TEMASEK
JUNIOR COLLEGE

Unit 06: Collisions

ASSESSMENT OBJECTIVES

Students should be able to

(a) recall that impulse is given by the area under the force-time graph for a body and use this to solve problems.

(b) state the principle of conservation of momentum.

(c) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required).

(d) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

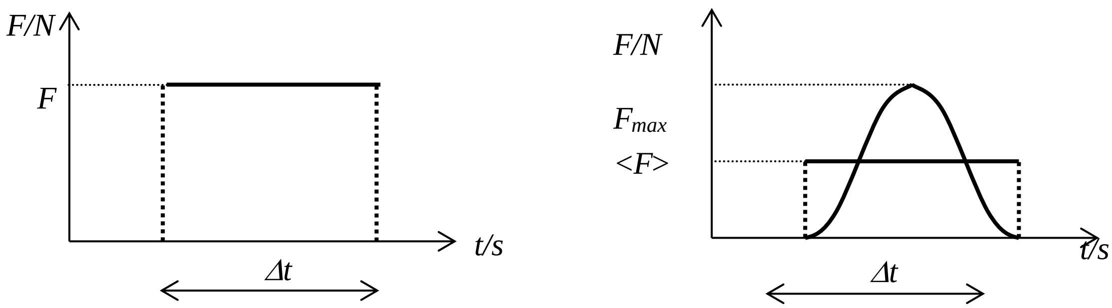
(e) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Collisions occur when one object strikes another. It is the event in which two or more bodies exert forces on each other in about a relatively short time.

1.1 Impulse *LO(a)*

Impulse is the product of the force acting on a body and the time interval during which the force is exerted.

Consider a constant force F acting on an object for a time interval Δt as shown by the force–time graph in (a). The impulse is given by product of force and time interval = $F\,\Delta t$.



(a) constant force (b) varying force

In general, the force need not be constant, as shown in (b).

Then impulse = $\int F \, dt$ = $\langle F \rangle \Delta t$, *where* **<*F*>** is the average force.

The average force <*F*> is defined as that constant force, which when acting over the same time interval Δt as the actual time-varying force, produces the same impulse and change in momentum. Graphically, the area under the average force–time graph is equal to the area under actual force–time graph.

By Newton's 2nd Law, $F = \dfrac{dp}{dt}$ => $\boldsymbol{F\,\Delta t = \Delta p = m\Delta v}$

The impulse of the force acting on a body is equal to the <u>change</u> in momentum of the body.

Impulse is a vector quantity. Its unit is the same as momentum : kg m s^{-1} or N s.

Example 1:

Use the concept of impulse to explain:
(a) "Follow through" when striking a tennis ball
(b) Use of a seat belt in a car
(c) Crumple zones of a car

(a) The "follow through" <u>increases</u> the impact time of the racket and the ball. The impulse and hence the change in momentum is <u>increased</u>. Thus the ball goes off with a <u>higher</u> velocity.

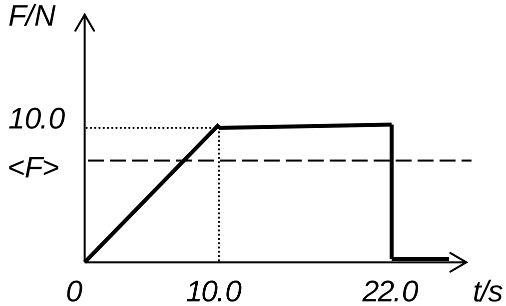
(b) *The seat belt extends slightly before arresting the forward lunge of the passenger. This* ***increases*** *the time at which the momentum of the passenger is brought to zero. The average force on the passenger is thus* ***reduced****.*

(c) *The crumple zones of the car helps to* ***increase*** *the impact time as the momentum of the crashing car is brought to* ***zero****, thus* ***reducing*** *the average force on the passengers in the passenger compartment.*

Read more on crumple zone of car :

Example 2:

An object moving with an initial velocity of 12.0 m s^{-1} is acted on by a force for 22.0 s as shown in the graph below. (a) Calculate the change in momentum of the object, and (b) the average force <*F*> acting on the object.



(a) $\Delta p = \int F \, dt = \text{area under the force - time graph}$

$= ½\,(10.0)(22.0 + 12.0) = 170 \text{ N s}$

(b) Let <*F*> be the average force.

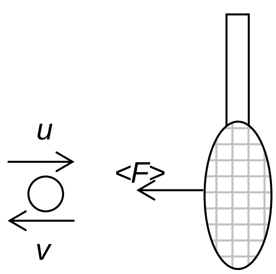
$\langle F \rangle \, \Delta t = \Delta p$

$\langle F \rangle \, (22.0) = 170$

=> $\langle F \rangle = 7.73$ N

Example 3:

A tennis ball of mass 0.060 kg and an initial speed of 30 m s^{-1} hits a racket. The ball rebounds from the bat with a speed of 40 m s^{-1} in the opposite direction. Given that the ball was in contact with the racket for 0.0020 s, calculate the average force on the tennis ball.



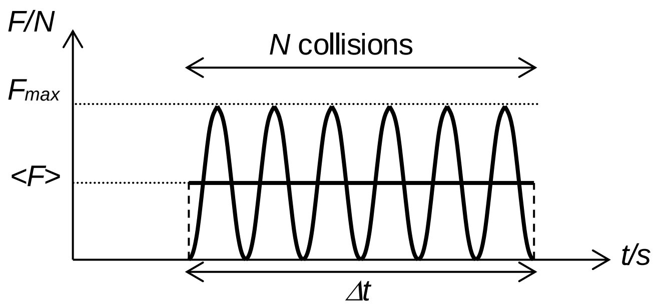
$+\leftarrow \quad \langle F \rangle \Delta t = \Delta p = m[v - (-u)]$

$\langle F \rangle 0.0020 = 0.060[40 - (-30)]$

$\langle F \rangle = 2.1 \times 10^3$ N to the left

1.2 Average force in multiple collisions

Consider a body being hit N times in time interval Δt as shown by the force–time graph below.

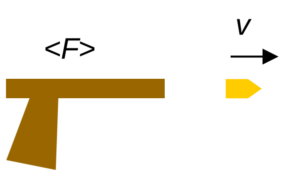


Then Impulse = $\langle F\rangle \Delta t = (\Delta p)N$ where Δt = time for N collisions

Examples of multiple collisions:
- a machine gun shooting a string of bullets,
- gas molecules bombarding walls of a vessel.

Example 4:

A machine gun fires 50.0 g bullets at a speed of 1.00 x 10^3 m s^{-1}. The gunner, holding the machine gun in his hands, can exert an average force of 180 N against the gun. Determine the maximum number of bullets he can fire per minute.



$$\langle F\rangle \Delta t = (\Delta p)N = (m\Delta v)N$$
$$180\Delta t = (0.0500 \times 1.00 \times 10^3)N$$
$$\Rightarrow \frac{N}{\Delta t} = 3.6\ s^{-1} = 216\ \text{min}^{-1}$$

How air bags works :

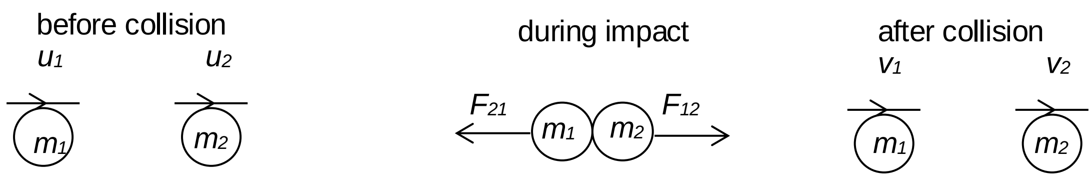
1.3 Principle of conservation of momentum *LO(b)*

The principle of conservation of momentum states that the momentum of a system of objects remains constant if no resultant external force acts on the system.

When two bodies collide, the forces F that they act on each other constitute an action-reaction pair. Therefore, each body receives an <u>equal</u> and <u>opposite</u> impulse, $F\Delta t$, where Δt is the time of collision. As a result, the change in momentum of one body is <u>equal</u> and <u>opposite</u> to the other body. The total momentum of the two colliding bodies remains constant, hence the change in momentum of the system is <u>zero</u>.

Proof:

Consider an isolated system (one which has no external force acting on it) of two objects m_1 and m_2 whereby m_1 is colliding with m_2.



Mathematical approach:

If the two bodies are in contact for a time of Δt,

From Newton's third law, $F_{21} = -F_{12}$

$$F_{21}\Delta t = -F_{12}\Delta t \quad \text{--(1)}$$

impulse on m_1 $\quad F_{21}\Delta t = m_1v_1 - m_1u_1$ --(2)

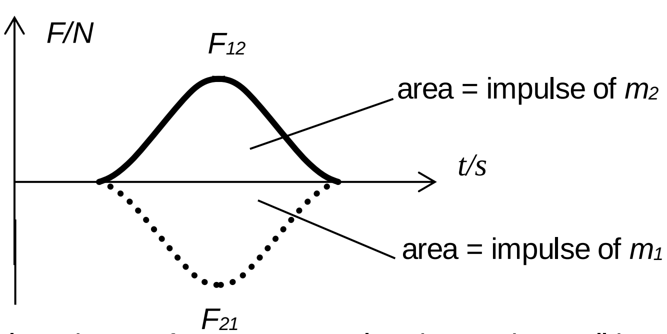
impulse on m_2 $\quad F_{12}\Delta t = m_2v_2 - m_2u_2$ --(3)

Sub (2),(3) into (1) $\quad m_1v_1 - m_1u_1 = -(m_2v_2 - m_2u_2)$

=> $\quad \boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$

Hence, the total momentum of the two objects before collision equals the total momentum after collision, which proves the principle of conservation of momentum.

Graphical approach: The solid curve in the graphs below shows the variation with time of the force exerted by m_1 on m_2 during impact while the dotted curve shows the force that m_2 exerts on m_1.



From Newton's third law, the two forces are equal and opposite at all instants. So the two curves are reflections of each other about the x-axis.

Thus, $\quad$ area under F_{12} = – area under F_{21}

impulse of m_2 = – impulse of m_1

change in momentum of m_2 = – change in momentum of m_1

=> $\quad$ change in momentum of m_2 + change in momentum of m_1 = 0

The result shows that the change in momentum of the system is zero, which again proves the principle of conservation of momentum.

Note:

- F_{12} and F_{21} are internal forces of the system. i.e. forces which arise between the interacting objects of the system.

- If there is an external force, for example, friction, then there will be an overall change in the momentum of the system caused by the external force.

1.4 Types of collisions *LO(d)(e)*

Generally, there are 3 types of collisions:

- *Elastic collision* in which both momentum and kinetic energy are conserved.
- *Inelastic collision* in which momentum is conserved but kinetic energy is not.
- *Completely inelastic collision* in which momentum is conserved and the particles stick together after collision so that their final velocities are the same.

Note:

- In all types of collision, momentum is always conserved (so long as there is no resultant external force acting on the system).
- In inelastic collisions, for example between a lump of plasticine and a hard surface, some of the kinetic energy is converted to internal energy and sound energy.

- Collisions between atomic and subatomic particles are elastic.

Head-on elastic collision *LO(j)*

Consider an elastic head-on collision between two atomic particles.

If the collision is elastic, both total momentum and kinetic energy are conserved.

By conservation of momentum, $mu_1 + Mu_2 = mv_1 + Mv_2$ --(1)

By conservation of kinetic energy, $\frac{1}{2}mu_1^2 + \frac{1}{2}Mu_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$ --(2)

From (1) $M(u_2 - v_2) = m(v_1 - u_1)$ --(3)

From (2) $M(u_2^2 - v_2^2) = m(v_1^2 - u_1^2)$ --(4)

(4)/(3) $u_2 + v_2 = v_1 + u_1$

Rearranging, $\boxed{v_2 - v_1 = u_1 - u_2}$ --(5)

Equation (5) tells us that **the relative velocity of separation is equal to the relative velocity of approach.** This equation is valid even for collision between bodies of different masses, so long as the collision is elastic.

From (5) $v_2 = u_1 - u_2 + v_1$ ---(6)

Sub into (3) $M[u_2 - (u_1 - u_2 + v_1)] = m(v_1 - u_1)$

Simplifying,
$$v_1 = \frac{m-M}{m+M}u_1 + \frac{2M}{m+M}u_2 \quad \text{---(7)}$$

and
$$v_2 = \frac{2m}{m+M}u_1 + \frac{M-m}{m+M}u_2 \quad \text{---(8)}$$

Note:

- If $m = M$ (two particles of equal mass):

  From (7) and (8), $v_1 = u_2$ and $v_2 = u_1$
  The particles exchange their velocities after the collision.

- If $u_2 = 0$ (collided particle *M* originally at rest) and :

  (i) $m = M$: $v_1 = 0$ and $v_2 = u_1$

  m comes to rest while *M* moves off with the velocity of the colliding particle *m*. i.e. The two particles exchange their velocities.

  (ii) $m << M$: $v_1 \approx -u_1$ and $v_2 \approx 0$

  m rebounds with almost the same speed, *M* remains stationary.

  (iii) $m >> M$: $v_1 \approx u_1$ and $v_2 \approx 2u_1$

  m continues with almost the same speed, *M* goes off with twice the speed of *m*.

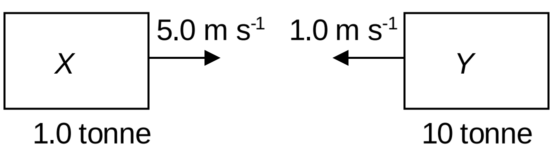
Newton cradle simulation :

1.5 Problem-solving using principle of conservation of momentum *LO(c)*

1. Draw the 'before' and 'after' diagrams for the system of colliding bodies, marking the masses and velocities involved.
2. Assign a **positive direction**.
3. Equate the **total momentum** before and after the collision.
4. If the collision is elastic, can also equate the **total kinetic energy** before and after the collision. Alternatively, apply the relationship that the **relative velocity of separation is equal to the relative velocity of approach**.

Example 5:

Two vehicles, traveling directly towards each other, collide. Initially, vehicle X of mass 1.0×10^3 kg is moving at 5.0 m s^{-1} and vehicle Y of mass 10 tonne is moving at 1.0 m s^{-1}. The vehicles become locked together on impact. What is the velocity of the vehicles immediately after impact?



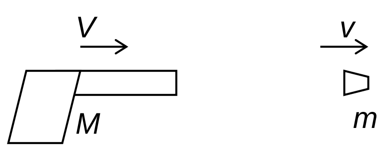
Let v be final velocity.
By conservation of momentum,

$\rightarrow+$ $1.0\,(5.0) - 10.0\,(1.0) = (1.0 + 10.0)\,v$

=> $v = -0.45$ m s^{-1}

Example 6:

Calculate the recoil velocity of a 4.0 kg rifle which shoots a 0.050 kg bullet at a speed of 2.8×10^2 m s^{-1}.



By conservation of momentum,

$\rightarrow+$ $0 = MV + mv$

$$V = -\frac{m}{M}v$$

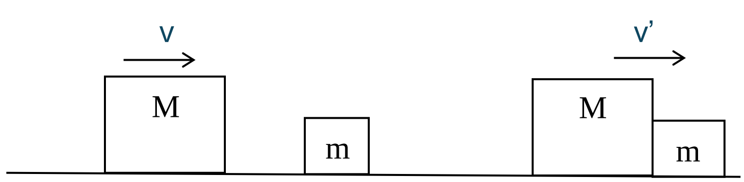
=>

$$= -\frac{0.050}{4.0}\left(2.8 \times 10^2\right) = -3.5 \ m\ s^{-1}$$

Thus the rifle recoils with 3.5 m s^{-1} in a direction opposite to the bullet.

Example 7:

On an air track, a rider of mass 200 g travelling at a speed of 3.0 m s^{-1} collides with a stationary rider of mass 100 g and sticks to it. Calculate the resultant speed of the riders and the loss in kinetic energy that occurs. What happens to this energy?



By conservation of momentum,

$\rightarrow +$ $Mv = (M + m)\, v'$

$0.200\,(3.0) = (0.200 + 0.100)\, v'$

=> $v' = 2.0 \text{ m s}^{-1}$

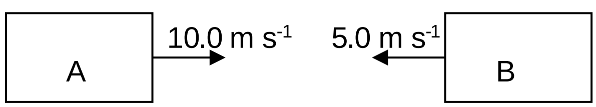
Loss in KE $= \frac{1}{2} Mv^2 - \frac{1}{2}(M + m)\, v'^2$

$= \frac{1}{2}(0.200)(3.0)^2 - \frac{1}{2}(0.300)\,(2.0)^2 = 0.30 \text{ J}$

This kinetic energy is converted into heat and sound energy.

Example 8:

An object A of mass 1.0 kg traveling with velocity 10.0 m s^{-1} collides elastically head-on with object B of mass 4.0 kg traveling with velocity –5.0 m s^{-1}. What are the velocities of each object after the collision?



Let v_A and v_B be the final velocities of objects A and B respectively.

By conservation of momentum,

$\rightarrow +$ $1.0\,(10.0) - 4.0\,(5.0) = 1.0\, v_A + 4.0\, v_B$

=> $v_A + 4.0\, v_B = -10.0$ ---(1)

Applying *relative velocity of separation is equal to the relative velocity of approach,*

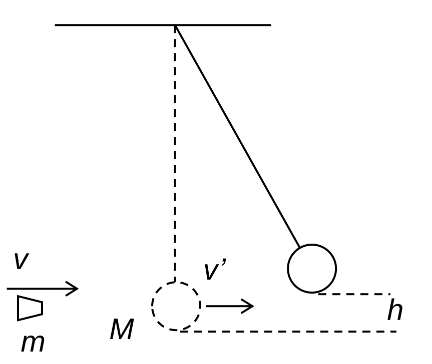
$v_B - v_A = 10.0 - (-5.0) = 15.0$ ---(2)

(1)+(2) $5.0\, v_B = 5.0$ => $v_B = 1.0 \text{ m s}^{-1}$

Sub into (1) $v_A = -14.0 \text{ m s}^{-1}$

Example 9:

The pellet from an air rifle is fired horizontally into a ball of plasticine suspended by a vertical thread, causing the ball to rise through a vertical height of 10 cm. Calculate the speed of the pellet, given that its mass is 1.2 g and the mass of the plasticine is 96 g.



By conservation of momentum,

$\rightarrow +$ $mv = (M + m)\, v'$ --(1)

By conservation of mechanical energy,

$\frac{1}{2}(M + m)\, v'^2 = (M + m)\, gh$

$$\Rightarrow v' = \sqrt{2gh} \qquad --(2)$$

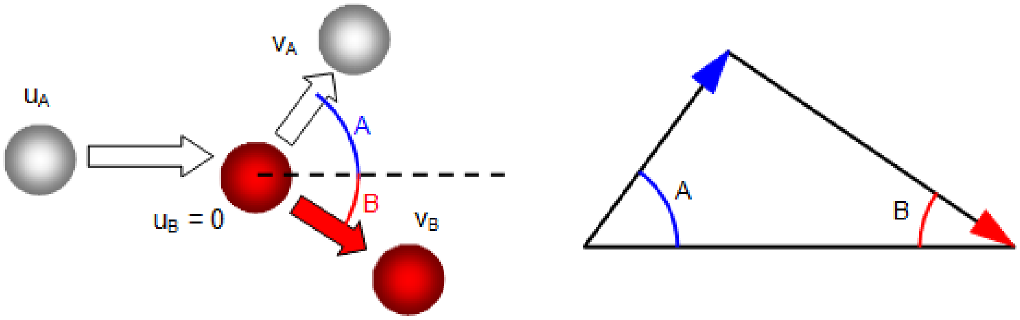
From (1) $v = \left(\frac{m + M}{m}\right) v' = \left(1 + \frac{M}{m}\right) v' \quad --(3)$

Sub (2) into (3) $v = \left(1 + \frac{M}{m}\right)\sqrt{2gh}$

SUPPLEMENTARY NOTES:

Glancing Collisions

Imagine Ball A colliding elastically with Ball B at an angle such that their centres of mass do not lie along the same line of action. Ball B has the same mass as Ball A and is initially at rest. Both balls will move off at an angle to the original direction after collision. The interesting thing is that the angle between the two balls after collision is exactly 90º.



Using a vector diagram for the momenta of the two balls before and after the collision we see that:

$$mu_A = mv_A + mv_B$$

Assuming the collision is elastic,

$$\frac{1}{2} mu_A^2 = \frac{1}{2} mv_A^2 + \frac{1}{2} mv_B^2$$

giving

$$u_A^2 = v_A^2 + v_B^2$$

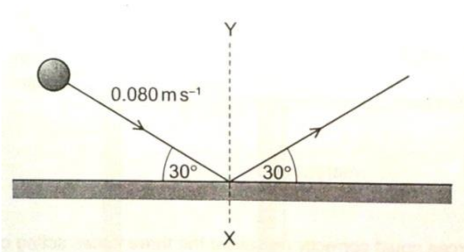
The vector triangle for velocity must be right angled.

Note since kinetic energy is a scalar, we can simply add the kinetic energies of the balls after impact without considering their different directions of motion.

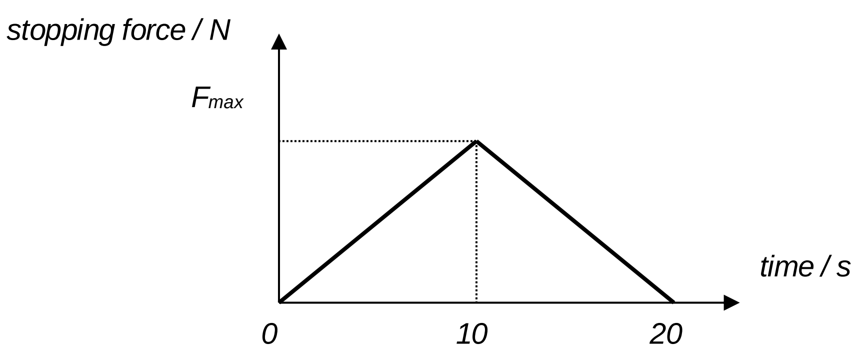
Tutorial (Solutions to Qns 3 and 6 are provided.)

Impulse

1. A snooker ball of mass 0.40 kg moving at 0.080 ms^{-1} collides with the side of a table at an angle of 30°. The ball is in contact with the side for 0.20 s. It bounces off at the same angle with the same speed. What is the average force on the wall from the side in the direction XY?
[0.16N]



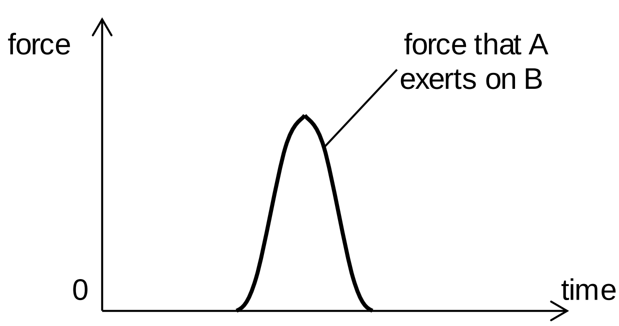
2. In order to stop a car of mass 1500 kg travelling at 30 m s^{-1}, the driver applies his brakes so that *F*, the stopping force, increases steadily to a maximum and then decreases to zero as shown in the figure below.



Calculate each of the following quantities:
(a) the momentum of the car when it is travelling at 30 m s^{-1},
(b) the change in momentum between t = 0 and t = 10 s,
(c) the value of F_{max},
(d) the maximum negative acceleration of the car.
[4.5 x 10^4 kg m s^{-1}, 2.25 x 10^4 kg m s^{-1}, 4.5 x 10^3 N, 3.0 m s^{-2}]

3. (a) Define *linear momentum*. State the relationship between the change in linear momentum of an object, the constant force acting on the object, and the time for which the force acts.

(b)



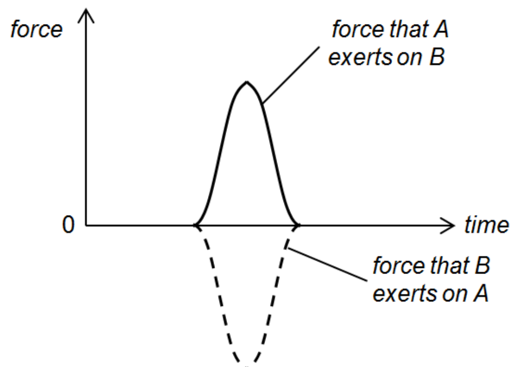
In a collision between two bodies A and B, the force that A exerts on B varies with time in the way shown. Copy and show on your sketch a graph of the force that B exerts on A. Explain your answer. Also explain how your answer is consistent with the principle of conservation of momentum.

(c) In a collision, when a truck of mass 12 tonnes runs into the back of a car of mass 1.2 tonnes, a constant force of 72 000 N acts for 0.25 s. Calculate the change in velocity of the car and the truck. Suggest **one** way in which the conditions are unrealistic. Discuss how seat belts and airbags in the car ensure greater safety. [15 m s^{-1}, – 1.5 m s^{-1}]

(d) In order to reduce the number of road traffic accidents, many countries conduct research into improving road safety. One area of research concerns braking. State **three** other factors that affect braking which might be considered by researchers. State **one** aspect of the car safety that could be researched, and suggest briefly how the research could be conducted.

(a) Linear momentum is defined as the product of the mass of a body and its velocity. The product of the constant force and the time for which the force acts equals the change in linear momentum of the object.

(b)



The 2 curves are reflections of each other about the time axis. This is because the 2 forces are equal and opposite at all instants and constitute an action-reaction pair. Area under force – time graph is impulse. So each of the 2 bodies receives an equal and opposite impulse and hence equal and opposite change in momentum. This means that the total change in momentum of the system is zero, consistent with the COM.

(c)

For car, $\rightarrow +$

$$F\,\Delta t = m_c\,\Delta v_c$$
$$72000\,(0.25) = 1200\,\Delta v_c$$
$$\Delta v_c = 15\text{ m s}^{-1}$$

For truck, $\rightarrow +$

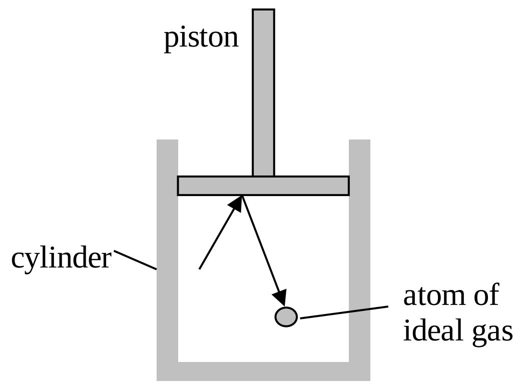
$$-F\,\Delta t = m_t\,\Delta v_t$$
$$-72000\,(0.25) = 12000\,\Delta v_t$$
$$\Delta v_t = -1.5\text{ m s}^{-1}$$

The condition is unrealistic as the force is unlikely to be constant.
Seat belt extends slightly before arresting the forward lunge of the passenger. Airbag inflates and absorbs the impact of the passenger. Both methods increase the time over which the momentum of the passenger is brought to zero. Average force on passenger is thus reduced.

(d) Three factors that affect braking: Construction and materials of tyres, construction and materials of road surface, efficiency and materials of brakes etc. One aspect of car safety could be anti-locking device. Research into how these devices can prevent tyres locking when a driver slams on the brake that result in skidding.

<u>Conservation Of Momentum</u>

4. State (a) two quantities that are conserved in any collision and (b) what is meant by *an elastic collision.*
An ideal gas is contained in a cylinder by means of a piston, as shown. An atom of the gas collides with the piston, as illustrated. State, with a reason, whether the momentum of the atom is conserved in the collision.



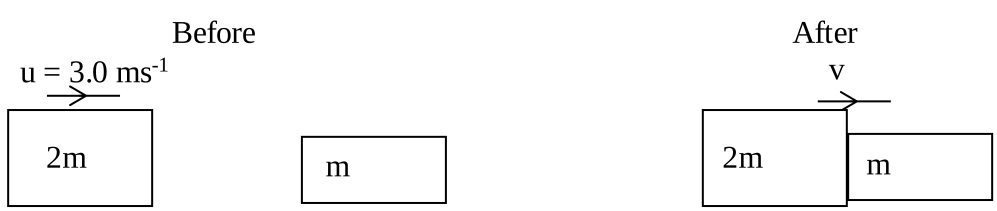
5. A lorry of mass 5000 kg collides with a car of mass 1500 kg traveling at 25 m s^{-1}. They are traveling in opposite directions. After the impact, the car rebounds backward with a speed of 4.0 m s^{-1} and the lorry travels in the same direction as before with a speed of 3.0 m s^{-1}.

(a) Find the velocity of the lorry before the collision.
(b) Is this an elastic collision? Why?
(c) If the time of impact is 0.20 s, calculate the force acting on each vehicle.

[11.7 m s^{-1}, 2.18 x 10^5 N]

6. An object of mass 2.0 kg, travelling at 3.0 m s^{-1}, collides with another object of mass 1.0 kg, which is initially at rest. They stick together after the collision.

(a) Find their common velocity.
(b) What fraction of KE is lost during the collision?
(c) What type of collision is this? [2.0 m s^{-1}, 33%]



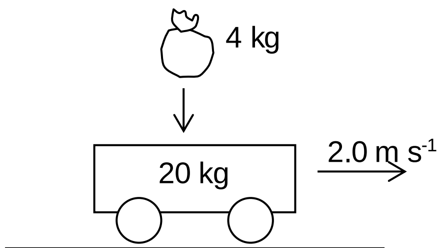
(a) COM: $2mu + 0 = (2m+m)v \Rightarrow v = \left(\frac{2}{3}\right)u = \left(\frac{2}{3}\right)3.0 = 2.0 \text{ m s}^{-1}$

(b) fraction of KE loss = $\frac{KE - KE'}{KE} = 1 - \frac{KE'}{KE} = 1 - \frac{\frac{1}{2}(3m)}{\frac{1}{2}(2m)}\left(\frac{v}{u}\right)^2 = 1 - \left(\frac{3}{2}\right)\left(\frac{2.0}{3.0}\right)^2 = 0.33$

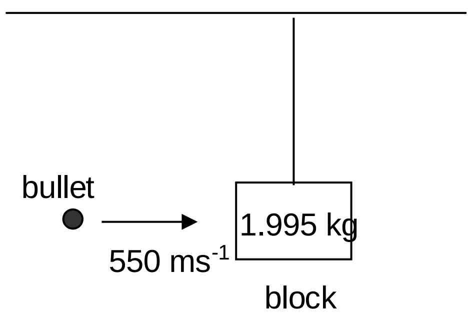
(c) *Perfectly inelastic collision.*

7. A trolley f mass 20 kg is traveling on a smooth horizontal surface with a speed of 2.0 m s^{-1}. Find the speed of the trolley when a load of mass 4.0 kg is dropped vertically onto it.

[1.67 m s^{-1}]



8. A bullet of mass 5.00 g moving at 550 ms^{-1} strikes a wooden block of mass 1.995 kg which is the bob of a ballistic pendulum.



Determine
(a) the speed at which the block and bullet leave the equilibrium position,
(b) the loss in kinetic energy of the bullet, and
(c) the height that the c.g. of the bullet block system reaches above the initial c.g.

(1.38 m s^{-1}, 756 J, 0.0964 m)

9 Two spheres A and B approach one another along a line joining their centres, as illustrated in Fig. 1.1.

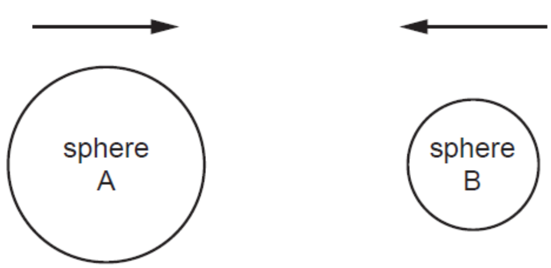


Fig. 1.1

When they collide, the average force acting on sphere A is F_A and the average force acting on sphere B is F_B. The forces act for a time t_A on sphere A and time t_B on sphere B.

(a) State the relationship between
 (i) F_A and F_B,
 (ii) t_A and t_B.

(b) Use your answers in (a) to show that the change in momentum of sphere A is equal in magnitude and opposite in direction to the change in momentum of sphere B.

(c) For the spheres, the variation with time of the momentum of sphere A before, during and after the collision with sphere B is shown in Fig. 1.2.

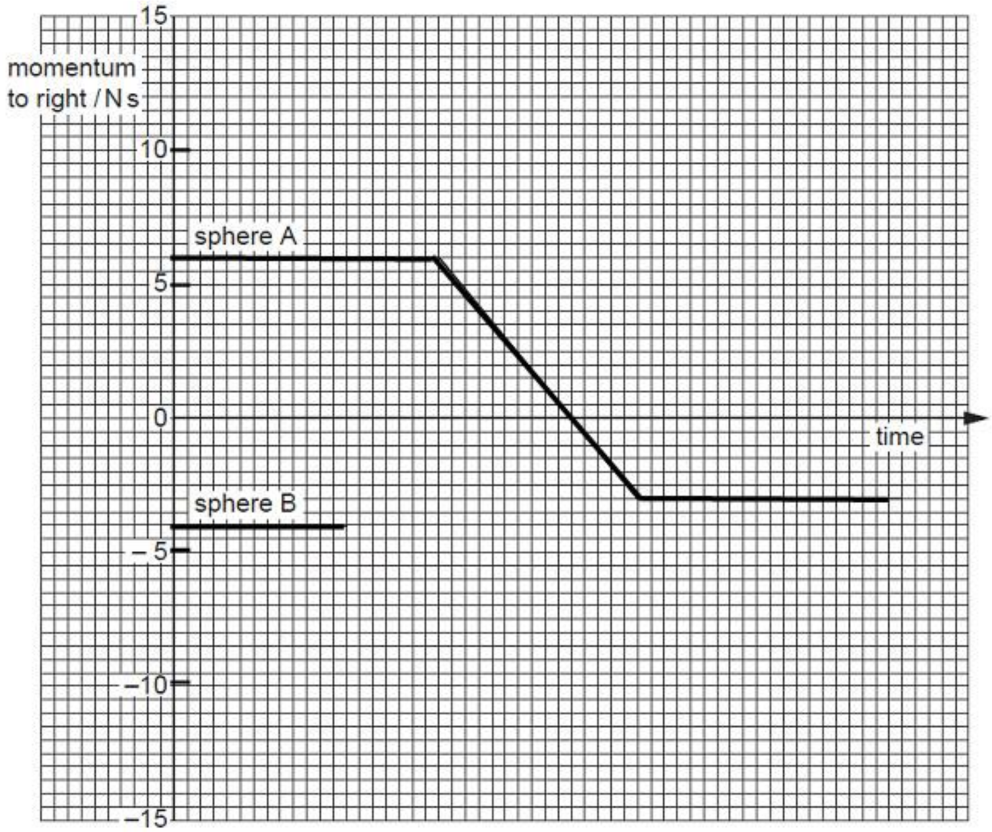


Fig. 1.2

The momentum of sphere B before the collision is also shown on Fig. 1.2.

(i) Complete Fig. 1.2 to show the variation with time of the momentum of sphere B during and after the collision with sphere A.

(ii) Explain why it is not possible for the two spheres to stop(or zero momentum) at the same instant.

(iii) If the spheres are in contact for a total time of 0.045 s, calculate the magnitude of the force acting on the spheres during the collision.

(iv) If the two spheres are of equal mass of 1.0 kg, state and explain whether or not this is an elastic collision.