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ANDERSON SECONDARY SCHOOL
Weighted Assessment 1 2024
Secondary Three Express

CANDIDATE NAME:

CLASS: /

INDEX NUMBER:

ADDITIONAL MATHEMATICS 4049

29 February 2024

45 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 30.

Setter: Mr Aaron Wong

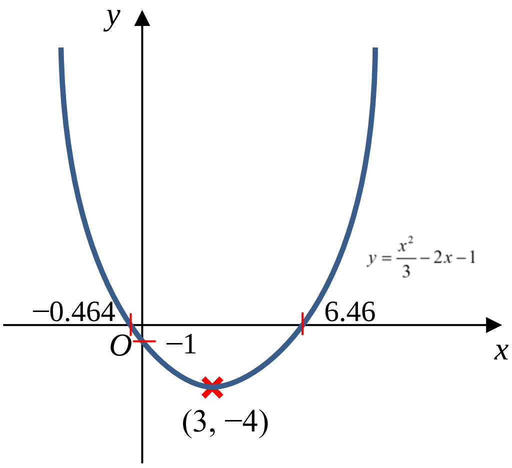
1 **(i)** Express $\frac{x^2}{3}-2x-1$ in the form $a(x-h)^2+k$, where a, h and k are constants. [3]

1(i)	$\frac{x^2}{3}-2x-1=\frac{1}{3}(x^2-6x)-1$ $=\frac{1}{3}\left(x^2-6x+3^2-3^2\right)-1$ $=\frac{1}{3}\left[(x-3)^2-9\right]-1$ $=\frac{1}{3}(x-3)^2-3-1$ $=\frac{1}{3}(x-3)^2-4$	M1 for factorizing $\frac{1}{3}$ M1 A1

(ii) Hence find the minimum value of $\frac{x^2}{3}-2x-1$ and the value of x at which the minimum occurs. [2]

1(ii)	Minimum value = -4 Value of x which minimum occurs = 3	B1 B1

(iii) Sketch the curve $y=\frac{x^2}{3}-2x-1$. [2]

1(iii)		B1 – Correct shape B1 – Turning point and y-intercept stated

2 Find the values of k for which the equation $(k+1)x^2 - 2kx = 2 - k$ has real roots. [4]

2	$(k+1)x^2 - 2kx = 2 - k$ $(k+1)x^2 - 2kx + k - 2 = 0$ Since the equation has real roots, discriminant ≥ 0. $(-2k)^2 - 4(k+1)(k-2) \geq 0$ $4k^2 - 4(k^2 - k - 2) \geq 0$ $4k^2 - 4k^2 + 4k + 8 \geq 0$ $4k + 8 \geq 0$ $4k \geq -8$ $k \geq -2$	M1 – Simplify quadratic equation M1 – correct discriminant ≥ 0 M1 – finding linear inequality A1 – range of k

3 The equation of a curve is $y = 2x^2 + (4-k)x - 2k$, where k is a constant.

(a) Show that the line $y = 7x - 18$ is a tangent to the curve when $k = 5$ and find the coordinates of the point of intersection. [3]

3(a)	When $k = 5$, $y = 2x^2 - x - 10$ ______(1) $y = 7x - 18$ ______(2) Substitute (1) into (2): $2x^2 - x - 10 = 7x - 18$ $2x^2 - 8x + 8 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $x = 2$ Since there is only one solution of x, $y = 7x - 18$ is tangent to the curve. OR Discriminant = $(-4)^2 - 4(1)(4)$ $= 0$ Since discriminant = 0, the line is a tangent to the curve. Substitute $x = 2$ into (2): $y = -4$ $\therefore$ Coordinates of the point of intersection is $(2, -4)$	M1 – correct substitution M1 – proof of tangency A1

(b) Explain why there is only one value of *k* for which *y* cannot be negative and state this value. [4]

3(b)	discriminant $= (4-k)^2 - 4(2)(-2k)$ $= 16 - 8k + k^2 + 16k$ $= 16 + 8k + k^2$ $= (k+4)^2$ Since $(k+4)^2 \geq 0$, the curve has real roots. Hence, when $k = -4$, the *x*-axis is a tangent to the curve and *y* cannot be negative. **OR** Since the discriminant is always positive, when $k = -4$, the curve intersects the *x*-axis at only 1 point, for which *y* cannot be negative.	M1 – correct discriminant M1 – factorized M1 – stating the nature of roots. A1 – proper reasoning/explanation

4 Find the set of values of the constant *a*, for which $(a-3)x^2 + 5x - 3 = 0$ is always negative for all real values of *x*. [3]

4	Since $(a-3)x^2 + 5x - 3 = 0$ is always negative, discriminant < 0 and $a - 3 < 0$ $5^2 - 4(a-3)(-3) < 0$ and $a < 3$ $25 + 12a - 36 < 0$ $12a - 11 < 0$ $12a < 11$ $a < \frac{11}{12}$ and $a < 3$ Hence $a < \frac{11}{12}$.	 M1 for $a - 3 < 0$ M1 for correct discriminant < 0 A1

5 Show that the equation $y = \frac{m}{4}x^2 + 4x + 6 - m$ has real and distinct roots for all values of m.

[3]

5	discriminant $= 4^2 - 4(\frac{m}{4})(6-m)$ $= 16 - m(6-m)$ $= 16 - 6m + m^2$ $= m^2 - 6m + 16$ $= (m-3)^2 - 3^2 + 16$ $= (m-3)^2 + 7$ Since $(m-3)^2 \geq 0$ for all real values of m, then $(m-3)^2 + 7 \geq 7 > 0$. Hence the curve has 2 real and distinct roots.	M1 – correct discriminant M1 – completed square form A1

6 The height, y metres, of a ball x seconds after it has been thrown from a cliff can be modelled by the equation $y = \frac{3}{2}(4 - x)(2x + 3)$.

(a) Find the height of the ball just before it was thrown. [2]

6(a)	When $x = 0$, $y = \frac{3}{2}(4-0)(0+3)$ $= 18$ The height of the ball was **<u>18m</u>**.	M1 A1

(b) Determine the maximum height reached by the ball. [3]

6(b)	**<u>Method 1: Completing the square</u>** $y = \frac{3}{2}(4-x)(2x+3)$ $= -3x^2 + \frac{15}{2}x + 18$ $= -3(x^2 - \frac{5}{2}x) + 18$ $= -3\left[x^2 - \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 18$ $= -3\left(x - \frac{5}{4}\right)^2 + 3\left(\frac{5}{4}\right)^2 + 18$ $= -3\left(x - \frac{5}{4}\right)^2 + \frac{363}{16}$ Maximum height is 22.6875 m	 M1 – complete the square M1 A1

	<u>Method 2: using x-intercepts.</u> At $y = 0$, $\frac{3}{2}(4-x)(2x+3)=0$ $(4-x)(2x+3)=0$ $\therefore x = 4 \quad \text{or} \quad x = -\frac{3}{2}$ $x - \text{coordinate of turning poin t} = \frac{4+\left(-\frac{3}{2}\right)}{2}$ $= \frac{5}{4}$ $\text{maximum height} = \frac{3}{2}(4-\left(\frac{5}{4}\right))(2\left(\frac{5}{4}\right)+3)$ $= 22.6875\text{m}$	M1 – finding x-intercepts M1 – x-coordinate of turning pt. A1

(c) Explain the significance of the x-intercept of the curve. [1]

(c)	When the curve intersects the x-axis, it is the time taken for the ball to reach the ground.	B1

End of Paper