2022 A Math 4E/5N PRELIM Paper 2 Marking Scheme

Solutions		
1a	$64m^{3x} = 27$ $m^{3x} = \frac{27}{64}$ $m^{3x} = \left(\frac{3}{4}\right)^3$ or $m^x = \sqrt[3]{\frac{27}{64}}$ $\therefore m^x = \frac{3}{4}$	
1b	$m > 0$, $m \neq 1$	
2a **2b**	$y = \frac{2x}{9-x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(9-x^2)(2)-(2x)(-2x)}{(9-x^2)^2}$ $= \frac{2x^2+18}{(9-x^2)^2}$ $\int \frac{2x^2+18}{(9-x^2)^2}\,\mathrm{d}x = \frac{2x}{9-x^2}+c$ $2\int \frac{9+x^2}{(9-x^2)^2}\,\mathrm{d}x = \frac{2x}{9-x^2}+c$ $\therefore \int \frac{9+x^2}{(9-x^2)^2}\,\mathrm{d}x = \frac{x}{9-x^2}+c$	
3	$2(e^z)^2 - e^z - 10 = 0$ $(2e^z - 5)(e^z + 2) = 0$ $e^z = \frac{5}{2}$ $z = \ln\left(\frac{5}{2}\right)$ $z = 0.92$ AND $e^z = -2$ (rejected as $e^z > 0$)	
4a	$f(-2) = -81$	
4b	$f(1) = 3(1)^3 - 11(1)^2 + 7(1) + 1$ $= 0$ $\therefore (x-1)$ is a factor. **Using Long division** $(x-1)(3x^2-8x-1) = 0$	

Solutions

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{8 \pm \sqrt{76}}{6}$$

$x = 2.79 \text{ or } -0.120 \qquad or \qquad x = 1$

5a

$$\cos\theta = \frac{DH}{3} \rightarrow DH = 3\cos\theta$$

$$\sin\theta = \frac{DK}{5} \rightarrow DK = 5\sin\theta$$

$$\therefore CF = DH + DK = 3\cos\theta + 5\sin\theta$$

5b

$$\text{Where } R = \sqrt{3^2 + 5^2}$$

$$= \sqrt{34}$$

$$\text{and } \alpha = \tan^{-1}\left(\frac{5}{3}\right)$$

$$= 59.036°$$

$$\therefore CF = \sqrt{34}\cos(\theta - 59.0°)$$

5c

$$\text{Max } CF = \sqrt{34} \text{ cm}$$

$$\text{When } \cos(\theta - 59.036°) = 1$$

$$\theta - 59.036° = 0°$$

$$\underline{\theta = 59.0°} \quad \text{(1 d.p.)}$$

6a

When $x = 0$, $y = 2$
A (0, 2)

6b

When $x = 4$, $y = 4$,
$4 = 4x + 2$
$m = 0.5$

6c

$$Area = \int_0^4 \frac{1}{2}x + 2\,dx - \int_0^4 2\sqrt{x}\,dx$$

$$= \int_0^4 \frac{1}{2}x + 2 - 2\sqrt{x}\,dx$$

$$= \left[\frac{1}{4}x^2 + 2x - \frac{4}{3}x^{\frac{3}{2}}\right]_0^4$$

$$= 1\frac{1}{3} \text{ units}^2$$

7a

x	10	20	30	40
$lg\ y$	-1.221	-0.420	0.374	1.167

7b

$$y = \frac{b^x}{10^A} \rightarrow \lg y = x\lg b - A$$

Accept $1.95 \le A \le 2.05$

Solutions		
	$\lg b = gradient = \frac{2-0.42}{20}$ Accept $1.16 \le b \le 1.24$	
7c	**Draw the graph:** $\lg y = -\frac{x}{4}$ $10^{A-\frac{x}{4}} = b^x$ $A-\frac{x}{4} = x\lg b$ $-\frac{x}{4} = x\lg b - A$ From graph, accept $6.0 \le x \le 6.5$	
8a	$v = \frac{ds}{dt} = 5e^{-t} - 0.4$ When $t = 0$, $v = 5e^{-0} - 0.4$ $= 4.6$ m/s	
8b	When v = 0, $0 = 5e^{-t} - 0.4$ $e^{-t} = 0.08$ $t \approx 2.53$	
8c	$a = \frac{dv}{dt} = -5e^{-t}$ When $t = 2.53$, $a \approx -0.4m/s^2$	
8d	t = 0, s = 0 m t = 2.53, s = 3.589 m t = 5, s = 2.966 m Total dist = 3.589 + (3.589 – 2.966) ≈ 4.21 m	
8e	t = 12, s = 0.199 m t = 13, s = −0.200 m Since the displacement s changes from a positive value (when t = 12) to a negative value (when t = 13), the particle passes through O at some instant during the thirteenth second.	
9a	$-x^2+5x-4<-10$ $-x^2+5x+6<0$ $x^2-5x-6>0$ $(x-6)(x+1)>0$ $\therefore x<-1 \, or \, x>6$	

Solutions		
9b	$-x^2+5x-4=-x+c$ $x^2-6x+4+c=0$ $Since\ \ b^2-4ac=0$ $(-6)^2-4(4+c)=0$ $\therefore c=5$	
10a	59049 + 98415x + 73811.25x^2 +….	
10b	x = 0.08 59049 + 98415(0.08) + 73811.25(0.08)^2 ≈ 67400	
10c	$T_{r+1}=\binom{9}{r}(x^2)^{9-r}\left(-\frac{k}{x}\right)^r$ $=\binom{9}{r}(x^{18-3r})(-k)^r$ $18-3r=0$ $\therefore \underline{r=6}$ x^0 term $=\binom{9}{6}(-k)^6$ $=84k^6$ $84k^6=5376$ $\therefore \underline{k=2}$	
11a **11b**	$A=(3,0),\qquad B=(0,4)$ $\therefore C=\left(\frac{3}{2},2\right)$ $\text{Radius}=\sqrt{\left(\frac{3}{2}-0\right)^2+(2-0)^2}$ $=\frac{5}{2}$ or 2.5 $\left(x-\frac{3}{2}\right)^2+(y-2)^2=\frac{25}{4}$ $x^2-3x+\frac{9}{4}+y^2-4y+4-\frac{25}{4}=0$ $x^2+y^2-3x-4y=0$	
11c	$\left(\frac{x_P+0}{2},\frac{y_P+0}{2}\right)=\left(\frac{3}{2},2\right)$ $(x_P,y_P)=(3,4)$ $m_{OP}=\frac{4}{3},\qquad m_T=-\frac{3}{4}$ $y-4=-\frac{3}{4}(x-3)$ $4y-16=-3x+9$	

Solutions		
	$4y = -3x + 25$	
11d	Since tangent // to y-axis $\Rightarrow x = c$ from centre of circle $x = \frac{3}{2} - \frac{5}{2} = -1$, $\quad x = \frac{3}{2} + \frac{5}{2} = 4$ $(x, y) = (-1, 7)$, $\left(4, \frac{13}{4}\right)$	
11e	Distance of R to centre $= \sqrt{(1.5-1)^2 + (2-4.3)^2}$ ≈ 2.35 Since 2.35 < 2.5, point R lies inside the circle.	