Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Integrate $\frac{3(1-x)^3-2}{1-x}$ with respect to x. [3]

2 If $p+q\sqrt{2}=\frac{3+2\sqrt{2}}{1-\sqrt{2}}$, where p and q are rational numbers, find the value of p and of q. [4]

3 Air is being pumped into a spherical balloon at a constant rate of 20 cm^3 per second. However, when the radius of the balloon stretches beyond 7 cm, air starts to leak out from the balloon at a rate of 4 cm^3 per second. Find the rate of change of the radius of the balloon when the radius is 11 cm. [4]

4 Find the coordinates of the points of intersection of the curve $2x^2 + 5xy - y^2 + 48 = 0$ and the line $y + 8 = 2x$. [5]

5 Express $\frac{8x^2+3x+1}{x(2x+1)^2}$ in partial fractions. [5]

6 **(a)** Express the equation $4^{x-1} = 3 + 2^{x+2}$ as a quadratic equation in 2^x. [2]

(b) Hence find the value(s) of x, giving your answer(s) correct to 2 decimal places. [4]

[Turn over

7 A manufacturing company modelled the average cost per chair, in dollars, using the function $\mathrm{C}(x) = 40x^2 - 480x + 1442$, where x is the number of chairs produced, in thousands.

(a) Express $\mathrm{C}(x) = 40x^2 - 480x + 1442$ in the form $a(x-h)^2 + k$, where a, h and k are constants. [2]

(b) Hence find the minimum value of C(x) and the corresponding value of x. [2]

(c) Interpret the meaning of your answers to **part (b)**. [2]

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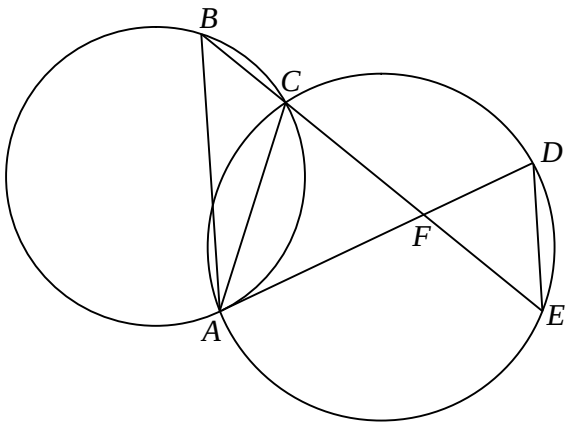


In the diagram, *AC* is a common chord of the two circles, *ABC* and *ACDE*.
AD is a tangent to the circle *ABC* at *A*. *AD* and *CE* intersect at *F*.

(a) Prove that *AB* is parallel to *ED*. [3]

(b) Prove that triangle *ACF* is similar to triangle *BAF*. [2]

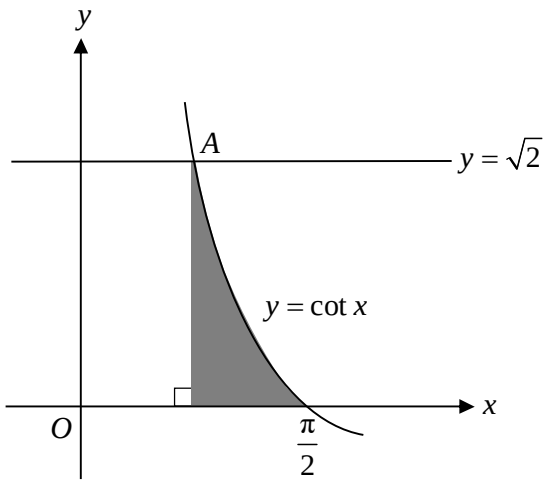
(c) Hence show that $AF^2 = BF \times CF$. [1]

9 **(a)** Prove that $\frac{\cos 2\theta + 1}{3 + 3\sin\theta} = \frac{2(1 - \sin\theta)}{3}$. [3]

(b) Hence solve the equation $\frac{\cos 2\theta + 1}{3 + 3\sin\theta} = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

10 **(a)** Differentiate $\ln(\sin x)$ with respect to x. [2]

(b)



The diagram shows part of the curve $y = \cot x$, cutting the x-axis at $\left(\frac{\pi}{2}, 0\right)$.

The line $y = \sqrt{2}$ intersects the curve at point A. Using your result from **part (a)**, find the area of the shaded region. [5]

[Turn over

11 The function $f(x)$ is defined by $f(x) = ax^3 + 8x^2 + bx + 18$ for all real values of x. It is given that $x + 2$ is a factor of $f(x)$ and when $f(x)$ is divided by $x + 1$, the remainder is 4.

(a) Find the value of a and of b. [5]

(b) Factorise $f(x)$ completely. [3]

12 *ABCD* is a parallelogram. The equation of *AB* and *AD* are $x = 10 - 3y$ and $y = x - 2$ respectively. The coordinates of *B* are (7, 1) and the diagonals of the parallelogram intersect at (7.5, 3.5).

Find

(a) the coordinates of *C*, [4]

(b) the equation of AC, [3]

(c) the area of the triangle ABC. [2]

13 The function f is defined, for all values of x, by

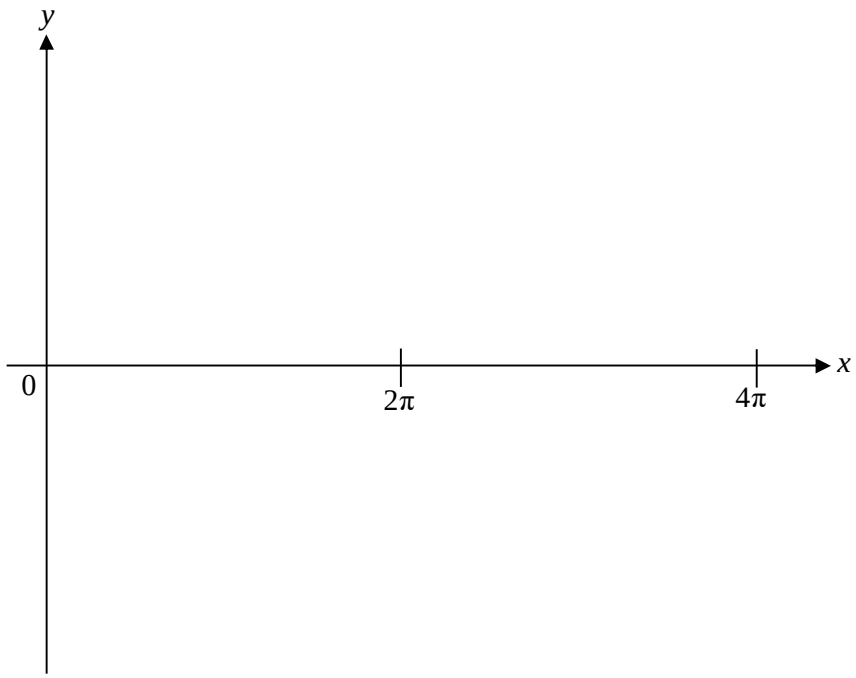
$$f(x) = a\sin\left(\frac{x}{b}\right) + c,$$

where a, b and c are constants.
The graph of $y = f(x)$ has a period 4π radians and passes through the points $(\pi, 2)$ and $(4\pi, -1)$.

(a) Show that $b = 2$. [2]

(b) Find the values of the integers a and c. [4]

(c) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \leq x \leq 4\pi$ radians. [2]



(d) Explain how you would use the graph of $y = \mathrm{f}(x)$ to solve the equation $\sin\left(\frac{x}{2}\right) = \frac{x}{12\pi} - \frac{1}{3}$. [2]

14 A sector OAB is formed by bending a piece of wire 150 cm long. It is given that O is the centre of the circle, $OA = r$ cm and θ, in radians, is the included angle.

(a) Show that the area, A cm^2, of the sector OAB is given by

$$A = 75r - r^2 .$$ [4]

(b) Given that r can vary, find the stationary value of A and determine its nature. [6]

End of Paper

Answer Key

1 $-(1-x)^3 + 2\ln(1-x) + c$

2 $p = -7,\ q = -5$

3 0.0105 cm/s

4 (–1, –10), (2, –4)

5 $\frac{1}{x} + \frac{2}{2x+1} - \frac{3}{(2x+1)^2}$

6 **(a)** $(2^x)^2 - 16(2^x) - 12 = 0$

(b) $x \approx 4.06$

7 **(a)** $C(x) = 40(x-6)^2 + 2$

(b) Minimum value of $C(x) = 2$

Corresponding value of $x = 6$

(c) Minimum average cost per chair

= \$2

Number of chairs produced = 6000

9 **(b)** $14.5^o,\ 165.5^o$

10 **(a)** $\cot x$

(b) 0.549 units^2

11 **(a)** $a = 1,\ b = 21$

(b) $f(x) = (x+2)(x+3)^2$

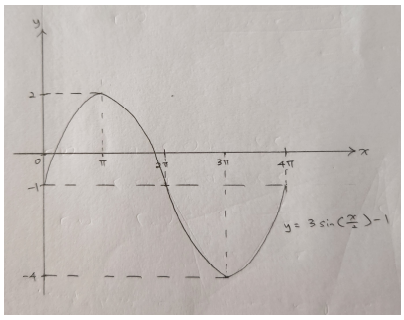
12 **(a)** (11, 5)

(b) $y = \frac{3}{7}x + \frac{2}{7}$ (or $7y = 3x + 2$)

(c) 8 units^2

13 **(b)** $a = 3,\ c = -1$

(c)



(d) The x-coordinates of the points of intersection of the graphs of $y = f(x)$ and $y = \frac{x}{4\pi} - 2$ give the solutions to the equation $\sin\left(\frac{x}{2}\right) = \frac{x}{12\pi} - \frac{1}{3}$.

14 **(b)** $A = 1406.25$, maximum