Name : ________________________________

Class : _________

TEMASEK JUNIOR COLLEGE

Unit 3: Motion & Forces

Learning Outcomes

Students should be able to:

(a) show an understanding of and use the terms position, distance, displacement, speed, velocity and acceleration.

(b) use graphical methods to represent distance, displacement, speed, velocity and acceleration.

(c) identify and use the physical quantities from the gradients of position-time or displacement-time graphs, and areas under and gradients of velocity-time graphs, including cases of non-uniform acceleration.

(d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.

(e) solve problems using equations which represent uniformly accelerated motion in a straight line, e.g. for bodies falling vertically without air resistance in a uniform gravitational field.

(f) show an understanding that mass is the property of a body which resists change in motion (inertia).

(g) define and use linear momentum as the product of mass and velocity.

(h) state and apply each of Newton's laws of motion:

1st law: a body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force;

2nd law: the rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force; and

3rd law: the force exerted by one body on a second body is equal in magnitude and opposite in direction to the force simultaneously exerted by the second body on the first body.

(i) recall the relationship that resultant force $F = ma$ for a body of constant mass, and use this to solve problems.

1 Introduction

Kinematics is the study of objects in motion. Motion occurs all around us. We see it in the everyday activity of people, of cars on the expressway, of javelins thrown by sportsmen, and even in the motion of comets. In Kinematics, we describe motion in terms of space and time while ignoring the agents that caused that motion. In the later part of this unit, we study the effects of forces on the motion of bodies.

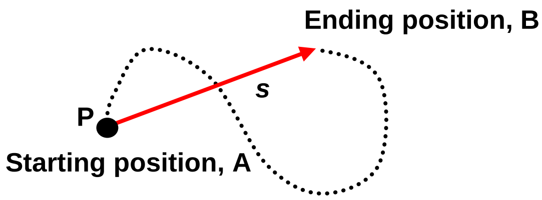
2 Definition of quantities *LO(a)*

2.1 Position, Distance and Displacement

Position is defined as the location of the object at any given time.

For one-dimensional motion, we often choose the *x* axis as the line along which the motion takes place. Then the position of the object at any moment is given by its x-coordinate. If the motion is vertical, as for a dropped object, we usually use the *y* axis.

Consider a particle, P, moving from position A to position B.



- Dotted lines represent the path taken by the particle P.
- The **distance** travelled by P between points A and B is the length of the dotted path.
- The **displacement** from point A is represented by the vector, ***s.***
- The <u>magnitude</u> of the displacement of P is the <u>length</u> of the displacement vector.

	Distance, *x*	**Displacement, *s***
Definition	Distance is the total length of the path taken.	Displacement is defined as the change in position of the object. It is the distance travelled in a straight line in a specified direction from a given reference point.
Type of quantity	scalar	vector
SI unit	metre (m)	metre (m)

2.2 Speed and velocity (v)

	Speed *v*	**Velocity *v***
Definition	Speed is the rate of change of distance.	Velocity is the rate of change of displacement.
Type	scalar	vector
SI unit	m s^{-1}	m s^{-1}

Speed:

* Instantaneous speed v is the rate of change of distance at that instant. $v = \frac{ds}{dt}$

* Average speed $\langle v \rangle = \frac{\text{total distance}}{\text{total time}} = \frac{\Delta s}{\Delta t}$

* A body that travels equal distances in equal time intervals is said to be moving with constant or uniform speed.

Velocity:

* Instantaneous velocity is the rate of change of displacement at that instant. $v = \frac{ds}{dt}$

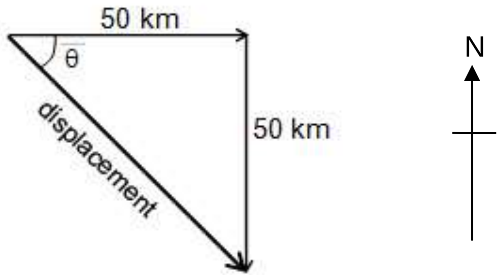
* Average velocity $\langle v \rangle = \frac{\text{total displacement}}{\text{total time}} = \frac{\Delta s}{\Delta t}$

* A body that travels equal displacement in equal time intervals is said to be moving with constant or uniform velocity.

Example 1

A car travels 50 km due East and a further 50 km due South in a total time of 1.0 h.
Determine

(a) the total distance travelled,
(b) the displacement,
(c) the average speed, and
(d) the average velocity, of the car.



Solution:

(a) total distance = 50 + 50 = 100 km

(b) displacement = $\sqrt{50^2 + 50^2}$ = 71 km

$\tan\theta = \frac{50}{50} = 1.0 \Rightarrow \theta = 45^o$

$\therefore$ displacement is 71 km at a bearing of 135^o or S45^{o}E

(c) $\langle speed \rangle = \frac{\text{total distance}}{\text{total time}} = \frac{100}{1.0} = 100 \text{ km h}^{-1} = 27.8 \text{ m s}^{-1}$

(d) $\langle velocity \rangle = \frac{\text{total displacement}}{\text{total time}} = \frac{71}{1.0} = 71 \text{ km h}^{-1} = 19.7 \text{ m s}^{-1}$

at a bearing of 135^o or S45^{o}E

2.3 Acceleration (*a*)

Acceleration is the rate of change of velocity.
SI unit : m s^{-2}. It is a vector quantity.

* A change in velocity can be due to:

1. a change in magnitude – for example: the speed changes while the direction remains the same.

2. a change in direction – for example: an object moving with constant speed in a circle.

3. a change in both magnitude and direction simultaneously.

* Instantaneous acceleration is the rate of change of velocity at that instant of time. $a = \frac{dv}{dt}$

* Average acceleration = $\frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta v}{\Delta t} = \frac{v-u}{t}$

2.3.1 Sign of Acceleration

In describing **acceleration**, it is necessary to consider the object's **direction of motion** and whether it is **speeding up** or **slowing down.**

* If an object is *speeding*, the **magnitude of velocity increases** with time. **Acceleration and velocity vectors must act in the same direction.**

* If an object is *slowing down* or decelerating**,** the **magnitude of velocity decreases** with time. **Acceleration and velocity vectors must act in opposite directions**.

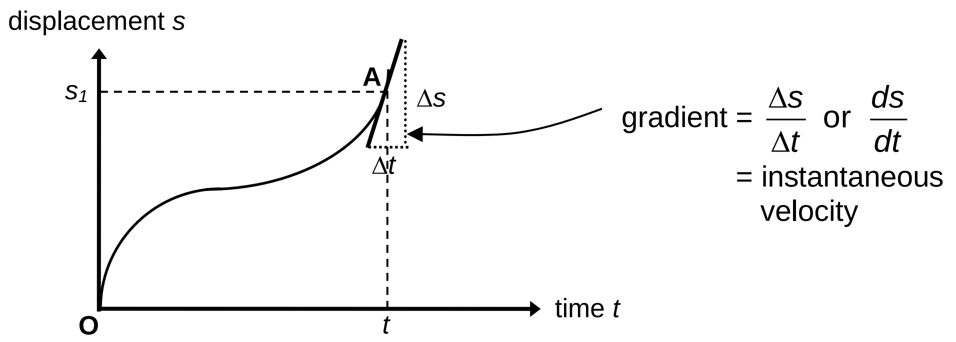
* Negative acceleration does not necessarily mean an object is slowing down. If both acceleration and velocity are negative, the object is speeding up.

3 Graphical Representation of Motion *LO(b), (c)*

The relationships between the three graphs can be summarised

Graph	**Gradient at a point**	**Area under the graph**
displacement-time, s-t	$\frac{ds}{dt}$ = instantaneous velocity	-
velocity-time, v-t	$\frac{dv}{dt}$ = instantaneous acceleration	change in displacement
acceleration-time, a-t	-	change in velocity

3.1 Displacement-time graph



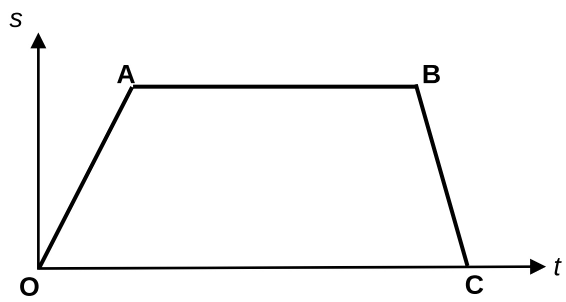
The displacement-time graph gives the following information:

1. Slope or gradient at **A** = $\frac{\Delta s}{\Delta t}$ = v, the instantaneous velocity at **A**

2. Average velocity between **O** and **A,** $\langle v \rangle = \frac{s_1}{t}$

Example 2

Consider a body moving along a straight line. Its displacement-time (*s*-*t*) graph is as shown. Describe the motion of the body from O to C.



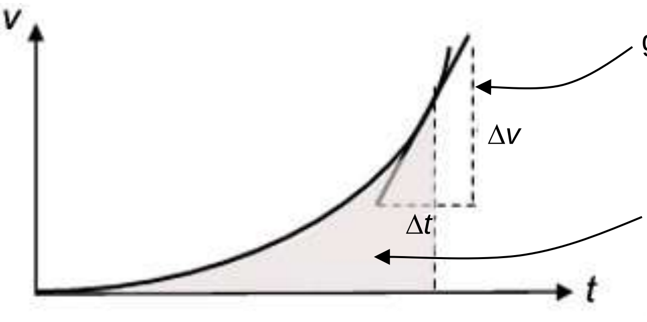
Solution:

O to **A** — *s* increases at a constant rate (constant gradient)
$\Rightarrow$ body is moving with constant velocity.

A to **B** — *s* is constant (zero gradient)
$\Rightarrow$ body is stationary or velocity is zero.

B to **C** — *s* decreases at a constant rate (constant negative gradient)
$\Rightarrow$ velocity is negative, meaning body is reversing/moving back towards **O**.
Velocity is constant but greater in magnitude than that of **OA**.

3.2 Velocity-time graph



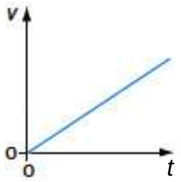
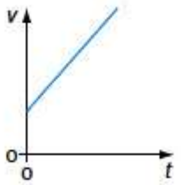
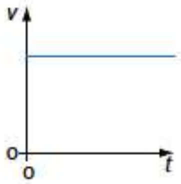
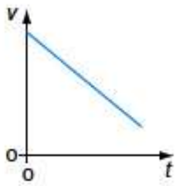
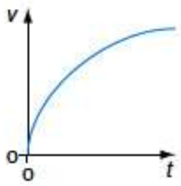
The velocity-time graph gives the following information:

1. Slope or gradient at any time = $\dfrac{\Delta v}{\Delta t}$ = *a*, the instantaneous acceleration

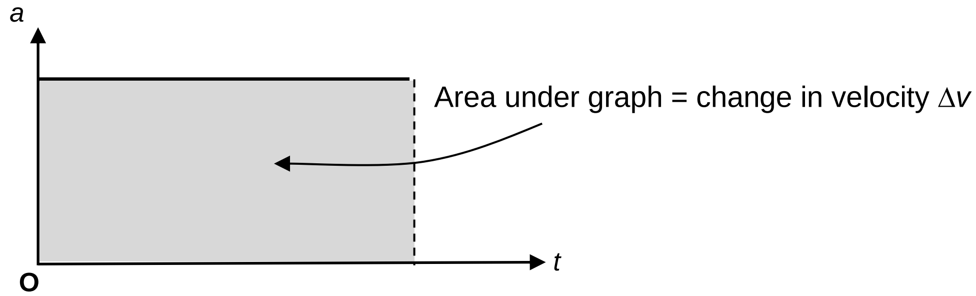
2. Area under the graph = $\int v\, dt$ = displacement of the body at *t*

Example 3

Describe the motion an object moving along a straight line using the following v-t graphs.

	Graph	Description of motion
(a)		Object starts from rest and moves with constant acceleration. Speed increases from zero uniformly.
(b)		Object moves with an initial speed and has constant acceleration. Speed increases uniformly.
(c)		Object moves with constant velocity. Acceleration is zero.
(d)		Object slows down with constant deceleration. (acceleration is negative while velocity is positive)
(e)		The body starts from rest and speeds up with decreasing acceleration. Eventually it moves at maximum constant velocity (terminal velocity).

3.3 <u>Acceleration-time graph</u>



Since $a = \dfrac{dv}{dt}$ $\Rightarrow$ change in velocity $\Delta v = \int a\, dt$ = area under *a-t* graph.

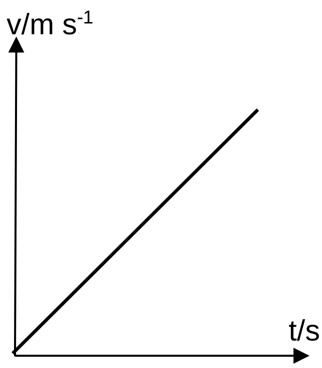
3.4 Free-fall

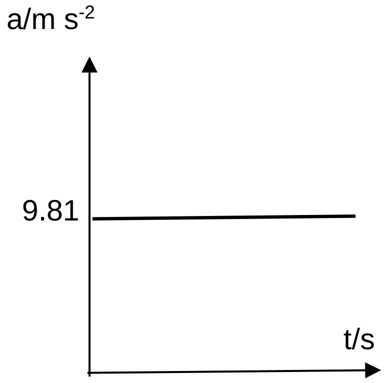
An important example of uniformly accelerated motion is the vertical motion of an object under the effects of gravity.

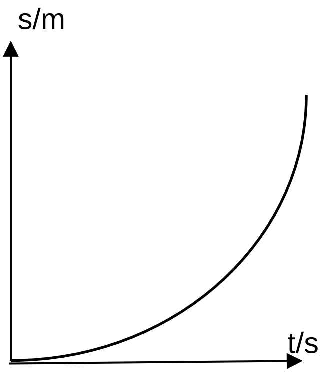
* In the **absence of air resistance**, an object is said to be in ***free fall*** if the only force acting on it is the **weight (gravitational force)** due to the Earth.

* The downward acceleration is known as the ***acceleration due to gravity*** (***g***) and is assumed to be a constant, equal to **9.81 m s^{-2}**, near the Earth's *surface.*

Taking ***downwards direction as positive***, we obtain the following graphs for an object in free fall:

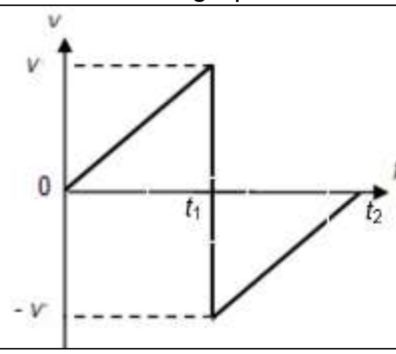
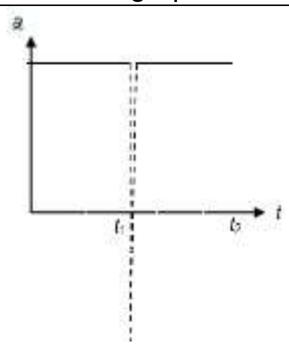






Example 4

An object is released from rest above ground, falls vertically and hits the ground with speed *v*. It bounces off the ground at the same speed *v*. Its velocity-time graph is shown in the table below. Complete the corresponding displacement-time and acceleration-time graphs.

s-t graph	*v-t* graph	*a-t* graph
		

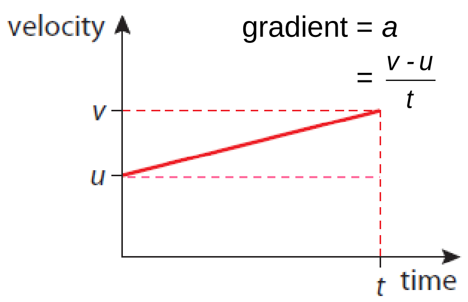
4 Equations of Motion *LO(d),(e)*

Suppose that a body is moving with **constant acceleration** *a* and at a time interval *t*, its velocity increases from *u* to *v* and its displacement increases from 0 to *s*.

Since $a = \dfrac{dv}{dt}$

$= \dfrac{v-u}{t}$

Thus $\boxed{v = u + at}$ ------------- [1]



As the velocity is increasing at a constant rate, the average velocity is

$$\langle v \rangle = \frac{u+v}{2}$$

In addition, displacement = average velocity × time

Thus $\boxed{s = \frac{1}{2}(u+v)\,t}$ ------------ [2]

Substituting [1] into [2], gives s = ½ (*u* + *u* + *at*)*t*

Thus $\boxed{s = ut + \frac{1}{2}at^2}$ ------------- [3]

From [1], $t = \dfrac{v-u}{a}$ and substituting into [2],

$$s = \frac{1}{2}(v+u) \times \left(\frac{v-u}{a}\right) = \frac{1}{2}\left(\frac{v^2-u^2}{a}\right)$$

Therefore $\boxed{v^2 = u^2 + 2as}$ ------------ [4]

Equations 1, 2, 3 and 4 are called *kinematics equations* and are applicable for **uniformly accelerated motion in a straight line**.

<u>Problem Solving Strategy</u>

1. If diagram of path is not given, sketch one, identifying start and end point.
2. Identify known quantities given and unknown quantities to be determined.
3. Identify the equation that will be used to determine unknown quantity from known information.
4. Assign a positive direction. Substitute known values with **correct sign** into the equation to solve for the unknown quantity.

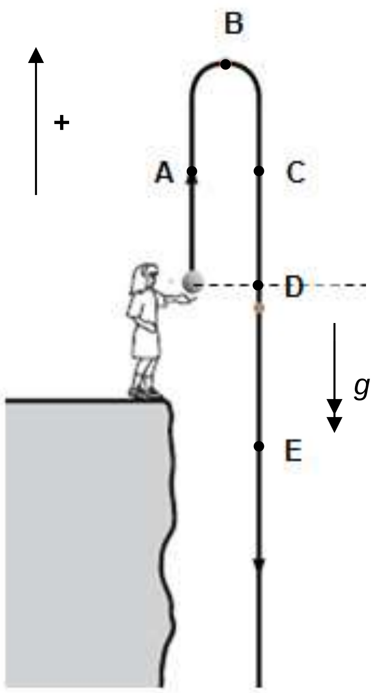
The sign of displacement, velocity and acceleration

Since *u, v, a* and *s* are vector quantities, **their directions must be taken into account** when solving problems. Appropriate signs must be assigned to their values when using these equations.

If the sign convention is such that upward direction is taken to be positive, then
- displacement is negative for points *below* the starting position,
- velocity of body moving *down* is negative, and
- *downward* directed acceleration is negative.

Example 5

A ball is projected vertically upwards with an initial speed from a girl's hand. Identify the signs of the displacement, velocity and acceleration during its flight, taking the ***upward direction as positive***. Give a brief description of its motion.

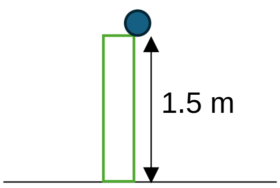


Path	*s*	*v*	*a*	Description of motion
A	+	+	–	The acceleration due to gravity is acting downwards. Since *v* and *a* are in opposite directions, the ball decelerates and slows down as it goes up.
B				At the highest point or maximum height, the ball is instantaneously at rest, v = 0. Note that gravity still acts.
C				Since both *v* and *a* are in the same direction, the ball accelerates downwards and its speed increases.
D				Here the ball returns to its initial position, so net displacement is zero.
E				The ball continues to accelerate on its downward journey. The displacement is negative since it is below its initial position.

Example 6

[2023 H2 P3 Q2 (part)]
A student throws a ball vertically upwards with a speed of 5.0 m s^{-1}. The ball is released at this speed at a height of 1.5 m above the ground.

Air resistance is negligible.



(a) Calculate:
  (i) the speed of the ball as it hits the ground [2m]

Taking upwards as positive direction,
v =?, u = 5.0 m s^{-1}, s = -1.5 m, a = - 9.81 m s^{-2}

$$v^2 = u^2 + 2as$$
$$v^2 = (5.0)^2 - 2(9.81)(-1.5)$$
$$v = 7.38 = 7.4 \text{ m s}^{-1}$$

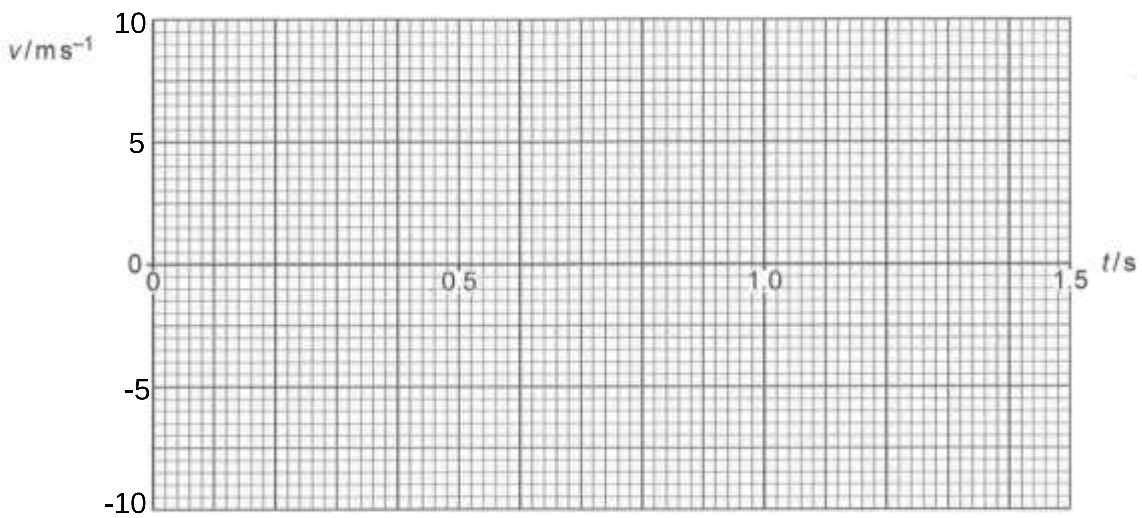
(ii) the time between the ball being released and the ball hitting the ground. [2m]

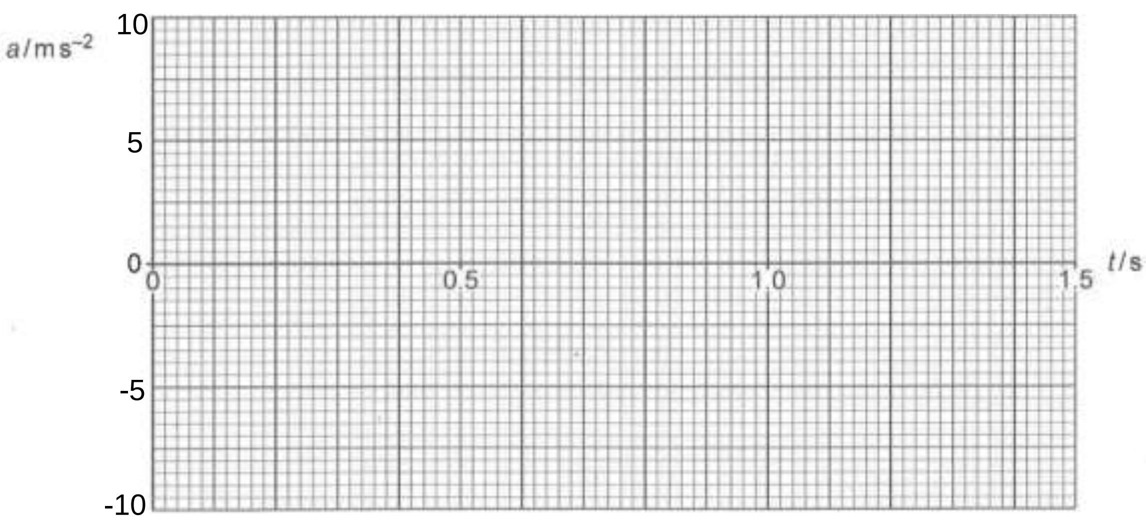
$$v = u + at$$
$$-7.38 = 5.0 + (-9.81)t$$
$$t = 1.26 = 1.3 \text{ s}$$

(b) Using your answers to (a)(i) and (a)(ii), draw the variation with time t of the velocity v and the acceleration a of the ball.

Add a suitable scale to each vertical axis.

Take upwards direction as positive. [3m]





5 Linear Momentum *LO(g)*

A body in straight line motion is said to have a linear momentum.
Momentum p is defined as the product of mass and velocity.

$$\boldsymbol{p} = m\boldsymbol{v}$$

SI unit of momentum : kg m s^{-1} or N s.

Momentum is a <u>vector</u> quantity. Its direction is the <u>same</u> as the direction of the velocity.

Example 7

A ball of mass 80 g collides with a vertical wall. The ball has a velocity of 23 m s^{-1} in a horizontal direction. After hitting the wall, the ball moves with a velocity of 23 m s^{-1} in the opposite direction.

(a) Calculate the momentum of the ball before colliding with the wall.
$p = mv = 80 \times 10^{-3}(23) = 1.8$ kg ms^{-1}

(b) Determine the change in momentum of the ball after the collision.

Momentum of the ball after collision is -1.8 kg ms^{-1}.
Change in momentum of the ball = $p_f - p_i$ = -1.8 -1.8 = -3.6 kg ms^{-1}.

6 Newton's Laws of Motion *LO(h)*

6.1 Newton's First Law of Motion *LO(f)*

Newton's first law of motion describes the state of motion of an object when there is no resultant force acting on the object:

A body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force.

- The first law implies that all matter has **inertia. Inertia is the property of a body to stay in a state of rest or uniform velocity and the tendency to resist changes in its state of motion.**

- **Mass is a measure of the inertia of the body**. The more mass a body has, the greater is its inertia and the harder it is to change its state of motion.

Example 8

Discuss how the property of inertia is demonstrated in the case of a box placed in a bus and the bus (a) starts to move, (b) stops suddenly, and (c) goes round a curve.

The box has mass and hence inertia.

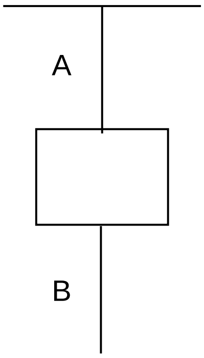
(a) When the bus starts to move, the box tends to <u>stay at rest</u>. Thus it slides <u>backward relative to bus</u>.

(b) When the bus stops suddenly, the box tends to <u>continue to move at its original velocity in a straight line</u>. Thus it slides <u>forward relative to bus.</u>

(c) When the bus goes round a curve, the box tends to continue its motion in a straight line. Thus it slides <u>tangentially</u> to the curve.

Example 9

A big block of mass is suspended by a thread A from a rigid support and a similar thread B is attached to the bottom of the mass. Use the concept of inertia to predict and explain what will happen if you pull the lower thread B (a) sharply, and (b) steadily.



(a) *When pulled sharply, the big block tends to resist change in its state of rest. Tension in thread B is larger than that in thread A. So thread B breaks.*

(b) *When pulled steadily, the tension in thread B is gradually transmitted to the upper thread A. Tension in thread A is larger than that in thread B. So thread A breaks.*

6.2 Newton's Second Law of Motion *LO (i)*

Newton's second law describes the motion of an object when a resultant force is acting on it:

The rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force.

$$F = \frac{dp}{dt}$$

For a body of constant mass,

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma$$

Force is a vector quantity. The SI unit of force is the **newton, N.**

1 newton is defined as the force which, when acting on a 1 kilogram mass, produces an acceleration of 1 m s^{-2}.

Note:

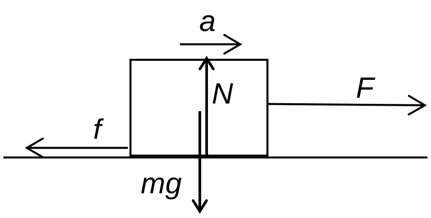
- Acceleration and the resultant force are always in the same direction.
- $F = \frac{dp}{dt}$ is a more general form of the second law equation. $F = ma$ is valid only for an accelerating body of constant mass m.
- If $F = 0$, then $a = 0$. This means that in the absence of any resultant force, a body is either at rest or travels at constant velocity.

6.3 Problem-solving using Free Body Diagram

1. Sketch a simple diagram of the objects in the system. Isolate the object of interest and identify all the forces acting on it. This is called a **free body diagram**.
2. Choose appropriate axis in the direction of the acceleration. **Resolve** all forces along this axis.
3. Apply $\boldsymbol{F_{net} = ma}$ along the chosen axis.

Example 10

(a) A block of mass 2.0 kg is being pulled on a horizontal bench by a force of 12 N. If the block accelerates at 5.0 m s^{-2}, what is (i) the frictional force on the block and (ii) the normal contact force between the block and the bench?



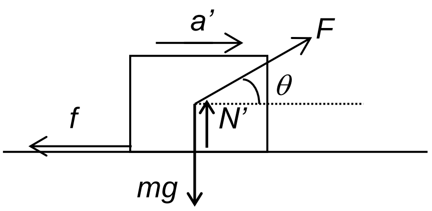
(i) Horizontally: $F - f = ma$

$12 - f = 2.0(5.0)$

=> $f = 2.0$ N

(ii) Vertically: $N = mg = 2.0(9.81) = 19.6$ N

(b) A 12 N force acting at 30° on a different block of mass 2.0 kg. It moves horizontally on the bench and the friction is 2 N, what is (i) the acceleration of the block and (ii) the normal contact force on the block?



Horizontally: $F\cos\theta - f = ma'$

$12\cos 30^{o} - 2.0 = 2.0\,a'$

=> $a' = 4.2$ m s^{-2}

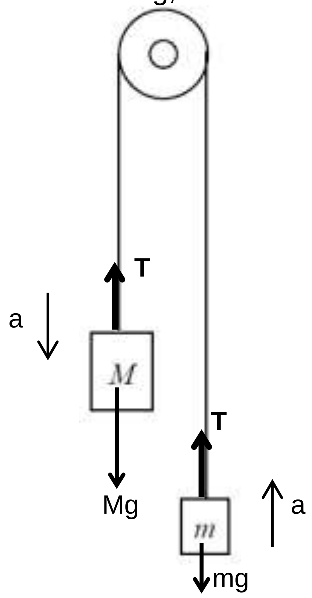
Vertically: $N' + F\sin\theta = mg$

$N' + 12\sin 30^{o} = 2.0(9.81)$

$N' = 13.6$ N

Example 11

Two masses are connected by a string that passes over a pulley, as shown below. If m = 3.1 kg and M = 4.4 kg, find the acceleration of the masses.



Applying Newton's second law to mass m alone,

$$T - mg = ma \quad \text{----(1)}$$

Applying Newton's second law to mass M alone,

$$Mg - T = Ma \quad \text{----(2)}$$

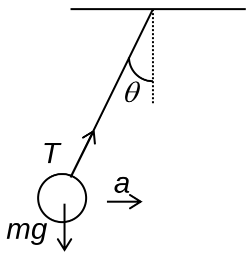
(1) + (2):

$$Mg - mg = Ma + ma$$

$$a = \left(\frac{M-m}{m+M}\right)g = \left(\frac{4.4-3.1}{3.1+4.4}\right)(9.81) = 1.7 \text{ m s}^{-2}$$

Example 12

An object of mass *m* is suspended in a bus which is accelerating at 2.0 m s^{-2}. Find the angle of inclination to the vertical θ.



$T \cos \theta = mg$ ---(1)

$T \sin \theta = ma$ ---(2)

$\frac{(2)}{(1)}$: $\tan\theta = \frac{a}{g} = \frac{2.0}{9.81}$

$\theta = 12^o$

6.4 Newton's Third Law of Motion

Newton's third Law describes how forces always occur in nature:

The force exerted by one body on a second body is equal in magnitude and opposite in direction to the force simultaneously exerted by the second body on the first body.

Examples of action-reaction pairs:

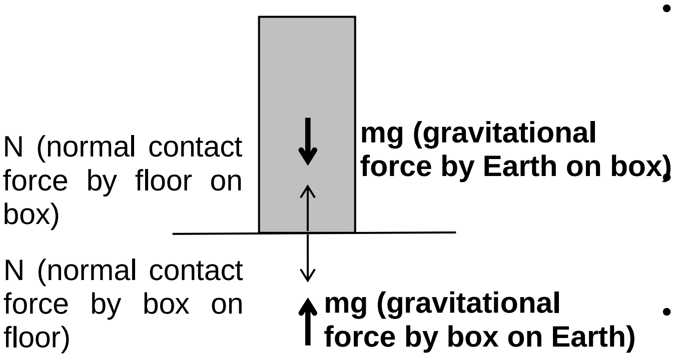
1. The gravitational force of attraction that the Earth exerts on the Moon and the force that the Moon exerts on the Earth.
2. The gravitational force of attraction that the Earth exerts on a book (i.e. weight of the book) and the force that the book exerts on the Earth.
3. Electric force of attraction between an electron and a proton.
4. Magnetic forces of repulsion between like poles of a magnet.
5. The backward force exerted by a person on the ground as he walks and the forward force exerted by the ground on the person.
6. The force exerted by a table on a book resting on it and the force exerted by the book on the table.
7. Force exerted by rocket on ejected gas and the force that the ejected gas exerts on the rocket (i.e. thrust).

Notes:

- The two forces must be of the same type, i.e. if one is an electrical force, then the other must be electrical too.
- Since action and reaction forces do not act on the same body, <u>an action-reaction pair does not balance to produce zero resultant force on any one object</u>.
- Examples 1 to 4 (gravitational and electromagnetic forces) are <u>action-at-a-distance</u> forces as they did not involve physical contact between objects but act from a distance even without physical contact. Examples 5 to 7 are contact forces as they arise because of physical contact between two objects.

Example 13

Consider a box resting on the floor. It experiences two forces: Its weight *mg* downward and the normal reaction force *N* acting on it upwards. Are these two forces an action-reaction pair according to the Newton's third law of motion?



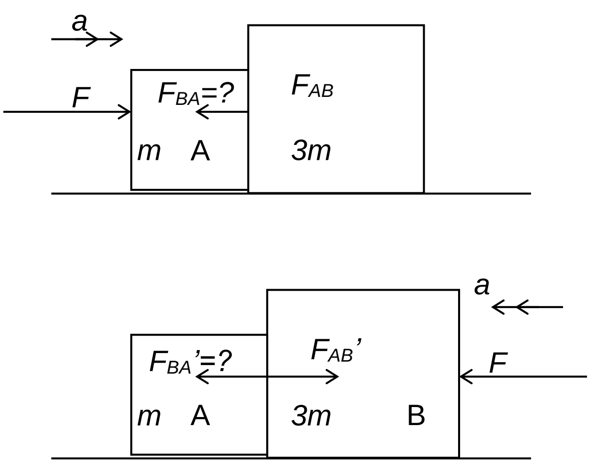
- The weight *mg* is the gravitational force by the Earth on the box. The reaction force is the gravitational force by the box on the Earth, acting on the centre of the Earth.

  N is the normal contact force that the floor acts upwards on the box. The reaction force is the normal contact force that the box acts on the floor.

- Hence weight mg and normal contact force *N* are not action-reaction pair because they both act on the same mass, and they are not the same type of force.

Example 14

Two bodies A and B are in contact and placed on a frictionless surface. A horizontal force, *F*, of 12 N is applied to body A which in turn pushes body B. If the mass of B is three times that of body A, what is the force that B acts on A? How would your answer differ if the force of 12 N is applied to body B instead?



A and B: $\rightarrow +$ $F = (m + 3m)a \Rightarrow a = \dfrac{F}{4m}$

B alone: $\rightarrow +$ $F_{AB} = 3ma = 3m\left(\dfrac{F}{4m}\right) = \dfrac{3}{4}F = 9N$

By Newton's third law, $F_{BA} = -F_{AB} = -9N = 9N$ *to the left*

If the force of 12 N is applied to body B instead, the acceleration a of the system is of the same magnitude but opposite in direction.

A alone: $+\leftarrow$ $F_{BA}' = ma = m\left(\dfrac{F}{4m}\right) = \dfrac{F}{4} = 3N$ to the left

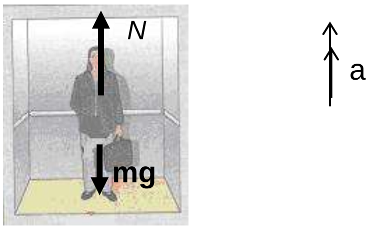
This force is smaller in magnitude, as a smaller force is needed to accelerate the smaller mass A.

Example 15

A man of mass *m* is in a lift. Find the force he exerts on the lift if

(a) it is accelerating upwards with acceleration *a*,
(b) it is accelerating downwards with acceleration *a*.
(c) it is moving with constant velocity and
(d) the cable breaks and the elevator "free falls".

(a) Lift is accelerating upwards



Consider forces acting on the Man :

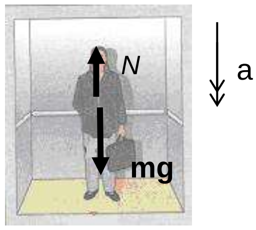
$N - mg = ma$ --(1)

$\Rightarrow$ $N = m(g + a)$ upwards

By Newton's third law,

he exerts an equal and opposite contact force $N = m(g + a)$ downwards on the floor of the lift. This force N is also called the **apparent weight** of the man. Note that the man feels heavier when lift is accelerating upwards.

(b) Lift is accelerating downwards



Consider forces acting on the Man :

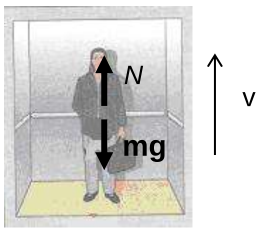
$mg - N = ma$ --(2)

$\Rightarrow$ $N = m(g - a)$ upwards

By Newton's third law,

he exerts an equal and opposite force $N = m(g - a)$ downwards on the floor of the lift. This force is also the apparent weight of the man. The man feels lighter when lift is accelerating downwards.

(c) Lift is at constant velocity, $a = 0$



Consider forces acting on the Man,

$N - mg = 0$ (constant v, zero a)

=> $N = mg$

By Newton's third law,

he exerts an equal and opposite contact force

$N = mg$ downwards on the floor of the lift.

The man weighs normally when the lift is moving at constant velocity.

(d) Cable breaks, $a = g$ downwards

From (b) $mg - N = mg$

=> $N = 0$

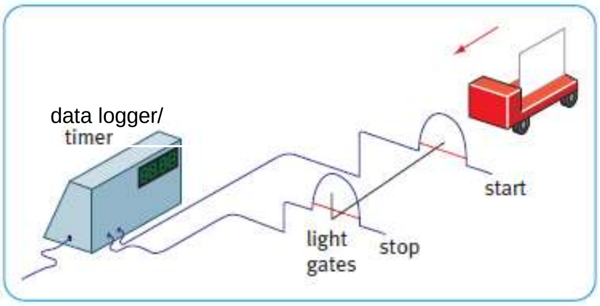
The man feels weightless!

Supplementary Notes:

Laboratory methods to determine motion of a body

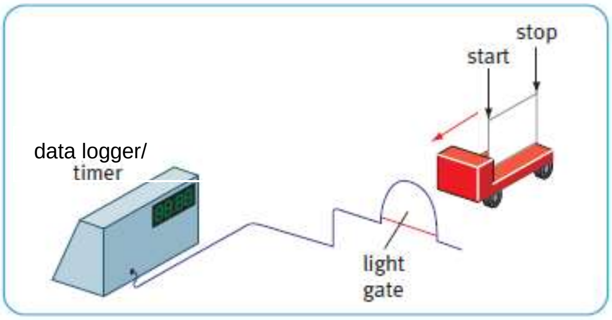
1. Using two light gates

The object(trolley) breaks the light beam as it passes the first light gate. This starts the timer. The timer stops when the object breaks the second beam. The trolley's speed is calculated from the distance between the light gates divided by the time interval.



2. Using one light gate

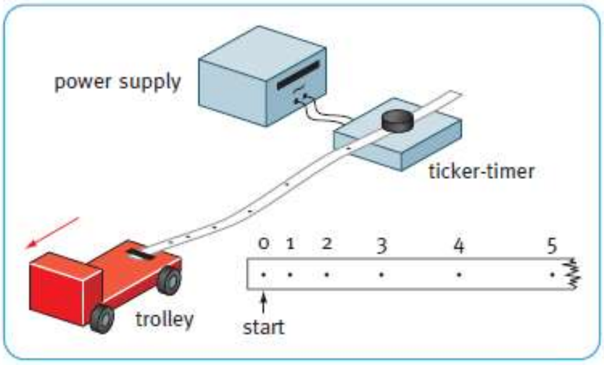
The timer starts when the leading edge of the card breaks the light beam. It stops when the trailing edge passes through. In this case, the time shown is the time taken for the trolley to travel a distance equal to the length of the card. The computer software can calculate the speed directly by dividing the distance by the time taken.



3. Using a ticker timer

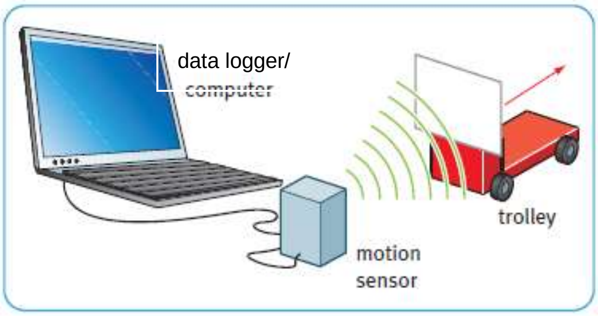
The ticker-timer marks dots on the tape at regular intervals, usually 1/50 s (i.e. 0.02 s). (in most countries the frequency of the a.c. mains is 50 Hz.) The pattern of dots acts as a record of the trolley's movement. Start by inspecting the tape. Identify the start of the tape. Then look at the spacing of the dots:

- even spacing – constant speed
- increasing spacing – increasing speed.



4. Using a motion sensor

The motion sensor transmits regular pulses of ultrasound at the trolley. It detects the reflected waves and determines the time they took for the trip to the trolley and back. From this, the computer or data logger can deduce the distance to the trolley from the motion sensor. It can generate a distance–time graph. The speed of the trolley can be determined from this graph.



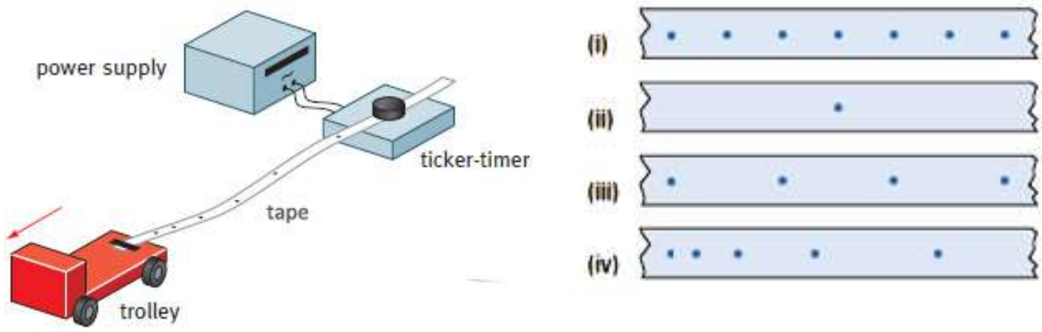
Unit 3: Motion & Forces Tutorial Questions

Solutions to Questions 8 and 18 provided.

Definitions of Displacement, Speed, Velocity, Acceleration

1 (a) Can a body have zero velocity and still be accelerating?
(b) Can a body move with a constant speed and still be accelerating?
(c) Can the direction of the velocity of a body change when its acceleration is constant?
(d) Can a body's speed be increasing as its acceleration decreases?

2 A trolley is attached to a ticker tape and a timer. As the trolley moves, the **timer** makes a series of dots on the **tape** at regular time intervals, usually 1/50 s (i.e. 0.02 s). Figs. (i) to (iv) shows four ticker-tapes.



Describe the motion of the trolley which produced them.

3 A stone is dropped off a cliff. When the stone has fallen 4.0 m, a second stone is dropped. Neglecting air resistance as the two stones continue to fall, which of the following statements is/are true?

A The velocity of the first stone increases faster than the velocity of the second.
B The velocities of both stones increases at the same rate.
C The separation of the two stones remains constant throughout the fall.
D The separation of the two stones increases as they continue to fall.

Graphical Representations of Motion

4 This question is about the motion of a Formula 1 racing car.

The graph below shows the velocity of a racing car in the first 4 seconds of a race as it accelerates and slows down to go around the first bend.

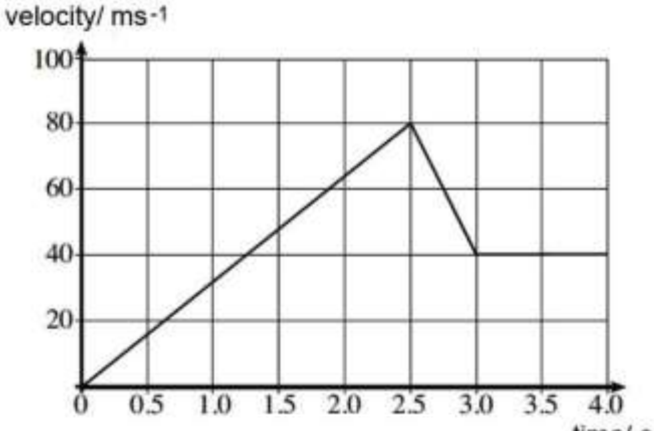
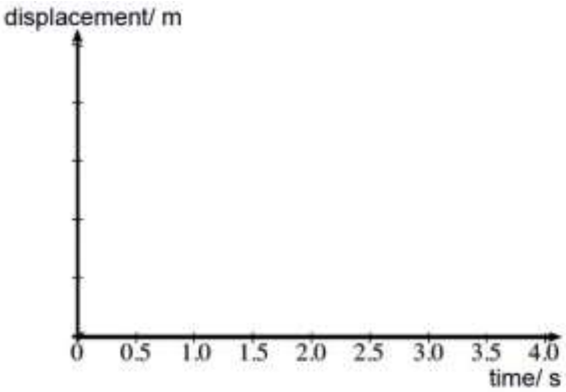


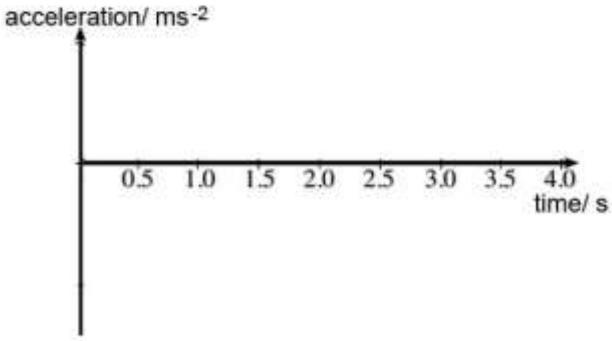
Fig. 1

(a) Using information from Fig. 1, determine
- [i] the acceleration from t = 0 to t = 2.5 s,
- [ii] the acceleration from t = 2.5 to t = 3.0 s,
- [iii] the displacement from t = 0 to t = 2.5 s,
- [iv] the displacement from t = 2.5 to t = 3.0 s,
- [v] the displacement from t = 3.0 to t = 4.0 s

(b) Calculate the average velocity of the car during the first 4 seconds.

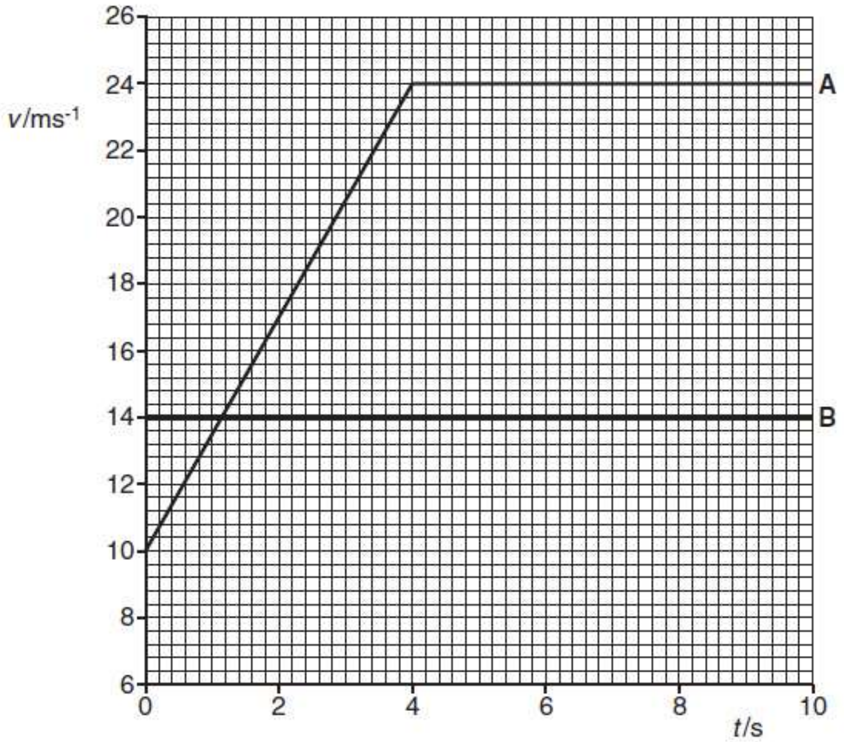
(c) Sketch the corresponding acceleration-time graph and displacement-time graph in the figures below.





[32 m s^{-2}, -80 m s^{-2}, 100 m, 30 m, 40 m, 42.5 ms^{-1}]

5 Figure shows graphs of velocity v against time t for two cars A and B travelling along a straight level road in the same direction.



At time t = 0, both cars are side-by-side.

(a) Describe the motion of car A from t = 0 to t =10 s.

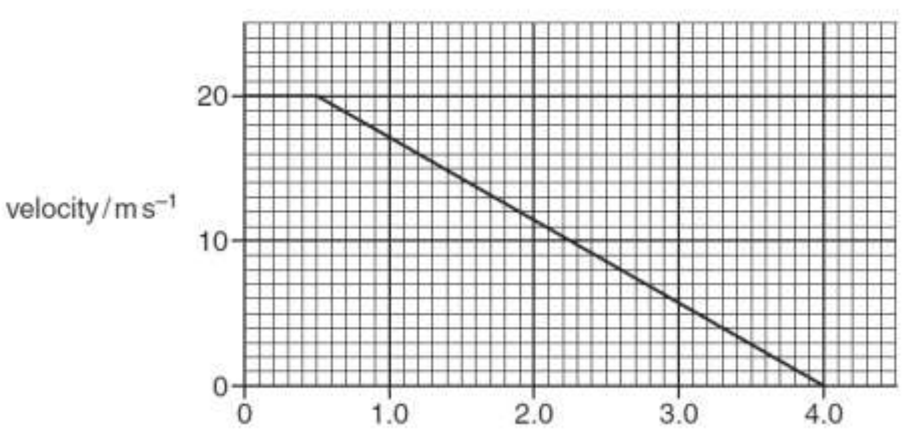
(b) State the time at which both cars have the same velocity.

(c) Determine the time at which car A overtakes car B. Explain your method.

[1.2 s, 2.4 s]

6 STOPPING DISTANCE PROBLEM

A driver travelling in a car on a straight level road sees an obstacle in the road ahead and immediately applies the brakes until the car stops. The initial speed of the car is 20 m s^{-1}. Figure shows the velocity - time graph of the car.



The thinking distance is defined as the distance travelled by the car during the driver's reaction time and the braking distance is the distance in which the car stops after the brakes have been applied.

(a) Use the graph to determine

(i) the reaction time of the driver,

(ii) the thinking distance,

(iii) the braking distance,

(iv) the magnitude of the deceleration of the car.

(b) For the same initial speed of the car, what would be the effects on the thinking distance and the braking distance if

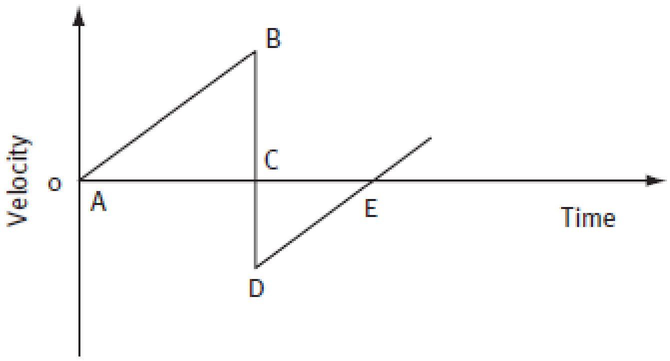
(i) the road is wet?

(ii) the driver is not fully alert?

[0.50 s, 10 m, 35 m, 5.7 m s^{-2}]

7 BOUNCING BALL PROBLEM

A ball is released from rest above a ground.
The ball is released at A and strikes the ground at B. The ball leaves the ground at D and reaches its maximum height at E. The effects of air resistance can be neglected.



(a) Explain

(i) why the velocity at D is negative,
(ii) why the gradient of the line AB is the same as the gradient of line DE,
(iii) why the speeds before and after impact with the ground are unequal,
(iv) why the area of triangle ABC is greater than the area of triangle CDE.

(b) Sketch the corresponding displacement-time and acceleration-time graphs.

Kinematics Equations

8 A metal ball is dropped from rest at a height above a bed of sand. It hits the sand bed one second later and makes an impression of maximum depth 8.0 mm in the sand.

(a) Neglecting air resistance, what is the speed of the ball just before hitting the ground?

(b) What is the average deceleration of the ball inside the sand?

[9.81 m s^{-1}, 6.0×10^3 m s^{-2}]

Ans: (a) $v = u + at$

$\downarrow^{+}\quad v = 0 + 9.81(1) = 9.81\, m\, s^{-1}$

(b) $v^2 = u^2 + 2as$

$\downarrow^{+}\quad 0 = 9.81^2 + 2a\left(8.0\times10^{-3}\right)$

$\Rightarrow a = -6.0\times10^{3}\, m\, s^{-2}$

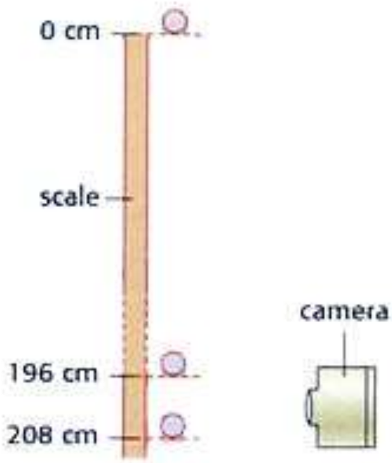
$Thus\ deceleration = 6.0\times10^{3}\, m\, s^{-2}$

9 (a) The distance *s* moved by an object in time *t* may be given by the expression

$$s = ½\, at^2$$

where *a* is the acceleration of the object.
State two conditions for this expression to apply to the motion of the object.

(a) In order to determine the shutter speed of a camera, a metal ball is held at rest at the zero mark of a vertical scale.
The ball is released. The shutter of a camera is opened as the ball falls



The photograph of the ball shows that the shutter opened as the ball reached the 196 cm mark and closed as it reached the 208 cm mark. Air resistance is negligible and the acceleration of free fall is 9.81 ms-^{2}.

(i) Calculate the time for the ball to fall from rest to the 196 cm mark.

(ii) Determine the time for which the shutter was open.

[0.20 s, 0.006 s, 20 ms^{-1}]

10 A balloonist drops a sandbag from a hot-air balloon that is rising at a constant velocity of 3.25 m s^{-1}.
It takes 8.75 s for the sandbag to reach the ground.

Determine

(a) the height of the balloon when the sandbag is dropped,

(b) the velocity with which the sandbag hits the ground.

(c) the path of the sandbag as seen by an observer on the ground.

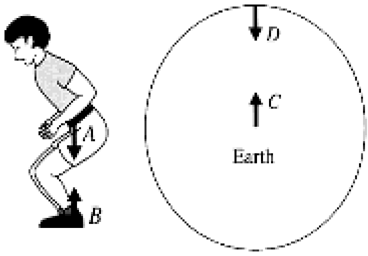
[347 m, 82.6 m s^{-1}]

Newton's Laws

11 Discuss whether the following situations are possible.

(a) An object with a net force is moving at constant speed.
(b) An object is at rest, therefore we conclude there is no force acting on it.
(c) An object is moving in one direction while the net force is in another direction.

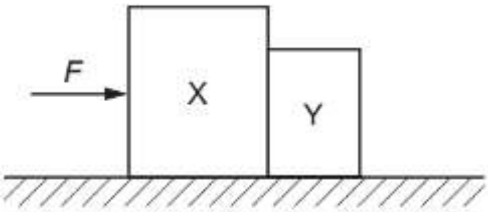
12 Check your understanding of Newton's Laws. A child is crouching at rest on the ground.
Opposite are the free-body force diagrams for the child and the Earth.
Complete the table describing forces A, B and C.



Force	Description of force	Body which exerts force	Body the force acts on
A	Gravitational	Earth	Child
B			
C			

All the forces A, B, C and D are of equal magnitude.
(a) Why are forces A and B equal in magnitude?
(b) Why must forces B and D be equal in magnitude?
(c) The child now jumps vertically upwards. With reference to the forces shown, explain what he must do to jump, and why he moves upwards.

13 A horizontal force *F* is applied to a block X which is in contact with a separate block Y as shown.



The blocks remain in contact as they accelerate along a horizontal frictionless surface. Air resistance is negligible. X has a greater mass than Y.

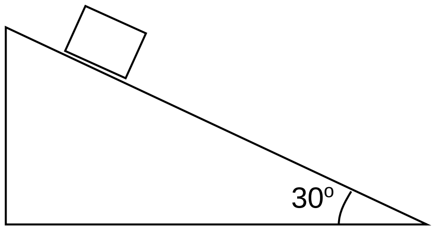
Which statement is correct?

A The acceleration of X is equal to force *F* divided by the mass of X.
B The force that X exerts on Y is equal to *F*.
C The force that X exerts on Y is less than *F*.
D The force that X exerts on Y is less than the force that Y exerts on X.

14 A spring balance carrying a mass of 20.0 kg in a lift registered 250 N. What was the acceleration of the lift? Calculate the balance readings during (a) free fall, (b) movement at constant velocity. [2.69 m s^{-2}, 0 N, 196 N]

15 A light spring has a mass of 0.20 kg suspended from its lower end. A second mass of 0.10 kg is suspended from the first by a thread. The arrangement is allowed to come into static equilibrium and then the thread is burned through. At this instant, what is the upward acceleration of the 0.20 kg mass? (Take g = 10 m s^{-2}.) [5.0 m s^{-2}]

16 A block of mass 2.0 kg moves down along a plane inclined at 30° to the horizontal. The frictional force between the block and the plane is 5.0 N.

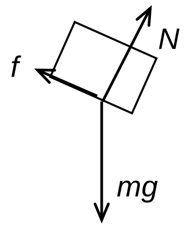


(a) Draw a free body diagram showing the forces acting on the block.

(b) Find the acceleration of the block along the inclined plane.

(c) If a force of 25 N pulls the block up the plane, what will be the acceleration of the block along the plane? [2.4 m s^{-2}, 5.1 m s^{-2}]

(a)



(b) ↘+ $mg\sin\theta - f = ma$

$2.0\,(9.81)\sin 30° - 5.0 = 2.0a$

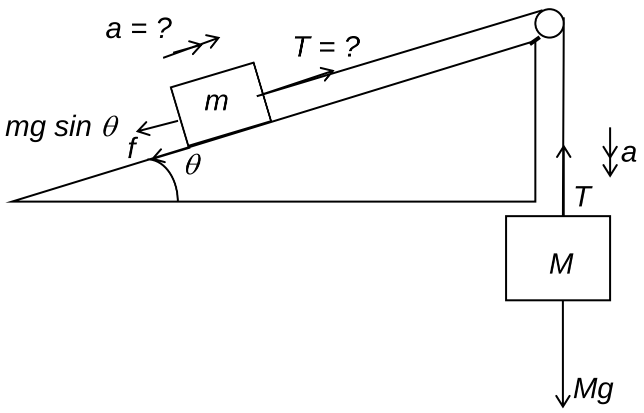
$a = 2.4\ ms^{-2}$

(c) ↖+ $F - f - mg\sin\theta = ma'$

$25 - 5.0 - 2.0\,(9.81)\sin 30° = 2.0a'$

$a' = 5.1\ ms^{-2}$

17 Two objects of masses 10 kg and 5 kg are connected by a light inextensible string that passes over a light frictionless pulley. The 5 kg object lies on a smooth incline of angle 30°. Find the acceleration of the two objects and the tension in the string.

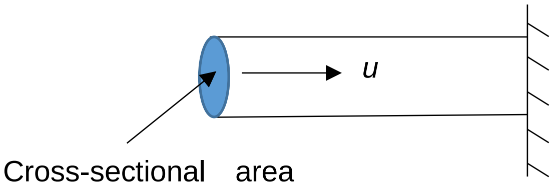


Force due to changing mass

18 A steady wind of 50 m s^{-1} blows against a rigid wall, which is in a plane perpendicular to the wind direction. Estimate the pressure exerted by the wind on the wall. Density of air is 1.25 kg m^{-3}. State any assumption you made in your calculations. [3.13 kN m^{-2}]

Assume no rebound, final velocity of wind = 0

Consider a cylindrical column of wind of cross-sectional area A and speed u hitting the wall, as shown :



In 1 s, the volume of air hitting the wall $V = A\,u$

and the mass of air hitting wall, $m = \rho A u$

By Newton's 2nd Law,

Force by wall on air, $F_{wa} =$ rate of change of momentum of air

$$= m\frac{\Delta v}{t} = (\rho A u)\times\frac{0-u}{1}$$

$$= -\rho A u^2$$

By Newton's 3rd Law, $F_{aw} = F_{wa} = u^2 \rho A$

$$P = \frac{F_{aw}}{A} = v^2\rho = 50^2(1.25) = 3.13 \text{ kNm}^{-2}$$