SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME	**Solution**										
CLASS	4					REGISTER NUMBER					
CENTRE NUMBER						INDEX NUMBER					

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

Wednesday **31 August 2022** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR EXAMINERS USE

Q1		**Q5**		**Q9**	
Q2		**Q6**		**Q10**	
Q3		**Q7**		**Q11**	
Q4		**Q8**		**Q12**	

/ 90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1. The equation of a curve is $y = -x^2 - kx + k - 8$, where k is a constant.

(a) Find the range of values of k for which the curve intersects the x-axis. [3]
(b) In the case where k = 2, show that the line $y = 4x + 3$ is a tangent to the curve. [2]

(a)	$-x^2 - kx + k - 8 = 0$ $(-k)^2 - 4(-1)(k-8) \geq 0$ $k^2 + 4k - 32 \geq 0$ $(k+8)(k-4) \geq 0$ $k \leq -8$ or $k \geq 4$
(b)	$-x^2 - 2x - 6 = 4x + 3$ $x^2 + 6x + 9 = 0$ Discriminant = $(-6)^2 - 4(-1)(-9)$ $= 0$ $\therefore$ line is a tangent to the curve OR $x^2 + 6x + 9 = 0$ $(x+3)^2 = 0$ line **touches the *x*-axis at only one point** where $x = -3$ $\therefore$ line is a tangent to the curve

2. [The volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$.]

An empty, inverted cone has a height of 60 cm and base radius 20 cm. The circular base is held horizontal and uppermost. Water is poured into the cone at a constant rate.

(a) When the depth of the water in the cone is h cm, show that the volume of the water in the cone is $\frac{\pi h^3}{27}$. [2]

The water level is rising at a rate of 3 cm per minute when the depth of the water is 12 cm.

(b) Find the rate at which water is being poured into the cone, leaving your answer in terms of π. [3]

(a)	$\frac{r}{20} = \frac{h}{60}$ $r = \frac{h}{3}$ $V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$ $= \frac{\pi h^3}{27}$ (shown)
(b)	$\frac{dv}{dh} = \frac{\pi h^2}{9}$ $\frac{dv}{dt} = \frac{dv}{dh} \times \frac{dh}{dt}$ $\frac{dv}{dt} = \frac{\pi(12)^2}{9} \times 3$ $= 48\pi$ cm^3/min

3. Solve the equation $\ln y + 1 = 2\log_y e$, giving your answers in terms of e. [4]

	$\ln y + 1 = 2\log_y e$ $\ln y + 1 = \frac{2}{\ln y}$ $(\ln y)^2 + \ln y - 2 = 0$ $(\ln y - 1)(\ln y + 2) = 0$ $\ln y = 1$ or $\ln y = -2$ $y = e$ or $\frac{1}{e^2}$

4. It is given that $y = \cos px + \sin\ qx$, where p and q are constants.

Given that $p + q = 6$ and $\frac{d^2y}{dx^2} + q^2 y = 12$ when $x = 0$. Calculate the values of p and q. [8]

$$\frac{dy}{dx} = -p\sin px + q\cos qx$$

$$\frac{d^2y}{dx^2} = -p^2 \cos px - q^2 \sin qx$$

When $x = 0$, $\frac{d^2y}{dx^2} = -p^2$

$$y = 1$$

$$-p^2 + q^2 = 12$$

$$(q - p)(q + p) = 12$$

$$q - p = 2 \text{ ---- } (1)$$

$$p + q = 6 \text{ ---- } (2)$$

(1) + (2) $q = 4$, $p = 2$

5. **(a)** By considering the general term of the binomial expansion $\left(\frac{x}{2}-\frac{k}{x^2}\right)^n$, where n is a positive integer, show that n is a multiple of 3 when the binomial expansion has a constant term. [3]

(b) Given that n = 9 and the constant term is $-\frac{2625}{2}$, find the value of k. [2]

(c) Hence, find the term independent of x in the expansion $\left(\frac{x^3}{5}+4\right)\left(\frac{x}{2}-\frac{k}{x^2}\right)^n$. [3]

(a)	General Term $=\binom{n}{r}\left(\frac{x}{2}\right)^{n-r}\left(-\frac{k}{x^2}\right)^r$ $n-3r=0$ $n=3r$ $\therefore$ n is a multiple of 3
(b)	r = 3 $\binom{9}{3}\left(\frac{1}{2}\right)^{9-3}(-k)^3=-\frac{2625}{2}$ k = 10
(c)	$9-3r=-3$ $r=4$ Coefficient of $x^{-3}=\binom{9}{4}\left(\frac{1}{2}\right)^{9-4}(-10)^4$ $= 39375$ Term independent of x $=\frac{1}{5}(39375)+4\left(-\frac{2625}{2}\right)$ = 2625

6. A curve is such that $\frac{d^2y}{dx^2} = kx - 2$, where k is a constant.

The curve has a minimum gradient at $x = \frac{1}{3}$.

(a) Show that $k = 6$. [1]

The normal to the curve at (1, 4) is $2y + x - 9 = 0$.

(b) Find the equation of the curve. [6]

(a)	$\frac{d^2y}{dx^2} = kx - 2$ $\frac{1}{3}k - 2 = 0$ $k = 6$
(b)	$\frac{dy}{dx} = 3x^2 - 2x + c$ $3(1)^2 - 2(1) + c = 2$ $c = 1$ $y = x^3 - x^2 + x + d$ $4 = 1 - 1 + 1 + d$ $d = 3$ Equation of the curve: $y = x^3 - x^2 + x + 3$

7. Given that $f(x) = 2 - 24\sin x \cos x$ and $g(x) = 10(1+\cos^2 x)$.

(a) Show that f (x) + g(x) = $5\cos 2x - 12\sin 2x + 17$. [3]

(b) By expressing f (x) + g(x) in the form R cos $(2x + \alpha) + q$ where $R > 0$, $q > 0$ and $0 < \alpha < \frac{\pi}{2}$, find the minimum value of $\frac{2}{f(x)+g(x)}$ and the corresponding values of x for $0 < x < \pi$. [6]

(a)	$2 - 24\sin x\cos x + 10\left(1+\cos^2 x\right)$ $= 10\cos^2 x - 24\sin x\cos x + 12$ $= 5\left(\cos 2x + 1\right) - 12\sin 2x + 12$ $= 5\cos 2x - 12\sin 2x + 17$
(b)	f (x) + g(x) $= \sqrt{5^2+12^2}\cos\left[2x + \tan^{-1}\left(\frac{12}{5}\right)\right] + 17$ $= 13\cos\left(2x + 1.18\right) + 17$ Min $\frac{2}{f(x)+g(x)} = \frac{2}{13+17} = \frac{1}{15}$ $\cos\left(2x + 1.1760\right) = 1$ $2x + 1.1760 = 2\pi$ $x = 2.55$

8. Given that $y = x\ln\sqrt{2x}$ for $x > 0$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Show that the y-coordinate of the turning point is $-\frac{1}{4e}$. [3]

(c) By considering the sign of $\frac{dy}{dx}$, determine the nature of the turning point. [2]

(a)

$$y = \frac{1}{2}x\ln 2x$$

$$\frac{dy}{dx} = \frac{1}{2}\ln 2x + \left(\frac{1}{x}\right)\left(\frac{x}{2}\right)$$

$$= \frac{1}{2}\ln 2x + \frac{1}{2} \quad \text{or} \quad \ln\sqrt{2x} + \frac{1}{2}$$

(b)

$$\frac{1}{2}(\ln 2x + 1) = 0$$

$$\ln 2x = -1$$

$$2x = \frac{1}{e}$$

$$x = \frac{1}{2e}$$

$$y = \frac{1}{4e}\ln\left(\frac{1}{e}\right)$$

$$= -\frac{1}{4e}$$

(c)

x	0.18394^-	0.18394	0.18394^+
$\frac{dy}{dx}$	$-$	0	$+$
slope	\	—	/

The stationary point is a minimum point.

9. **(a)** Given that $y=\dfrac{x-1}{\sqrt{2x-3}}$, show that $\dfrac{dy}{dx}=\dfrac{x-2}{\sqrt{(2x-3)^3}}$. [3]

(b) Hence find $\displaystyle\int \frac{3x}{\sqrt{(2x-3)^3}}\,dx$. [4]

(a)	$\dfrac{dy}{dx}=\dfrac{(2x-3)^{\frac{1}{2}}-\frac{1}{2}(2x-3)^{-\frac{1}{2}}(2)(x-1)}{2x-3}$ $=\dfrac{(2x-3)^{-\frac{1}{2}}\left[2x-3-(x-1)\right]}{2x-3}$ $=\dfrac{x-2}{\sqrt{(2x-3)^3}}$ (shown)
(b)	$\displaystyle\int \frac{x-2}{\sqrt{(2x-3)^3}}\,dx=\frac{x-1}{\sqrt{(2x-3)}}+c$ $\displaystyle\int \frac{x}{\sqrt{(2x-3)^3}}\,dx-\int \frac{2}{\sqrt{(2x-3)^3}}\,dx=\frac{x-1}{\sqrt{(2x-3)}}+c$ $\displaystyle\int \frac{x}{\sqrt{(2x-3)^3}}\,dx=\frac{x-1}{\sqrt{(2x-3)}}+\int 2(2x-3)^{-\frac{3}{2}}\,dx+c$ $=\dfrac{x-1}{\sqrt{(2x-3)}}+\dfrac{2(2x-3)^{-\frac{1}{2}}}{-\frac{1}{2}(2)}+c$ $=\dfrac{x-3}{\sqrt{(2x-3)}}+c$ $\displaystyle\int \frac{3x}{\sqrt{(2x-3)^3}}\,dx=\frac{3x-9}{\sqrt{(2x-3)}}+d$

10. A circle, centre C, has a chord AB where A is the point (7, 27) and B is the point (–11, 15). The normal at A passes through the point D (–3, 3).

(a) Showing all your working, find the equation of the circle. [7]
(b) Show that D lies on the circle. [2]

(a)	$m_{AD} = \dfrac{27-3}{7-(-3)}$ $= \dfrac{12}{5}$ Equation of AD, $y-3=\dfrac{12}{5}(x+3)$ $y=\dfrac{12}{5}x+\dfrac{51}{5}$ $m_{AB} = \dfrac{27-15}{7-(-11)}$ $= \dfrac{2}{3}$ Gradient of $\perp$ bisector of AB $=-\dfrac{3}{2}$ Midpoint of AB = (–2, 21) Equation of perpendicular bisector of AB, $y-21=-\dfrac{3}{2}(x+2)$ $y=-\dfrac{3}{2}x+18$ At C, $\dfrac{12}{5}x+\dfrac{51}{5}=-\dfrac{3}{2}x+18$ $\dfrac{12}{5}x+\dfrac{3}{2}x=18-\dfrac{51}{5}$ $\dfrac{39}{10}x=\dfrac{39}{5}$ $x=2$ $y=15$ Radius $= BC$ $= 13$ units Equation of circle, $(x-2)^2+(y-15)^2=169$
(b)	$CD = \sqrt{(2+3)^2+(15-3)^2}$ = 13 units = radius of circle Hence, D lies on the circle.

11. The fitness index, F, of a person is related to the total number of hours, t, of exercise this person does per week. The variables F and t are related by the formula, $F = 100 - ae^{bt}$, where a and b are constants. Measured values of F and t are given in the following table.

t	4	6	8	10
F	71.4	78.4	83.7	87.7

(a) On graph paper, plot ln (100 – F) against t. the vertical ln (100 – F) axis should start at 2.4 and have a scale of 2 cm to 0.2. [3]

Use the graph to estimate

(b) the value of a and of b. [4]

(c) the number of hours of exercise a person needs to do each week to reach a fitness index of 80. [2]

(a)

t	4	6	8	10
$\ln(100 - F)$	3.35	3.07	2.79	2.51

Line

(b)

$F = 100 - ae^{bt}$

$100 - F = ae^{bt}$

$\ln(100 - F) = \ln a + bt$

$\ln a$ = vertical-intercept = 3.90

$a = e^{3.90\pm0.1}$

$= 44.7$ to 54.6

b = gradient

$= \dfrac{3.9-2.4}{0-10.8}$

$= -0.139$ (accept -0.135 to -0.150)

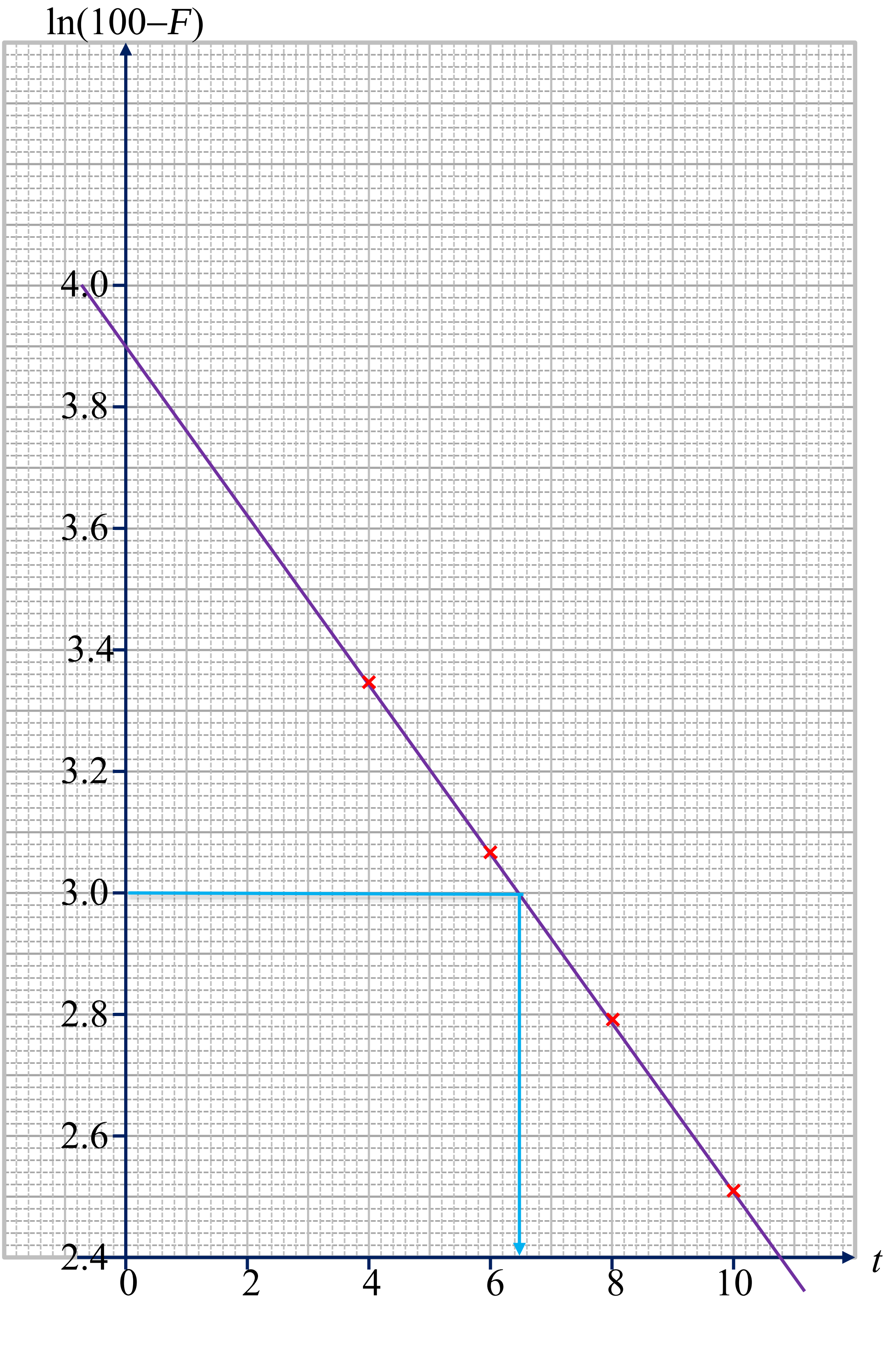
(c)

When $F = 80$, $\ln(100 - F) = \ln 20$

$= 3.00$

$t = 6.5$

He must complete 6.5 h of exercise.


ln(100−F)
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
0
2
4
6
8
10
t

9. A particle is moving in a straight line passes through a fixed point O with velocity of -15 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O, is given by $a = 2e^{2t} - 4e^{t}$.

(a) Find the initiation acceleration of the particle. [1]

(b) Explain what the sign of the acceleration indicates. [1]

(c) Find the value of t when the particle is at instantaneous rest, leaving your answer in exact form. [4]

(d) Find the displacement of the particle from the fixed point O when it is at instantaneous rest. [4]

(e) Explain what the sign of displacement indicates with reference to the fixed point O. [1]

(a)	When $t = 0$, $a = 2e^0 - 4e^0$ $= -2 \text{ ms}^{-2}$
(b)	The negative/minus sign in the answer for acceleration means that the insect is slowing down.
(c)	$v = \int (2e^{2t} - 4e^{t})\,\mathrm{d}t$ $= e^{2t} - 4e^{t} + c$ When $t = 0$, $v = -15$, $-15 = 1 - 4 + c$ $c = -12$ $\therefore v = e^{2t} - 4e^{t} - 12$ When $v = 0$, $e^{2t} - 4e^{t} - 12 = 0$ Let $y = e^{t}$. $y^2 - 4y - 12 = 0$ $(y+2)(y-6) = 0$ $e^t = -2$ (NA) or $e^t = 6$ $t = \ln 6$
(d)	$s = \int (e^{2t} - 4e^{t} - 12)\mathrm{d}t$ $= \dfrac{e^{2t}}{2} - 4e^{t} - 12t + d$ When $t = 0$, $s = 0$, $0 = \dfrac{1}{2} - 4 + d$ $d = \dfrac{7}{2}$ or 3.5 $\therefore s = \dfrac{e^{2t}}{2} - 4e^{t} - 12t + 3.5$ When $t = \ln 6$,

	$s=\frac{1}{2}(36)-4(6)-12\ln 6+3.5$ $=-24.0011$ The displacement of the insect from *O* at instantaneous rest is -24.0 cm
(e)	The negative / minus sign in the answer for displacement means that the insect is at a position to the left of *O*.

End of Paper 2