CHIJ ST. THERESA'S CONVENT

PRELIMINARY EXAMINATION 2022

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/2**

Paper 2

26 Aug 2022
2 hours 15 minutes

Candidates answer on the Question Paper as well as on the graph paper provided.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of 16 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

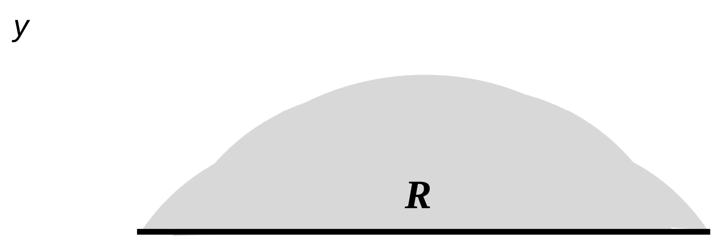
1 **(a)** Express $\dfrac{\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$ in the form $a\sqrt{6}+b$, where a and b are rational numbers. [2]

(b) Solve the equation $\sqrt{x-1}+5 = 2x$. [4]

2 The equation of a quadratic curve is $y = -\frac{1}{2}x^2 + 2x - 1$.

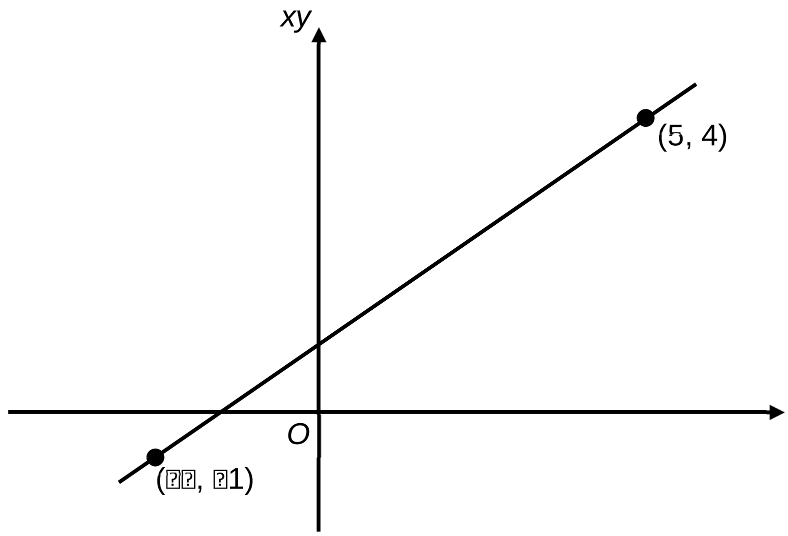
(i) Express y in the form $a(x+b)^2 + c$, where a, b and c are constants. [2]

The graph of $y = -\frac{1}{2}x^2 + 2x - 1$ is shown in the diagram. The region R consists of the shaded area as well as the perimeter surrounding the shaded area.



(ii) Find the range of values of x and the range of values of y corresponding to the region R. [4]

3



The diagram shows the straight line graph obtained by plotting xy against x^3. The line passes through the points $(-2, -1)$ and $(5, 4)$.

(i) Obtain an expression for y in terms of x. [3]

A second line is drawn on the same diagram above. This second line is parallel to the first line and it passes through the point with coordinates (0, 1).

(ii) Obtain an expression for y, for this second line, in terms of x. [2]

4 Two of the angles of a triangle are represented by x and y. It is given that

$$\sin(x+y)+\sin(x-y) = \frac{1}{4} \qquad \text{--- (1)}$$

and $$\cos(x+y)+\cos(x-y) = -\frac{1}{6} \qquad \text{--- (2)}$$

(i) Show that $\tan x = -\frac{3}{2}$. [3]

(ii) Explain why angle y is acute. [2]

(iii) Find the exact value of $\cos y$. [3]

5 **(i)** Differentiate $e^{-x}\sqrt{2x+7}$ with respect to x, giving your answer in the form $\dfrac{(a+bx)\,e^{-x}}{\sqrt{2x+7}}$ where a and b are constants. [3]

The curve $y = e^{-x}\sqrt{2x+7}$ has a stationary value at the point C.

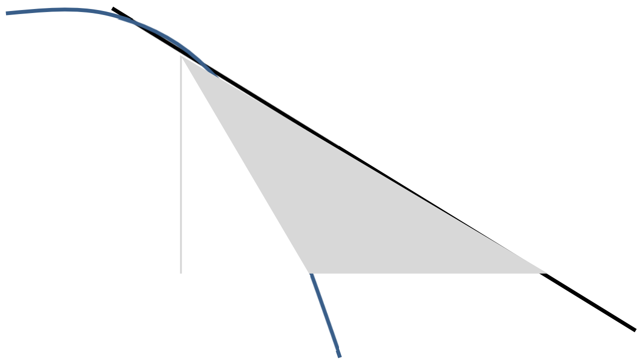
(ii) Find the x-coordinate of C. [2]

(iii) Determine if point C is a maximum or a minimum point. [2]

6(a) The polynomial f(x) is given by $24x^3+14x^2+ax+b$ where a and b are constants. It is given that f(x) is exactly divisible by $4x + 3$ and it leaves a remainder of –6 when it is divided by $x + 1$.
Find the value of a and of b. [4]

(b) Express $\dfrac{x^2+x+6}{(x+1)\left(2x^2+1\right)}$ in partial fractions. [5]

7



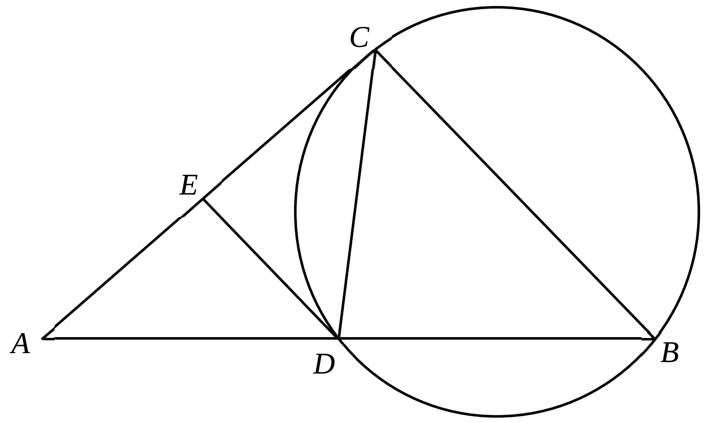
The diagram shows part of the curve $y = -x^3 + x^2 + 4$ which meets the x-axis at Q. The tangent to the curve at P meets the x-axis at R. The x-coordinates of P and Q are 1 and 2 respectively.

Find

(i) the equation of the tangent, [4]

(ii) the area of the shaded region PQR. [5]

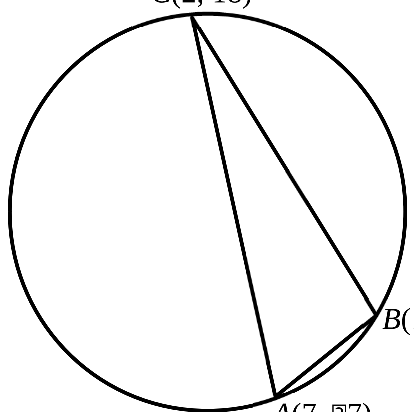
8 In the diagram, B, C and D are points on a circle. DE is a tangent to the circle. D and E are midpoints of AB and AC respectively.



(i) Show that BCD is an isosceles triangle. [3]

(ii) Show that angle $ACB = 90°$. [4]

9

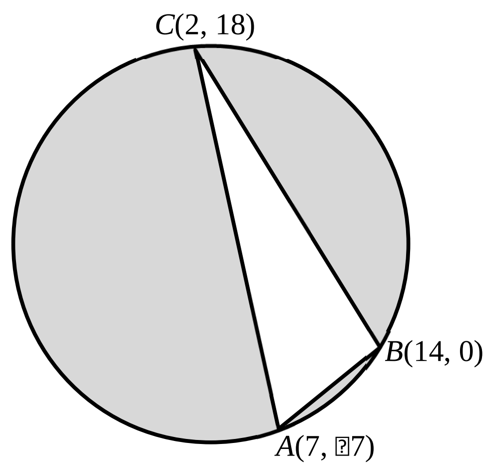


The coordinates of the points A, B and C are $(7, -7)$, $(14, 0)$ and $(2, 18)$ respectively. The equation of the perpendicular bisector of the line segment AB is $y = -x + 7$.

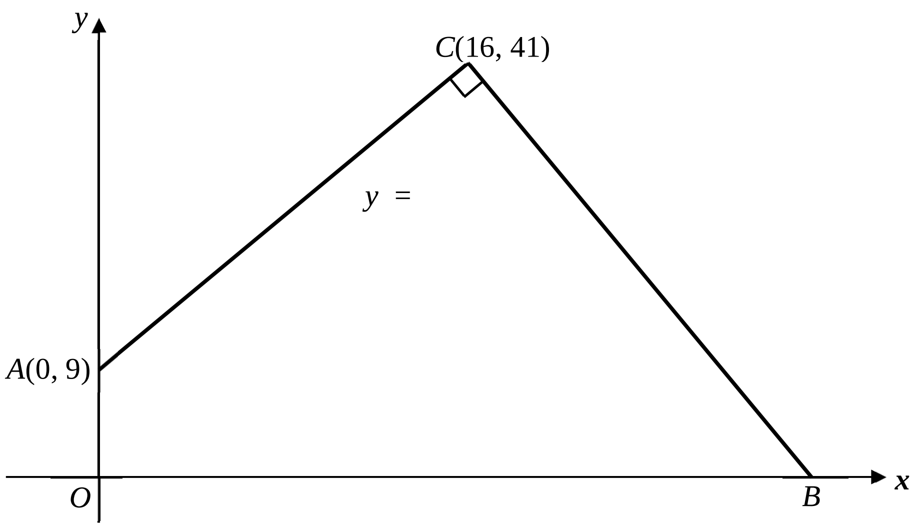
(i) Find the equation of the perpendicular bisector of the line segment BC. [3]

(ii) Hence find the equation of the circle that passes through the three points A, B and C. [4]

(iii) Find the total area of the shaded regions shown in the diagram. [3]



10 The diagram below shows the side view of a building. The line *OA* represents the vertical wall and the perpendicular lines *AC* and *BC* represent the slanted ceilings of the building.



(i) Find the equation of the line *AC*. [2]

(ii) Find the equation of the line *BC*. [2]

A particle is projected from the origin *O* as shown. The path taken by the particle inside the building is given by the equation $y = -x^2 + kx$.

(iii) Briefly explain why $k > 0$. [1]

For the particle not to touch the ceiling represented by *AC*, the value of k must be between 0 and 8.

(iv) Show that the particle will not touch the ceiling represented by *BC* if it does not touch the ceiling represented by *AC*. [5]

11 A particle moves in a straight line from a point O so that, t seconds after leaving O, its velocity, v m/s, is given by

$$v = \begin{cases} 5t - 5 & \text{for } 0 \le t \le 5 \\ 20e^{5-t} & \text{for } t > 5 \end{cases}$$

(i) Sketch the graph of v against t for t ε 0. [3]

(ii) Determine the time at which the particle comes to instantaneous rest. [1]

(iii) Show that the particle is 37.5m from O when $t = 5$. [3]

(iv) Determine the time when the deceleration of the particle is $0.5\ \text{m/s}^2$. [3]

(v) Find the displacement of the particle from O at the time found in part **(iv)**. [3]

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