$ ≈ 0.02293	M1 M1 A1		

7(i)	$y = 10 + Ab^x$ $y - 10 = Ab^x$ Take log to the base 10 on both sides, $\lg(y - 10) = \lg b(x) + \lg A$ Plot $\lg(y - 10)$ against x to obtain a straight line. <table border="1" data-bbox="323 467 1005 786"> <tr> <td>x</td><td>0.5</td><td>1.0</td><td>1.5</td><td>2.0</td></tr> <tr> <td>y</td><td>15.9</td><td>19.1</td><td>23.4</td><td>30.2</td></tr> <tr> <td>$y - 10$</td><td>5.9</td><td>9.1</td><td>13.4</td><td>20.2</td></tr> <tr> <td>$\lg(y - 10)$</td><td>0.77</td><td>0.96</td><td>1.13</td><td>1.31</td></tr> </table>	x	0.5	1.0	1.5	2.0	y	15.9	19.1	23.4	30.2	$y - 10$	5.9	9.1	13.4	20.2	$\lg(y - 10)$	0.77	0.96	1.13	1.31	M1, G1	1 mark for drawing the straight line	
x	0.5	1.0	1.5	2.0																				
y	15.9	19.1	23.4	30.2																				
$y - 10$	5.9	9.1	13.4	20.2																				
$\lg(y - 10)$	0.77	0.96	1.13	1.31																				
(ii)	$\lg A =$ vertical intercept of the graph $= 0.60$ (Accept ± 0.02) $A = 10^{0.60}$ ≈ 3.98 $\lg b =$ Gradient of the graph $= \frac{1.31 - 0.60}{2 - 0}$ $= 0.355$ (Accept ± 0.02) $b = 10^{0.355}$ ≈ 2.26	M1 A1 M1 A1																						

(i)	$3t^2 - 10t + 18$ $\lg a + x \lg b = 2x$ $\therefore \text{draw } \lg(y-10) = 2x$ $= 6t + k$ From the graph, $x \approx 0.375$ (Accept ± 0.01) When $t = 1$,		M1 M1 A1A1		
	$6(1) + k = -9$ $k = -15$ (shown)				
(ii)	$3t^2 - 15t + 18 = 0$ $t^2 - 5t + 6 = 0$ $(t - 2)(t - 3) = 0$ $t = 2$ or $t = 3$		M1 A1 for both answers		

(iii)	$s = \int v \, dt$ $= \int (3t^2 - 15t + 18) \, dt$ $= t^3 - \frac{15}{2}t^2 + 18t + C$ When $t = 0, s = 0, C = 0$ $\therefore s = t^3 - \frac{15}{2}t^2 + 18t$ when $t = 2 \, \text{s}, s = 14 \, \text{m}$ when $t = 3 \, \text{s}, s = 13.5 \, \text{m}$  Distance travelled in first 3 s = $14 + (14 - 13.5)$ = 14.5 m	M1		
		M1		
		M1 A1		

(iii)	Area of shaded region = $\int_{-\ln 2}^0 (4 - e^{-2x}) \, dx + \frac{1}{2}(6)(3)$ $= \left[4x + \frac{1}{2}e^{-2x} \right]_{-\ln 2}^0 + 9$ $= \left(0 + \frac{1}{2} \right) - \left(-4\ln 2 + \frac{1}{2}e^{2\ln 2} \right) + 9$ $= 7\frac{1}{2} + 4\ln 2$ $\approx 10.3 \text{ units}^2$	M1, M1		
		M1		
		M1		
		A1		

10(i)	$(x-3)^2 + (y+2)^2 = 12 + 9 + 4$ $(x-3)^2 + (y+2)^2 = 25$ $C(3, -2)$ Radius = $\sqrt{25}$ = 5 units	B1 B1		
(ii)	Normal of a circle passes through centre of a circle. $3(-2) = m - 4(3)$ $m = 6$	M1 A1		

(iii)	$3y = 6 - 4x$ $y = -\frac{4}{3}x + 2 \text{ ----- (1)}$ $(x-3)^2 + (y+2)^2 = 25 \text{ ----- (2)}$ $(x-3)^2 + \left(-\frac{4}{3}x + 2 + 2\right)^2 = 25$ $(x-3)^2 + \left(4 - \frac{4}{3}x\right)^2 = 25$ $x^2 - 6x + 9 + \left(\frac{16}{9}x^2 - \frac{32}{3}x + 16\right) = 25$ $\frac{25}{9}x^2 - \frac{50}{3}x = 0$ $25x^2 - 150x = 0$ $25x(x-6) = 0$ $x = 6 \text{ (rejected) or } x = 0$ $\text{when } x = 0,$ $y = -\frac{4}{3}(0) + 2$ $y = 2$ $\therefore A(0, 2) \text{ (shown)}$	M1		
(iv)	$(3, -2) \text{ is midpoint of } AB \text{ and the required coordinates } B \text{ be } B(e, f)$ $\left(\frac{0+e}{2}, \frac{2+f}{2}\right) = (3, -2)$ $B(6, -6)$ $\text{Equation of tangent at } B:$ $y + 6 = \frac{3}{4}(x - 6)$ $y = \frac{3}{4}x - \frac{21}{2}$	M1 M1 A1		

