2023 Additional Mathematics Preliminary Examination Paper 1 Solutions

1 $\dfrac{2x^2+3x+4}{(2-x)(2+x^2)} = \dfrac{A}{2-x} + \dfrac{Bx+C}{2+x^2}$

Multiplying by $(2-x)(2+x^2)$,

$2x^2+3x+4 = A(2+x^2)+(Bx+C)(2-x)$

Sub $x = 2$: $2\times4 + 3\times2 + 4 = A(2+4)$

$6A = 18 \Rightarrow A = 3$

Sub $x = 0$: $4 = 2A + 2C$

$2C = 4 - 6 \Rightarrow C = -1$

Comparing coefficients of x^2,

$A - B = 2 \Rightarrow B = 1$

$\therefore \dfrac{2x^2+3x+4}{(2-x)(2+x^2)} = \underline{\underline{\dfrac{3}{2-x} + \dfrac{x-1}{2+x^2}}}$ [4]

2 $\displaystyle\int_2^4 \frac{(x+1)^2}{x^2}\,dx = \int_2^4 \frac{x^2+2x+1}{x^2}\,dx$

$\displaystyle = \int_2^4 \left(1+\frac{2}{x}+\frac{1}{x^2}\right)dx$

$= \left[x + 2\ln x - \dfrac{1}{x}\right]_2^4$

$= \left[4+2\ln 4-\dfrac{1}{4}\right]-\left[2+2\ln 2-\dfrac{1}{2}\right]$

$= \left[\dfrac{15}{4}+2(2\ln 2)\right]-\left[\dfrac{3}{2}+2\ln 2\right]$

$= \dfrac{9}{4}+2\ln 2$ [4]

3 $y = a + 2\tan bx$

(a) When $x = 0$, $y = -4$

$a = -4$

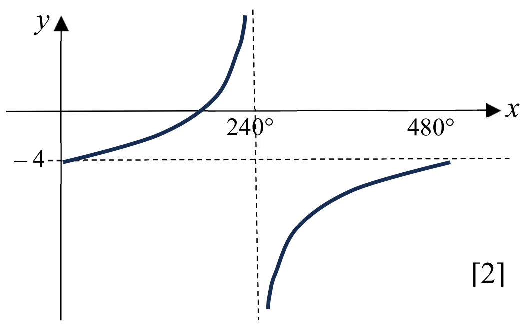
Since period of $y = 480°$,

$\dfrac{180°}{b} = 480°$

$b = \dfrac{180°}{480°} = \dfrac{3}{8}$

$\therefore\ \underline{\underline{a = -4,\ b = \dfrac{3}{8}}}$ [3]

(b) Graph of $y = -4 + 2\tan\dfrac{3}{8}x$



[2]

4 **(a)** Equation of the straight line is

$Y - 5 = \dfrac{9-5}{8-6}(X-6)$

$Y = 2X - 7$

$\therefore \ln y = 2\ln x - 7$

$\ln y - 2\ln x = -7$

$\ln\dfrac{y}{x^2} = -7$

$\therefore \dfrac{y}{x^2} = e^{-7} \Rightarrow \underline{\underline{y = e^{-7}x^2}}$ [4]

(b) When $Y = -1$, $X = m$,

$2m - 7 = -1$

$\therefore \underline{\underline{m = 3}}$ [1]

5 **(a)** $y = \left(3+\sqrt{5}\right)x^2 - 8\sqrt{5}x + 60$

$\dfrac{dy}{dx} = 2\left(3+\sqrt{5}\right)x - 8\sqrt{5}$

For stationary points, $\dfrac{dy}{dx} = 0$.

$2\left(3+\sqrt{5}\right)x - 8\sqrt{5} = 0$

$x = \dfrac{8\sqrt{5}}{2\left(3+\sqrt{5}\right)}$

$x = \dfrac{4\sqrt{5}}{\left(3+\sqrt{5}\right)} \times \dfrac{3-\sqrt{5}}{3-\sqrt{5}}$

$x = \dfrac{4\sqrt{5}\left(3-\sqrt{5}\right)}{9-5}$

$\underline{\underline{x = 3\sqrt{5}-5}}$ [3]

(b) $\left(3+\sqrt{5}\right)x^2 = \left(3+\sqrt{5}\right)\left(3\sqrt{5}-5\right)^2$

$= \left(3+\sqrt{5}\right)\left(45-30\sqrt{5}+25\right)$

$= \left(3+\sqrt{5}\right)\left(70-30\sqrt{5}\right)$

$= 210-150+\sqrt{5}(70-90)$

$= 60-20\sqrt{5}$

$\therefore y = \left(60-20\sqrt{5}\right)-8\sqrt{5}\left(3\sqrt{5}-5\right)+60$

$= \left(60-20\sqrt{5}\right)-\left(120-40\sqrt{5}\right)+60$

$= \underline{\underline{20\sqrt{5}}}$ [2]

6 $y = x\sin 4x$

(a) $\dfrac{dy}{dx} = \sin 4x + x(4\cos 4x)$

$= \underline{\underline{\sin 4x + 4x\cos 4x}}$ [2]

(b) Integrating,

$$[y]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin 4x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x\cos 4x\,dx$$

$$[x\sin 4x]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[-\frac{\cos 4x}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x\cos 4x\,dx$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x\cos 4x\,dx = \left[x\sin 4x + \frac{\cos 4x}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x\cos 4x\,dx = \left[\frac{\pi}{3}\left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{4}\left(-\frac{1}{2}\right)\right] - \left[\frac{1}{4}(-1)\right]$$

$$= -\frac{\pi\sqrt{3}}{6} - \frac{1}{8} + \frac{1}{4}$$

$$= \underline{\underline{\frac{1}{8} - \frac{\pi\sqrt{3}}{6}}}$$ [4]

7 Since $x - 2$ is a factor, $P(2) = 0$

$\therefore\ 8a - 36 + 2b - 6 = 0$

$\therefore\ 4a + b = 21 \quad \ldots (1)$

Since $P(x)/x - 3$, $R = 66$, $P(3) = 66$

$\therefore\ 27a - 81 + 3b - 6 = 66$

$\therefore\ 9a + b = 51 \quad \ldots (2)$

$(2) - (1)$: $5a = 30 \Rightarrow a = 6$

(1): $b = 21 - 24 \Rightarrow b = -3$

$\therefore\ \underline{\underline{a = 6,\ b = -3}}$ [4]

(b) $P(x) = 6x^3 - 9x^2 - 3x - 6$

$= (x - 2)(6x^2 + kx + 3)$

Comparing coefficients of x:

$3 - 2k = -3$

$k = 3$

$P(x) = (x - 2)(6x^2 + 3x + 3)$

$= \underline{\underline{3(x - 2)(2x^2 + x + 1)}}$ [2]

(c) $ax^3 + 9x^2 + bx + 6 = 0$

$-ax^3 - 9x^2 - bx - 6 = 0$

$a(-x)^3 - 9(-x)^2 + b(-x) - 6 = 0$

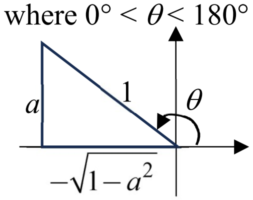
$\Rightarrow P(-x) = 0$

Since $P(x) = 0 \Rightarrow x = 2$,

Then $P(-x) = 0 \Rightarrow -x = 2$

$\therefore\ \underline{\underline{x = -2}}$ [2]

8 **(a)** $\sin\theta = a$ where $0° < \theta < 180°$



(i) $\cos\theta = \underline{\underline{-\sqrt{1 - a^2}}}$ [2]

(ii) $\cot\theta = \underline{\underline{\dfrac{-\sqrt{1 - a^2}}{a}}}$ [1]

(iii) $\sin(90° - \theta) = \cos\theta$

$= \underline{\underline{-\sqrt{1 - a^2}}}$ [1]

(b) $\frac{1}{2}\sec\left(2\phi + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ for $-\pi < \phi < \pi$

$\sec\left(2\phi + \frac{\pi}{4}\right) = \frac{2}{\sqrt{3}}$

$\cos\left(2\phi + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$

basic angle $= \frac{\pi}{6}$

$-2\pi + \frac{\pi}{4} < 2\phi + \frac{\pi}{4} < 2\pi + \frac{\pi}{4}$

$2\phi + \frac{\pi}{4} = -\frac{\pi}{6}, \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$

$2\phi = -\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$\phi = -\frac{5\pi}{24}, -\frac{\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}$ [4]

9 **(a)** $(a - x)(2 - x)^7$

$= (a - x)\left[2^7 + \binom{7}{1}2^6(-x) + \binom{7}{2}2^5(-x)^2 + \ldots\right]$

$= (a - x)\left[128 - 448x + 672x^2 + \ldots\right]$

Since coefficient of $x^2 = 616$,

$672a + 448 = 616$

$672a = 168$

$\underline{\underline{a = \frac{1}{4}}}$ [4]

(b) $\left(x^2 - \dfrac{1}{2x^4}\right)^n$

$$T_6 = \binom{n}{5}\left(x^2\right)^{n-5}\left(-\frac{1}{2x^4}\right)^5$$

Since T_6 is a constant,

then power of $x = 0$,

$\therefore\ 2(n-5) + 5(-4) = 0$

$2n = 30$

$n = 15$

$$\therefore T_6 = \binom{15}{5}\left(x^2\right)^{10}\left(-\frac{1}{2x^4}\right)^5$$

$$= 3\,003\left(x^{20}\right)\left(-\frac{1}{32x^{20}}\right)$$

$$= -\frac{3\,003}{32}$$ [4]

10 $A = 150\,000e^{-pt}$

(i) When $t = 0$, $A = 150\,000$

Amount paid for the car = $150 000 [1]

(ii) When $t = 24$, $A = 120\,000$,

$$\therefore\ 150\,000e^{-24p} = 120\,000$$

$$e^{-24p} = \frac{120\,000}{150\,000} = \frac{4}{5}$$

$$-24p = \ln\frac{4}{5}$$

$$p = -\frac{1}{24}\ln\frac{4}{5} = 0.009\,297$$

After 40 months,

$$A = 150\,000e^{-0.009\,297\times 40}$$

$= 103\,415.545$

$= 103\,416$ (nearest dollar)

$\therefore$ The value of the car = $ 103 416. [3]

(iii) When the value drops to $ 60 000,

$$150\,000e^{-0.009\,297t} = 60\,000$$

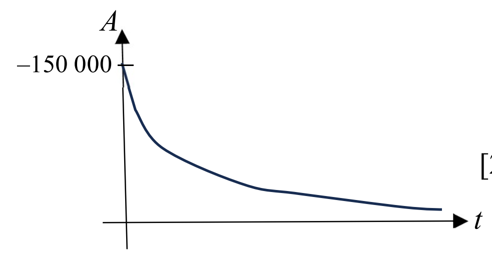
$$e^{-0.009\,297t} = \frac{60\,000}{150\,000} = \frac{2}{5}$$

$$-0.009\,297\,t = \ln\frac{2}{5}$$

$$t = -\frac{1}{0.009\,297}\ln\frac{2}{5} = 98.55$$

$\therefore$ The car is 99 months. [2]

(iv) Graph of A against t



[2]

11 **(a)** $y = \ln\left(\dfrac{1-x}{x^2}\right)$ for $0 < x < 1$

$$y = \ln(1-x) - 2\ln x$$

$$\frac{dy}{dx} = -\frac{1}{1-x} - \frac{2}{x}$$

$$\frac{dy}{dx} = \frac{-x - 2(1-x)}{(1-x)x}$$

$$= \frac{x-2}{(1-x)x}$$

For $0 < x < 1$, $x - 2 < 0$

$1 - x > 0$

$x > 0$

$$\therefore\ \frac{dy}{dx} = \frac{(-)}{(+)(+)}$$

$$\therefore\ \frac{dy}{dx} < 0$$

$\therefore$ y is a decreasing function. [5]

(b) Surface area of cylindrical ice block,

$A = 2\pi r h + 2\pi r^2$

$= 2\pi r(2r) + 2\pi r^2$

$= 6\pi r^2$

$$\frac{dA}{dr} = 12\pi r$$

Using chain rule, $\dfrac{dA}{dt} = \dfrac{dA}{dr}\times\dfrac{dr}{dt}$

$$\therefore\ -24 = 12\pi r \times \frac{dr}{dt}$$

$h = 8 \Rightarrow r = 4$

$$\therefore\ \frac{dr}{dt} = -\frac{24}{12\pi(4)}$$

$$= -\frac{1}{2\pi}$$

$= -0.159$ (3 s.f.)

Thus, r is decreasing at 0.159 cm/s. [4]

12 $L_2: \dfrac{x}{12}+\dfrac{y}{9}=1$

(a) When $x = 0$, $y = 9$.

When $y = 0$, $x = 12$.

$\therefore$ $C = (0, 9)$ and $D = (12, 0)$

Midpoint of CD, $A = \left(\dfrac{0+12}{2}, \dfrac{9+0}{2}\right)$

$= \left(6, \dfrac{9}{2}\right)$

Gradient of $CD = \dfrac{9-0}{0-12} = -\dfrac{3}{4}$

$\therefore$ gradient of $L_1 = \dfrac{4}{3}$

$\therefore$ equation of L_1, is

$$y-\frac{9}{2}=\frac{4}{3}(x-6)$$

When $x = 0$, $y = -8+\dfrac{9}{2}=-\dfrac{7}{2}$

$\therefore$ $B = \underline{\underline{\left(0, -\dfrac{7}{2}\right)}}$ [3]

(b) Since gradient of $L_1 = \dfrac{4}{3}$, $\tan\theta = \dfrac{4}{3}$

$\therefore$ $\underline{\underline{\theta = 53.1^\circ}}$ [2]

(c) $A = \left(6, \dfrac{9}{2}\right)$, $B = \left(0, -\dfrac{7}{2}\right)$ and $D = (12, 0)$

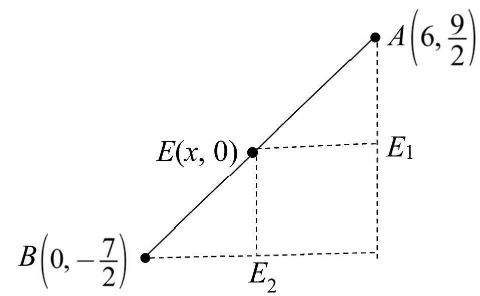
Area of triangle ABD

$$= \frac{1}{2}\begin{vmatrix} 12 & 6 & 0 & 12 \\ 0 & \frac{9}{2} & -\frac{7}{2} & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left[(54-21+0)-(0+0-42)\right]$$

$= \underline{\underline{37.5}}$ units^2 [2]

(d) E lies on the x-axis, $\therefore y = 0$.



By similar triangles,

$$\frac{AE}{EB}=\frac{AE_1}{EE_2}=\frac{\frac{9}{2}}{\frac{7}{2}}=\frac{9}{7}$$

$\therefore$ $AE : EB = \underline{\underline{9 : 7}}$ [2]

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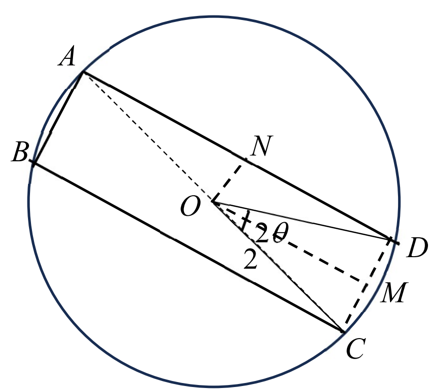

(a) $AB = CD = 2CM$

ΔOCM: $\sin\theta = \dfrac{CM}{2}$

$\therefore$ $CM = 2\sin\theta$

$AB = 4\sin\theta$

$AD = 2AN$

ΔOAN: $\sin\left(\dfrac{\pi-2\theta}{2}\right) = \dfrac{AN}{2}$

$AN = 2\sin\left(\dfrac{\pi}{2}-\theta\right)$

$= 2\cos\theta$

$AD = 4\cos\theta$

$\therefore P = AB + 2BC + 2OC$

$= 4\sin\theta + 2(4\cos\theta) + 2(2)$

$= 4(2\cos\theta + \sin\theta + 1)$ (shown) [3]

(b) Let $2\cos\theta + \sin\theta = R\cos(\theta-\alpha)$

$\therefore$ $R = \sqrt{2^2+1^2} = \sqrt{5}$

$\tan\alpha = \dfrac{1}{2} \Rightarrow \alpha = 26.565^\circ$

$\therefore$ $\underline{\underline{2\cos\theta + \sin\theta = \sqrt{5}\cos(\theta - 26.6^\circ)}}$ [2}

(c) $\therefore$ $P = 4\left[\sqrt{5}\cos(\theta - 26.565^\circ)+1\right]$

$\therefore$ $P = 12.5$

$\Rightarrow 4\sqrt{5}\cos(\theta-26.565^\circ)+4 = 12.5$

$\therefore \cos(\theta - 26.565^\circ) = \dfrac{12.5-4}{4\sqrt{5}}$

basic angle $= 18.134^\circ$

$\theta - 26.565^\circ = \pm 18.134^\circ$

$\therefore \theta = 44.699^\circ$ or 8.431°

$\therefore$ $\underline{\underline{\theta = 44.7^\circ \text{ or } 8.4^\circ}}$ [3]

(d) max $P = 4\left(\sqrt{5}+1\right)$

$= 12.944$

< 13

$\therefore$ Nicholas' claim is correct. [2]