Candidate Name Paper 1 Solutions

Class	Index Number

ST ANDREW'S SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

MATHEMATICS 4048/01

Paper 1

MONDAY 15 AUGUST 2022 **2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 Mrs Ram bought a washing machine for \$390 during a sale.
The shop was having a 35% discount on all appliances.

Find the usual selling price of the washing machine before the sale.

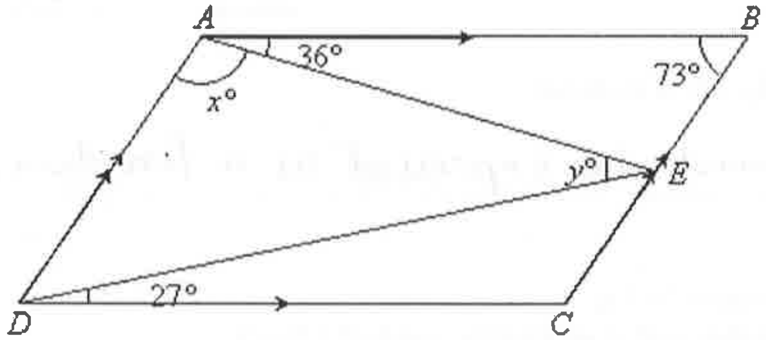
65% — \$390

100% — $\frac{390}{65} \times 100$ or $\frac{100}{65} \times 390$

$= \$600$

Answer \$ 600 .... [2]

2



In the diagram, *ABCD* is a parallelogram.
Given that angle *BAE* = 36°, angle *ABE* = 73° and angle *EDC* = 27°.

Find the value of

(a) x,

$x = 180 - 36 - 73$

$= 71$

Answer $x =$ 71 .... [1]

(b) y.

$y = 27 + 36$

$= 63$

Answer $y =$ 63 .... [1]

[Turn over

3 (a) A light on a computer comes on for 26 700 microseconds.
One microsecond is 10^{-6} seconds.

Work out the length of time, in seconds, that the light is on in standard form.

26700 microseconds

$= 26700 \times 10^{-6}$ seconds

$= 2.67 \times 10^{-2}$ seconds

Answer 2.67×10^{-2} s [1]

(b) Explain why $\sqrt{3}$ is irrational.

$\sqrt{3}$ cannot be expressed as a fraction. [1]

---

4 The length of a road is 385 m.
Maria runs along this road at an average speed of 3.9 m/s.
This speed is correct to 1 decimal place.

Calculate the greatest possible time taken by Maria.

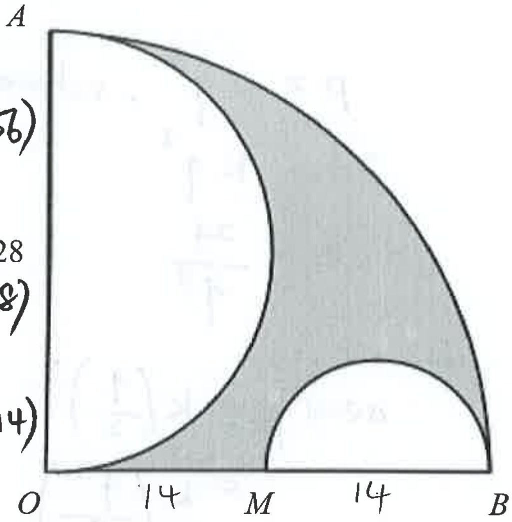
$= \dfrac{385 \text{ m}}{3.85 \text{ m/s}}$

$= 100$ seconds

Answer 100 s [2]

5 The diagram shows a quadrant of a circle, centre O and radius 28 cm.
M is the midpoint of OB and a semi-circle is drawn with MB as diameter.
The other semicircle in the quadrant has diameter OA.

Find the perimeter of the shaded region.
Give your answer in the form $a + b\pi$.



Arc length $AB = \frac{1}{4} \times 2\pi(28)$ $\frac{1}{4}\pi(56)$
$= 14\pi$

Arc length $OA = \frac{1}{2} \times 2\pi(14)$ $\frac{1}{2}\pi(28)$
$= 14\pi$

Arc length $MB = \frac{1}{2} \times 2\pi(7)$ $\frac{1}{2}\pi(14)$
$= 7\pi$

Perimeter of shaded region
$= 14\pi + 14\pi + 7\pi + 14$
$= (14 + 35\pi)$ cm

Answer $14 + 35\pi$ cm [2]

---

6 **(a)** Solve the inequality $2(5 - x) \leq 3x < 37 - 5x$.

$2(5-x) \leq 3x$ and $3x < 37 - 5x$
$10 - 2x \leq 3x$ $\quad 8x < 37$
$10 \leq 5x$ $\quad x < 4\frac{5}{8}$ $\quad \frac{37}{8}$
$2 \leq x$

$\therefore 2 \leq x < 4\frac{5}{8}$

$\frac{37}{8}$

$2 \leq x < 4.625$

Answer $2 \leq x < 4\frac{5}{8}$ [2]

(b) Write down the largest integer x which satisfies the inequality
$$2(5 - x) \leq 3x < 37 - 5x.$$

Answer 4 [1]

---

7 **(a)** p is proportional to q^3.
It is known that $p = 24$ for a particular value of q.

Find the value of p when this value of q is halved.

$p = kq^3$, where k is a constant

$24 = kq^3$

$\therefore k = \frac{24}{q^3}$

new $p = k\left(\frac{q}{2}\right)^3$

$= k\left(\frac{q^3}{8}\right)$

$= \frac{24}{q^3} \times \frac{q^3}{8}$

$= 3$

Answer $p =$ 3 [1]

(b) y is inversely proportional to x^2.
$y = 4$ when $x = 3$.

Find y when $x = 10$.

$y = \frac{k}{x^2}$, where k is a constant

$4 = \frac{k}{3^2}$

$\therefore k = 36$

$y = \frac{36}{x^2}$

$= \frac{36}{10^2}$

$= 0.36 \left(\text{or } \frac{9}{25}\right)$

Answer $y =$ 0.36 [2]

8 A map is drawn to a scale of 1 : 50 000.

(a) An airport runway is represented by a line of length 4.6 cm on the map.

Calculate the actual length of the runway, giving your answer in kilometres.

1 : 50 000

1 cm : 50 000 cm

1 cm : 0.5 km

4.6 cm : 0.5 × 4.6

= 2.3 km

Answer ....2.3........ km [1]

(b) The actual area of the airport is 3.5 km^2.

Calculate the area on the map which represents the airport, giving your answer in square centimetres.

1 cm : 0.5 km

$(1\ \text{cm})^2 : (0.5\ \text{km})^2$

$1\ \text{cm}^2 : 0.25\ \text{km}^2$

$\therefore$ area on map $= \dfrac{3.5}{0.25}$

$= 14\ \text{cm}^2$

Answer ....14........ cm^2 [2]

9 Which one of the following could be the graph of

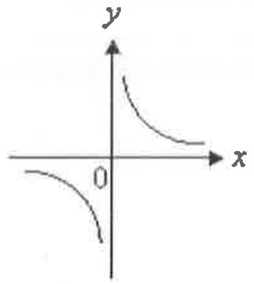


Figure 1

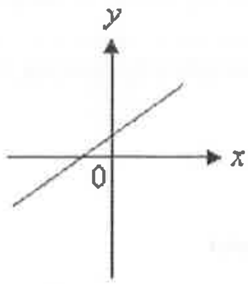


Figure 2

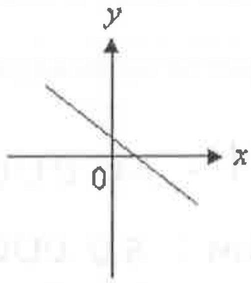


Figure 3

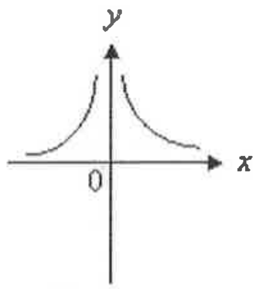


Figure 4

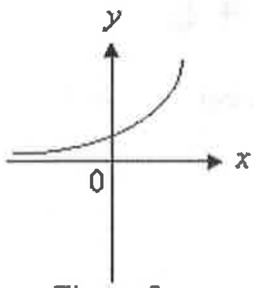


Figure 5

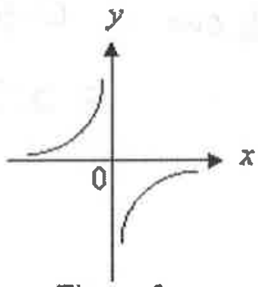


Figure 6

(a) $y = 2^x$,

Answer Figure 5 .... [1]

(b) $y = \dfrac{3}{x^2}$,

Answer Figure 4 .... [1]

(c) $x + y = 1$?

Answer Figure 3 .... [1]

10 Express as a single fraction in its simplest form

$$\frac{2x}{x^2-4}+\frac{5}{2-x}.$$

$$= \frac{2x}{(x-2)(x+2)} - \frac{5}{x-2}$$

$$= \frac{2x-5(x+2)}{(x-2)(x+2)}$$

$$= \frac{2x-5x-10}{(x-2)(x+2)}$$

$$= \frac{-3x-10}{(x-2)(x+2)}$$

or $\dfrac{3x+10}{(2-x)(x+2)}$

or $-\dfrac{(3x+10)}{(x-2)(x+2)}$

Answer $\dfrac{-3x-10}{(x-2)(x+2)}$ [3]

or $\dfrac{3x+10}{(2-x)(x+2)}$

11 **(a)** Express 60 as the product of its prime factors.

2 | 60
2 | 30
3 | 15
5 | 5
  | 1

Answer 60 = $2^2 \times 3 \times 5$ .... [1]

(b) A school buys packs of pens and packs of rulers.
There are 60 pens in each pack of pens.
There are 42 rulers in each pack of rulers.
The school wants to buy exactly the same number of pens and rulers.

Work out the smallest number of each pack the school should buy.

$42 = 2 \times 3 \times 7$

Lcm of 42 and 60

$= 2^2 \times 3 \times 5 \times 7$

$= 420$

$\frac{420 \text{ pens}}{60 \text{ pens}} = 7$ packs

$\frac{420 \text{ rulers}}{42 \text{ rulers}} = 10$ packs

Answer ....7............. packs of pens

.............10............. packs of rulers [2]

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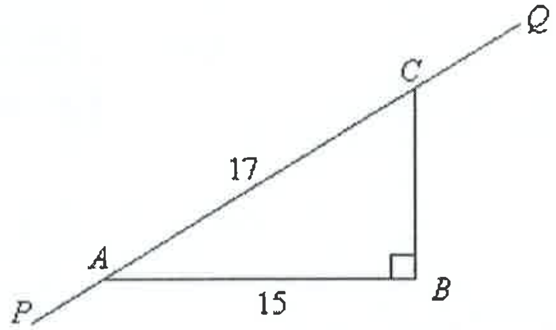
12 In the diagram, angle $ABC = 90°$, $AB = 15$ cm, $AC = 17$ cm and $PACQ$ is a straight line.



(a) Calculate the length of BC.

By Pythagoras' Theorem,

$BC = \sqrt{17^2 - 15^2}$

$= \sqrt{64}$

$= 8$

Answer BC = ...8............. cm [1]

(b) Write down, as a fraction, the value of

(i) $\tan C\hat{A}B$,

Answer $\tan C\hat{A}B$ = ...$\frac{8}{15}$......... [1]

(ii) $\sin P\hat{A}B$,

Answer $\sin P\hat{A}B$ = ...$\frac{8}{17}$......... [1]

(iii) $\cos Q\hat{C}B$.

Answer $\cos Q\hat{C}B$ = ...$-\frac{8}{17}$......... [1]

13 Factorise completely

(a) $3ab - 6ac + 4cd - 2bd$,

$= 3a(b-2c) + 2d(2c-b)$

$= 3a(b-2c) - 2d(b-2c)$

$= (b-2c)(3a-2d)$

Answer $(b-2c)(3a-2d)$ .... [2]

(b) $y^2 - 6xy + 5x^2$.

$y - 5x$	$-5xy$
$y - x$	$-xy$
$y^2 + 5x^2$	$-6xy$

$(y-5x)(y-x)$

Answer $(y-5x)(y-x)$ .... [2]

14 **(a)** **(i)** Show that the point (–1, 0.5) lies on the line $2y = 3x + 4$.

Answer

sub $x = -1$,

$2y = 3(-1) + 4$

$2y = 1$

$\therefore y = 0.5$

$\therefore (-1, 0.5)$ lies on the line $2y = 3x + 4$. [1]

(ii) Find the coordinates of the y-intercept of the line $2y = 3x + 4$.

$2y = 3x + 4$

$\Rightarrow y = 1.5x + 2$

y-intercept is (0, 2).

Answer (0 , 2) [1]

(b) A is the point (2, 4) and B is the point (m, n), where m and n are both positive integers less than 10.

The gradient of the line AB is $\frac{2}{3}$. $\frac{\text{change in } y}{\text{change in } x}$

Find the coordinates of one of the possible positions for B.

Case 1

$\frac{n-4}{m-2} = \frac{2}{3}$

$n = 2+4 = 6$

$m = 3+2 = 5$

B is (5, 6)

Case 2

$\frac{n-4}{m-2} = \frac{4}{6}$

$n = 4+4 = 8$

$m = 6+2 = 8$

B is (8, 8)

$3n - 12 = 2m - 4$

$3n = 2m + 8$

Answer (5 , 6) [2]

or (8 , 8)

15 Container A and container B are geometrically similar.

The surface area of container A is $\frac{9}{16}$ of the surface area of container B.

(a) Container B has a diameter of 24 cm.

Calculate the diameter of container A.

$$\frac{d_A}{d_B} = \sqrt{\frac{9}{16}}$$

$$\frac{d_A}{24} = \frac{3}{4}$$

$$\therefore d_A = 18 \text{ cm}$$

Answer ....18........ cm [2]

(b) Each container has chocolates in it.
Container A costs \$10.80.

Calculate the cost of container B.

$$\frac{C_B}{C_A} = \frac{V_B}{V_A}$$

$$\frac{\text{Cost of B}}{\$10.80} = \left(\frac{4}{3}\right)^3$$

$$\therefore \text{cost of B} = \frac{64}{27} \times 10.80$$

$$= \$25.60$$

Answer \$ ....25.60........ [2]

16 **(a)** Given that $\dfrac{x^5 \times x^0}{\sqrt{x}} = x^k$, find k.

$x^k = x^{5-\frac{1}{2}}$

$= x^{\frac{9}{2}}$

$\therefore k = \dfrac{9}{2}$

Answer $k =$ $\dfrac{9}{2}$ [1]

(b) Simplify, expressing your answer in positive index,

(i) $\left(\dfrac{x^{27}}{27}\right)^{\frac{2}{3}}$,

$= \dfrac{x^{27 \times \frac{2}{3}}}{27^{\frac{2}{3}}}$

$= \dfrac{x^{18}}{9}$

Answer $\dfrac{x^{18}}{9}$ [1]

(ii) $\dfrac{27}{7} \div \left(\dfrac{p^2}{3}\right)^{-1}$.

$= \dfrac{27}{7} \div \dfrac{3}{p^2}$

$= \dfrac{27}{7} \times \dfrac{p^2}{3}$

$= \dfrac{9p^2}{7}$

Answer $\dfrac{9p^2}{7}$ [2]

17 The stem-and-leaf diagram shows the distribution of the lengths of fish caught by a fisherman.
The numbers x and y are non-negative digits.
The mode and range of the data are 24 cm and 22 cm respectively.

Stem	Leaf							
1	x	4	5	7	8			
2	0	1	2	y	4	4	5	8
3	0	0	3					

Key: 2|0 means 20 cm.

(a) Find the value of x and the value of y.

range = max − min
22 = 33 − min
∴ min = 11
∴ x = 1

mode = 24
∴ y = 4.

Answer x = 1

y = 4 [2]

(b) If two of the fishes caught are selected at random, find the probability that at least one of them is longer than or equal to 25 cm.

$1 - P(<25\text{cm}, <25\text{cm})$

$= 1 - \left(\frac{11}{16} \times \frac{10}{15}\right)$

$\left(\frac{5}{16} \times \frac{4}{15}\right) + 2\left(\frac{5}{16} \times \frac{11}{15}\right)$

$= \frac{13}{24}$

Answer $\frac{13}{24}$ [2]

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18 **(a)** $\mathscr{E} = \{x : x \text{ is a positive integer}, \ x \leq 12\}$
$X = \{x : x \text{ is a prime number}\}$ $X = \{2, 3, 5, 7, 11\}$
$Y = \{x : \ x \leq 6\}$ $Y = \{1, 2, 3, 4, 5, 6\}$

(i) Find $X \cap Y$.

Answer { 2, 3, 5 } [1]

(ii) A number, k, is chosen at random from $\mathscr{E}$.

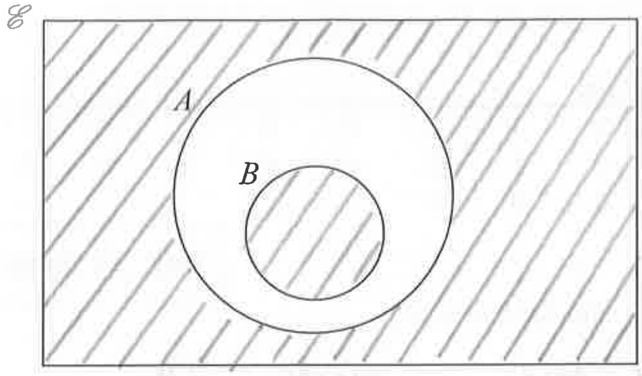
Find the probability that $k \notin (X \cup Y)$.
Give your answer as a fraction in the lowest terms.

$X \cup Y = \{1, 2, 3, 4, 5, 6, 7, 11\}$
$(X \cup Y)' = \{8, 9, 10, 12\}$

$\frac{4}{12} = \frac{1}{3}$

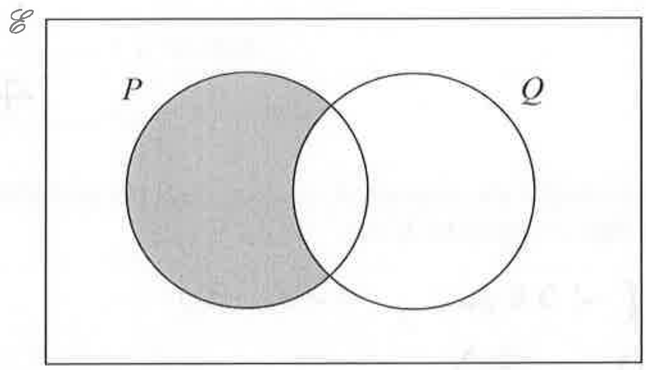
Answer $\frac{1}{3}$ [1]

(b) **(i)** On the Venn diagram, shade the region which represents $A' \cup B$.



[1]

(ii) Use set notation to describe the shaded region.



$Q' \cap P$

Answer $P \cap Q'$ [1]

$(P' \cup Q)'$

19 The scale drawing in the answer space below shows the positions of three shops A, B and C.
A is due North of B.

(a) Find the bearing of C from B.

Answer ....118°........ [1]

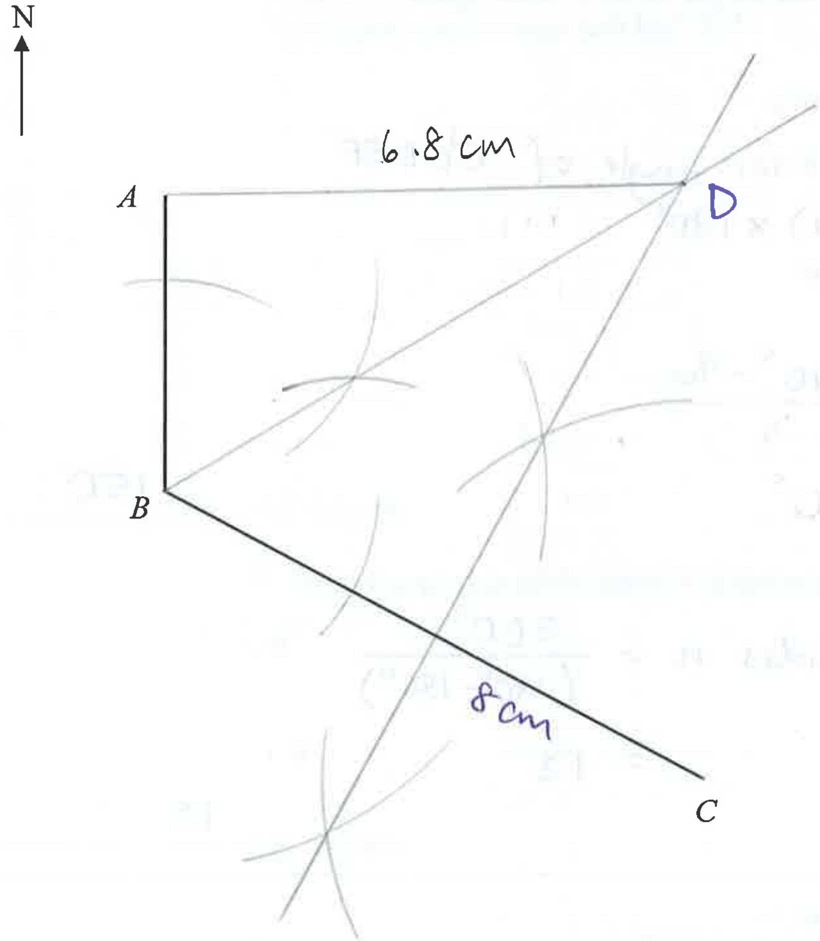
(b) A new shop D is to be built equidistant from AB and BC, and equidistant from B and C.

By constructing bisectors, find and label the position of the new shop D. [2]

(c) Given that $BC = 1600$ m, calculate the actual distance from A to the new shop D.

Answer ....1340........ m [1]

(Accept 1320 m to 1360 m



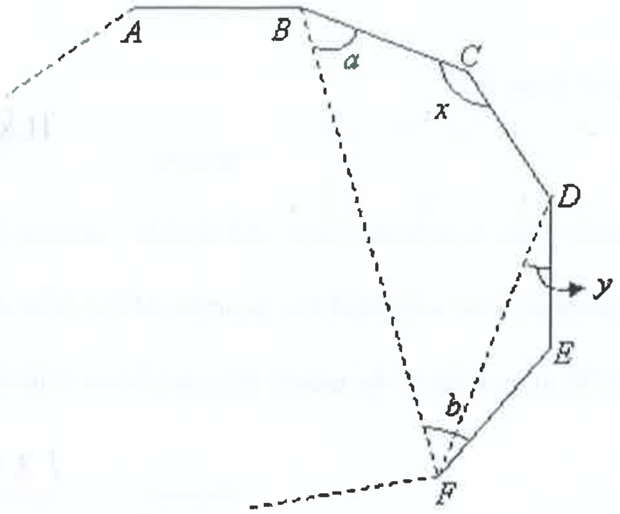
8 cm : 1600 m

1 cm : 200 m

AD 6.7 cm : 6.8 × 200

= 1340 m

20



The figure *ABCDEF* shows part of an n-sided regular polygon.
It is given that $\angle a + \angle b = 90°$ and that each interior angle is $x°$.

(a) Find the value of x.

Total interior angle of BCDEF

$= (5-2) \times 180°$

$= 540°$

$$x = \frac{540° - 90°}{3}$$

$= 150°$

Answer $x =$ 150 .... [2]

(b) Hence, find the number of sides of the regular polygon.

$$\text{No. of sides } n = \frac{360°}{(180° - 150°)}$$

$= 12$

Answer $n =$ 12 .... [1]

(c) Find the value of y.

$$y = \frac{180° - 150°}{2}$$

$= 15°$

Answer $y =$ 15 .... [1]

21 Khalid has savings of $15 000 which he would like to deposit in a bank for 5 years. He is deciding between the schemes offered by two banks.

Bank A: Simple interest of 1.2% per annum.
Bank B: Compound interest compounded monthly at 0.85% per annum.

Which bank should he leave his money with? Explain briefly.

Answer

Bank A: Interest $= \dfrac{15\,000 \times 1.2 \times 5}{100}$

$= \$900$

Bank B: Total amount $= 15000\left(1 + \dfrac{\frac{0.85}{12}}{100}\right)^{60}$

$= \$15651.00537$

$\therefore$ Interest $= 15651.00537 - 15000$

$= \$651.00537$

$= \$651$ (3 s.f.)

Khalid should leave his money with Bank A, as he would earn more interest ($900 > $651).

[5]

[Turn over

22 **(a)** The expression $x^2 - 10x + 17$ can be written in the form $(x-5)^2 + n$.

(i) Find the value of n.

$x^2 - 10x + 17 = (x-5)^2 + 17 - (5)^2$
$= (x-5)^2 - 8$

$\therefore n = -8$

Answer $n =$ -8 [1]

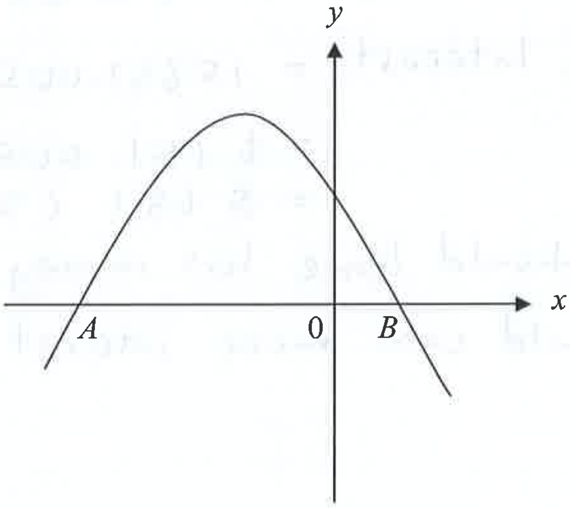
(ii) Explain why when $x = 5$, the expression $x^2 - 10x + 17$ has its minimum value.

$(x-5)^2$ has a min/smallest value since $(x-5)^2 \geq 0$. As such, when $(x-5)^2$ is min, $(x-5)^2 - 8$ is a min.

As $(x-5)^2$ is never negative, then $(x-5)^2 \geq 0$.

$\therefore x^2 - 10x + 17$ has its min. value when $x=5$. [1]

(b) The diagram shows the graph of $y = (1-x)(x+4)$.
The curve passes through the x-axis at point A and point B.



(i) Find the coordinates of A and of B.

x-axis $\Rightarrow y = 0$

$(1-x)(x+4) = 0$

$x = 1$ or $x = -4$

Answer A (-4 , 0) [1]

B (1 , 0) [1]

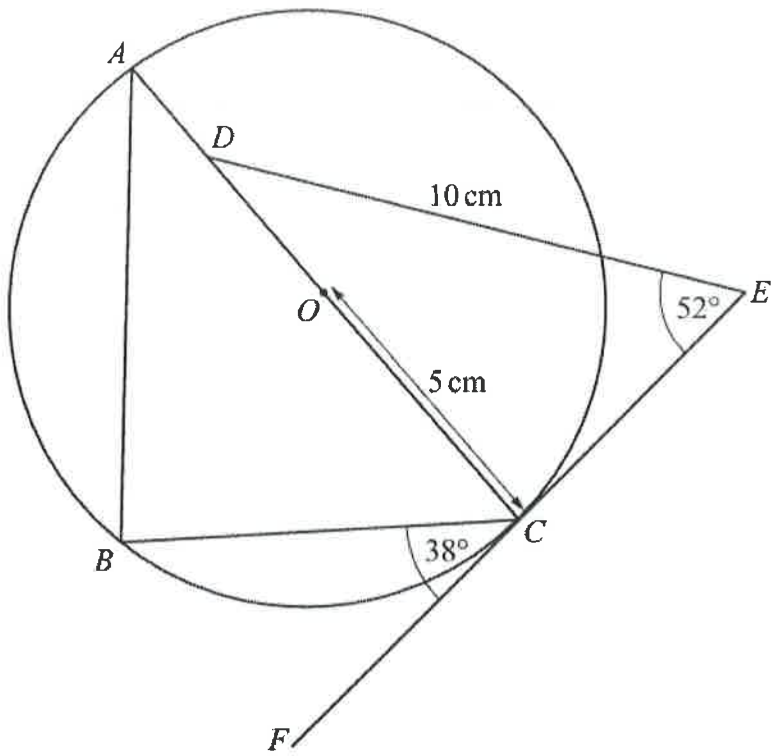
(ii) Write down the equation of the line of symmetry of the graph $y = (1-x)(x+4)$.

$x = \dfrac{1 + (-4)}{2}$

$\therefore x = -1.5$

Answer $x = -1.5$ [1]

23



A, B and C are points on a circle, centre O, with radius 5 cm.
AC is a diameter of the circle and point D lies on AC.
EF is a tangent to the circle at C.
$DE = 10$ cm.

(a) Show that triangle ABC is congruent to triangle DCE.
Give a geometrical reason for each statement you make.

Answer

① $\angle ABC = 90°$ ($\angle$ in a semicircle)
$\angle DCE = \angle OCE = 90°$ (tangent $\perp$ radius)
$\therefore \angle ABC = \angle DCE = 90°$

or adj. $\angle$s on a straight line

② $\angle ACB = 90° - 38° = 52°$ (tangent $\perp$ radius)
$\therefore \angle ACB = \angle DEC = 52°$

must be seen ③ $AC = DE = 10$ cm (given)

$\therefore \triangle ABC \equiv \triangle DCE$ (AAS)

[3]

(b) Calculate AD.

$\sin 52° = \frac{DC}{10}$

$DC = 10 \sin 52° = 7.8801$

$\therefore AD = 10 - (10 \sin 52°)$
$= 2.11989$
$= 2.12$ cm (3 s.f.)

Answer $AD =$ ......2.12...... cm [3]