ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4049/02

PAPER 2

13 September 2022
2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

This question paper consists of **21** printed pages (including this cover page)

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

1 **(a)** By using suitable substitution or otherwise, solve the equation $\mathrm{e}^{-x}\left(3-\mathrm{e}^{x}\right)=5\mathrm{e}^{x}$, giving your answer correct to 2 decimal places. [4]

(b) Solve $\sqrt{x+7}=x-5$. [3]

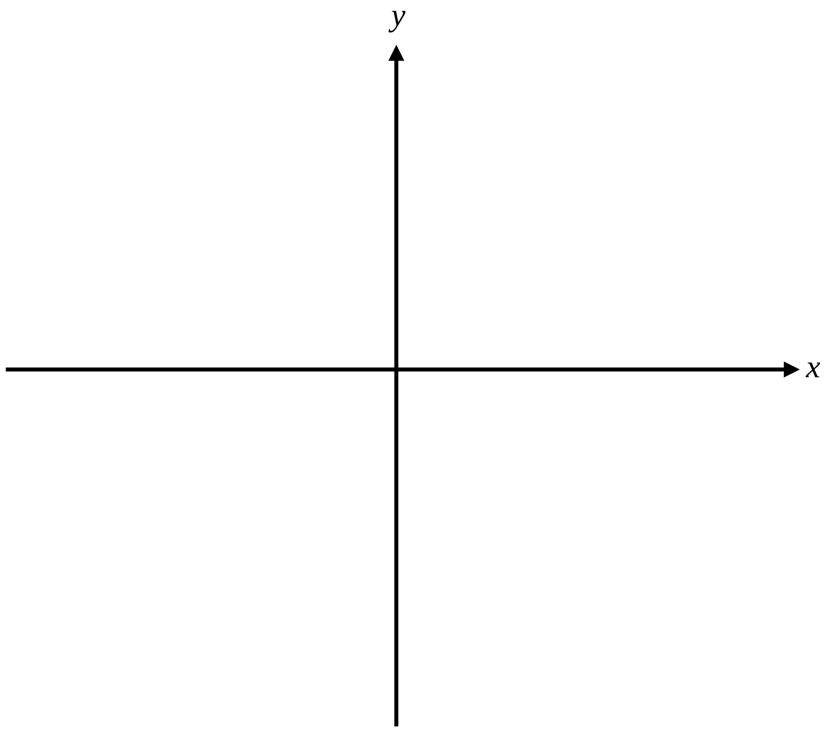
2 Given that $\tan\theta = p$ and that θ is acute, find in terms of *p*,

(a) $\sin(-\theta)$ [1]

(b) $\tan\left(\frac{\pi}{2}-\theta\right)$ [1]

3 **(a)** Express $-3x^2 - 36x - 106$ in the form $a(x+b)^2 + c$. [2]

(b) Hence, sketch the graph of $y = -3x^2 - 36x - 106$ on the axes below.
Indicate clearly the coordinates of the points where the graph crosses the y-axis and the turning point on the curve. [3]

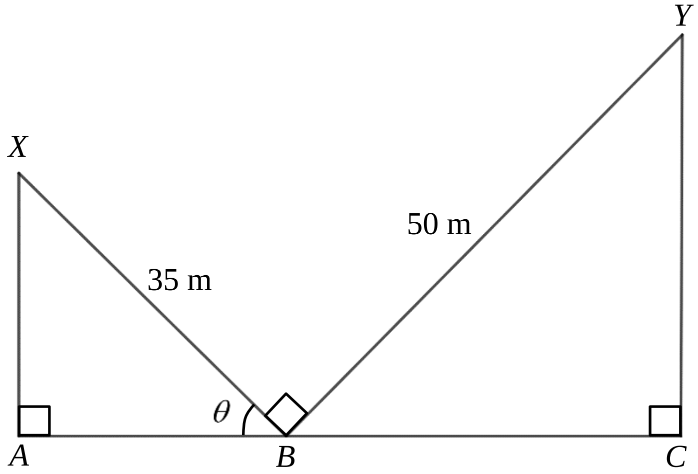


4 **(a)** Given that $y = \dfrac{2+\sin x}{\cos x}$, show that $\dfrac{dy}{dx} = \dfrac{1+2\sin x}{\cos^2 x}$. [3]

(b) Hence, evaluate $\int_0^{\frac{\pi}{4}} \dfrac{2+2\sin x}{\cos^2 x}\, dx$ in exact form. [4]

(c) Find the y-coordinate of the maximum turning point of $y = \dfrac{2 + \sin x}{\cos x}$ when $0 < x < 2\pi$. [6]

5



The diagram shows a straight coastline *ABC*. A turtle, at point *X*, at sea, swam 35 m in a straight line to point *B*, where John was throwing food to feed the turtles.
The turtle then turned $90°$ and swam 50 m in a straight line to point *Y* at sea.
Angle $XBY = 90°$, angle $XBA = \theta$ and *XA* and *YC* are perpendicular to *ABC*.

(a) Show that $AC = 35\cos\theta + 50\sin\theta$. [2]

(b) Express *AC* in the form of $R\cos(\theta-\alpha)$, where $R>0$ and $0°<\alpha<90°$. [4]

(c) John's father claims that the coastline *AC* is 70 m. Without measuring, John said that it was incorrect. Explain, with clear workings, how John arrived at this conclusion. [1]

6 **(i)** Write down the general term in the binomial expansion of $\left(x - \dfrac{p}{x^3}\right)^{12}$, where p is a constant. [1]

(ii) Write down the power of x in this general term. [1]

(iii) The constant term in the binomial expansion of $\left(x - \dfrac{p}{x^3}\right)^{12}$ is -27500. Find the value of p. [3]

(iv) Hence, show that every term in the expansion of $\left(1+\frac{x^{12}}{525}\right)\left(x-\frac{p}{x^3}\right)^{12}$ is dependent on x. [4]

[Turn over

7 **(a)** The equation of a curve is $y = 3x^2 + 5x - 16$.

Find the range of values for which the curve lies below the line $y = 12$. [3]

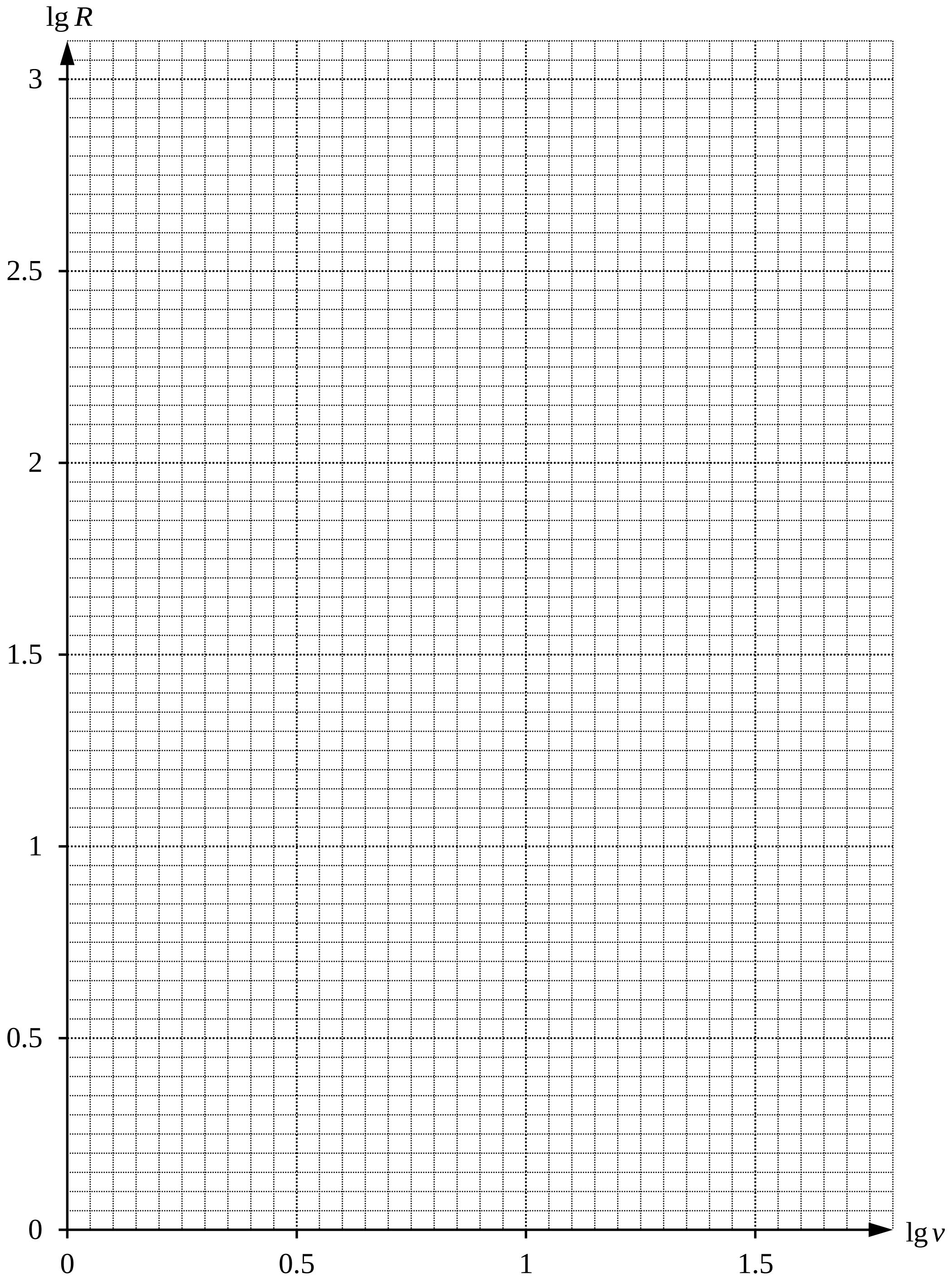
(b) Given that the line $y = 3x + k$ is a tangent to the curve $y^2 = bx$, where k and b are positive constants, show that b is a multiple of k. [3]

8 **(a)** A formula used in a science experiment is $ax^2 + by^2 = 1$, where a and b are constants. Values of x for different values of y have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [4]

(b) A particle, moving in a certain medium with speed v m/s, experiences a resistance to motion of R N (Newton). It is known that R and v are related by the equation $R = kv^a$, where k and a are constants. The table below shows experimental values of the variables v and R.

v	5	10	15	20	25	30
R	32	96	180	290	410	550

(i) On the grid on the next page, plot $\lg R$ against $\lg v$ and draw a straight line graph.
Hence, estimate the value for each of the constants k and a. [6]



(ii) Use your graph to estimate the resistance for which the speed is 12 m/s. [3]

9 A car moves on a straight road, from a point *O*, so that *t* seconds after leaving *O*, its velocity *v* m/s is given by $v = 20\left(e^{0.2t} + 4\right)$.

(a) Find the initial acceleration of the car. [3]

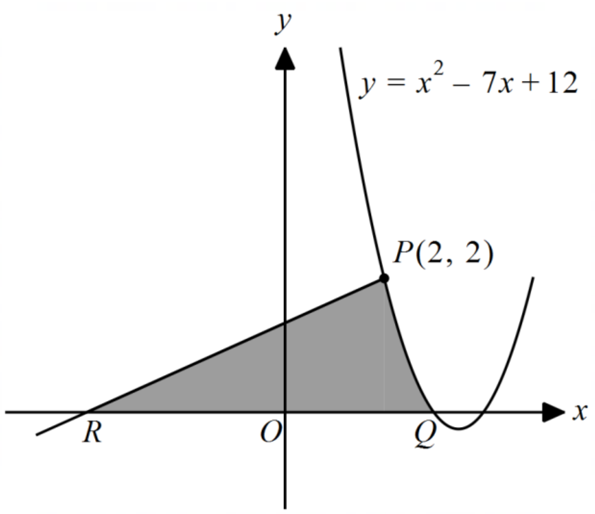
(b) Find the displacement of the car from *O* when $t = 5$. [4]

(c) Will the car return to O? Explain and show your working clearly. [2]

10 **(a)** A circle with equation $x^2 + y^2 - 2x + Ay = B$ touches the x-axis at 1 point.
Find the value of B. [4]

(b) Find the equation of the smallest possible circle which passes through the point $P(4,7)$ and has its centre C lying on the line $4y+3x-15=0$. [6]

11 The diagram shows part of the curve $y = x^2 - 7x + 12$, cutting the *x*-axis at *Q*. The normal to the curve at $P(2, 2)$ meets the *x*-axis at *R*.



Show that the area of the shaded region bounded by the *x*-axis, the line *PR* and the curve is $6\frac{5}{6}$ units^2. [10]

Continuation of working space for question 11.

End of paper