ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4049/01

PAPER 1

12 September 2022
2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Express $\dfrac{2x^3-4x^2+x-18}{(x-2)(x^2+4)}$ in partial fractions. [6]

2 **Without using a calculator,**

(i) show that $\tan 285° = \dfrac{\sqrt{3}+1}{1-\sqrt{3}}$, [3]

(ii) express $\sec^2 285°$ in the form $a+b\sqrt{3}$, where a and b are integers. [4]

3 Find the range of values of k for which the curve $y = kx^2 + 8x$ lies entirely above the line $y = x - k$. [4]

4 A curve is such that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\dfrac{4}{x^3} - \dfrac{15}{(3x-2)^2}$ and the point $P(1, -6)$ lies on the curve.

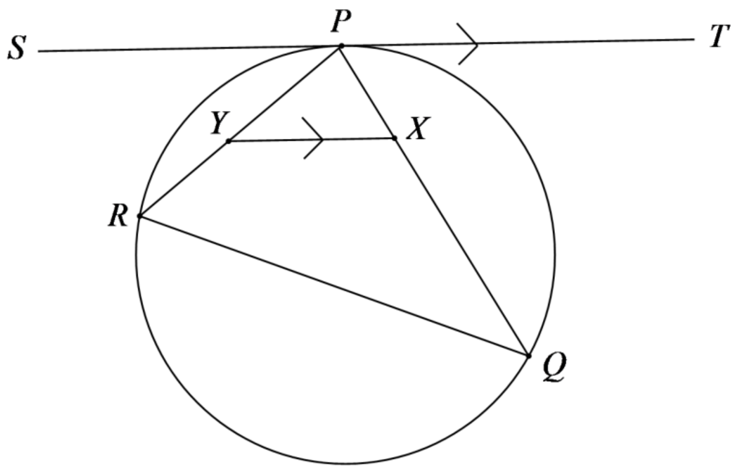
The gradient of the curve at P is 3. Find the equation of the curve. [6]

5 The equation of a polynomial is given by $p(x)=3x^3-5x^2-3x+2$.

(a) Show that $(x-2)$ is a factor of $p(x)$. [1]

(b) Hence, solve the equation $p(x)=0$, expressing non-integer roots in surd form. [3]

6



The diagram shows a circle passing through the points *P*, *Q* and *R*. The points *X* and *Y* lie on *QP* and *RP* respectively. The tangent *ST* to the circle at *P* is parallel to *YX*.

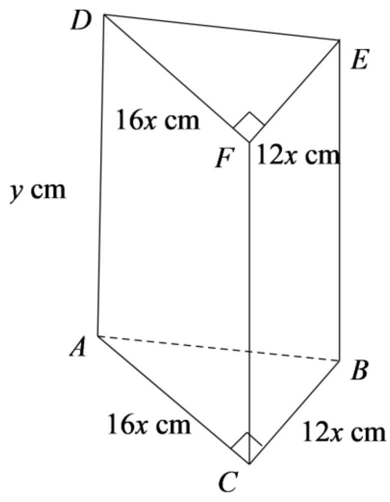
(a) Prove that *Q*, *R*, *Y* and *X* lie on a circle. [5]

6 **(b)** The line RP is extended to Z such that angle $PZT = 90°$.
Explain why a circle passing through the points P, T and Z has its centre at the midpoint of PT. [2]

7 **(a)** The equation of a curve is $y = x(2x-5)^3$. Find the range of values of *x* for which *y* is decreasing. [4]

(b) The equation of a curve is $y = \dfrac{2x}{x-1} - 2x^2$ where $x > 1$. Find the value of the maximum gradient on the curve. [4]

8



The diagram shows a solid prism with triangular base and height y cm. The triangles have right angles at C and F. The edges AC and DF are each of length $16x$ cm and the edges BC and EF are each of length $12x$ cm. The volume of the prism is 12 000 cm^3.

(a) Express y in terms of x . [2]

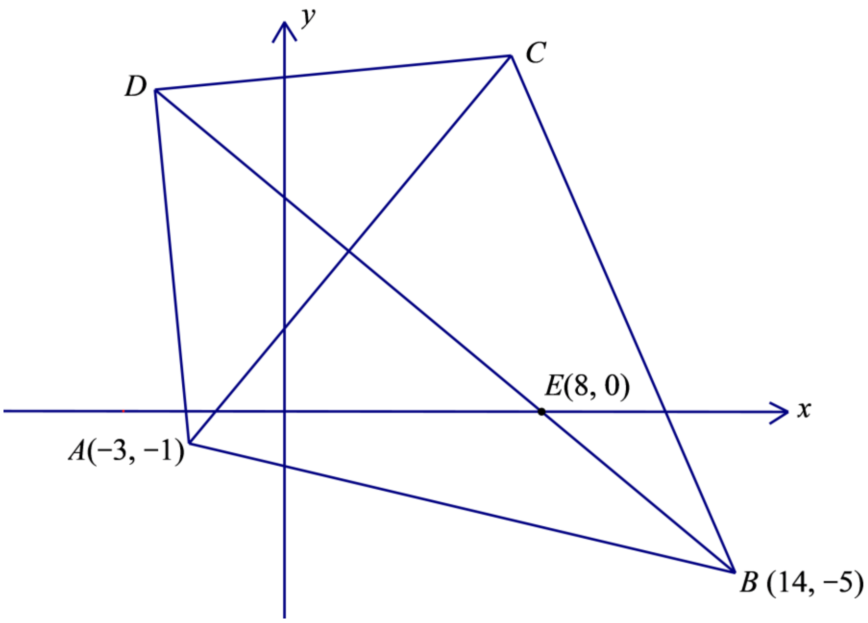
(b) Show that the total surface area, S cm^2, is given by [3]

$$S = 192x^2 + \frac{6000}{x}.$$

Continuation of working space for question 8(b).

(c) Given that x varies, find the stationary value of S and determine its nature. [5]

9



ABCD is a kite. The coordinates of *A* and *B* are $(-3, -1)$ and $(14, -5)$. The point $E(8, 0)$ lies on the diagonal of the kite.

(a) Find the equation of *AC*. [3]

(b) Find the coordinates of *C*. [5]

Continuation of working space for question 9(b).

(c) Given that $BD = 3BE$. Find the coordinates of D. [2]

10 At a yacht club, the depth of water, d metres, at time t hours after 08 00 is modelled with a sinusoidal function $d = 2 + 1.25\sin\left(\frac{\pi}{6}t\right)$.

(a) Sketch the graph of d against t over a 24-hour period. [3]

10 Sailing from the yacht club is permitted only when the depth of water is at least 0.9 metres.

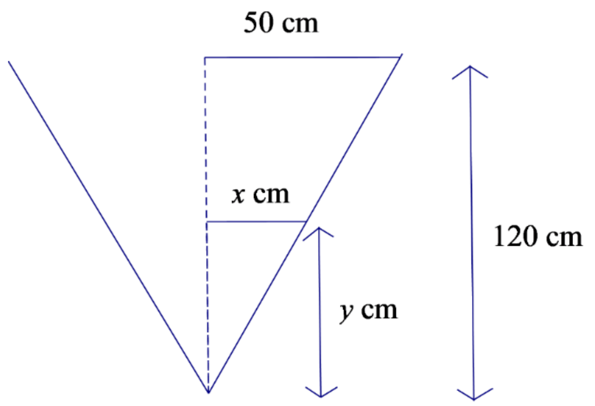
(b) Find the range(s) of values of t between which sailing is **not** permitted over a 24-hour period. [4]

11 [The volume of a cone with base radius r and vertical height h is given by $\frac{1}{3}\pi r^2 h$.]

In the industrial process, water flows at a constant rate of p cm^3/s into a conical funnel. The water flows out through a small hole of negligible radius in the vertex of the cone at a constant rate of q cm^3/s, where $q < p$. The axis of the cone is vertical. The volume of water in the cone at time t s is V cm^3. Initially the funnel is empty.

(a) Express $\frac{\mathrm{d}V}{\mathrm{d}t}$ in terms of p and q. [1]

The diagram shows the vertical cross-section of the funnel. The open top of the funnel is a horizontal circle of radius 50 cm which is 120 cm above the vertex.
At time t s, the water surface has radius x cm and is at a vertical height of y cm above the vertex.



(b) Express V in terms of y. [2]

In the case where $p = 80$ and $q = 20$,

(c) find the vertical height of the water surface after one minute, [2]

(d) **(i)** Find the rate of change of the vertical height of the water surface after one minute. [2]

(ii) Determine, with working whether this rate will increase or decrease as t increases. [2]

12 **(a)** State the set of values of x for which $\sin^{-1}\left(\frac{1}{5}x\right)$ is defined. [1]

(b) Prove the identity $\dfrac{(\cos A - \sin A)(1 + \cos A \sin A)}{\sin^3 A} = \cot^3 A - 1$. [4]

13 **(a)** Without using a calculator, find the exact value of x in the form of $k\lg\frac{4}{5}$ in [3]

$$80^{x}\times\frac{16}{25}=20^{3x}.$$

[Turn over

(b) Solve the equation $\left(\log_2 y\right)^3 - \dfrac{12}{\log_{16} y^3} = 0$. [4]

End of Paper

Answer Key

1. $2-\dfrac{2}{(x-2)}-\dfrac{-2x+3}{(x^2+4)}$

2 (b) $8+4\sqrt{3}$

3 . $k>\dfrac{7}{2}$

4. $y=-\dfrac{2}{x}+\dfrac{5\ln(3x-2)}{3}-4x$

5. $x=2$ or $x=\dfrac{-1\pm\sqrt{13}}{6}$

6 $(2x-5)^2\geq 0$, $x<\dfrac{5}{8}$

7 -10

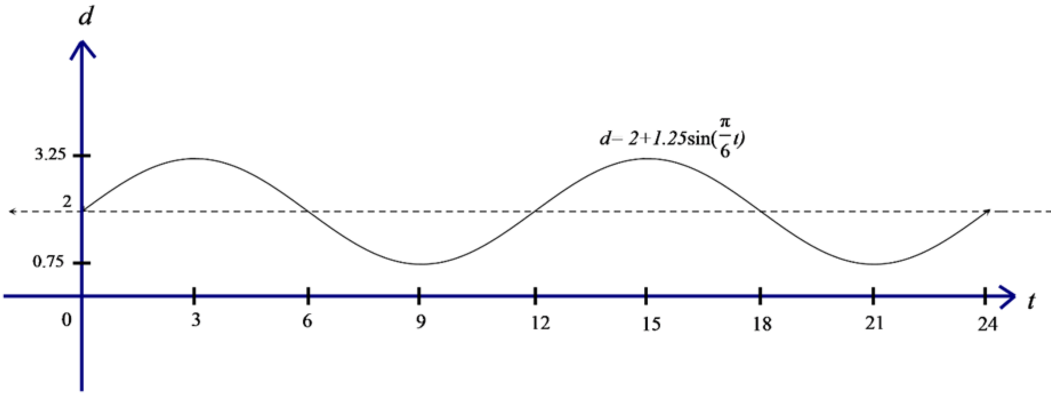
8 3600, minimum

9 (a) $y=\dfrac{6}{5}x+\dfrac{13}{5}$

(b) $C=(7,\ 11)$

(c) $(-4,\ 10)$

10 (a)



(b) $8.05<t<9.95$ or $20.1<t<21.9$

11 (a) $p-q$

(b) $V=\dfrac{25\pi y^3}{432}$ (c) 27.1

12 (a) $-5\leq x\leq 5$

13 (a) $\lg\dfrac{4}{5}$ **(b)** $y=4$ or $y=\dfrac{1}{4}$