



CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

Marking Scheme

CANDIDATE
NAME

CLASS

INDEX
NUMBER

MATHEMATICS

4048/2

Paper 2

23 August 2022

2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answers in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks is 100.

Mathematical Formulae**Compound interest**

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 (a) Simplify $\left(\frac{32a^{10}}{b^{15}}\right)^{-\frac{2}{5}}$.

$$\left(\frac{b^{15}}{2^5 a^{10}}\right)^{\frac{2}{5}} \quad [\text{M1}]$$

$$= \frac{b^6}{4a^4} \quad [\text{A1}]$$

Answer [2]

(b) Express $\frac{y}{(3y-5)^2} + \frac{2}{5-3y}$ as a single fraction in its simplest form.

$$\frac{y}{(3y-5)^2} + \frac{2}{5-3y}$$

$$= \frac{y-2(3y-5)}{(3y-5)^2} \quad [\text{M1}]$$

$$= \frac{10-5y}{(3y-5)^2} \quad \text{or} \quad \frac{5(2-y)}{(3y-5)^2} \quad [\text{A1}]$$

Answer [2]

(c) Simplify $\frac{3x^2-108}{2x^2+5x-42}$.

$$\frac{3x^2-108}{2x^2+5x-42}$$

$$= \frac{3(x^2-36)}{(x+6)(2x-7)} \quad [\text{M1}]$$

$$= \frac{3(x+6)(x-6)}{(x+6)(2x-7)} \quad [\text{M1}]$$

$$= \frac{3(x-6)}{2x-7} \quad [\text{A1}]$$

Answer [3]

- (d) Solve the inequality $\frac{3x-2}{4} \leq \frac{4+5x}{3}$.

$$\begin{aligned}\frac{3x-2}{4} &\leq \frac{4+5x}{3} \\ 3(3x-2) &\leq 4(4+5x) && \text{[M1]} \\ 9x-6 &\leq 16+20x \\ -11x &\leq 22 \\ x &\geq -2 && \text{[A1]}\end{aligned}$$

Answer [2]

- (e) Solve the simultaneous equations.

$$4x - 3y = 5$$

$$6x + 5y = -2$$

$$\begin{aligned}4x - 3y &= 5 && \text{---- (1)} \\ 6x + 5y &= -2 && \text{---- (2)} \\ (1) \times 3 - (2) \times 2: \\ 12x - 9y &= 15 && \text{----- (3)} && \text{[M1 – Elimination/Substitution Method]} \\ 12x + 10y &= -4 && \text{----- (4)} \\ (3) - (4): \\ -19y &= 19 \\ y &= -1 && \text{[A1]} \\ \text{Sub } y = -1 \text{ into (1)} \\ 4x - 3(-1) &= 5 \\ x &= \frac{1}{2} && \text{[A1]} \\ \therefore x &= \frac{1}{2}, y = -1\end{aligned}$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [3]

- 2 A bag contains 10 red balls and 8 green balls.
Anna takes two balls from the bag, at random, without replacement.

- (a) Anna says that the probability that she takes one red ball and one green ball is $\frac{40}{81}$.
Explain why Anna is wrong and provide the correct solution.

.... She does not take into consideration that the balls are taken **without replacement** hence [B1]

.... P(RG or GR)

.... $= \frac{10}{18} \times \frac{8}{17} + \frac{8}{18} \times \frac{10}{17}$ and correct answer is $\frac{10}{18} \times \frac{8}{17} + \frac{8}{18} \times \frac{10}{17}$

.... $= \frac{40}{81}$ $= \frac{80}{153}$ [B1]

.....[2]

- (b) Anna puts back the two balls into the bag.
 n red balls are removed from the bag.

The probability that she takes two green balls is $\frac{4}{13}$.

Write down an equation to represent this information and show that it simplifies to

$$n^2 - 35n + 124 = 0. \quad [3]$$

Answer

$$\begin{aligned} \text{P(two green balls)} &= \frac{4}{13} \\ \frac{8}{18-n} \times \frac{7}{17-n} &= \frac{4}{13} & [M1] \\ 4(18-n)(17-n) &= 13(8)(7) & [M1] \\ 4(306 - 35n + n^2) &= 728 \\ 4n^2 - 140n + 1224 &= 728 \\ n^2 - 35n + 124 &= 0 & [AG1] \end{aligned}$$

- (c) Solve the equation $n^2 - 35n + 124 = 0$.

Use factorisation or quadratic formula

$$(n-4)(n-31) = 0 \quad [\text{M1}] \quad \text{OR} \quad n = \frac{-(-35) \pm \sqrt{(-35)^2 - 4(1)(124)}}{2(1)} \quad [\text{M1}]$$

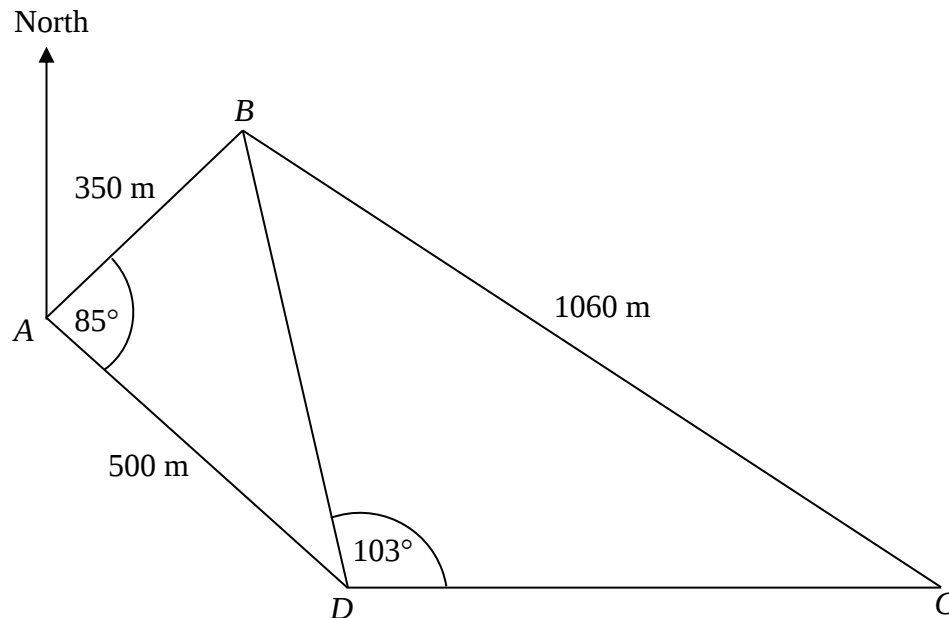
$$n = 4 \text{ or } n = 31 \quad [\text{A1, A1}] \quad n = \frac{35 \pm 27}{2} = 4 \text{ or } 31 \quad [\text{A1, A1}]$$

Answer $n = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- (d) Explain why one of the solutions in part (c) must be rejected.

... When $n = 31$, number of red balls $= 18 - n = 18 - 31 = -13 < 0$ hence
need to reject $n = 31$. [B1]
OR[1]
... Since there are only 10 red balls hence it is impossible to remove 31
red balls from the bag. [B1]

- 3 A, B, C and D are four points on horizontal ground.
 The points are connected by straight paths AB, BC, BD, AD and DC .
 C is due east of D .
 $AB = 350$ m, $AD = 500$ m and $BC = 1060$ m.
 Angle $BAD = 85^\circ$ and angle $BDC = 103^\circ$.



- (a) Show that $BD = 585$ m, correct to 3 significant figures. [3]

Answer

Using Cosine Rule,

$$BD^2 = 500^2 + 350^2 - 2(500)(350)\cos 85^\circ \quad [\text{M2}]$$

$$BD \approx 584.80 = 585 \text{ m (3 s.f.)} \quad [\text{A1}]$$

- (b) Calculate the bearing of B from C .

Using Sine Rule,

$$\frac{\sin \angle BCD}{584.80} = \frac{\sin 103^\circ}{1060} \quad [\text{M1}]$$

$$\angle BCD = \sin^{-1} \left(\frac{584.80 \sin 103^\circ}{1060} \right)$$

$$\approx 32.518^\circ \text{ or } 32.530^\circ \quad [\text{A1}]$$

Bearing of B from C
 $= 270^\circ + 32.519^\circ$
 $= 302.5^\circ \text{ (1 d.p.)} \quad [\text{A1}]$

Answer $^\circ$ [3]

- (c) John is a Grab delivery partner. He cycles at an average speed of 12 km/h. He starts at point C , cycles along path CB to collect food at point B then path BA to collect food at point A , and delivers the food collected to point D . He spends 5 minutes to collect the food at each collection point.

Calculate the time it takes him to complete the food delivery.
 Give your answer in minutes and seconds.

Total distance travelled
 $= (1060 + 350 + 500)/1000 = 1.91 \text{ km} \quad [\text{B1}]$

Total time taken
 $= \frac{1.91}{12} \times 60 + 10 \quad [\text{M1}]$
 $= 19.55 \text{ min}$
 $= 19 \text{ min } 33 \text{ sec} \quad [\text{A1}]$

Answer minutes seconds [3]

- 4 $ABCD$ is a trapezium with AB parallel to CD .
 A is the point $(-1, 3)$ and B is the point $(7, 9)$.

$$\overrightarrow{AD} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}.$$

- (a) Find the length of the line AB .

Length of AB	
$= \sqrt{(-1-7)^2 + (3-9)^2}$	[M1]
$= \sqrt{100}$	
$= 10 \text{ units}$	[A1]

Answer [2]

- (b) Find the equation of the line CD .

Gradient of CD = gradient of AB	
$= \frac{9-3}{7-(-1)}$	
$= \frac{3}{4} \text{ or } 0.75$	[M1]
Since $\overrightarrow{AD} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}$,	
$\overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}$	
$\overrightarrow{OD} - \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}$	[B1 – coord. of D]
$\overrightarrow{OD} = \begin{pmatrix} 1 \\ -7 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$	
Sub $m = \frac{3}{4}$, $x = 0$ and $y = -4$ into $y = mx + c$	
$-4 = \frac{3}{4}(0) + c$	[M1]
$c = -4$	
Equation of CD is $y = \frac{3}{4}x - 4$.	[A1]

Answer [4]

- (c) X is the point on AB such that $\frac{\text{area of triangle } AXD}{\text{area of triangle } BXD} = \frac{1}{3}$.

- (i) Find $\overrightarrow{AX}$.

$$\frac{\text{area of triangle } AXD}{\text{area of triangle } BXD} = \frac{1}{3} \Rightarrow \frac{AX}{XB} = \frac{1}{3} \quad [\text{B1}]$$

$$\overrightarrow{AX} = \frac{1}{4} \overrightarrow{AB}$$

$$\overrightarrow{AX} = \frac{1}{4} (\overrightarrow{OB} - \overrightarrow{OA})$$

$$\overrightarrow{AX} = \frac{1}{4} \left[\begin{pmatrix} 7 \\ 9 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right] = \frac{1}{4} \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix} \quad [\text{B1}]$$

Answer $\overrightarrow{AX} = \begin{pmatrix} \quad \\ \quad \end{pmatrix} [2]$

- (ii) Find the position vector of X .

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$

$$\overrightarrow{OX} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

$$\overrightarrow{OX} = \begin{pmatrix} 1 \\ 4.5 \end{pmatrix} \quad [\text{B1}]$$

Answer $\overrightarrow{OX} = \begin{pmatrix} \quad \\ \quad \end{pmatrix} [1]$

- 5 The variables x and y are connected by the equation $y = 3x + \frac{8}{x} - 11$.

Some corresponding values of x and y , correct to 1 decimal place, are given in the table below.

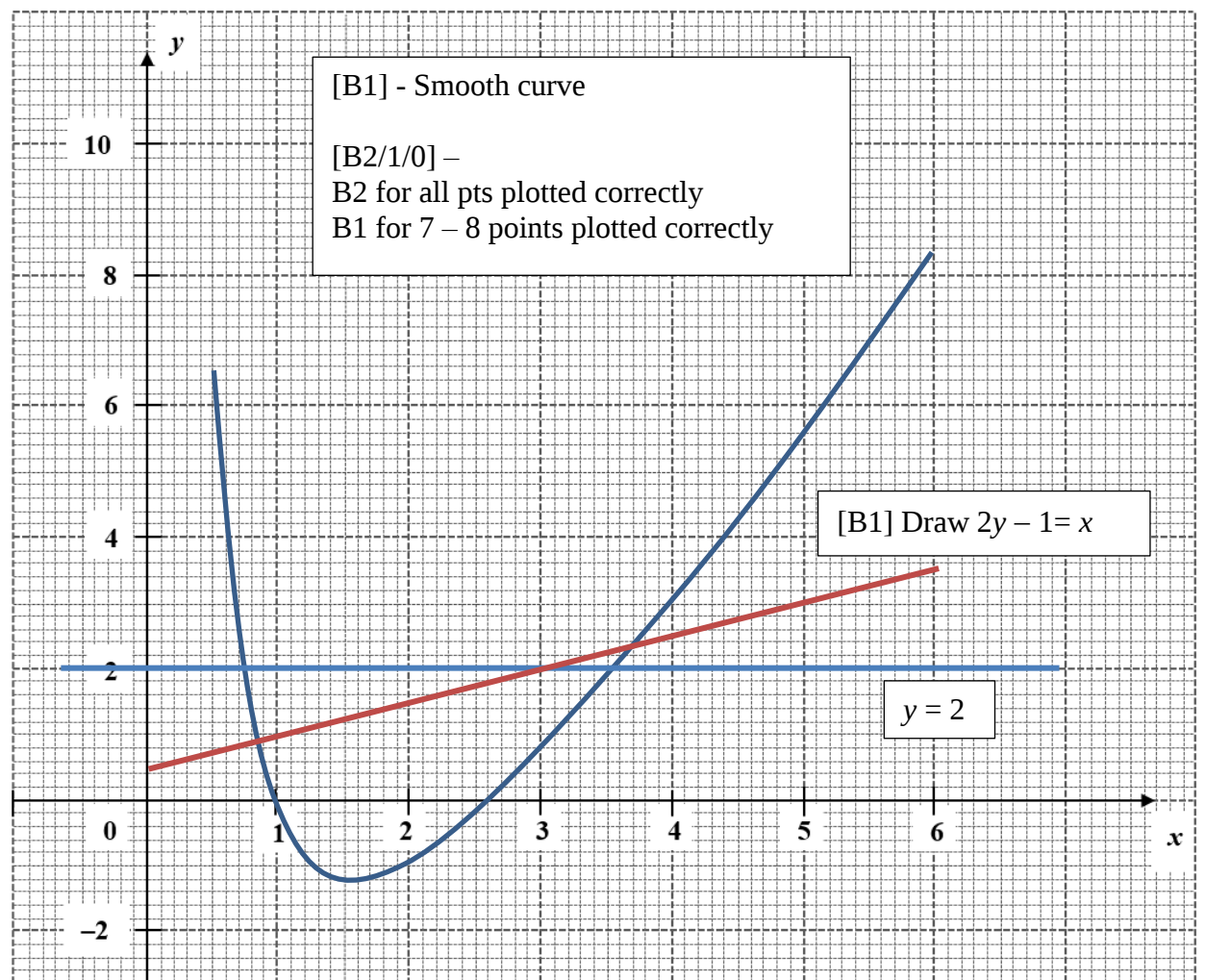
x	0.5	1	1.5	2	2.5	3	4	5	6
y	6.5	0	-1.2	-1	-0.3	0.7	3	5.6	p

- (a) Calculate the value of p .

$$p = 8.3 \text{ (1 d.p.)} \quad [\text{B1}]$$

Answer $p = \dots\dots\dots$ [1]

- (b) On the grid, draw the graph of $y = 3x + \frac{8}{x} - 11$ for $0.5 \leq x \leq 6$. [3]



- (c) Use your graph to find the solutions to the equation $3x + \frac{8}{x} = 13$ in the range $0.5 \leq x \leq 6$.

$$3x + \frac{8}{x} - 11 - 2 = 0$$

$$3x + \frac{8}{x} - 11 = 2$$

Draw $y = 2$ [B1]

Read the point where the graph intersects/cuts the $y = 2$.
 $x = 0.7 - 0.8$ or $3.5 - 3.7$ [B1]

Answer $x = \dots\dots\dots$ [2]

- (d) (i) On the same axes, draw the graph of $2y-1=x$ for $0 \leq x \leq 6$. [2]

Draw $2y-1=x$ [B1]			
x	0	3	6
y	0.5	2	3.5

- (ii) Write down the x -coordinates of the points where this line intersects the curve.

Read the point where the graph intersects/cuts the line.
 $x = 0.8 - 0.9$ or $3.7 - 3.8$ [B1, B1]

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (iii) These values of x give the solutions of the equation $5x^2 + Ax + B = 0$.
 Find the value of A and the value of B .

$$\text{Sub } 2y-1=x \Rightarrow y = \frac{x+1}{2} \text{ into } y = 3x + \frac{8}{x} - 11$$

$$3x + \frac{8}{x} - 11 = \frac{x+1}{2} \quad [\text{M1}]$$

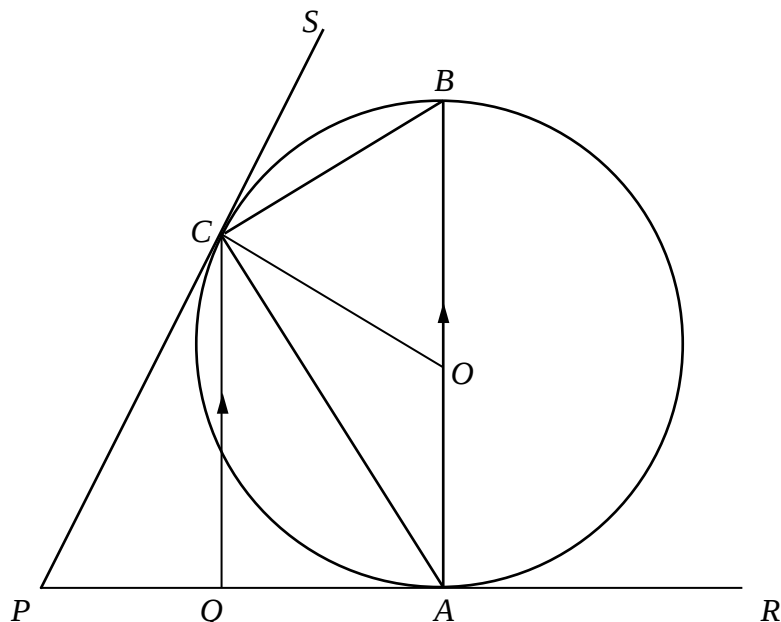
$$5x^2 - 23x + 16 = 0$$

Hence, $A = -23$ and $B = 16$. [A1, A1]

Answer $A = \dots\dots\dots$

$B = \dots\dots\dots$ [3]

6 (a)



A , B and C are points on the circle with centre O .

AOB is a straight line.

PCS and PAR are tangents to the circle.

Q is on PAR and QC is parallel to AB .

- (i) Show that triangle AQC is similar to triangle BCA .
Give a reason for each statement you make.

In $\triangle AQC$ and $\triangle BCA$,

$\angle BAP = \angle BAR = 90^\circ$ (radius $\perp$ tangent)

$\angle CQA = 90^\circ$ (corr. /int. $\angle$ s, $QC \parallel AB$)

$\angle ACB = 90^\circ$ ($\angle$ in semicircle)

$\therefore \angle CQA = \angle ACB$

$\angle QCA = \angle CAB$ (alt. $\angle$ s, $QC \parallel AB$)

By **AA similarity test**,
triangle AQC is similar to triangle BCA .

Alt. Method:

$\angle CAQ = \angle ABC$ ($\angle$ s in alt. segment)

$\angle QCA = \angle CAB$ (alt. $\angle$ s, $QC \parallel AB$)

[B2, 1, 0]

[2]

- (ii) Angle $COB = x^\circ$.
Find angle APC in terms of x .

$$\angle OBC = \frac{180^\circ - x^\circ}{2} \text{ (base angles, isosceles } \triangle)$$

$$\angle CAQ = \frac{180^\circ - x^\circ}{2} \text{ (corr. angles in similar } \triangle\text{s)} \quad [\text{B1}]$$

$PA = PC$ (tangents from external point P are equal)

$$\angle APC = 180^\circ - 2\left(\frac{180^\circ - x^\circ}{2}\right) = x^\circ \text{ (}\angle \text{ sum of } \triangle) \quad [\text{B1}]$$

Alt. Method:

In $OAPC$, $\angle AOC = 180^\circ - x^\circ$

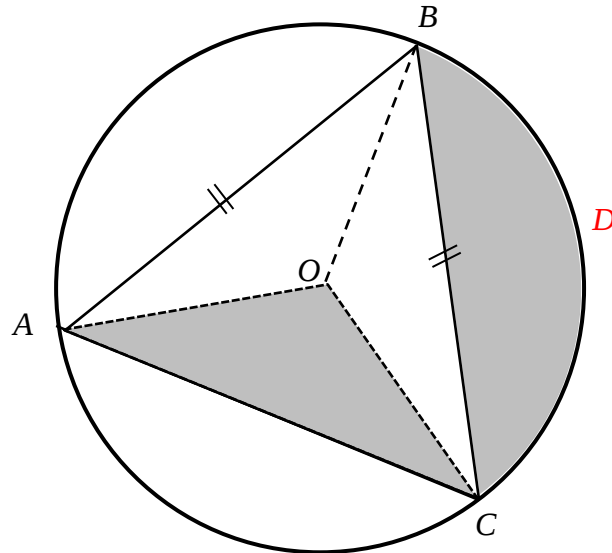
$\angle OCP = \angle OAP = 90^\circ$ (radius $\perp$ tangent)

$\angle APC + \angle AOC + 90^\circ + 90^\circ = 360^\circ$ [B1]
(sum of angles in quadrilateral)

$$\begin{aligned} \angle APC &= 180^\circ - (180^\circ - x^\circ) \\ &= x^\circ \end{aligned} \quad [\text{B1}]$$

Answer $^\circ$ [2]

6 (b)



A, B and C are points on a circle with centre O and radius 10 cm.
 $BA = BC$ and angle AOC is 2.4 radians.

- (i) Calculate angle COB , in radians, in terms of π .

$$\begin{aligned}\angle COB &= \frac{2\pi - 2.4}{2} \quad (\angle AOB = \angle COB \text{ as } BA = BC) \\ &= \pi - 1.2 \text{ radians} \quad [\text{B1}] [\text{Accept } \frac{5\pi - 6}{5}]\end{aligned}$$

Answer radians [1]

- (ii) Calculate the length of major arc AC .

$$\begin{aligned}\text{Reflex } \angle AOC &= 2\pi - 2.4 \approx 3.883185307 \approx 3.8832 \\ \text{Length of major arc } AC &= r\theta \\ &= 10(2\pi - 2.4) \quad [\text{M1}] \\ &\approx 38.831853071 \\ &= 38.8 \text{ cm (3 s.f.)} \quad [\text{A1}]\end{aligned}$$

$$\begin{aligned}\text{Alt. Method:} \\ \text{Length of major arc } AC &= \text{circumference} - \text{minor arc } AC \\ &= 2\pi(10) - 10(2.4) \quad [\text{M1}] \\ &\approx 38.831853071 \\ &= 38.8 \text{ cm (3 s.f.)} \quad [\text{A1}]\end{aligned}$$

Answer cm [2]

- (iii) Calculate the total area of the shaded regions.

Area of triangle OAC

$$= \frac{1}{2} \times 10^2 \times \sin 2.4$$

$$\approx 33.773 \text{ cm}^2 \quad [\text{M1}]$$

Area of segment BCD = Area of sector OBC – Area of triangle OBC

$$= \frac{1}{2} \times 10^2 \times (\pi - 1.2) - \frac{1}{2} \times 10^2 \times \sin(\pi - 1.2)$$

$$= 97.079632679 - 46.601954298$$

$$\approx 50.478 \text{ cm}^2 \quad [\text{M1}]$$

Area of shaded region

$$= \text{Area of triangle } OAC + \text{Area of segment } BCD$$

$$\approx 33.773 + 50.478$$

$$\approx 84.251$$

$$= 84.3 \text{ cm}^2 \text{ (3 s.f.)} \quad [\text{A1}]$$

Answer cm² [3]

- 7 In Café X, an Americano (A) costs \$5.20, a Latte (L) costs \$6.60 and a Mocha (M) costs \$7.30. In Café Y, an Americano costs \$0.30 more, a Latte costs \$0.20 less and a Mocha costs \$0.10 more than Café X.

The matrix **P** shows the price of a cup of coffee in each café.

$$\mathbf{P} = \begin{pmatrix} & \text{A} & \text{L} & \text{M} \\ \begin{pmatrix} 5.2 & 6.6 & 7.3 \\ 5.5 & 6.4 & 7.4 \end{pmatrix} & \text{X} \\ & & & \text{Y} \end{pmatrix}$$

- (a) Belinda and Carol are project group leaders in a company. They want to give a treat to their group members to welcome them for the first meeting in February.

Belinda intends to buy 2 cups of Americano, 4 cups of Latte and 3 cups of Mocha. Carol intends to buy 5 cups of Latte and 4 cups of Mocha.

Represent their intended purchases using a 3×2 matrix **Q**.

$$\mathbf{Q} = \begin{pmatrix} & \text{B} & \text{C} \\ \begin{pmatrix} 2 & 0 \\ 4 & 5 \\ 3 & 4 \end{pmatrix} & \text{A} \\ & & \text{L} \\ & & \text{M} \end{pmatrix} \quad [\text{B1}]$$

Answer **Q** = [1]

- (b) Evaluate the matrix **R** = **PQ**.

$$\begin{aligned} \mathbf{R} = \mathbf{PQ} &= \begin{pmatrix} 5.2 & 6.6 & 7.3 \\ 5.5 & 6.4 & 7.4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 4 & 5 \\ 3 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 5.2 \times 2 + 6.6 \times 4 + 7.3 \times 3 & 5.2 \times 0 + 6.6 \times 5 + 7.3 \times 4 \\ 5.5 \times 2 + 6.4 \times 4 + 7.4 \times 3 & 5.5 \times 0 + 6.4 \times 5 + 7.4 \times 4 \end{pmatrix} \quad [\text{M1}] \\ &= \begin{pmatrix} 58.7 & 62.2 \\ 58.8 & 61.6 \end{pmatrix} \quad [\text{A1}] \end{aligned}$$

Answer **R** = [2]

- (c) State what each element of matrix **R** represents.

The two elements in row 1 of **R** represent total amount spent on the drinks by Belinda and Carol respectively if they bought them at Café X.

The two elements in row 2 of **R** represent total amount spent on the drinks by Belinda and Carol respectively if they bought them at Café Y.

OR

Amount Belinda would have spent is \$58.70 in Café X and \$58.80 in Café Y.

Amount Carol would have spent is \$62.20 in Café X and \$61.60 in Café Y. [B1]

- (d) Belinda and Carol each bought their drinks from the café that provided a better offer.

For the latest meeting in July, Belinda and Carol gave their members another treat to celebrate the completion of the project. Each of them bought the same set of drinks from the same café they patronised in February. However, the prices of the drinks have changed by the following percentages in both cafés.

The price of a cup of Americano has increased by 10%.

The price of a cup of Latte has increased by 15%.

The price of a cup of Mocha has decreased by 5%.

Calculate the percentage change in the combined total spent in February and July.

Total amount for latest meeting in July

$$= \$ (5.2 \times 1.1 \times 2 + 6.6 \times 1.15 \times 4 + 7.3 \times 0.95 \times 3) + (6.4 \times 1.15 \times 5 + 7.4 \times 0.95 \times 4)$$

$$= \$ (62.605 + 64.92)$$

$$= \$ 127.525$$

[B1]

Percentage change

$$= \frac{(127.525 - 58.70 - 61.60)}{(58.70 + 61.60)} \times 100\% \quad [\text{M1}]$$

$$\approx 6.005818786\%$$

$$= 6.01\% \text{ (3 s.f.)}$$

[A1]

Alt. Method:

$$\text{New price} = \begin{pmatrix} 5.2 & 6.6 & 7.3 \\ 5.5 & 6.4 & 7.4 \end{pmatrix} \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.15 & 0 \\ 0 & 0 & 0.95 \end{pmatrix} = \begin{pmatrix} 5.72 & 7.59 & 6.935 \\ 6.05 & 7.36 & 7.03 \end{pmatrix}$$

Total cost of drinks latest meeting in July in Cafe X and Y:

$$= \begin{pmatrix} 5.72 & 7.59 & 6.935 \\ 6.05 & 7.36 & 7.03 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 4 & 5 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 62.605 & 65.69 \\ 62.63 & 64.92 \end{pmatrix} \quad [\text{M1}]$$

Percentage change

$$= \frac{(62.605 + 64.92) - (58.70 + 61.60)}{(58.70 + 61.60)} \times 100\% \quad [\text{M1}]$$

$$\approx 6.005818786\%$$

$$= 6.01\% \text{ (3 s.f.)}$$

[A1]

Answer % [3]

- 8** A class of 40 Secondary Four students were asked how much time they have spent on social media in the last week.
The results are shown in the table.

Time spent (x hours)	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$
Frequency	p	11	10	q	3

The upper quartile of the time spent on social media was 30 hours.

- (a)** Show that $p = 9$ and hence state the value of q .

Answer

Upper quartile = 30 hours
 $\Rightarrow$ 75% of 40 students or 30 students spent 30 hours or less on social media,
 $\Rightarrow p + 11 + 10 = 30$ [M1]
 $p = 9$ [A1]
 $q = 40 - 30 - 3 = 7$ [B1]

Answer $q = \dots\dots\dots$ [3]

- (b)** Estimate the mean number of hours spent by the students.

Mean number of hours
 $= \frac{(9 \times 5) + (11 \times 15) + (10 \times 25) + (7 \times 35) + (3 \times 45)}{40}$
 $= \frac{840}{40}$ [M1]
 $= 21$ [A1]

Answer $\dots\dots\dots$ [2]

- (c)** Estimate the standard deviation.

Using the calculator, $SD = 12.2$ (3sf) [B1]
 OR
 $SD = \sqrt{\frac{23600}{40} - 21^2} \approx 12.206555615 = 12.2$ (3 s.f.)

Answer $\dots\dots\dots$ [1]

- (d) A class of 40 Secondary Three students were asked how much time they have spent on social media in the last week.

The results are summarised in the table.

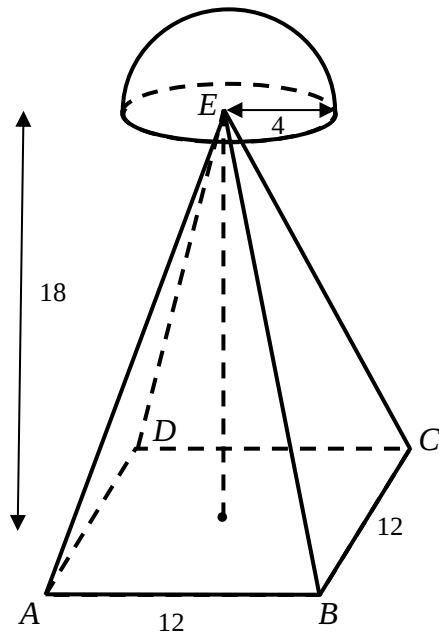
Mean	15.8
Standard Deviation	10.5

Make two comparisons between the number of hours spent on social media by the Secondary Three students and by the Secondary Four students.

1 .. Secondary Four students spent **more time** on social media than
 Secondary Three students as its **mean (21) is higher** than the mean (15.8)
 of Secondary Three students. [B1]

2 .. The time spent on social media by Secondary Four students **was less**
 **consistent or had a wider spread** than Secondary Three students as its
 **standard deviation (12.2) is higher** than the standard deviation (12.2) of
 Secondary Three students. [B1] ..[2]

- 9 The diagram shows a metal trophy which is made up of a hemisphere and a pyramid. The radius of the hemisphere with centre E is 4 cm. The base $ABCD$ of the pyramid is a square of side 12 cm. E is vertically above the centre of the square base. The vertical height of the pyramid is 18 cm.



- (a) Show that $AE = 19.9$ cm, correct to three significant figures. [3]

Answer

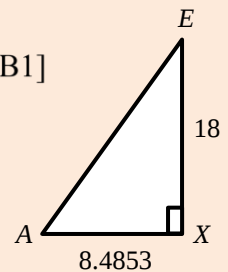
Let X be the centre of the square base.

Since vertical height of pyramid is 18 cm, $EX = 18$ cm.

$$AX = \frac{1}{2} AC \Rightarrow AX = \frac{1}{2} (\sqrt{12^2 + 12^2}) = \sqrt{72} = 8.4853 \text{ cm (5 s.f.)} \quad [\text{B1}]$$

By Pythagoras' Theorem,

$$\begin{aligned} AE &= \sqrt{18^2 + 8.4853^2} \quad \text{or} \quad \sqrt{18^2 + 72} & [\text{M1}] \\ &= 19.899756684 \quad \text{or} \quad \sqrt{396} \\ &= 19.9 \text{ cm (3 s.f.) (Shown)} & [\text{AG1}] \end{aligned}$$



- (b) Calculate angle BAE .

Let M be the midpoint of AB . $AM = 12 \div 2 = 6$ cm [B1]

Since $EA = EB$, AEB is an isosceles triangle hence angle $EMA = 90^\circ$.

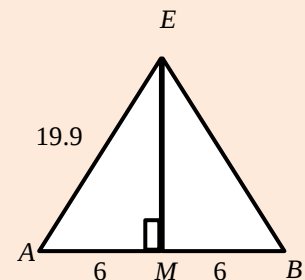
$$\cos \angle BAE = \frac{6}{19.900} \quad \text{or} \quad \frac{6}{\sqrt{396}}$$

$$\begin{aligned} \angle BAE &= \cos^{-1} \left(\frac{6}{19.900} \right) \quad \text{or} \quad \cos^{-1} \left(\frac{6}{\sqrt{396}} \right) & [\text{M1}] \\ &\approx 72.452 = 72.5^\circ (1 \text{ d.p.}) & [\text{A1}] \end{aligned}$$

Alt. Method:

$$\cos \angle BAE = \frac{AE^2 + AB^2 - BE^2}{2(AE)(AB)} = \frac{(\sqrt{396})^2 + 12^2 - (\sqrt{396})^2}{2(\sqrt{396})(12)} \quad [\text{M2}]$$

$$\angle BAE = \cos^{-1} \left(\frac{1}{\sqrt{11}} \right) \approx 72.452^\circ = 72.5^\circ (1 \text{ dp}) \quad [\text{A1}]$$



- (c) Calculate the total surface area of the trophy.

$$\begin{aligned} &\text{Total surface area of hemisphere} \\ &= \text{curved surface area hemisphere} + \text{base area} \\ &= 2\pi r^2 + \pi r^2 = 2\pi(4^2) + \pi(4^2) = 32\pi + 16\pi = 48\pi \text{ cm}^2 \quad [\text{B1}] \end{aligned}$$

$$\begin{aligned} &\text{Total surface area of pyramid} \\ &= (4 \times \text{area of lateral face}) + \text{base area} \\ &= 4\left(\frac{1}{2}ab \sin C\right) + (l^2) \\ &= 4\left(\frac{1}{2} \times 12 \times 19.900 \times \sin 72.452\right) + (12^2) = 455.37 + 144 = 599.37 \text{ cm}^2 \quad [\text{B2}] \end{aligned}$$

$$\begin{aligned} &\text{Total surface area of the trophy} \\ &\approx 150.80 + 599.37 \\ &\approx 750.17 \\ &= 750 \text{ cm}^2 \text{ (3 s.f.)} \quad [\text{B1}] \end{aligned}$$

Answer cm² [4]

- (d) The whole trophy is melted to make paper weights each in the shape of a cube of side 5 cm.
What is the maximum number of paper weights that can be made from the metal trophy?

$$\begin{aligned} &\text{Volume of trophy} \\ &= \text{Volume of hemisphere} + \text{Volume of pyramid} \\ &= \frac{2}{3}\pi r^3 + \frac{1}{3} \times \text{base area} \times \text{height} \\ &= \frac{2}{3}\pi(4^3) + \frac{1}{3} \times 12^2 \times 18 \\ &\approx \frac{128\pi}{3} + 864 \approx 998.04 \text{ cm}^3 \quad [\text{B1}] \end{aligned}$$

$$\begin{aligned} &\text{Maximum number of paper weights} \\ &= \frac{998.04}{5^3} \quad [\text{M1}] \\ &\approx 7.98432 = 7 \quad [\text{A1}] \end{aligned}$$

Answer [3]

- 10 Mr Tan plans to bring his wife and two children for a free-and-easy 5-day holiday in Jeju, South Korea during the school holidays in September 2022.

The tables below give information that they can use to work out their travelling costs.

Cost of Pre-departure PCR/ART Test (per person) in Singapore	Cost of PCR Test (per person) in South Korea												
<table><tr><td>Clinic</td><td>SGST</td><td>NM</td><td>Swabstation</td></tr><tr><td>PCR Test</td><td>\$88</td><td>\$98</td><td>\$78</td></tr><tr><td>ART Test</td><td>\$28</td><td>\$25</td><td>\$28</td></tr></table>	Clinic	SGST	NM	Swabstation	PCR Test	\$88	\$98	\$78	ART Test	\$28	\$25	\$28	70,000 won through the Korean National Tuberculosis Association (KNTA).
Clinic	SGST	NM	Swabstation										
PCR Test	\$88	\$98	\$78										
ART Test	\$28	\$25	\$28										
Estimated expenditure in South Korea	Cost of return air tickets from SINGAPORE to JEJU for 2 adults and 2 children												
<table><tr><td>Hotel (Family Suite)</td><td>\$ 2903</td></tr><tr><td>Food, Shopping & Sight-seeing</td><td>\$ 400 per day</td></tr><tr><td>Car rental for 5 days</td><td>\$ 464</td></tr></table>	Hotel (Family Suite)	\$ 2903	Food, Shopping & Sight-seeing	\$ 400 per day	Car rental for 5 days	\$ 464	<table><tr><td>Airlines</td><td>Scoot</td><td>Jeju Air</td></tr><tr><td>Adult / Child</td><td>\$467.13 per person</td><td>\$697.49 per person</td></tr></table>	Airlines	Scoot	Jeju Air	Adult / Child	\$467.13 per person	\$697.49 per person
Hotel (Family Suite)	\$ 2903												
Food, Shopping & Sight-seeing	\$ 400 per day												
Car rental for 5 days	\$ 464												
Airlines	Scoot	Jeju Air											
Adult / Child	\$467.13 per person	\$697.49 per person											

Summary of Travel Insurance Plans (Family plan for 2 adults and 2 children)

Travel Insurance Company	Singlife by Aviva*			FWD#		
Coverage/Type of Plan	Travel Lite: S\$110.66	Travel Plus: S\$159.65	Travel Prestige: S\$222.63	Premium: \$59.54	Business: S\$93.89	First: S\$119.08
Overseas medical expenses (per adult below 70 years old)	S\$250,000	S\$2,000,000	Unlimited	S\$200,000	S\$500,000	S\$1,000,000
Travel delay (S\$100 for every six hours of delay overseas and S\$100 after six hours of delay in Singapore)	S\$500	S\$1000	S\$2000	S\$300	S\$500	S\$1000
Trip cancelled before you leave Singapore for Family Plans	S\$10,000	S\$20,000	S\$30,000	S\$7,500	S\$10,000	S\$15,000
Emergency Medical Cover And Repatriation Due To COVID-19 (Free extension)	S\$50,000	S\$100,000	S\$200,000	# Extra Coverage for COVID-19 with an additional cost of \$93.28 - Up to S\$5,000 for trip cancellation and loss of deposit - Trip disruption: Up to S\$5,000 - Overseas hospital cash: S\$100 per day up to 14 days - Overseas quarantine allowance: S\$50 per day up to 14 days - Emergency medical evacuation & repatriation: up to S\$200,000 - Hospital cash while in Singapore: S\$100 per day up to 14 days - Medical expenses: Up to S\$200,000 - Extended policy cover in some situations: Up to 21 days		
(1) Overseas Medical Expenses due to COVID-19	S\$250,000	S\$2,000,000	Unlimited			
(2) Emergency Medical Evacuation due to COVID-19	S\$30,000	S\$500,000	Unlimited			
(3) Repatriation due to COVID-19	Not covered	S\$1,000 (S\$50 for every 24 hours)	S\$2,000 (S\$100 for every 24 hours)			
ii. Overseas Quarantine Allowance Due To COVID-19	S\$500	S\$1500	S\$2000			
iii. Trip Cancellation/Postponement Due To COVID-19	S\$500	S\$1500	S\$2000			
iv. Trip Interruption Due To COVID-19	S\$500	S\$1500	S\$2000			

*30% discount off your travel quote if you or your immediate family member(s) has an active MINDEF & MHA Group Term Life / Group Personal Accident policy or POGIS Group Term Life policy with Singlife.

- (a) The exchange rate between South Korean won (KRW) and Singapore dollars (SGD) changes each day.

The graph shows the daily exchange rate for the period 4 June – 4 July 2022.

Lowest rate on 5 June 2022	1 SGD = 908.724 KRW
Highest rate on 23 June 2022	1 SGD = 939.752 KRW



(Source: <https://www.xe.com/currencycharts/?from=SGD&to=KRW&view=1M>)

All travellers are required to take a PCR test within three days of arriving in South Korea.

Use the information to work out the difference between the greatest and least possible cost, in Singapore dollars, for Mr Tan's family to book their appointments to take PCR tests in South Korea.

Give your answer correct to the nearest ten cents.

Difference between the greatest and least possible cost

$$= \frac{70000 \times 4}{908.724} - \frac{70000 \times 4}{939.752} \quad [\text{M1}]$$

$$\approx 308.12 - 297.95$$

$$\approx 10.17$$

$$= \text{S\$}10.20 \text{ (nearest ten cents)} \quad [\text{A1}]$$

Answer S\$ [2]

- (b) Mr Tan intends to buy travel insurance with some coverage for COVID-19 for the whole family. The plan should have coverage of at least \$300,000 per person for overseas medical expenses. Mr Tan has an active POGIS Group Term Life policy with Singlife. He wants to spend not more than \$150 on the travel insurance.

The family will need to take a pre-departure PCR or ART test in Singapore before their trip.

Mr Tan adds an extra 10% to the estimated total cost of the trip to cover any extra costs.

Mr Tan finds this information about exchange rates over the past two years.

Lowest rate in Dec 2020	1 SGD = 811.551 KRW
Highest rate on Jun 2022	1 SGD = 939.752 KRW



Estimate the total cost, in Singapore dollars, for the 5-day trip for Mr Tan and his family.

Justify any decisions you make and show your calculations clearly.

[8]

Answer

Total cost of pre-departure ART tests = $$(4 \times 25) = \100
[ART is cheaper than PCR and \$25 is cheapest]

Estimated total cost of PCR tests in South Korea in Sep

$$= \frac{70000 \times 4}{811.551} = S\$345.02 \text{ (5 s.f.)}$$

[Use lowest exchange rate in 2 yrs]

Cost of Travel Plus Plan by Singlife = $0.7 \times \$159.65 = S\111.755
[Mr Tan qualifies for 30% discount hence cost is less than \$150 and overseas coverage expenses is S\$2,000,000]

Total cost of air tickets = $S\$ (4 \times 467.13) = S\1868.52
[Cheaper to travel by Scoot]

Total estimated expenditure in South Korea

$$= S\$ [2903 + (400 \times 5) + 464] = S\$5367$$

Estimated total cost for the 5-day trip

$$= S\$ [100 + 345.02 + 111.755 + 1868.52 + 5367] \times 1.1$$

$$= S\$ (7792.295 \times 1.1) \approx S\$8328.995 = S\$8329.00 \text{ (nearest cents)}$$

[M4 – total cost,
B2 – any 2 valid justifications]

[M1]

[A1]

Additional space for working
(b)

~~ End of Paper 2 ~~