TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2022
Secondary 4

MARKING SCHEME

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1 **Thursday 11 August 2022**

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

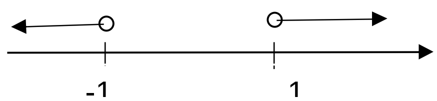
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

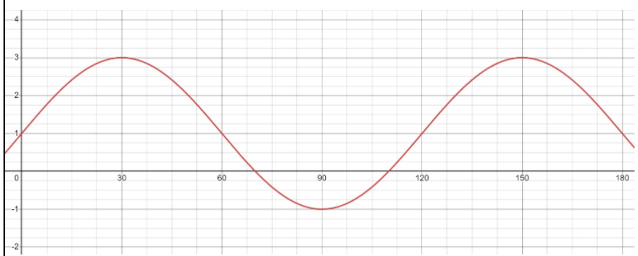
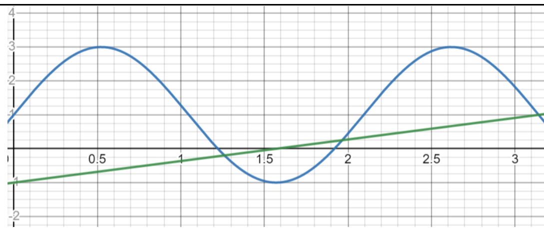
$$\Delta = \frac{1}{2} bc \sin A$$

1	$x^2-3x(2m-x)+(2m-x)^2+1=0$ $x^2-6mx+3x^2+4m^2-4mx+x^2+1=0$ $5x^2-10mx+4m^2+1=0$ $D>0$ $(-10m)^2-4(5)(4m^2+1)>0$ $20(m^2-1)>0$ $20(m+1)(m-1)>0$ m < −1 or m > 1 	M1 B1 M1 A1 B1	Remove / substitute to 1 variable factorisation
		Total:5m	

2(i)	$y=x^3+ax^2+bx+c$ $\frac{dy}{dx}=3x^2+2ax+b$	B1	
(ii)	For decreasing function, $\frac{dy}{dx}<0$ $3x^2+2ax+b<0$ From $-1<x<3$, $3(x+1)(x-3)<0$ $3x^2-6x-9<0$ By comparison, a= −3 and b = −9	B1 M1 B1 AG	Their (i) < 0 Form eqn with factors (x+1)(x-9) ignore 3 Both correct
		Total: 4m	

3a	$\left[\left(\frac{100}{6}\right)^{-2}\times\sqrt{5^3}\right]\div\frac{5}{2}$ $=\left[\left(\frac{2^2\times5^2}{2\times3}\right)^{-2}\times5^{\frac{3}{2}}\right]\times\frac{2}{5}$ $=\left[\left(\frac{2\times3}{2^2\times5^2}\right)^{2}\times5^{\frac{3}{2}}\right]\times\frac{2}{5}$ $=2^{-1}\times3^2\times5^{\frac{3}{2}-5}$ $=2^{-1}\times3^2\times5^{-\frac{7}{2}}$ a = -1, b = 2, c = -3.5	M1 B1, B1	Attempt to reduce to base 2, 3, 5 3 correct B1, B1 2 correct B1
3b	$\left[50000\left(1+\frac{5}{100}\right)^{12}\right]$ = \$89792.82	M1 A1	Form exponential function 2 dp
		Total: 5m	

4	$\frac{2x^2+x+1}{x^2-x-2}$ $=2+\frac{3x+5}{(x-2)(x+1)}$ $\frac{3x+5}{(x-2)(x+1)}=\frac{A}{(x-2)}+\frac{B}{(x+1)}$ $3x+5=A(x+1)+B(x-2)$ $A=\frac{11}{3},B=\frac{-2}{3}$ $\frac{2x^2+x+1}{x^2-x-2}=2+\frac{11}{3(x-2)}-\frac{2}{3(x+1)}$	M1 B1 M1 A1 B1	Long division seen Correct partial fractions Use comparing coefficient or substitution of *x* √ From correct PF
		Total: 5m	

5(i)	$1 = p\sin 3(\pi) + q$ $1 = q$ Since y max = 3, p = 3-1 = 2	B1 B1	Substitution seen 2 – 1 seem
(ii)		G1 P1	Correct shape, 1.5 cycles seen Max point at y =3, min at y = -1
(iii)	 Between $0 \le x \le 2\pi$, there will be **3 cycles** of the graph $y = p\sin 3x + q$. For line $y = \frac{2}{\pi}x - 1$, when $x = 2\pi, y = 3$ For graph $y = p\sin 3x + q$, when $x = 2\pi, y = 1$ The line will intersect the graph at 5 points within the domain.	L1 B1	Straight line passing through (0, -1) and (π, 1) o.e. Accept sketch
(iv)	0	B1	
		Total: 7m	

6	$y=(x+2)(x-1)^2$ $\frac{dy}{dx}=(1)(x-1)^2+(x+2)2(x-1)$ $=(x-1)[(x-1)+2x+4]$ $=(x-1)(3x+3)$ $=3(x-1)(x+1)$ $\frac{dy}{dx}=0$ $3(x-1)(x+1)=0$ $x=1,-1$ $y=0,4$ $\frac{d^2y}{dx^2}=(1)(x+1)+(x-1)(1)=2x$ At (1, 0) $\frac{d^2y}{dx^2}=2(1)>0$ (1,0) is a minimum point At (-1, 4) $\frac{d^2y}{dx^2}=2(-1)<0$ (-1, 4) is a maximum point	M1 A1 M1 M1 B1 B1	Product rule seen o.e. Solve their derivative = 0 Use of 1st / 2nd derivative
		Total: 6m	

7(i)	*LHS* $=\dfrac{\cot x-\tan x}{\cot x+\tan x}$ $=\dfrac{\frac{1}{\tan x}-\tan x}{\frac{1}{\tan x}+\tan x}$ $=\dfrac{1-\tan^2 x}{1+\tan^2 x}$ $=\dfrac{1-\frac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$ $=\dfrac{1-\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$ $=\cos^2 x-\sin^2 x$ $=\cos 2x$ $=RHS$ *LHS* $=\dfrac{\cot x-\tan x}{\cot x+\tan x}$ $=\dfrac{\frac{\sin x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x}+\frac{\sin x}{\cos x}}$ $=\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x+\sin^2 x}$ $=\cos^2 x-\sin^2 x$ $=\cos 2x$ $=RHS$	B1 B1 B1	$\cot x=\dfrac{1}{\tan x}\ or\ \dfrac{\sin x}{\cos x}$ Rewrite in terms of sin x or cos x / simplify fraction Double angle formula to arrive at AG
(ii)	$\cos 2x=\cos x$ $2\cos^2 x-1-\cos x=0$ $(2\cos x+1)(\cos x-1)=0$ $\cos x=-\dfrac{1}{2},\quad \cos x=1$ $x=0°,120°,240°,360°$	B1 M1 A1, A1	Use identity For both answers in each pair 0, 360 and 120, 240
		Total: 4m	

8(i)	$x^2+4x-5=(x-1)(x+5)$ if $(x-1)$ is a factor, $2(1)^3-9(1)^2+3(1)+m=0$ $m=4$ if $(x+5)$ is a factor, $2(-5)^3-9(-5)^2+3(-5)+m=0$ $m=490$	M1 B1 B1	Substitute the value of 1 or 5 to form eqn =0
(ii)	When m = 4, (x – 1) is a factor (By Factor Theorem) $2x^3-9x^2+3x+4=(x-1)(2x^2+ax-4)$ *compare coeff* x $3=-4-a$ $a=-7$ $2x^3-9x^2+3x+4=0$ $(x-1)(2x^2-7x-4)=0$ $(x-1)(2x+1)(x-4)=0$ $x=1,4,-0.5$	B1 M1 M1 B1	(x-1) as a factor soi Find quadratic factor Factorise completely = 0
(iii)	$2-9y^2+3y^4+4y^6=0$ $x=\frac{1}{y^2}$ $y=\pm1,\pm\frac{1}{2}$	M1 A1	soi √
		Total: 9m	

9(i)	At $A, y=-2$ $x=1$ $A=(1,-2)$ $Period = \frac{2\pi}{\left(\frac{\pi}{4}\right)} = 8$ By symmetry $B=(4,0)$ *OR* At $B, y=0$ $3\sin\frac{\pi x}{4}=0$ $x=4$	B1 B1	
(ii)	Area X $=\int_1^4 3\sin\frac{\pi x}{4}dx$ $=3\left[-\frac{4}{\pi}\cos\frac{\pi x}{4}\right]_1^4$ $=-\frac{12}{\pi}\left[\cos\pi-\cos\frac{\pi}{4}\right]$ $=-\frac{12}{\pi}\left(-1-\frac{\sqrt{2}}{2}\right)$ $=\frac{12}{\pi}\left(1+\frac{\sqrt{2}}{2}\right)$ $=\frac{6}{\pi}\left(2+\sqrt{2}\right)(shown)$	B1 M1 B1 AG	Correct integral Seen (ignore limits) Substitution of limits Evaluate special angles leading to answer
(iii)	$AreaY$ $=-\int_1^4 -\frac{2}{x}dx$ $=2\left[\ln x\right]_1^4$ $=2[\ln 4-\ln 1]$ $=2\ln 4$ $=2.773$ $\frac{AreaX}{AreaY}=\frac{\frac{6}{\pi}\left(2+\sqrt{2}\right)}{2\ln 4}=2.35$	B1 B1 B1 o.e. B1	Negative integral
		Total: 9m	

10(i)	$0 = 3(7)^2 + k(7) - 7$ $k = -20$	M1 A1	substitution
(ii)	$v = 3t^2 + kt - 7$ $a = 6t - 20$ $when\ t = 4$ $a = 4$	M1 A1	$a = \frac{dv}{dt}$ seen
(iii)	$v = 3t^2 - 20t - 7$ $s = t^3 - 10t^2 - 7t + c$ $At\ A, t = 0, S = 0,$ $c = 0$ $When\ S = 0,$ $t^3 - 10t^2 - 7t = 0$ $t(t^2 - 10t - 7) = 0$ $t = 0,$ $t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-7)}}{2} = 10.66s$	B1 M1 A1, A0	Correct integral with +c -1m if c=0 is not seen DM awarded only when quadratic formula or completed square is seen Minus 1 mark if t = 0 not shown / t is divided
(iv)	$when\ v = 0,$ $(3t + 1)(t - 7) = 0$ $t = 7$ $At\ B, t = 7$ $= [(7^3 - 10(7)^2 - 7(7))]$ $= -196m$ $At\ t = 8,$ $(8^3 - 10(8)^2 - 7(8)$ $= -184$ Total distance travelled = 196 + (196-184) = 208m	M1 M1 A1	Soi Find S at either t=7 or t=8
		Total: 10m	

11(a)	$16-6x-x^2$ $=-(x^2+6x-16)$ $=-(x+6x+3^2-16-3^2)$ $=-(x+3)^2+25$	M1 A1	Add / subtract 3 square
	$(x+3)^2 \geq 0$ $-(x+3)^2 \leq 0$ $-(x+3)^2+25 \leq 25$ $y \leq 25$ Since graph has a maximum value of y = 25, the graph will never meet the line y = 30. OR $-(x+3)^2+25=30$ $(x+3)^2=-5$ Since there is no solution, there is no intersection of the line and graph OR $-(x+3)^2+25=30$ $(x+3)^2=-5$ $x^2+6x+14=0$ $D=(6)^2-4(1)(14)$ $D<0$ Since there are no real roots, the line and graph do not intersect.	M1 A1	Accept any acceptable method

11b(i)	(A+B) is an acute angle	B1	Reject: lies in quadrant 1 Accept: lies between 0 to 90
(ii)	$\sin(A+B)=\frac{12}{13}$	B1	
(iii)	$\sin(A+B)=\frac{12}{13}$ $\sin A\cos B+\cos A\sin B=\frac{12}{13}$ $\left(\frac{4}{5}\right)\cos B+\left(\frac{3}{5}\right)\sin B=\frac{12}{13}$	B1 B1	$\cos A=\frac{3}{5}$ seen Eqn 1
	$\cos(A+B)=\frac{5}{13}$ $\cos A\cos B-\sin A\sin B=\frac{5}{13}$ $\left(\frac{3}{5}\right)\cos B-\left(\frac{4}{5}\right)\sin B=\frac{5}{13}$ Eqns $4\times(1)+3\times(2)$: $5\cos B=\frac{63}{13}$ $\cos B=\frac{63}{63}$	B1 M1 A1	Eqn 2 Solve simultaneous eqns
		Total: 7m	

12(i)	$x^2+y^2-14x+2y+\ 46=0$ $centre=(7,-1)$ $radius=\sqrt{7^2+(-1)^2-(46)}=2units$	M1A1 M1A1	Method SOI
(ii)	$M=(1,\ -1)$	B1	
(iii)	Eqn of C_2: $(x-1)^2+(y+1)^2=4$ $x^2+y^2-2x+2y-2=0$	B1	o.e.
(iv)	Equation of line *l*: $y-(-1)=1(x-1)$ $y=x-2$	B1	√
(v)	Gradient of normal: $\frac{0.6-(-1)}{8.2-7}=\frac{4}{3}$ Eqn of tangent: $y-0.6=-\frac{3}{4}(x-8.2)$ $4y+3x=27\quad ----------\text{Eqn1}$ $y=x-2\quad ----------\text{Eqn2}$ $4(x-2)+3x=27$ $4x-8+3x=27$ $7x=35$ $x=5$ $y=3$ $Q=(5,3)$	M1 M1 M1 A1 A1	 Solve simultaneous eqns Find *x* or *y* coordinates
		Total: 7m	