MATHS (Help sheet)

"Knowing is not enough; we must apply.
Willing is not enough; we must do."
~Johann Wolfgang Von Goethe

CHAPT 1 & 2 — FACTORS & MULTIPLES; REAL NUMBERS

① Prime factors / Index notation

↳ e.g.

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$12 = 2^2 \times 3$

⇐ Note: We only use prime numbers to divide.

② Prime Nos.

↳ Numbers that are divisible by 1 and itself only. → e.g. 2, 3, 5, 7, ...

③ Rational Nos.

↳ Numbers that can be expressed as a fraction. → e.g. $\frac{2}{3}$, 15, $0.\dot{3}$, ...

④ Irrational Nos.

↳ Numbers that cannot be expressed as a fraction. → e.g. π, $\sqrt{3}$, $\sqrt{2}$, ...

⑤ Find LCM.

↳ E.g. Find the LCM of 90 and 24.

Mtd 1: $90 = 2 \times 3^2 \times 5$

$24 = 2^3 \times 3$

$LCM = 2^3 \times 3^2 \times 5$ (Note: Take higher power and EXTRA numbers)

$= 360$

Mtd 2:

$$\begin{array}{r|l} 2 & 24, 90 \\ 3 & 12, 45 \\ 2 & 4, 15 \\ 2 & 2, 15 \\ 3 & 1, 15 \\ 5 & 1, 5 \\ & 1, 1 \end{array}$$

⇐ Note: STOP at 1, 1.

$LCM = 2^3 \times 3^2 \times 5$

$= 360$

⑥ Find HCF.

↳ E.g. Find the HCF of 90 and 24.

Mtd 1: $90 = 2 \times 3^2 \times 5$

$24 = 2^3 \times 3$

$HCF = 2 \times 3$ (Note: Take the lower power and don't include EXTRA No.)

$= 6$

Mtd 2:

$$\begin{array}{r|l} 2 & 24, 90 \\ 3 & 12, 45 \\ & 4, 15 \end{array}$$

⇐ Note: STOP when there is no more common divisor.

⑦ Find the smallest integer value of p such that $\square$p is a MULTIPLE of ____.

↳ Steps: (i) Find LCM

(ii) p = LCM divide by $\square$

Note: Number that is together with the algebra.

⑧ Find the smallest integer k such that $\square$k is a perfect square.

↳ Aim: Make the POWER of the Factors to be MULTIPLES of 2.

↳ Multiply the additional factors that are needed to make the power into multiples of 2.

(Note: For Perfect CUBE, make it into multiples of 3.)

⑨ Word Problem on HCF and LCM

↳ LCM (Together, meet), "Smallest, least".

↳ HCF (Divide, pack), "Highest, greatest".

⑩ Temperature qns

↳ Temp. diff. = Highest Temp. − (Lowest Temp.)

↳ Temp. at Mid point = $\frac{\text{Highest temp.} + \text{Lowest temp.}}{2}$

CHAPT 3 — APPROXIMATION & ESTIMATION

① Decimal place(s)

↳ Start counting the decimal place(s) after the decimal point.

② Significant figure(s)

↳ Start counting the significant figure(s) from the first NON-ZERO digit.

e.g. 0.05010 (4 s.f.)

2.01 (3 s.f.)

CHAPT 4, 5, 6 — ALGEBRA

① Expand and simplify an expression

↳ e.g. Expand and simplify $2(x-1)+3$

$2(x-1)+3$

$= 2x - 2 + 3$

$= 2x + 1$

② Solve an equation

↳ e.g. Solve $2x+3 = 5x$.

$2x + 3 = 5x$

$2x - 5x = -3$

$-3x = -3$

$x = 1$

③ Word problem on Algebra

↳ Form an equation and solve for the unknown.

CHAPT 7 — ANGLES & PARALLEL LINES

① Types of Angles

↳ obtuse, acute and reflex angles

obtuse ⇓ more than 90° but less than 180°

acute ⇓ Less than 90°

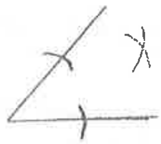
reflex ⇓ More than 180° but less than 360°

"There are no secrets to success. It is the result of preparation, hardwork and learning from failure."
~Colin Powell

② Reasons
↳ Adj. ∠s on a st. line
↳ ∠s at a point
↳ Vert. opp. ∠s
↳ Corr. ∠s
↳ Alt. ∠s
↳ int. ∠s
} For diagrams with parallel lines.

③ Construction ⇒ Note: Label the diagram and don't erase the arcs.
↳ Angles: use protractor
↳ Measurements (__ cm): Use Compasses & Ruler
↳ Perpendicular Bisector
↳ Steps: (i) Open the compasses to more than half the length of the line.
(ii) Draw 4 arcs ⇒

↳ Angle Bisector
↳ Steps: (i) Open the compasses to any length and draw 2 arcs on the 2 lines that are beside the angle.
(ii) From the 2 arcs, draw another 2 arcs.



↳ Complete the sentence.
↳ The point X is equidistant from the points ____ and ____ and equidistant from the lines ____ and ____.
↳ Steps: (i) Go back to the qns and look for angle bisector and perpendicular bisector.
e.g. Angle bisector of ∠CBD.
Perpendicular bisector of AC.
(ii) From AC, we have the points A and C. (Answer)
(iii) From $\boxed{C\boxed{B}D}$, we have the lines CB and BD.

The point X is equidistant from the points A and C and equidistant from the lines CB and BD.

CHAPT 8 — TRIANGLES & POLYGONS

① Refer to the last page for the properties of special quadrilateral (e.g. Parallelogram, rectangle, rhombus, square, trapezium, kite)

② Polygons
↳ Types of polygons: Triangle (3 sided)
Quadrilateral (4 sided)
Pentagon (5 sided)
Hexagon (6 sided)
Heptagon (7 sided)
Octagon (8 sided)
Nonagon (9 sided)
Decagon (10 sided)

↳ Sum of ext. ∠s of any polygon = 360°

↳ Sum of int. ∠s of a polygon
$= (n-2)(180°)$
Note: n is the number of sides of the polygon

↳ An int. ∠ of a REGULAR polygon
$= \dfrac{(n-2)(180°)}{n}$
Note: n is the no. of sides of the polygon.

↳ An int. ∠ + An ext. ∠ = 180°

CHAPT 11 — NUMBER PATTERN

① Sequence with same difference
↳ General term (nth term)
= JUMP × PATTERN NO. + or − WHATEVER NEEDED
↳ e.g. 3, 7, 11, 15, ...
general term $= 4n - 1$

CHAPT 12 — LINEAR GRAPH

① Axes: x axis (Horizontal →)
y axis (vertical ↑)
② Origin ⇒ (0, 0)
③ Coordinates ⇒ (x, y)
④ Gradient
↳ Negative slope: grad. $= \dfrac{-\text{Rise}}{\text{Run}}$
↳ Positive slope: grad. $= \dfrac{\text{Rise}}{\text{Run}}$
⑤ Graph of $x =$ __ ⇒ vertical line |||
Graph of $y =$ __ ⇒ horizontal line ≡
⑥ Area bounded ⇒ Area that is being "surrounded".

CHAPT 13 — INEQUALITIES

① Solve the inequality → ● ⩾ or ⩽
→ ○ > or <
e.g. $-2x > 1$
$-1 > 2x \rightarrow -\frac{1}{2} > x$ ← bring the negative x term to the positive side.
② Find the largest/smallest value
↳ solve the inequality and use a number line to find the value.

CHAPT 14 & 15 — AREA, PER., VOL., SURFACE AREA

① Formula, units, mathematical statement.
② Answers that are not exact must be in 3 s.f.
③ Refer to cover page for value of π.

Summary (CHAPTER 1 TO 3·2) =}

"You become stronger with every problem you face and every obstacle you overcome!"

Chapter 1 (Proportion)

- Scale ⇒ Make sure that we have the same unit on both sides of the ratio, before we remove the units.

  e.g. 5cm : 1km
  5cm : 100000 cm
  5 : 100 000
  1 : 20 000 (Map scale)

  * Always reduce the ratio to the simplest form.

- Map scale ⇒ Ratio form (e.g. 1 : 20 000)
  ⇒ Fraction form (e.g. $\frac{1}{20\ 000}$)
- Map scale ⇒ use this to find length / distance.
- Area scale ⇒ use this to find Area.

Map scale $\xrightarrow{\text{square}}$ Area scale
Map scale $\xleftarrow{\text{square root}}$ Area scale

E.g. Given that 1cm represent 4km. How to find the area scale?
(Map scale) 1cm : 4km
(Area scale) 1^2 cm^2 : 4^2 km^2
$1cm^2$: $16km^2$

E.g. Given that $1cm^2$ represent $16 km^2$. How to find the map scale?
(Area scale) $1cm^2$: $16 km^2$
(Map scale) $\sqrt{1}$ cm : $\sqrt{16}$ km
1cm : 4km

* Note: When given an Area scale, always check that the ratio is in its simplest form before taking square root, to find the map scale. Make sure that when you take square root of the area scale, the result is a whole number.

e.g. Given that $8cm^2 : 200km^2$. What is the map scale?
Wrong ⇒ (map scale) $\sqrt{8}$ cm : $\sqrt{200}$ km
Correct ⇒ (Area scale) $8cm^2 : 200km^2$
$1cm^2 : 25km^2$
(Map scale) $\sqrt{1}$ cm : $\sqrt{25}$ km
1cm : 5km

* Reduce the ratio to the simplest form first!

- Conversion:

1km = 1000m
= 100 000 cm
1m = 100 cm
1cm = 10mm
} Distance

$1km^2 = 1 000 000 m^2$
$= 100 000\ 00 000 cm^2$
$1m^2 = 10 000 cm^2$
$1cm^2 = 100mm^2$
} Area

* Note: No. of "zero" is doubled!!! (as compared to the above)

- Direct proportion ⇒ when x↑, y↑. When x↓, y↓.
When x and y are directly proportional, we have $\boxed{y = kx}$ or $\boxed{x = ky}$, where k is a constant.
↳ not changing.

Inverse proportion ⇒ when x↑, y↓. When x↓, y↑.
When x and y are inversely proportional, we have $y = \frac{k}{x}$ or $x = \frac{k}{y}$, where k is a constant.

e.g. y is inversely proportional to the square root of x+3. x is 6 when y is 18.
(a) Express y in terms of x.
(b) Find the value of y when x is 1.

* To answer (a), you need to know that they are asking for the equation. Another way of asking is: "Find an equation connecting y and x", or "Find an equation in terms of x and y" or anything similar to these!

Steps: To find the equation!!! Answer to (a).

$y = \frac{k}{\sqrt{x+3}}$ ① Write the general equation first! Note: This is not what qns (a) is asking for.

$18 = \frac{k}{\sqrt{6+3}}$ ② Put a pair of values for x and y, to find the value of the constant k.

$18 = \frac{k}{3}$
$k = 18 \times 3$
$= 54$

Equation ⇒ $y = \frac{54}{\sqrt{x+3}}$ (answer) ③ Rewrite the general equation but replace the "k" with the numeric value of k that we have found.

Note: For qns like (a), when we need to find the equation, make sure that we only replace "k" by the numeric value. "y" and "x" will still be "y" and "x" in the equation! Don't replace "y" and "x" by the numeric values given in the qns. Because "y" and "x" are numbers that are changing.

* To answer (b), we need to make use of the equation that we have found in (a) to solve!

$y = \frac{54}{\sqrt{x+3}}$ ⇒ Equation from (a)
$= \frac{54}{\sqrt{1+3}}$ ⇒ Replace "x" by "1" and find the new y value.
$= 27$

Note: General equation is very important!!! If it is wrong, everything will be wrong. Always read the qns carefully to make sure that you have the correct general equation!

Chapter 2 (Expansion and factorisation of Algebraic expressions)

- Expansion ⇒ Always simplify even if qns never state!

⇒ Type ①: Term × Term (e.g. expand (2ab)(3a²b))
$(2ab)(3a^2b) = 2 \times a \times b \times 3 \times a^2 \times b$
$= 2 \times 3 \times a \times a^2 \times b \times b$
$= 6 \times a^3 \times b^2$
$= 6a^3b^2$

⇒ Type ②: Term × Expression (e.g. expand (2ab)(3a²b−1))
$(2ab)(3a^2b-1) = 6a^3b^2 - 2ab$

* 1 expression ⇒ apply single rainbow!

⇒ Type ③: Expression × Expression
(e.g. expand (2ab+a)(3a²b−1+b))

$(2ab+a)(3a^2b-1+b)$

* 2 expressions ⇒ apply double rainbow!

$= 6a^3b^2 - 2ab + 2ab^2 + 3a^3b - a + ab$
$= 6a^3b^2 - ab + 2ab^2 + 3a^3b - a$

⇒ Type ④: Expand using special product formula!
ⓐ $(a+b)^2 = a^2 + 2ab + b^2$
ⓑ $(a-b)^2 = a^2 - 2ab + b^2$
ⓒ $(a+b)(a-b) = a^2 - b^2$

e.g. Expand $(2x+5)^2$.

$(2x+5)^2 = (2x)^2 + 2(2x)(5) + 5^2$

$= 4x^2 + 20x + 25$

Note: If you see any qns with any of the special product format, you can apply ⓐ, ⓑ or ⓒ directly. If you can't recognise it, don't worry. You can still use the double rainbow method (Type ③), to find the same result.

e.g. $(2x+5)^2 = (2x+5)(2x+5)$

$= 4x^2 + 10x + 10x + 25$

$= 4x^2 + 20x + 25$

⇒ Type ⑤: Rojak qns (combination of everything)

e.g. Expand $2x - 5 + 3(x+2)(x-2)$

$= 2x - 5 + 3(x^2 - 4)$ ⇒ special product ⓒ

$= 2x - 5 + 3x^2 - 12$ ⇒ Multiplication before addition!

$= 2x - 17 + 3x^2$

e.g. Expand $\underbrace{(2x-1)^2}_{\text{apply Type ④, ⓑ}} - \underbrace{(x+1)(2x-3)}_{\text{double rainbow}}$

$(2x-1)^2 - (x+1)(2x-3)$ ⇐ Bracket is important here! Need to distribute the minus sign later!

$= [(2x)^2 - 2(2x)(1) + 1^2] - [2x^2 - 3x + 2x - 3]$

$= (4x^2 - 4x + 1) - (2x^2 - x - 3)$

$= 4x^2 - 4x + 1 - 2x^2 + x + 3$

$= 2x^2 - 3x + 4$

Note: Always remember to expand the bracket first, then followed by multiplication & division, and lastly addition and subtraction! Don't be scared by the complicated expression. Split them and do the expansion step by step separately.

- Factorisation ⇒ reverse process of expansion.

⇒ Type ①: Factorise an expression with 2 terms.

Steps: (1) Check whether there is any common factor that we can take out from the 2 terms.

(2) Check whether it is a special product of the form $a^2 - b^2$.

e.g. Factorise (a) $2x^2y - 18y^3$

(b) $3a^2b - ba$

(a) $2x^2y - 18y^3$

$= 2y(x^2 - 9y^2)$ ⇒ Apply Step (1).

$= 2y(x-3y)(x+3y)$ ⇒ Apply Step (2). STOP! (Factorised completely)

(b) $3a^2b - ba$

$= ab(3a-1)$ ⇒ Apply Step (1). STOP! (Factorised completely)

⇒ Type ②: Factorise an expression with 3 terms.

Steps: (1) Check whether there is any common factor that can be taken out from the 3 terms.

(2) Check whether it is a special product of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$.

Can also skip (2)

(3) Use cross method, when you see expression with "one" or "two" terms with power of 2. (Quadratic expression) e.g. $x^2 + 4xy + 3y^2$; $x^2 - 4x + 3$ etc.

(4) Check that everything in the bracket is completely factorised!

e.g. Factorise $8x^2y - 24xy^2 + 18y^3$.

Mtd 1: $8x^2y - 24xy^2 + 18y^3$

$= 2y(4x^2 - 12xy + 9y^2)$ ⇒ Apply step (1).

$= 2y(2x-3y)^2$ ⇒ Apply step (2).

Mtd 2: $8x^2y - 24xy^2 + 18y^3$

$= 2y(4x^2 - 12xy + 9y^2)$ ⇒ Apply step (1).

$= 2y(2x-3y)(2x-3y)$ ⇒ Apply step (3) directly.

$= 2y(2x-3y)^2$

$+2x$	╳	$-3y$	$-6xy$
$+2x$		$-3y$	$-6xy$
$+4x^2$		$+9y^2$	$-12xy$

e.g. Factorise $x(x-5)+6$

$x(x-5)+6 = x^2 - 5x + 6$

$= (x-3)(x-2)$

⇒ Apply step (3).

$+x$	╳	-3	$-3x$
$+x$		-2	$-2x$
$+x^2$		$+6$	$-5x$

⇒ Type ③: Factorise an expression with 4 terms.

Steps: (1) Check whether there is any common factor that we can take out from the 4 terms.

(2) Group the terms 2 by 2 and factorise them.

(3) Take out the common expression and put the remaining terms in the second bracket.

(4) Check that everything in the bracket is factorised completely.

Note: You need to rearrange the terms when you see that there is no similar expressions when you are at step (2). Sometimes you might have to factorise "1".

e.g. Factorise $10ax + 5by - 2ay - 25bx$

Wrong ⇒ $10ax + 5by - 2ay - 25bx$

$= 5(2ax + by) + 1(2ay - 25bx)$ ⇒ No similar expressions to proceed to step (3).

Correct ⇒ $10ax + 5by - 2ay - 25bx$

$= 10ax - 2ay + 5by - 25bx$ ← rearrange the terms.

$= 2a(5x - y) + 5b(y - 5x)$

$= 2a(5x - y) - 5b(-y + 5x)$ ← Step (2).

$= (5x - y)(2a - 5b)$ ← Step (3).

-TRICK: When we factor out a minus sign, the sign in the bracket will reverse.

e.g. $(2a - b) = -(-2a + b)$.

- How do we know what sign to put when we group the terms 2 by 2 and factorise them?

Note: Always put the "+" sign after the first bracket. The sign in the second bracket will follow the 3rd and 4th term.

e.g. Factorise $9x^2y + x - 7y - 63xy^2$

$9x^2y + x - 7y - 63xy^2$ — sign of the terms in the 2nd bracket will follow the sign of the 3rd & 4th terms.

$= x(9xy + 1) + 7y(-1 - 9xy)$

$= \ldots\ \ldots$

Always have a "+" sign here!

e.g. Factorise $5cd - 25c^2 + 10d - 50c$.

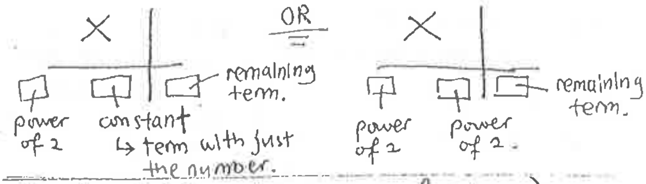
$5cd - 25c^2 + 10d - 50c$ — sign of the terms in 2nd bracket will follow the sign of the 3rd & 4th terms.

$= 5c(d - 5c) + 10(d - 5c)$

$= \ldots\ \ldots$

Always "+" sign

- Cross method (for expressions with 1 or 2 terms with power of 2) — quadratic expression.



Chapter 3 (Simplify algebraic fractions)

- Cancel Numerator with denominator.
- Factorise expressions completely before performing cancellation.

e.g. Simplify $\frac{x^2-1}{x-1}$.

Wrong ⇒ $\frac{x^{\cancel{2}}-1}{\cancel{x}-1} = \frac{x-1}{1}$

correct ⇒ $\frac{x^2-1}{x-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = x+1$

- Change "÷" with "×", but remember to invert the second fraction. e.g. $\frac{2x}{3} \div \frac{x}{2} = \frac{2x}{3} \times \frac{2}{x}$

$= \ldots\ldots$

Summary (CHAPTER 3·3 To 5) =]

It is hard to fail, but it is worse never to have tried to succeed. ~ Theodore Roosevelt.

Chapter 3: Simple Algebraic Fractions

- Add or Subtract fractions

Steps: ① Check whether the denominator can be factorised.
② Factorise the denominator (if possible)
③ Find the LCM of the denominator.
↳ Make the denominator common.
④ Combine the fractions into one single fraction.
⑤ Simplify the numerator. (Don't cancel yet!)
⑥ Check whether we can perform any simplification (cancellation) to the numerator and denominator.

e.g. $\frac{3}{x^2-10x+25} - \frac{8}{x-5}$

$$\begin{array}{c|c|c} +x & -5 & -5x \\ +x & -5 & -5x \\ \hline +x^2 & +25 & -10x \end{array}$$

$= \frac{3}{(x-5)(x-5)} - \frac{8}{(x-5)}$

$= \frac{3}{(x-5)^2} - \frac{8(x-5)}{(x-5)(x-5)}$

Common mistake: To get $(x-5)^2$ as the denominator, it is wrong to do this: $\frac{8^2}{(x-5)^2}$

$= \frac{3-8(x-5)}{(x-5)^2}$

$= \frac{3-8x+40}{(x-5)^2}$

Remember that we cannot square the fraction. We can only multiply something to it!

$= \frac{43-8x}{(x-5)^2}$

Stop here! Don't need to perform any cancellation! (Already in simplified form.)

Important Trick:
What do we do when we notice that the sign of the terms (all) are reversed?

e.g. $\frac{1}{t-2} - \frac{1}{-t+2} = \frac{1}{t-2} - \frac{1}{-(t-2)}$

$= \frac{1}{t-2} + \frac{1}{t-2}$

$= \frac{2}{t-2}$

> TRICK: $(-t+2) = -(t-2)$
> * Factor out a "minus sign" and flip all the sign of the terms.

- Make SOMETHING as the subject.
↳ Let that SOMETHING be left alone at one side of the equation.

e.g. $E = \frac{1}{2}mv^2 + mgh$. Make v the subject.

$E - mgh = \frac{mv^2}{2}$

$2(E-mgh) = mv^2$

$\frac{2(E-mgh)}{m} = v^2$

$\pm\sqrt{\frac{2(E-mgh)}{m}} = v$ ⇐ Note: We need to add "$\pm$" sign in front of "$\sqrt{\ }$" whenever we need to take square root.

e.g. Make "a" the subject.

$\sqrt{a} = \sqrt{2ba+c}$ ⇐ Square the equation to remove the $\sqrt{\ }$ sign.

$a = 2ba + c$

$a - 2ba = c$ ← Factorise to group the "a" together.

$a(1-2b) = c \Rightarrow a = \frac{c}{1-2b}$

Chapter 4: Congruence and Similarity

- Congruent ($\equiv$) figures
↳ Same shape (corr. ∠s are equal)
↳ Same size (corr. sides are equal)

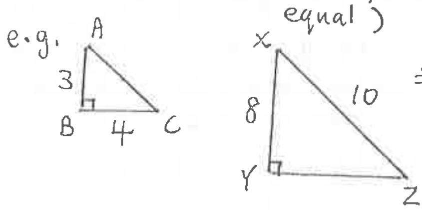
e.g. $PQRS \equiv CBAD$ (Note: Order is important. We can find the corr./matching point here!)

$PQ = CB$; $RQ = AB$; $SP = DC$; $RS = AD$.
$\angle RSP = \angle ADC$; $\angle RQP = \angle ABC$; ......

* Congruent figures ⇒ exact copy of one another. (Exactly the same!)

- Similar figures
↳ same shape (corr. ∠s are equal)
↳ same ratios (Ratios of corr. sides are equal)

e.g.



⇒ Given that $\triangle ABC$ is similar to $\triangle ZYX$

$\triangle ABC$ similar to $\triangle ZYX$.

Qns: How to find AC?

$\frac{AC}{ZX}$ ⇒ ① Find the corr. side of AC. Put it in ratio / fraction form!

$\frac{AC}{ZX} = \frac{BC}{YX}$ ⇒ ② Equate $\frac{AC}{ZX}$ to another set of corr. sides. (fraction)

(a) Why we choose $\frac{BC}{YX}$ and not $\frac{AB}{ZY}$?
- When choosing the other set of corr. sides, we have to make sure that the measurements or values are given so that we can solve the equation with one unknown.

(b) Why not $\frac{YX}{BC}$ instead of $\frac{BC}{YX}$?
- We must be clear that since we have $\frac{AC}{ZX}$ $\left(\frac{\text{side of } \triangle ABC}{\text{corr. side of } \triangle ZYX}\right)$, we must make sure that this order of which on top and which below is consistent! Thus we must also have $\frac{BC}{YX}$ $\left(\frac{\text{side of } \triangle ABC}{\text{corr. side of } \triangle ZYX}\right)$.

$\frac{AC}{10} = \frac{4}{8}$ ③ ⇒ Replace "ZX", "BC" and "YX" by the given values.

$AC = \frac{4}{8} \times 10$

$= 5$ cm

Note: To find the angles of similar figures, we still apply the same method as how we do it for congruent figures. Because the corr. angles of similar figures will still be equal.

* Similar figures ⇒ Enlarged / Reduced copy of the original figure.

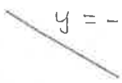
Remember to state the reason (e.g. ∠s sum of △, adj. ∠s on str. line, etc.) when they are being used to solve the angle.

Chapter 5: Linear Equations in two unknowns

Solve simultaneous equations (Two equations)
↳ Find the value of the 2 unknowns.

Mtd 1: Solve simultaneous Eqns by Graphical mtd.

① Copy and complete the tables.
② Plot all the points for the two equations
③ Join up all the points with a str. line.
↳ There should be 2 str. lines since there are two eqns.
④ Label the lines with the equation.

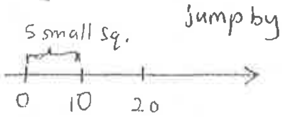


* If the points that we get from the tables do not give us a str. line, we need to go back and check our table of values. There might be some mistakes in the calculation of the values.

⑤ Point of intersection will be the answer (solution) to the question.
e.g. (2, 5) is the point of intersection.
Ans: $x = 2$ and $y = 5$.

*Basic information that we need to know:

- Scale (5 small squares on the graph paper is equal to 1cm)
↳ 1cm to 10 units (After 5 small squares, jump by 10.)



- Horizontal axis ⇒ x-axis ⟶
- Vertical axis ⇒ y-axis ↑
- origin ⇒ (0, 0)
- Coordinates ⇒ (x, y)

Mtd 2: Solve Simultaneous Eqns by Substitution mtd.

① Choose one equation and make one of the unknown in that equation as the subject.
② Sub. the new equation into the other equation.

e.g. Solve $2x + 3y = 7$ ——— (1)
$-3x + 2y = -4$ ——— (2)

From (1): $3y = 7 - 2x$
$y = \frac{7-2x}{3}$ ——— (3) ⇒ Make "y" the subject.

Sub. (3) into (2): ⇒ (3) is taken from (1), so we sub. (3) into the other eqn (2).

$-3x + 2\left(\frac{7-2x}{3}\right) = -4$

$-3x + \frac{14-4x}{3} = -4$

$-\frac{9x}{3} + \frac{14-4x}{3} = -4$

$\frac{-9x + 14 - 4x}{3} = -4$

$-9x + 14 - 4x = -12$

$-13x = -26$

$x = 2$ ——— (4)

Sub. (4) into (3): $y = \frac{7-2(2)}{3}$

$= \frac{7-4}{3}$

$= 1$

Ans: $x = 2$ and $y = 1$.

Mtd 3: Solve Simultaneous Eqns by Elimination mtd.

① Make the coeff. of one of the unknown to be the same.
↳ fractional coeff. ⇒ Find LCM of the denominator and multiply it to the equation to get rid of the denominator.
② Add / subtract the equations to eliminate one of the unknowns.
↳ coeff. of same sign ⇒ subtract the eqns
↳ coeff. of diff. sign ⇒ add the eqns.

Solve $2x + 3y = 7$ ——— (1)
$-3x + 2y = -4$ ——— (2)

(1) × 3: $6x + 9y = 21$ ——— (3)
(2) × 2: $-6x + 4y = -8$ ——— (4)
} Make the coeff. of x the same.

(3) + (4): $(6x + 9y) + (-6x + 4y) = 21 + (-8)$

$6x + 9y - 6x + 4y = 21 - 8$

$13y = 13$

$y = 1$ ——— (5)

* Add the eqns, because the coeff. of "x" here are of diff. sign.

Sub. (5) into (1):

$2x + 3(1) = 7$

$2x + 3 = 7$

$2x = 4$

$x = 2$

Ans: $x = 2$ and $y = 1$.

* Use Graphical mtd when qns ask you to solve the simultaneous eqns graphically.

* When qns ask you to solve simultaneous eqns (but did not mention by which mtd), then you DECIDE which mtd is easier for that qns or which mtd you prefer!!!

Maths (Help sheet) – Chapt 6 to 8.

Try, try, try and keep on trying is the rule that must be followed to become an expert in anything.
~ W. Clement Stone.

Chapt 6 : Quadratic functions and equations

– How to identify whether an equation is quadratic?

↳ When there's a term with power of "2", the equation will be quadratic. ⇒ e.g. $y = 3x^2 + 2x - 1$
↑ power of 2!

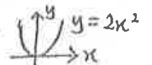
– Quadratic graph

↳ A curve, either ∪ or ∩ depending on the coefficient (number) of the x^2 term.

e.g. $y = 2x^2$ ⇒ positive 2, so we have ∪.
$y = -2x^2$ ⇒ negative 2, so we have ∩.

* Positive ⇒ happy ⇒ ☺ ⇒ so ∪-shape curve.

* Negative ⇒ sad ⇒ ☹ ⇒ so ∩-shape curve.

↳ after drawing the curve, remember to write the equation beside the curve. e.g. 

↳ DON'T shade / sketch the curve! Make sure it is a smooth curve. No sharp point.

↳ y-intercept ⇒ the point where the curve cuts / intersects the y-axis.

* For the y-intercept ⇒ x value is always 0.

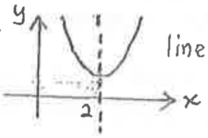
↳ x-intercept ⇒ the point(s) where the curve cuts / intersects the x-axis.

* For the x-intercept ⇒ y value is always 0.

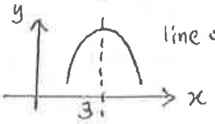
↳ Line of Symmetry ⇒ the line that will divide the curve into two EQUAL parts. Line of symmetry will pass through the min./max. point of the curve.

e.g. For ∪-shape curve, the line of symmetry will pass through the minimum point.
For ∩-shape curve, the line of symmetry will pass through the maximum point.

* Equation of the line of symmetry ⇒ $x = ???$, where ??? takes the x-value that the line passes through. e.g.



line of symmetry : $x = 2$

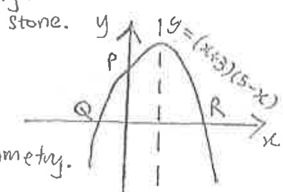


line of symmetry : $x = 3$

Since line of symmetry is always a vertical line for ∪-shape / ∩-shape curve, the equation will always be $x = ???$.

* The line of symmetry can also be calculated from a pair of symmetrical points. The midpoint (middle point) of the symmetrical points will give us the value of line of symmetry.

E.g. The diagram shows the graph of the curve $y = (x+3)(5-x)$. The dotted line is the line of symmetry of the curve. The curve meets the y-axis at P, and the x-axis at Q and R as shown.



(a) Find the coordinates of P.
(b) Find the coordinates of Q and R.
*(c) Find the equation of the line of symmetry.

Solution:

(a) P ⇒ y-intercept ⇒ x-value = 0,
when $x = 0$, $y = (0+3)(5-0)$
$= 15$
$\therefore P = (0, 15)$

(b) Q and R ⇒ x-intercepts ⇒ y-value = 0,
when $y = 0$, $0 = (x+3)(5-x)$
$x + 3 = 0$ or $5 - x = 0$
$x = -3$ $\quad$ $x = 5$
$\therefore Q = (-3, 0)$ and $R = (5, 0)$.

(c) Line of symmetry ⇒ midpoint of Q and R!!!
Midpoint $= \frac{-3+5}{2} = 1$
Equation of line of symmetry ⇒ $x = 1$.

↳ Using the graph, solve the quadratic equation.

E.g. After drawing the graph of $y = 2x^2 + 3x$, you are asked to solve $2x^2 + 3x + 1 = 0$ from your graph.

↳ This just mean that we need to find the value(s) of x from the graph! DON'T calculate x but read off the x values from the graph!

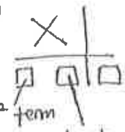
↳ How? $2x^2 + 3x + 1 = 0$
$\underbrace{2x^2 + 3x}_{y} = -1$

So, when $y = -1$, what are the values of x from the graph? Read off the x values from graph.

– Solving Quadratic Equations ⇒ Roots are solutions or answers of the equation!!!

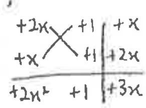
↳ Step 1: Make sure that one side of the equation is 0.
If the equation does not have a zero at any side of the equation, then bring all the terms to one side so that we get a zero.
e.g. $2x^2 + 3x = -1$
$2x^2 + 3x + 1 = 0$ ⇐ Create a '0' at one side of the equation!

↳ Step 2: Factorise!!!
– 2 terms ⇒ Bring out common factor OR
⇒ Apply $a^2 - b^2 = (a+b)(a-b)$
– 3 terms ⇒ Apply cross method ⇒



↳ Apply Zero product rule ⇘ Either one must be zero, in order to get a '0'.

– e.g. $2x^2 + 3x + 1 = 0$
3 terms ⇙
$(2x+1)(x+1) = 0$
$2x + 1 = 0$ or $x + 1 = 0$
$x = -\frac{1}{2}$ $\quad$ $x = -1$



– e.g. $2x^2 - 8 = 0$
2 terms ⇙
$2(x^2 - 4) = 0$ ⇐ $a^2 - b^2 = (a+b)(a-b)$
$2(x+2)(x-2) = 0$
$2(x+2) = 0$ or $(x-2) = 0$
$x + 2 = 0$ $\quad$ $x = 2$
$x = -2$

– e.g. $3x^2 - 5x = x$
2 terms ⇙
$3x^2 - 6x = 0$
$3x(x-2) = 0$ ↗ $3x = 0$ or $x - 2 = 0$
$x = 0$ $\quad$ $x = 2$

↳ Find the other roots of the equation.

e.g. One solution of $2x^2+kx-15=0$ is $x=3$. Find

(a) the value of k.

(b) the other solution of the equation.

Solution: (a) when $x=3$, $2(3)^2+3k-15=0$

$18+3k-15=0$

$3k+3=0$

$k=-1$

(b) when $k=-1$, $2x^2+(-1)x-15=0$

$2x^2-x-15=0$

$(2x+5)(x+3)=0$

$2x+5=0$ or $x+3=0$

$x=-\frac{5}{2}$ $\quad$ $x=-3$

$x=-2.5$

$+2x$	5	$+5x$
$+x$	3	$-6x$
$+2x^2$	-15	$-x$

The other solution is $x=-2.5$.

- Solving word problem involving Quadratic Equation

↳ Step 1: Form an equation/show an equation.

↳ Step 2: Solve the equation. Check if there's any x values that we can reject (based on the context of the question).

↳ Step 3: Use the x value (remaining x value) for the next part of the question.

E.g. A tank holds 300 ℓ of water. The tank can be filled with water by Pipe A or Pipe B.

(a) Pipe A pumps x litres of water per minute into the tank. Write an expression, in terms of x, for the number of minutes it takes for Pipe A to fill up the tank.

(b) Pipe B pumps $(x+6)$ litres of water per minute into the tank. Write an expression, in terms of x, for the number of minutes it takes for pipe B to fill up the tank.

(c) It takes 5 minutes longer for Pipe A than for Pipe B to fill up the tank. Write down an equation in x, and shows that it simplifies to $x^2+6x-360=0$.

Solution:

(a) $x\ell$ — 1 min (Pipe A)

300ℓ — $\frac{300}{x}$ mins

(b) $(x+6)\ell$ — 1 min (Pipe B)

300ℓ — $\frac{300}{x+6}$ mins

(c) Pipe A took 5 mins longer than Pipe B!!!

$\frac{300}{x}-\frac{300}{x+6}=5$ ⇒ * Use the HIGHER value Minus the LOWER value and equate to their difference.

$\frac{300(x+6)}{x(x+6)}-\frac{300x}{x(x+6)}=5$

*Check that all the terms are in the same unit.

$\frac{300(x+6)-300x}{x(x+6)}=5$

$\frac{300x+1800-300x}{x(x+6)}=5$

$\frac{1800}{x(x+6)}=5$

$1800=5x(x+6)$

$1800=5x^2+30x$

$0=5x^2+30x-1800$

$0=x^2+6x-360$

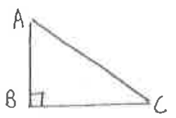
(shown)

*If you cannot show the equation in (c), you can still use the equation as given in the question to solve for the next part of the question. Don't give up and stop at (c)!!!

Chapt 7: Pythagoras theorem ⇒ Must be Right angled triangle.

- How to identify the longest side?

↳ The two sides that form the right angle will not be the longest side.



$AC^2 = AB^2 + BC^2$

longest side

* For any topic, final answers that are not exact must be in 3s.f. and use 5s.f. for intermediate workings.

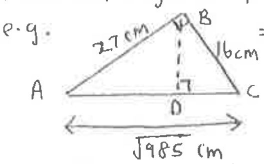
- Finding the length of a right-angled triangle.

↳ By pythagoras theorem OR

↳ Using the Area of the triangle to find the length.

* Shortest length ⇒ perpendicular length.

e.g.



⇒ BD is the shortest length from B to AC.

(a) Find the area of △ABC.

(b) Calculate the length of BD.

Solution

(a) Area of $\triangle ABC = \frac{1}{2}\times 27\times 16 = 216\text{cm}^2$

(b) Area of $\triangle ABC = 216$ ⇐ Have to use the area to solve for the missing length. (cannot apply pythagoras thm to find the length because there are a few missing info.

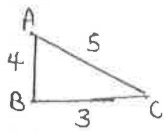
$\frac{1}{2}\times\sqrt{985}\times BD = 216$

$BD \approx 13.765$

$= 13.8\text{cm}$ (3s.f.)

- How to prove that a triangle is right angle?

e.g. Is △ABC a right-angled triangle?



Solution: AC = 5cm is the longest side!

$AC^2 = 5^2 = 25$

$AB^2+BC^2 = 4^2+3^2$

$=25$

$\therefore AC^2 = AB^2+BC^2$ ⇒ By pythagoras thm, △ABC is a right-angled triangle.

Chapt 8: Mensuration of pyramids, cones and spheres.

- Pyramids ⇒ vol. = $\frac{1}{3}\times$ base area $\times$ height (perpendicular)

⇒ total surface area = base area + area of all the lateral faces.

- Cones ⇒ vol. = $\frac{1}{3}\times\pi r^2\times$ height (perpendicular)

⇒ total surface area = base area + curved surface area

$= \pi r^2 + \pi r\ell$ ⇒ ℓ is the slant height!!!

- Spheres ⇒ vol. = $\frac{4}{3}\pi r^3$

⇒ total surface area = $4\pi r^2$

- Hemispheres ⇒ vol. = $\frac{2}{3}\pi r^3$

(Half the sphere.) ⇒ total surface area

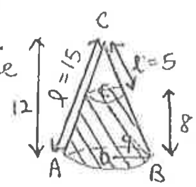
= base area + curved surface area

$= \pi r^2 + 2\pi r^2$

*For cones and pyramids questions, we might need to use the concept on pythagoras thm and similar figures to solve.

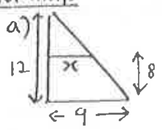
e.g. (a) Find the radius of the small cone.

(b) Find the curved surface area of the shaded region.



Solution:

a)



By similar figures,

$\frac{x}{9} = \frac{12-8}{12}$

$\frac{x}{9} = \frac{4}{12}$

$x = \frac{4}{12}\times 9$

$x = 3\text{cm}$

Radius = 3cm

(b) Curved surface area

$=\pi(9)(15)-\pi(3)(5)$

≈ 376.99

$= 377\text{cm}^2$ (3s.f.)

* For cones and pyramids, always try to find right-angled triangle so that you can apply Pythagoras theorem to find either the radius, height or the slant height.

* Right pyramid/cone ⇒ Slant heights will be EQUAL!!! ⇒ Vertex (top) is directly above the center of the base

* Regular pyramid ⇒ All the lateral faces will be the same ⇒ Sides of the base are equal.

↳ same area.

- When π is not given, use calculator π value. Refer to cover page.

Summary (CHAPTER 1 TO 2) (Sec 3)

"Everything is hard before it's easy."
~ Johann Wolfgang Von Goethe.

CHAPT 1 : Indices

① $a^m \times a^n = a^{m+n}$

② $a^m \div a^n = a^{m-n}$ or $\frac{a^m}{a^n} = a^{m-n}$

③ $(a^m)^n = a^{mn}$

④ $(ab)^n = a^n b^n$

⑤ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

⑥ $a^0 = 1$, $a \neq 0$

⑦ $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

⑧ $a^{\frac{1}{n}} = \sqrt[n]{a}$

⑨ $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$

e.g. simplify $\frac{2(\sqrt{b}) \times 3b^2}{(2b^2)^3}$

$= \frac{2b^{\frac{1}{2}} \times 3b^2}{8b^6}$ ⇐ apply ⑧, ③ & ④

$= \frac{6b^{\frac{1}{2}+2}}{8b^6}$ ⇐ apply ①

$= \frac{6b^{\frac{5}{2}}}{8b^6}$ ⇐ apply ②

$= \frac{6}{8} b^{\frac{5}{2}-6} = \frac{3}{4} b^{-\frac{7}{2}}$

e.g. simplify the following and leave it in positive index form.

(a) $5t^{-2} = \frac{5}{t^2}$

(b) $\frac{3}{t^{-2}} = 3t^2$

(c) $(2t)^{-3} = \left(\frac{1}{2t}\right)^3 = \frac{1}{8t^3}$

* Positive index form / indices ↳ Positive power!!!

* Negative power ⇒ FLIP ONLY the 'term' with the negative power.

⇒ If $a^y = a^x$ then $y = x$.

e.g. $3^x = 81$
$3^x = 3^4$ ⇒ Change to the same base before we equate the power!
$x = 4$

e.g. $2^x \times 2^3 = 4$

$2^x \times 2^3 = 2^2$ ⇒ Change to the same base. Cannot equate the power yet because we have more than 1 base on the LHS!!! Combine the base first!

$\underbrace{2^{x+3}}_{\text{Combine into 1 base!}} = 2^2$

$x + 3 = 2$
$x = -1$

Standard form → $A \times 10^n$ where $1 \leq A < 10$ and n is an integer.

Easier way to remember standard form → (The decimal point should be placed right after the first significant figure.)

e.g. $0.00305 = 3.05 \times 10^{-3}$
$235.56 = 2.3556 \times 10^2$

> Special Names for power of 10.

10^3 ⇒ kilo / Thousand (k)	10^{-3} ⇒ milli (m)
10^6 ⇒ mega / Million (M)	10^{-6} ⇒ micro (μ)
10^9 ⇒ Giga / Billion (G)	10^{-9} ⇒ nano (n)
10^{12} ⇒ tera / Trillion (T)	10^{-12} ⇒ pico (p)

CHAPT 2 : More about quadratic equations.

$ax^2 + bx + c = 0$ (Power of 2!!!)

Mtd 1: Factorisation (cross-mtd!) (Cannot be used to solve for certain quadratic equations!)

$x^2 + 3x + 2 = 0$
$(x+2)(x+1) = 0$
$x + 2 = 0$ or $x + 1 = 0$
$x = -2$ $\quad x = -1$

$+x$	$+2$	$+2x$
$+x$	$+1$	$+x$
$+x^2$	$+2$	$+3x$

Mtd 2: Graphical.
↳ only use this method when we are asked to draw the graph.

Mtd 3: General formula (The Song!!!)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b and c are taken from the equation $ax^2 + bx + c = 0$.
(a: coeff. with "x^2"; b: coeff. with "x"; c: constant.)

Mtd 4: Completing the square

$$\boxed{x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2}$$

↑ coefficient of x must be 1!!!
If it is not 1, we have to divide the entire eqn by the number. e.g. $2x^2 + x - 3 = 0$) $\div 2$
$x^2 + \frac{x}{2} - \frac{3}{2} = 0$

e.g. (a) Express $x^2 - 2x - 2$ in the form $(x-a)^2 + b$.
(b) Hence solve $x^2 - 2x - 2 = 0$.

(a) $x^2 - 2x - 2$ ⇐ Expression!!! Not solving for x!
$= \left(x + \left(\frac{-2}{2}\right)\right)^2 - \left(\frac{-2}{2}\right)^2 - 2$ ⇐ b = −2 here!
$= \left(x + \left(\frac{-2}{2}\right)\right)^2 - 2 - 1$
$= (x-1)^2 - 3$

(b) $x^2 - 2x - 2 = 0$
$(x-1)^2 - 3 = 0$ ⇐ from (a), we have the completing the square form.
$(x-1)^2 = 3$
$x - 1 = \pm\sqrt{3}$ ⇐ Always take square root when we use completing the square mtd to solve!
* Remember: '±' sign!!!
$x - 1 = \sqrt{3}$ or $x - 1 = -\sqrt{3}$
$x = 2.73$ (3 s.f.) $\quad x = -0.732$ (3 s.f.)
* Round off ans that are not exact to 3 s.f.

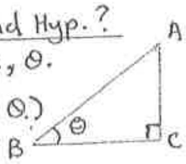
Trigonometry

TOA $\Rightarrow \tan\theta = \frac{Opp.}{Adj.}$
CAH $\Rightarrow \cos\theta = \frac{Adj.}{Hyp.}$
SOH $\Rightarrow \sin\theta = \frac{Opp.}{Hyp.}$

} θ is an angle (Reference angle)

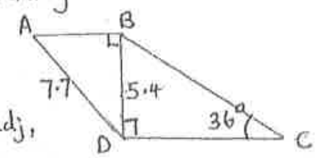
* TOA, CAH, SOH can only be applied to right angled triangle. There can only be one unknown that we are finding. Either one unknown angle or unknown side.
* We only use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ when we want to find the unknown angle.

$\Rightarrow$ How to identify the Opp., Adj. and Hyp.?



- Take an angle as the reference angle, θ.
  ↳ e.g. $\angle ABC$
- Opp. side will be AC. (side facing θ.)
- Hyp. will be AB. (side facing the right angle ⊾ sign.)
- Adj. will be BC. (the remaining side.)

E.g. Calculate (a) BC,
(b) $\angle ADB$.



Soln:
(a) Steps: ① Identify opp., adj, hyp.
② We take 36° as the reference angle. Thus, we have 5.4 (opp.) and BC (hyp.). So is it TOA, CAH or SOH do we use? Which of these will involve both opp. and hyp.? It will be 'SOH'. Hence we use 'sin'.

$\sin 36° = \frac{5.4}{BC}$

$BC \sin 36° = 5.4$

$BC = \frac{5.4}{\sin 36°}$
≈ 9.1870
$= 9.19\text{ cm}$ (3s.f.)

(b) Steps: ① Since we need to find $\angle ADB$, we use that as the reference angle.
② Identify opp., adj. and hyp.. So BD (adj.) and AD (hyp.). Then, is it TOA, CAH or SOH involves both adj. and hyp.? It will be CAH. Hence we use 'cos'.

$\cos\angle ADB = \frac{5.4}{7.7}$

$\angle ADB = \cos^{-1}\left(\frac{5.4}{7.7}\right)$
≈ 45.469
$= 45.5°$ (1d.p.)

* Angles qns: If the ans is not exact, round off to 1 d.p.

* For any right angle triangle, always think of using Pythagoras's thm or TOA, CAH, SOH to solve for the unknown.

Inequality $\Rightarrow$ One final line

E.g. $\boxed{3x < \boxed{x-1} \leq 2x+1}$

$3x < x-1$
$2x < -1$
$x < -\frac{1}{2}$

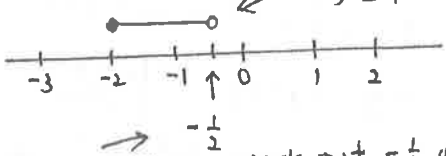
and

$x - 1 \leq 2x + 1$
$-x \leq 2$
$-2 \leq x$

$\therefore \boxed{-2 \leq x < -\frac{1}{2}}$

* Must write this final statement for the combined solution.

only 1 final line.



* Make sure you mark out $-\frac{1}{2}$ on the number line.

* Checklist: ① Flip the sign when you multiply/divide by a negative number.
→ e.g. $-2x < 4$
$x > -2$

Alternatively: To avoid this, always make x (unknown) positive.
e.g. $-2x < 4$
$-4 < 2x$
$-2 < x$

② Final statement
↳ combined soln
$(-2 \leq x < -\frac{1}{2})$

③ $>$ or $<$ → ○
$\geq$ or $\leq$ → ●

MATHS SUMMARY (CHAPT 4, 7 & 8.1)

CHAPT 4: Conditions of Congruence & Similarity

Congruent ⇒ Same shape and size (exactly the same)

Congruency test:

① SAS (2 pairs of equal sides and 1 included angle.)

② SSS (3 pairs of equal sides)

③ AAS/ASA/SAA (2 pairs of equal angles and 1 pair of equal side)

④ RHS (1 right angle, 1 pair of equal hypotenuse side and 1 pair of equal adjacent/opposite side)

Similarity ⇒ Same shape BUT diff. size.

Similarity test:

① SAS (2 ratio of sides that are equal and 1 included angle.)

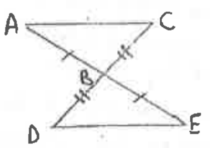
② SSS (3 ratio of sides that are equal.)

③ AA (2 pairs of equal angle.)

* Note: The 'S' in congruency test refers to the side BUT the 'S' in similarity test refers to the ratio of the sides.

* Always state reason to support your prove.

e.g. Show that △ABC is congruent to △EBD.



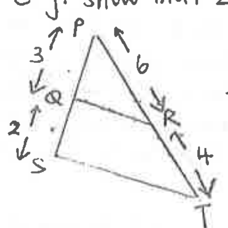
∠ABC = ∠EBD (vert. opp. ∠s)

AB = EB (given)

BC = BD (given)

∴ △ABC ≡ △EBD (SAS)

e.g. Show that △PQR is similar to △PST.



∠QPR = ∠SPT (common ∠)

$\frac{PQ}{PS} = \frac{3}{5}$, $\frac{PR}{PT} = \frac{6}{10} = \frac{3}{5}$

$\frac{PQ}{PS} = \frac{PR}{PT} = \frac{3}{5}$ (side ; side)

included angle

∴ △PQR is similar to △PST (SAS)

⇒ How do you know when to prove using congruency test and not similarity test? (Especially when qns did not mention anything about similar or congruent...)

Qns asking you to show whether...

① the area of 2 △s are equal,

② the corresponding side of the 2△s are equal,

③ the corresponding angle of the 2 △s are equal,

④ the line is an angle bisector,

⑤ the line bisects another line.

⑥ the figure is a parallelogram/rhombus... Etc !!!

Prove using Congruency test.

"They can because they think they can." ~ Virgil

For similar figures...

① (Ratio of the sides)^2 = Area ratio.

② (Ratio of the sides)^3 = Vol. ratio. / Mass ratio.

③ $\sqrt{\text{Area ratio}}$ = side ratio

④ $\sqrt[3]{\text{Vol. ratio}}$ = side ratio.

* Side ratio ⇒ e.g. radius, side, circumference, perimeter etc.

* Area ratio ⇒ e.g. area, base area, surface area etc.

* Vol. ratio ⇒ e.g. volume, mass, etc.

3 Tricks to use for Qns involving $\frac{\text{Area}\ldots}{\text{Area}\ldots}$:

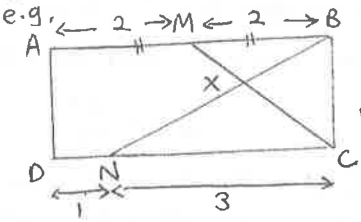
① If the figures are similar, (ratio of sides)^2 = Area ratio.

② Using the equal height.

③ Comparing ratio.

If you cannot show that the figures are similar !!!

e.g.



Given that ABCD is a rectangle, find

(a) $\frac{\text{Area } \triangle XMB}{\text{Area } \triangle XCN}$,

(b) $\frac{\text{Area } \triangle BXC}{\text{Area } \triangle BNC}$,

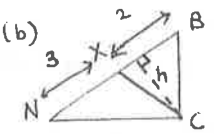
(c) $\frac{\text{Area } \triangle BXC}{\text{Area } ABCD}$.

Soln:

(a) △XMB similar to △XCN (can be proved using AA.)

$\frac{\text{Area } \triangle XMB}{\text{Area } \triangle XCN} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ (using Trick ①)

(b)

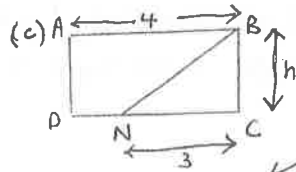


Since △XMB similar to △XCN, all the sides ratio will be $\frac{2}{3}$.

∴ $\frac{MX}{CX} = \frac{BX}{NX} = \frac{MB}{CN} = \frac{2}{3}$.

$\frac{\text{Area } \triangle BXC}{\text{Area } \triangle BNC} = \frac{\cancel{\frac{1}{2}} \times 2 \times \cancel{h}}{\cancel{\frac{1}{2}} \times 5 \times \cancel{h}} = \frac{2}{5}$ (using Trick ② ↳ common height)

(c)



$\frac{\text{Area } \triangle BNC}{\text{Area } ABCD} = \frac{\frac{1}{2} \times 3 \times \cancel{h}}{4 \times \cancel{h}} = \frac{3}{8}$

↳ using Trick ② ↳ common height.

Using comparing ratio...

$\underbrace{\frac{\text{Area } \triangle BXC}{\text{Area } \triangle BNC} = \frac{2}{5}}_{\text{from (b)}}$ and $\frac{\text{Area } \triangle BNC}{\text{Area } ABCD} = \frac{3}{8}$

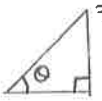
Area △BXC	:	Area △BNC	:	Area ABCD
2	:	5		
		3	:	8
6	:	15	:	40

∴ $\frac{\text{Area } \triangle BXC}{\text{Area } ABCD} = \frac{6}{40} = \frac{3}{20}$

CHAPT 7: Trigonometry

Right angle △ Qns
↳ Solve using TOA, CAH, SOH OR pythagoras's thm.

TOA CAH SOH...



⇒ Opposite/facing the reference angle θ
↳ Opposite side (opp.)
⇒ Opposite / facing the right angle
↳ hypotenuse side (hyp.)

⇒ The last remaining side → adjacent side (adj.)

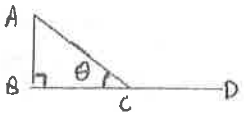
$\tan\theta = \frac{opp.}{adj.}$; $\cos\theta = \frac{adj.}{hyp.}$; $\sin\theta = \frac{opp.}{hyp.}$

* Use $\tan^{-1}$, $\cos^{-1}$ or $\sin^{-1}$ when you need to solve for the angle θ.

Obtuse & Acute relationship...

$\left.\begin{array}{l} * \sin\theta = \sin(180^\circ-\theta) \\ * \cos\theta = -\cos(180^\circ-\theta) \\ * \tan\theta = -\tan(180^\circ-\theta) \end{array}\right\}$ θ is the acute angle. 180°−θ is the obtuse angle.

e.g. Given than $\sin\theta = \frac{3}{5}$, find $\cos\angle ACD$.



$\sin\theta = \frac{AB}{AC} = \frac{3}{5}$

$BC^2 + AB^2 = AC^2$
$BC^2 + 3^2 = 5^2$
$BC = 4$

$\therefore \cos\angle ACD$
$= -\cos\angle ACB$
$= -\frac{4}{5}$

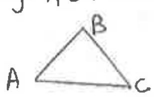
e.g. $\sin x = \sin 135^\circ$, $0 < x \leq 180^\circ$.
$x = 180^\circ - 135^\circ$ or $x = 135^\circ$
$= 45^\circ$

e.g. $\cos 25^\circ = -\cos x$, $0 < x \leq 180^\circ$.
$x = 180^\circ - 25^\circ$
$= 155^\circ$

Area formula (Applicable to any △)

Area of △ = $\frac{1}{2}$(side)(side) sin (included angle)

e.g. Area △ABC = $\frac{1}{2}(AB)(BC)\sin\angle ABC$.



Sine Rule (Applicable to any △)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

OR $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

} "a" & "A" are opposite pair of ∠ and side. Similarly for "b" & "B" and "c" & "C".

⇒ a, b, c are the sides of the △.
⇒ A, B, C are the angles of the △.

⇒ When to use the sine rule?
↳ Use it when we have 1 pair of opposite angle and side given.
↳ Solving for the unknown side/angle of another opposite pair.

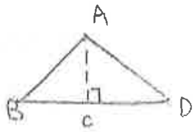
Cosine Rule (Applicable to any △)

$a^2 = b^2 + c^2 - 2bc\cos A$.

⇒ a, b, c are the sides of the △.
⇒ A is the angle of the △.
⇒ "a" and "A" are the opposite side and angle. (opposite pair.)

⇒ When to use the cosine rule?
↳ At least 2 sides must be given.
↳ Solving for the unknown side/angle of the opposite pair.
↳ Unknown will always be from the opposite pair. e.g. $\underline{a}^2 = b^2 + c^2 - 2bc\cos \underline{A}$.
opposite pair
(Either one of them will be the unknown).
↳ Case 1: Given 3 sides, we need to solve for the unknown opp. angle.
↳ Case 2: Given 2 sides and the angle, we solve for the unknown side (opp. the angle given).

⇒ Shortest dist?
↳ Perpendicular height.
e.g. Shortest dist. from A to BD.
↳ AC (perpendicular distance)



⇒ longest side ⇒ facing the largest angle.
smallest side ⇒ facing the smallest angle.

⇒ To find the shortest dist./perpen
↳ ① Find the Area of △ / TOA CAH SOH Mtd.
↳ ② Use the Area of △ ÷ $\frac{1}{2}$ ÷ largest base.

* To find the longest perpendicular height, use the area divided by $\frac{1}{2}$ and the shortest base.

Note: Intermediate workings to 5 s.f. Final answer, if not exact, will be 1 d.p. (angles) and 3 s.f. (any other qns).

Important: Remember that $\sin\theta = \sin(180^\circ-\theta)$, calculator will always give the acute answer. So you have to refer to the diagram and see if you need to use $180^\circ-\theta$ to find the obtuse angle. Whenever you use sine rule, you have to check your answer with the given diagram.

CHAPT 8.1: Bearing

① Measured from North point
② Turn clockwise direction
③ 3 digits (e.g. 020°, 125.7°, 045.1°)

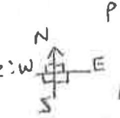
e.g. Bearing of B from A.

Note: ↑N and ↑N from diff. points are parallel.

e.g.

← Parallel North point! (can apply alt. ∠s, corr. ∠s, int. ∠s to solve qns!)

Note:



Any point that is due west or east, will be perpendicular to the North and South.

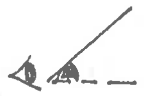
MATHS HELP SHEET =}

(CHAPT 8, 9 &5) — SEC 3

CHAPT 8: Applications of Trigonometry

⇒ Angle of...

Elevation



Depression

(For **BOTH** cases, the angle is made with the horizontal)

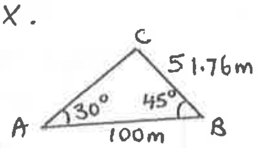
⇒ 3D Trigo

↳ To find the greatest angle of elevation/depression.

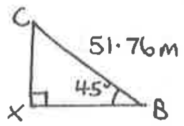
Step 1: Find the shortest dist. from the observer to the position of the object.

Step 2: Look for the standing right angled △ from the shortest dist. to the object. Find the angle of elevation/depression.

Example:
D is a point at the top of a building, 25m, vertically above C. A, B and C are three points on a horizontal land. X is a point on AB, find the greatest angle of elevation of D from X.



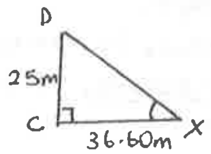
Step 1: Find the shortest dist.



$\sin 45° = \frac{CX}{51.76}$

$CX = 51.76 \sin 45°$
$\approx 36.60\text{ m}$

Step 2: Find the greatest angle of elevation/depression.



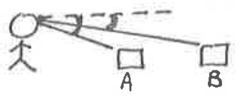
$\tan \angle DXC = \frac{25}{36.60}$

$\angle DXC = \tan^{-1} \frac{25}{36.60}$

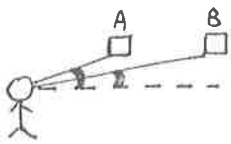
≈ 34.335
$= 34.3°$ (1d.p.)

Greatest angle of elevation, $\angle DXC = 34.3°$ (1d.p.).

⇒ Greater/smaller angle of elevation & depression?



* Nearer object ⇒ Greater angle of depression.



* Nearer object ⇒ Greater angle of elevation.

"Persistent people begin their success where others end in failure." ~ Edward Eggleston

CHAPT 9: Coordinate Geometry

⇒ Grad. of a line $= \frac{y_2 - y_1}{x_2 - x_1}$

⇒ Dist. $= \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$

⇒ Eqn of a line: $y = mx + c$

m, gradient
c, y-intercept

* 2 lines that are parallel ⇒ same gradient
* 2 lines cut the y-axis at the same point
  ↳ same y-intercept.
* A line is being reflected on the y-axis
  ↳ y-intercept is the same
  ↳ gradient has the opposite sign. (+ve ⇄ −ve)
* A line intersect a curve/line. To find the points of intersection, solve the simultaneous eqns.
* A point on a line/curve will satisfy (FIT) the equation of line/curve.

Example:
Find the equation of a line that is parallel to $y + 2x = 6$ and passing through the point (4, 2).

parallel ⇒ same gradient.

$y + 2x = 6$
$y = 6 - 2x$
∴ grad. $= -2$

Thus, we have $y = -2x + c$
(So what is c?)

* Use the point (4, 2):

$y = -2x + c$
$2 = -2(4) + c$
$c = 10$

} A point on the line will satisfy (FIT) the equation of the line $y = -2x + c$.

∴ Eqn of line ⇒ $y = -2x + 10$

CHAPT 5: Graphs and Functions

⇒ Turning point (max./min. pt.)

How to determine whether there is a max./min. pt.?

For the curve, $y = ax^2 + bx + c$,

if 'a' is POSITIVE ⇒ ∪ min. pt. (Happy)

OR

if 'a' is NEGATIVE ⇒ ∩ max. pt. (Sad)

To sketch a curve (Quadratic Eqn)
↳ power 2!!!

$\boxed{\text{Mtd 1:}}$ Completing the square form

$$x^2+bx = \left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

*NOTE: Coefficient of x^2 MUST be +ve !!!

Step 1: Find max./min. pt.
↳ TRICK: Make the $(\ )^2$ expression ZERO.

Step 2: Find x-intercept $(y=0)$.

Step 3: Find y-intercept $(x=0)$.

Example:
Express $-x^2+3x-5$ in the form $-(x-p)^2+q$ and sketch the curve $y=-x^2+3x-5$.

$$\begin{aligned}-x^2+3x-5 &= -[x^2-3x+5]\\ &= -\left[\left(x+\left(-\tfrac{3}{2}\right)\right)^2-\left(-\tfrac{3}{2}\right)^2+5\right]\\ &= -\left[\left(x-\tfrac{3}{2}\right)^2-\tfrac{9}{4}+5\right]\\ &= -\left[\left(x-\tfrac{3}{2}\right)^2+2\tfrac{3}{4}\right]\\ &= -\left(x-\tfrac{3}{2}\right)^2-2\tfrac{3}{4}\end{aligned}$$

$\therefore y = -x^2+3x-5$
$= -\left(x-\frac{3}{2}\right)^2-2\frac{3}{4}$ } Negative coeff. of $x^2 \Rightarrow$ ∩ max. pt.

Step 1: Max. pt. $\Rightarrow \left(\frac{3}{2}, -2\frac{3}{4}\right)$

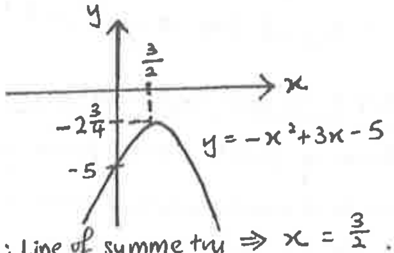
Step 2: x-intercept $(y=0)$

$$\begin{aligned}0 &= -\left(x-\tfrac{3}{2}\right)^2-2\tfrac{3}{4}\\ 2\tfrac{3}{4} &= -\left(x-\tfrac{3}{2}\right)^2\\ -2\tfrac{3}{4} &= \left(x-\tfrac{3}{2}\right)^2\\ \pm\sqrt{-2\tfrac{3}{4}} &= x-\tfrac{3}{2}\end{aligned}$$

No solution $\Rightarrow$ No x-intercept !!!

Step 3: y-intercept $(x=0)$

$$\begin{aligned}y &= -\left(0-\tfrac{3}{2}\right)^2-2\tfrac{3}{4}\\ &= -2\tfrac{1}{4}-2\tfrac{3}{4}\\ &= -5\end{aligned}$$



*NOTE: Line of symmetry $\Rightarrow x = \frac{3}{2}$.

$\boxed{\text{Mtd 2:}}$ Factorised form

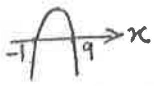
Step 1: Find x-intercept $(y=0)$
↳ Find the line of symmetry $\Rightarrow x = \frac{a+b}{2}$
↳ Find min./max. point

Step 2: Find y-intercept $(x=0)$

Example:
Sketch the curve $y=(x+1)(-x+9)$.

Step 1: x-intercept $(y=0)$
$0=(x+1)(-x+9)$
$x+1=0$ or $-x+9=0$
$x=-1$ $\quad$ $x=9$

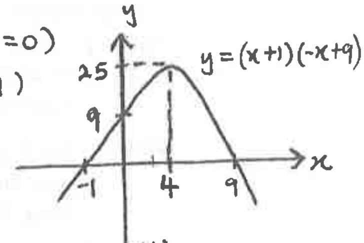
NOTE: "$-x^2$" when we expand. Thus, ∩ max. point!!!



Line of symmetry $\Rightarrow x = \frac{-1+9}{2}$ (Find the mid point)
$x = 4$

Max. pt. $\Rightarrow$ when $x=4$,
$y=(4+1)(-4+9)$
$= 25$
$\therefore (4, 25)$

Step 2: y-intercept $(x=0)$
$y=(0+1)(0+9)$
$= 9$



Qns on GRAPH PAPER (graphical mtd!)

How to use your graph to solve an equation?

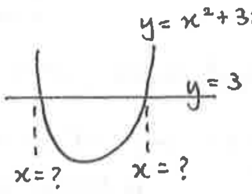
e.g. Draw $y=x^2+3x-5$ on the graph paper.
Hence, solve $x^2+3x-6=2$.

Step 1: Find a relationship between $y=x^2+3x-5$ and $x^2+3x-6=2$.

$$\begin{aligned}x^2+3x-6 &= 2\\ x^2+3x-6\underline{+1} &= 2\underline{+1}\\ \underbrace{x^2+3x-5}_{y} &= 3\end{aligned}$$

Step 2: Draw an additional line, $y=3$ on your graph paper.

Step 3: Read off the solutions (x-values) from the INTERSECTIONS of the curve and the line.

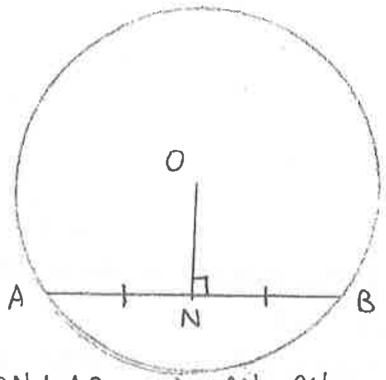


MATHS HELP SHEET =} (SEC3)

"Little by little, one walks far."
~ Peruvian Proverb.

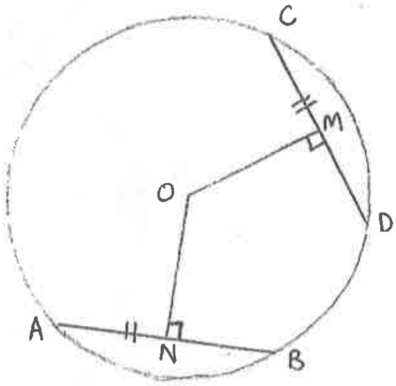
CHAPT 6 : Properties of circles

Property 1 : ($\perp$ from centre bisects chord)



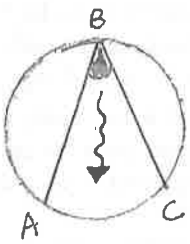
$\Rightarrow$ ON $\perp$ AB $\longrightarrow$ AN = BN.
$\longleftarrow$

Property 2 : (Equal chords, equidistant from centre)



$\Rightarrow$ AB = CD $\rightleftarrows$ OM = ON

$\Rightarrow$ Given an angle, how do you find the reference arc? (The angle is subtended by which arc?)

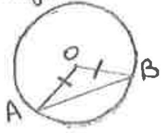


Think :
- How to trap the angle ($\angle$ABC) in an enclosed area?
- Do you draw the MINOR ARC or the MAJOR ARC AC?

Ans : We need to enclose the angle ABC by the MINOR arc AC.
$\therefore$ Reference arc $\Rightarrow$ MINOR arc AC!!!

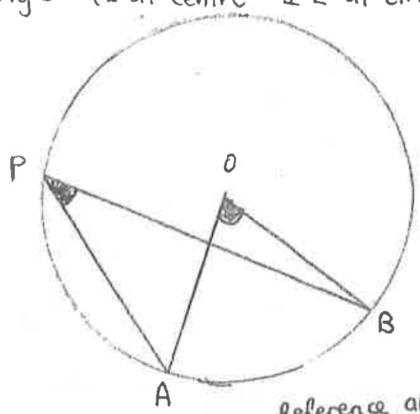
↳ MINOR arc AC

$\Rightarrow$ Common TRICK for circles qns :
↳ look for the Radius of the circle!!! There might be an isosceles $\triangle$.



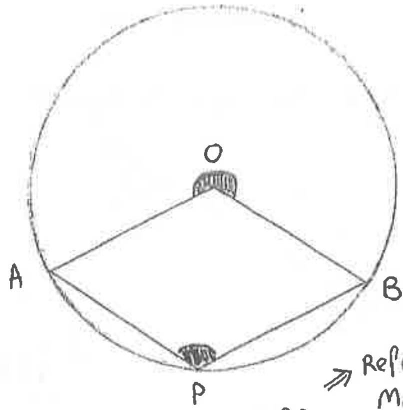
$\Rightarrow$ OA = OB (radius)
$\triangle$OAB $\Rightarrow$ isos. $\triangle$.

Property 3 : ($\angle$ at centre = 2 $\angle$ at circumference)



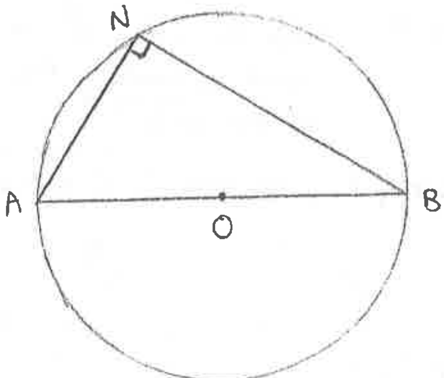
↳ $\angle AOB = 2\angle APB$ $\Rightarrow$ Reference arc : Minor AB.

*** RULE : $\angle$ at centre and $\angle$ at circumference must share the same ARC!



↳ Reflex $\angle AOB = 2\angle APB$ $\Rightarrow$ Reference arc : Major AB.

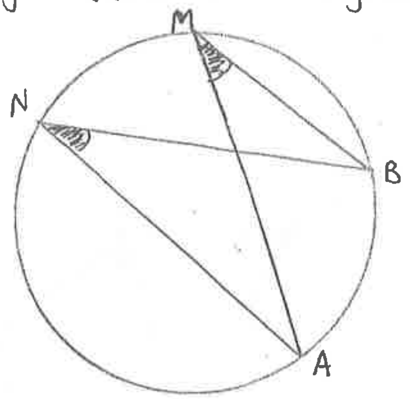
Property 4 : ($\angle$ in a semicircle)



↳ If AOB is the diameter of the circle, then $\angle ANB = 90°$

*** RULE : One side of the triangle must be the diameter
↳ diameter will pass through the centre of the circle.

Property 5: ($\angle$s in the same segment)

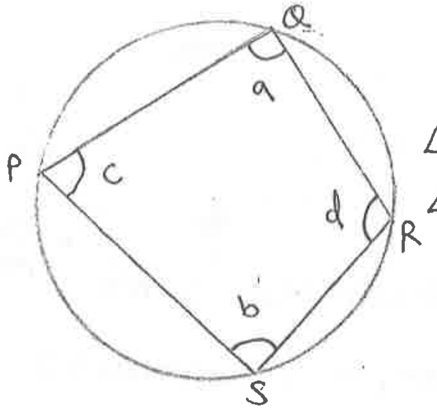


$\angle ANB = \angle AMB$ ⇒ Reference arc: Minor AB.

*** RULE: - The two angles are on the circumference.

- The two angles have the same reference arc.

Property 6: ($\angle$s in opposite segments)

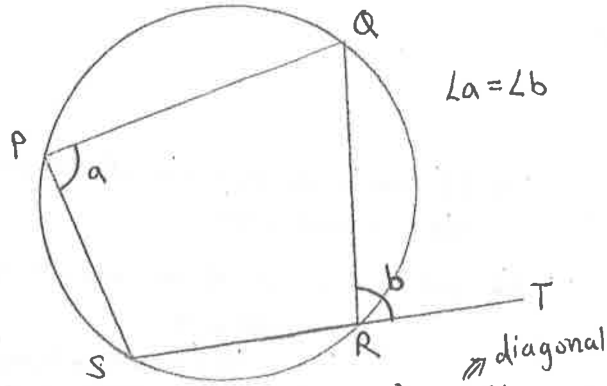


$\angle a + \angle b = 180°$

$\angle c + \angle d = 180°$

*** RULE: All four sides of the quadrilateral must be touching the circumference of the circle.

Property 7: (exterior $\angle$ of cyclic quadrilateral)



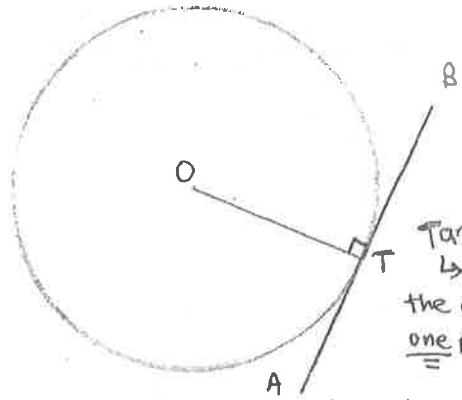
$\angle a = \angle b$

Exterior angle = Inside (interior) opposite angle.

*** RULE: - All four sides of the quadrilateral touching the circumference of the circle.

- Exterior angle formed by extending one side of the quadrilateral.

Property 8: (tangent perpendicular to radius)

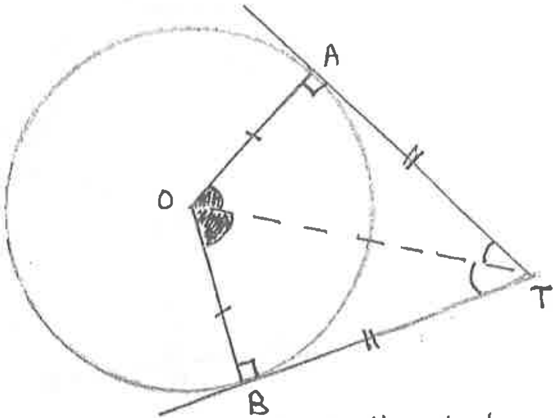


Tangent
↳ touching the circle at one point only.

↳ If AB is tangent to the circle, then $OT \perp AB$.

*** RULE: Tangent is ONLY perpendicular to the Radius.

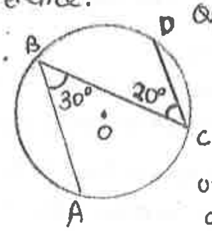
Property 9: (tangents from external point)



↳ If TA and TB are tangents to the circle, then $TA = TB$, $\angle AOT = \angle BOT$ and $\angle ATO = \angle BTO$.

⇒ How to answer qns "inside, outside or on" the circumference."

E.g.



Qns: A point S is to be marked on the diagram such that $\angle ASC = 32°$. Does the point S lie inside the circle, outside the circle or on the circumference? Explain your answer.

Ans: If S is on the circumference, $\angle ASC = 30°$ ($\angle$s in the same segment). Thus, to get a bigger angle, S must be inside the circle.

*Note: - Angle formed outside the circle ⇒ smaller than angle on the circumference.

- Angle formed inside the circle ⇒ bigger than angle on the circumference.

1.1(a) Numbers, fractions, decimals and percentages.

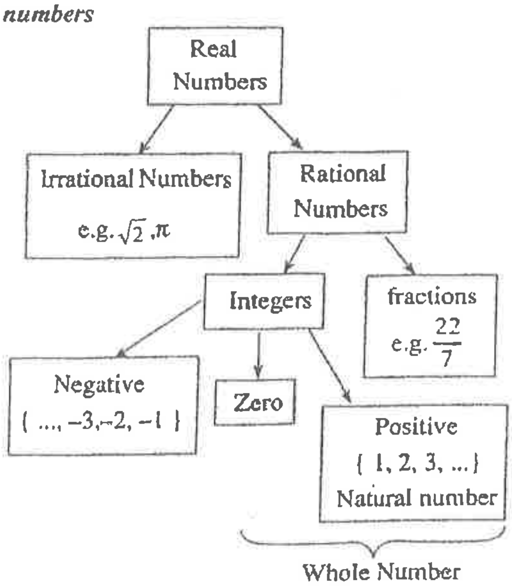
Number

The set of natural numbers, $N = \{ 1, 2, 3, \dots \}$.
The set of whole numbers, $W = \{ 0, 1, 2, 3, \dots \}$
The set of integers, $Z = \{ \dots, -2, -1, 2, 0, 1, 2, \dots \}$.
The set of positive integers, $\mathbb{Z}^+ = \{ 1, 2, 3, \dots \}$.
The set of negative integers, $\mathbb{Z}^- = \{ -1, -2, -3, \dots \}$.
The set of rational numbers, $Q = \{ \frac{a}{b}, a, b, \in \mathbb{Z}, b \neq 0 \}$.
An irrational number is a number which cannot be expressed in the form $\frac{a}{b}$ where a, b are integers and $b \neq 0$,

e.g. $\sqrt{2}, \sqrt{5}, \pi$, etc.

The set of real numbers $\mathbb{R}$ is the set of rational and irrational numbers.
The set of odd numbers = { 1, 3, 5, 7, . . . }.
The set of even numbers = { 0, 2, 4, 6, . . . }.
The set of prime numbers = { 2, 3, 5, 7, 11, . . . }.
The set of perfect square numbers = { 1, 4, 9, 16, 25, 36, . . . }.
The set of perfect cube numbers = { 1, 8, 27, 64, 125, 216, . . . }.
The set of triangular number = { 1, 3, 6, 10, 15, . . . }.
0 is an even number.
1 is not a prime number.

Real numbers



Vulgar and decimal fraction

The vulgar fraction of 0.35 is $\frac{35}{100} = \frac{7}{20}$.

The decimal fraction of $\frac{7}{8}$ is 0.375.

Percentage

A percentage is a fraction with denominator 100.

x % means $\frac{x}{100}$.

To convert a fraction to a percentage, multiply the fraction by 100%.

$$\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$$

1.1(b) Approximation and estimation. Exact values. Significant figures. Decimal places and standard form. Squares and square roots. Cubes and cube roots.

Significant figures

(i) All non-zero digits are significant.
(ii) A zero (zeros) between non-zero digits is (are) significant.
(iii) In a whole number, zeros after the last non-zero digit may or may not be significant.
(iv) In a decimal, zeros before the first non-zero digit are not significant.
(v) In a decimal, zeros after the last non-zero digit are significant.

Examples:	7 006 = 7 000	(corr. to 1 sig. fig.)
	7 006 = 7 000	(corr. to 2 sig. fig.)
	7 006 = 7 010	(corr. to 3 sig. fig.)
	7 436 = 7 000	(corr. to 1 sig. fig.)
	7 436 = 7 400	(corr. to 2 sig. fig.)
	7 436 = 7 440	(corr. to 3 sig. fig.)
	7 436 = 7 436	(corr. to 4 sig. fig.)
	0.006 09 = 0.006	(corr. to 1 sig. fig.)
	0.006 09 = 0.006 1	(corr. to 2 sig. fig.)
	6.009 = 6.01	(corr. to 3 sig. fig.)

Decimal places

Include one extra figure for consideration. Simply drop the extra figure if it is less than 5. If it is 5 or more, add 1 to the previous figure before dropping the extra figure.

Examples:	0.737 4 = 0.74	(corr. to 2 dec. places)
	5.030 2 = 5.030	(corr. to 3 dec. places)

Standard form

Very large or small numbers are usually written in standard form $A \times 10^n$ where $1 \leq A < 10$ and n is an integer.

Examples:	$1\ 350\ 000 = 1.35 \times 10^6$	$7 = 7 \times 10^0$
	$0.000\ 875\ 6 = 8.756 \times 10^{-4}$	$10 = 1 \times 10^1$