Name : ______________________________

Class : ________

TEMASEK JUNIOR COLLEGE

Unit 9: Oscillations

Learning Outcomes

Candidates should be able to:

(a) describe simple examples of free oscillations, where particles periodically return to an equilibrium position without gaining energy from or losing energy to the environment

(b) investigate the motion of an oscillator using experimental and graphical methods

(c) show an understanding of and use the terms amplitude, period, frequency, angular frequency, phase and phase difference and express the period in terms of both frequency and angular frequency

(d) show an understanding that $a = -\omega^2 x$ is the defining equation of simple harmonic motion, where acceleration is (directly) proportional to displacement from an equilibrium position and acceleration is always directed towards the equilibrium position

(e) recognise and use $x = x_o \sin\omega t$ as a solution to the equation $a = -\omega^2 x$

(f) recognise and use the equations $v = v_o \cos\omega t$ and $v = \pm\omega\sqrt{x_o^2 - x^2}$

(g) describe, with graphical illustrations, the relationships between displacement, velocity and acceleration during simple harmonic motion

(h) describe the interchange between kinetic and potential energy during simple harmonic motion

(i) describe practical examples of damped oscillations with particular reference to the effects of the degree of damping and to the importance of critical damping in applications such as a car suspension system

(j) describe graphically how the amplitude of a forced oscillation changes with driving frequency, resulting in maximum amplitude at resonance when the driving frequency is close to or at the natural frequency* of the system

(k) show a qualitative understanding of the effects of damping on the frequency response and sharpness of the resonance

(l) describe practical examples of forced oscillations and resonance, and show an appreciation that there are some circumstances in which resonance is useful, and other circumstances in which resonance should be avoided

*Natural frequency refers to the frequency when the system is in free oscillation.

1 Introduction

An **oscillation** is a motion in which a system moves to-and-fro along the same path repeatedly.

The oscillation of a pendulum of a clock is apparent, but the oscillation of atoms within a solid is hidden. The oscillation associated with wave motion can be appreciated when a body floating in a liquid rises and falls as a wave travels past it, but the oscillation associated with light waves cannot be perceived by our senses. In fact, all communication by sight and by hearing makes use of wave oscillation.

1.1 Free oscillations *LO (a)*

In a **free oscillation,** the particle periodically returns to an equilibrium position without gaining energy from or losing energy to the environment.

The only external force acting on it is the restoring force and it oscillates at its natural frequency with constant amplitude and constant total energy.

Simple harmonic oscillations are free oscillations.

1.2 Motion of an oscillator *LO (b)*

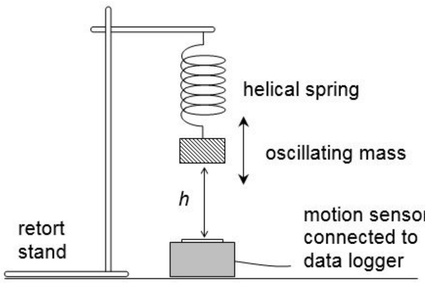


Fig (a)

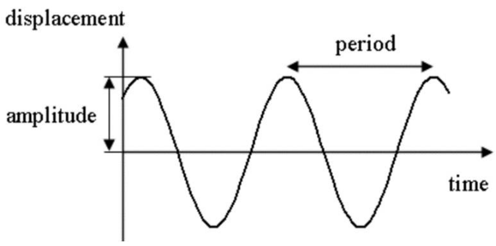


Fig (b)

Fig (a) shows an oscillating **mass-spring system**. The motion is easily monitored with a motion sensor connected to a data logger to record the variation with time of the position of the mass.

Fig (b) shows a typical displacement–time graph generated by a motion sensor. The curve is sinusoidal and the motion is said to be sinusoidal. An oscillation that follows a sinusoidal function of time is called a **simple harmonic motion**. In practice, dissipative forces exists and the energy and hence amplitude of the oscillations diminish over time.

1.3 Important terms *LO (c)*

In the context of an oscillating system,

1. **Displacement** x is the distance of an oscillating particle from its equilibrium position in a specific direction.

2. **Amplitude** x_o is the maximum magnitude of displacement of the oscillating particle from the equilibrium position.

3. **Period** T is the time for one complete oscillation.

4. **Frequency** f is the number of complete oscillations per unit time.
   SI unit is hertz (Hz) where 1 hertz = 1 cycle per second = 1 s^{-1}. And $\boxed{f = \frac{1}{T}}$

5. **Angular frequency** ω is defined as the product of 2π and the frequency.

   i.e. $\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$

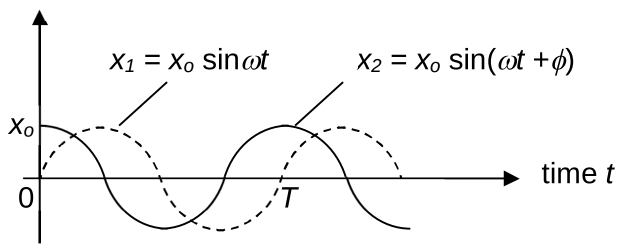
   SI unit is rad s^{-1}.

   As ω is a constant, T is a constant and is independent of the amplitude x_o of the oscillation. This is an important characteristic of simple harmonic motion.

6. **Phase** $\theta = \omega t$ refers to the stage or position reached within a cycle of an oscillation with respect to a reference (zero) position, usually expressed as a fraction of a cycle or a period or as an angle. i.e. phase $\theta = \omega t = \left(\frac{2\pi}{T}\right)t$.

   Phase difference ϕ is an angular measure of the fraction of a cycle that two oscillations having the same frequency are out of step.

   Consider an oscillation of displacement $x_1 = x_o \sin\omega t$ and a second oscillation of displacement $x_2 = x_o \sin(\omega t + \phi)$ as shown.

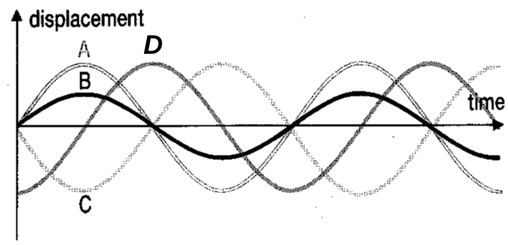


   Both oscillations have the same amplitude x_o, same angular frequency ω and hence same frequency f and period T. But they have a phase difference of $\phi = \frac{\pi}{2}$rad = 90°.

   Note:
   $x_2 = x_o \sin(\omega t + \phi)$ reaches its peak first.
   We say that $x_2 = x_o \sin(\omega t + \phi)$ leads $x_1 = x_o \sin\omega t$ by $\phi = \frac{\pi}{2}$rad or 90°.

Example 1:

The figure below shows four different oscillations A, B, C and D on the same displacement-time graph. Complete the sentences below with regards to their phase difference.



Phase difference between A and B = _________, they are ___________________.

Phase difference between A and C = _________, they are ___________________.

Phase difference between A and D = _________, they are ___________________.

1.4 Defining equation of SHM *LO (d)*

Simple harmonic motion is an oscillatory motion of a particle whose acceleration is (directly) proportional to displacement from an equilibrium position and acceleration is always directed towards the equilibrium position

Mathematically, the defining equation of SHM is

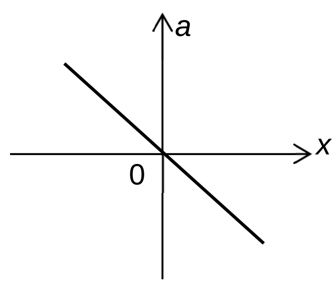
$$\boxed{a = -\omega^2 x}$$

where a is acceleration, x is displacement from equilibrium position and ω is the angular frequency.

The negative sign implies that the acceleration points in the opposite direction to the displacement vector.

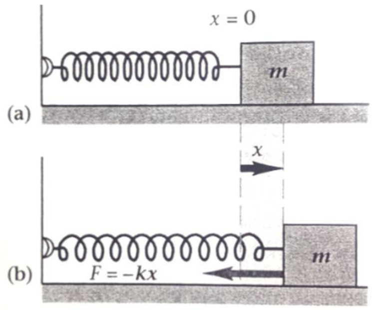
From Newton's second Law, force $F = ma = -m\omega^2 x$. Thus, $\boxed{F \propto -x}$

Graphically, a motion is said to be simple harmonic only if a **straight line of negative slope passing through the origin** is observed for the acceleration versus displacement graph as shown.



To prove that a motion is simple harmonic, it is sufficient to **establish that $a \propto -x$.**

For example, consider a mass-spring system.



In Fig (a), a body of mass m attached to a light spring of force constant k rests on a frictionless table. In Fig (b), the body is displaced by an amount x from its equilibrium position ($x = 0$).

The spring is extended and exerts a **restoring force** F on the mass in the direction opposite to displacement x.

If the spring obeys Hooke's law, $F = -kx$ where k is the force constant. The negative sign indicates that the restoring force on the body due to the spring is opposite to displacement.

From Newton's second law, $F = ma = -kx$

$\Rightarrow a = -\dfrac{k}{m}x$ and hence $a \propto -x$

Thus the motion is simple harmonic.

Compare with the defining equation of SHM $a = -\omega^2 x$,

$\boxed{\omega^2 = \dfrac{k}{m}}$ and $\omega = 2\pi f \Rightarrow f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

Practice of science:
How do astronauts weigh themselves in space?

Example 2:
Values of the acceleration a of a particle moving in simple harmonic motion as a function of its displacement x are given in the table below:

a/cm s^{-2}	16	8	0	– 8	– 16
x/cm	– 4	– 2	0	2	4

Find the period of the motion.

1.5 Solution to SHM equation *LO (e)(f)(g)*

Any SHM can be described in terms of a sinusoidal function. We can analyse the motion with respect to time or displacement.

1.5.1 Variation with time

For variation with time, the expressions depend on the position of the particle when the timing begins.

If the particle **<u>starts at amplitude at t = 0,</u>** solutions for displacement x, velocity v, and acceleration a in terms of time t could be expressed as

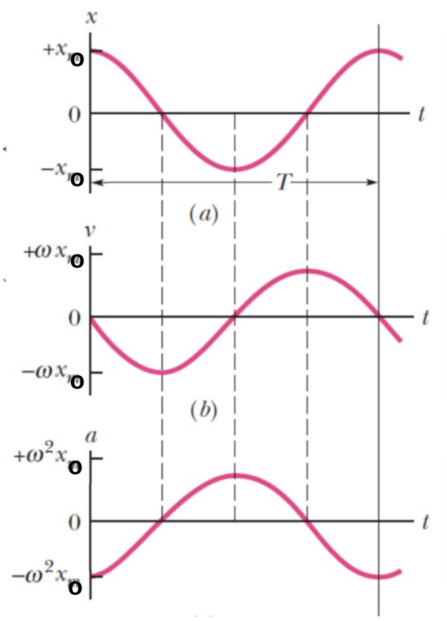
$\boxed{x = x_o \cos\omega t}$	x_o is the amplitude
$v = -\,\omega x_o \sin\omega t$	$\omega x_o = v_o = v_{max}$ is the maximum velocity
$a = -\,\omega^2 x_o \cos\omega t$	$\omega^2 x_o = a_o = a_{max}$ is the maximum acceleration

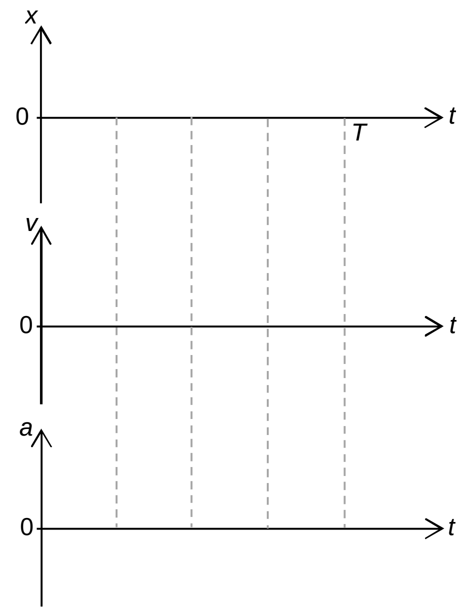
If the particle **<u>starts at equilibrium position at t = 0,</u>** solutions for displacement x, velocity v, and acceleration a in terms of time t could be expressed as

$\boxed{x = x_o \sin\omega t}$	x_o is the amplitude
$v = \omega x_o \cos\omega t$	$\omega x_o = v_o = v_{max}$ is the maximum velocity
$a = -\,\omega^2 x_o \sin\omega t$	$\omega^2 x_o = a_o = a_{max}$ is the maximum acceleration

Deduce that both sets of results agree with SHM equation $a = -\,\omega^2 x$!!

Graphically, for a particle that <u>starts at amplitude at t = 0,</u> the variation with time t of x, v, a are as shown. Sketch on the right side the corresponding graphs for a particle that <u>starts at equilibrium position at t = 0.</u>





1.5.2 Variation with displacement

Velocity could be expressed in terms of displacement x as $\boxed{v = \pm\omega\sqrt{x_o^2 - x^2}}$

Proof:

Consider an oscillation with displacement $x = x_o \sin\omega t$,

Then $v = \omega x_o \cos\omega t$

$= \pm\omega x_o \sqrt{1 - \sin^2\omega t}$ since $\sin^2\omega t + \cos^2\omega t = 1$.

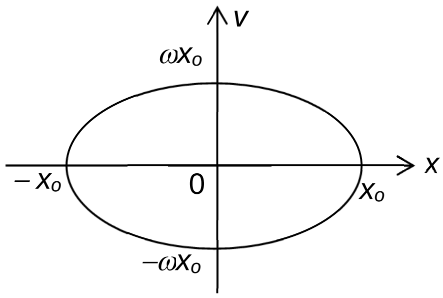
$\Rightarrow v = \pm\omega\sqrt{x_o{}^2 - x_o^2\sin^2\omega t} = \pm\omega\sqrt{x_o^2 - x^2}$

Graphically, the variation with displacement x of v and a are as shown.

velocity $v = \pm\omega\sqrt{x_o^2 - x^2}$

at $x = 0$, $\boxed{v_{max} = \omega x_o}$

at $x = \pm x_o$, $v = 0$

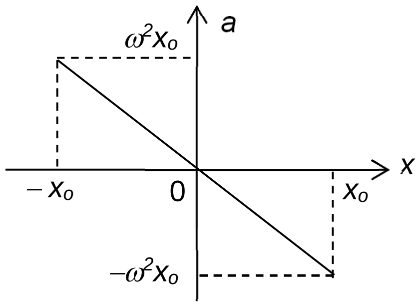


i.e. velocity is maximum when the particle is passing through the equilibrium position but is zero when the particle is at the amplitude.

acceleration $a = -\omega^2 x$

at $x = 0$, $a = 0$

at $x = \pm x_o$, $\boxed{a_{max} = \omega^2 x_o}$



i.e. acceleration is zero when the particle is passing through the equilibrium position but maximum when the particle is at the amplitude.

Example 3:

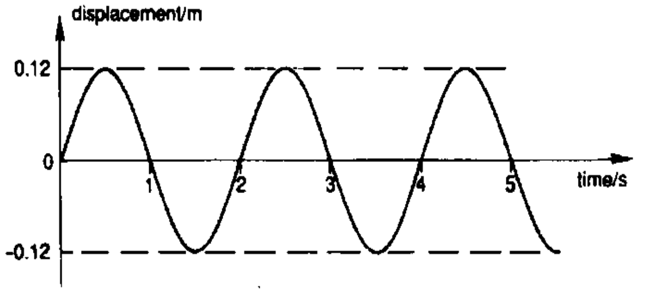
The displacement x at the time t of a particle moving in simple harmonic motion is given by $x = 0.25 \cos 7.5\, t$, where x is in meters and t is in seconds.

(a) Use the equation to find the amplitude, frequency and period for the motion.

(b) Find the displacement when the $t = 0.50$ s.

Example 4: 1995P2Q2

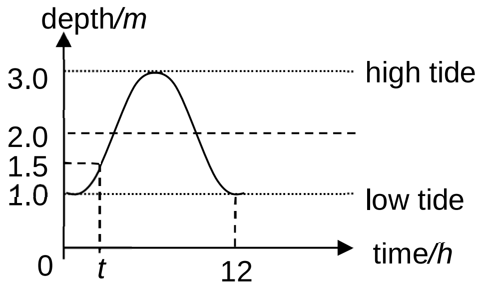
The pendulum bob in a particular clock oscillates so that its displacement from a fixed point is as shown in the diagram. By taking the necessary readings from the graph, determine for these oscillations,



(a) the amplitude, period, frequency and angular frequency;
(b) the acceleration (i) when the displacement is zero, and (ii) when the displacement is at its maximum;
(c) the maximum velocity of the pendulum bob.

Example 5:

The rise and fall of water in a harbour is simple harmonic. The depth varies between 1.0 m at low tide and 3.0 m at high tide. The time between successive low tides is 12 hours. A boat, which requires a minimum depth of water of 1.5 m, approaches the harbour at low tide. How long will the boat have to wait before entering?



2 Energy in SHM

LO (h)

In the absence of external dissipative forces, the total energy of the oscillating system is a constant and there is just an interchange of kinetic and potential energies. We can analyse the variation of energies with respect to time or with displacement.

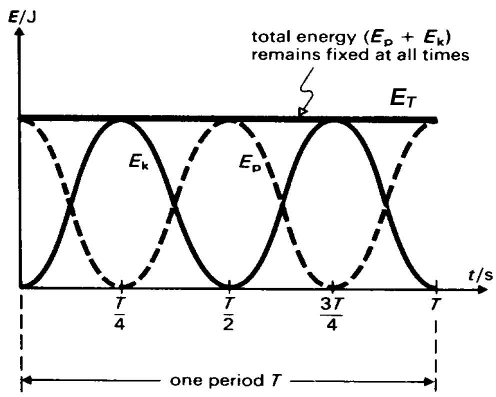
2.1 Variation with time

Consider $x = x_o \cos\omega t$, then $v = -\omega x_o \sin\omega t$

$E_k = ½mv^2 = ½m\omega^2 x_o^2 \sin^2\omega t$

$E_p = ½kx^2 = ½m\omega^2 x^2 = ½m\omega^2 x_o^2 \cos^2\omega t$

$E_T = E_k + E_p = ½m\omega^2 x_o^2 (\sin^2\omega t + \cos^2\omega t) = ½m\omega^2 x_o^2$



The variation with time t of kinetic energy E_k, potential energy E_p and total energy E_T are as shown. Both E_k and E_p are **sinusoidal curves** showing an **interchange of E_k and E_p** with time. However, the total energy E_T at any point is constant.

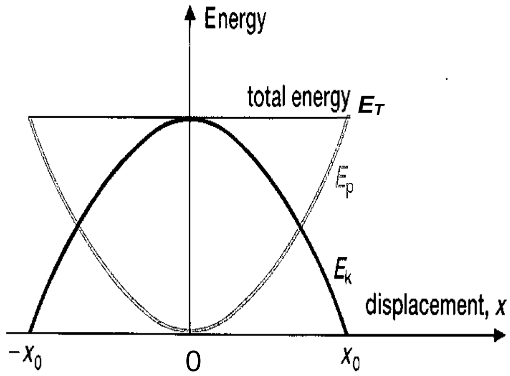
2.2 Variation with displacement

Using $v = \pm\omega\sqrt{x_o^2 - x^2}$

$E_k = ½mv^2 = ½m\omega^2(x_o^2 - x^2)$

$E_p = ½kx^2 = ½m\omega^2x^2$

$E_T = E_k + E_p = ½m\omega^2(x_o^2 - x^2) + ½m\omega^2x^2 = ½m\omega^2x_o^2$



The variation with displacement x of kinetic energy E_k, potential energy E_p and total energy E_T are as shown. Both E_k and E_p are **parabolic curves** showing an **interchange of E_k and E_p** with displacement. However, the total energy E_T at any point is constant.

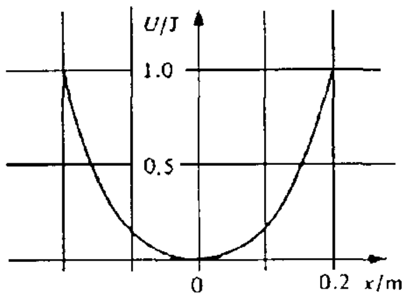
Deduce that $\boxed{\text{maximum } E_p = \text{maximum } E_k = ½m\omega^2x_o^2 = E_T}$

Graphs vs displacement and vs time

Example 6:

A particle of mass 4.0 kg moves with simple harmonic motion and its potential energy U varies with position x as shown in the figure.

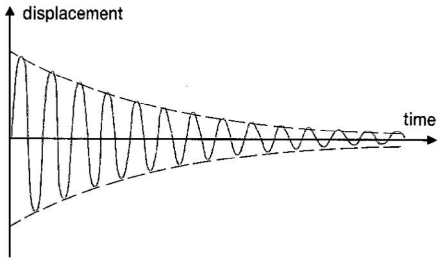
What is the period of oscillation of the mass?



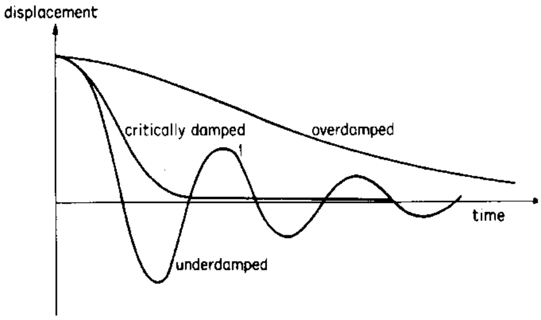
3 Damped oscillations

A **damped oscillation** is an oscillation that loses energy due to dissipative forces like friction and air resistance.

The total energy and hence the amplitude of the oscillation decreases **exponentially** with time.



3.1 Degree of damping



The degree of damping are classified as follows:

1. **Light damping (underdamped)** is when the dissipative force is small so that definite oscillations still occur but the amplitude of oscillation decreases exponentially with time.

2. **Critical damping** is when the dissipative force is of a critical value so that the system returns to equilibrium position in the shortest possible time without oscillation.

3. **Heavy damping (overdamped)** is when the dissipative force is large so that the system returns very slowly to equilibrium position without oscillation.

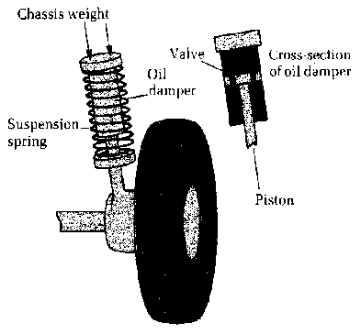
3.2 Importance of critical damping *LO (i)*

Critical damping is important in many practical applications. For example,

1. Shock absorber in car suspension system

   The car suspension system links the wheels and the body of the car. It consists of springs and oil dampers (shock absorbers).

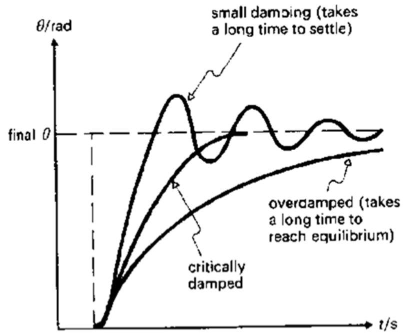
   The oil damper provides **critical damping** such that the car does not bounce after hitting a bump but returns to vertical equilibrium position in the shortest time to respond to further humps in the road, thus ensuring a comfortable ride.



   If the oil dampers are badly worn, damping is reduced and the car becomes too bouncy, making the ride uncomfortable.

2. Measuring instruments

Instruments such as voltmeters and ammeters are **critically damped** by electromagnetic effect so that the pointer moves quickly to the correct position in the shortest time without oscillating.



4 Forced oscillations and resonance

Although we cannot eliminate the loss of mechanical energy from a damped oscillation, we can replenish the energy from some sources. Usually an external periodic force (driver) is applied to do work against the dissipative forces. I.e. To replenish the energy lost to the dissipative forces. We call this forced oscillation.

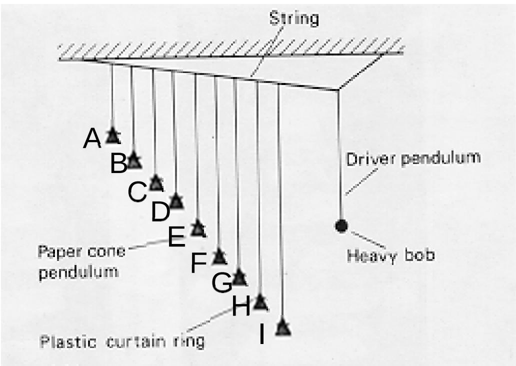
Natural frequency is the characteristic frequency at which a system oscillates when it is set into motion.

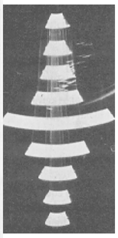
Driving frequency is the frequency of the external periodic force which replenishes the energy of an oscillating system.

Forced oscillation occurs when an oscillating system is subjected to an external periodic force. It oscillates at the driving frequency of the applied force.

Resonance occurs when the driving frequency of the external periodic force equals the natural frequency of the oscillating system. There is maximum energy transfer and the system oscillates with maximum amplitude.

The Barton's pendulum can be used to demonstrate forced oscillation and resonance.





When the heavy bob oscillates perpendicularly to the plane of the paper, energy is being transferred from it to the light paper cones.

<u>Cone E</u> will show the largest amplitude. This is because it has the same length as the heavy bob and thus its natural frequency is the same as the driving frequency of heavy bob. Resonance occurs for this cone with maximum energy transfer from the bob to this cone.

4.1 Frequency response curves

LO (j)(k)

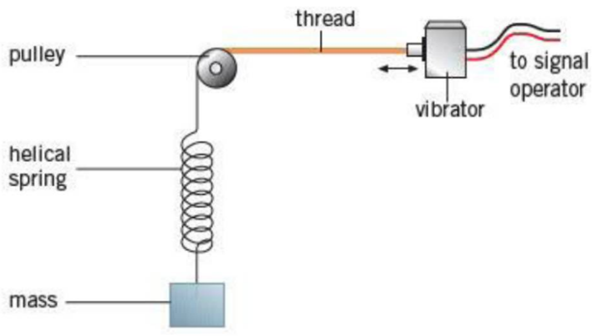


Fig (a)

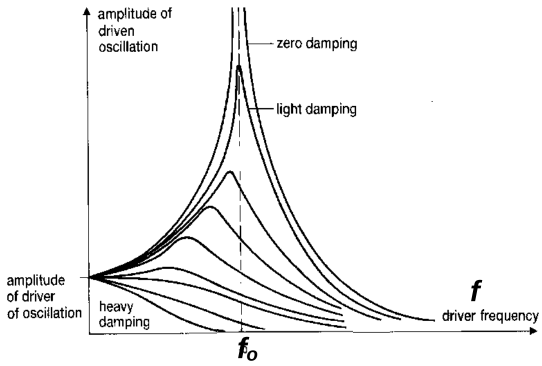


Fig (b)

Fig (a) shows the set-up for demonstration of forced oscillation. The vibrator provides the driving frequency *f* and has a fixed amplitude of oscillation. As the driving frequency *f* approaches the natural frequency f_o, the amplitude of the oscillation increases and peaks when resonance occurs at $f = f_o$. As *f* is increased further, the amplitude of oscillations decreases.

Fig (b) shows the variation with driving frequency of the amplitude of oscillation. These graphs are known as the frequency response curves or resonance curves. For no damping, the amplitude tends to infinity.

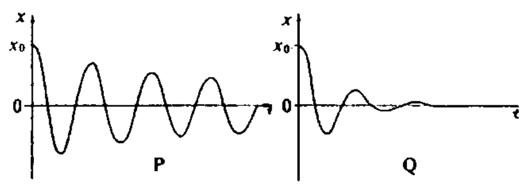
Effects of damping on resonance:

1. **Amplitude at all frequency is reduced and the peak becomes flatter or broader.** The more the damping, the lower and broader the resonance curve.
2. **Maximum amplitude occurs at a lower driving frequency** than the natural frequency. The greater the damping, the more the downward shift from the natural frequency.

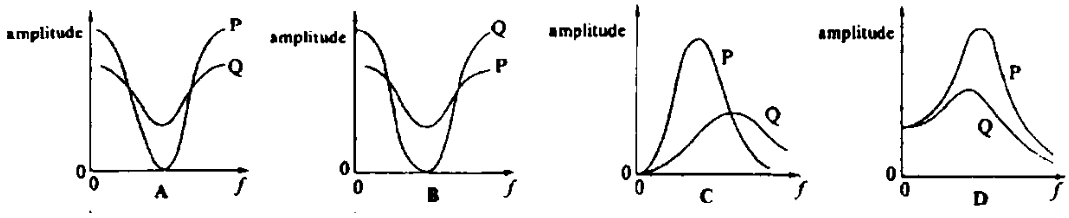
Thus, damping is useful if we want to reduce the damaging effects of resonance,

Example 7: N1993P1Q7

Two bodies P and Q are given an initial displacement x_o and then released. The graphs show how their displacements vary with time.



P and Q are then subjected to a driving force of constant amplitude and of variable frequency *f*. Which graph best represents the way in which the amplitudes of P and Q vary with *f* ?

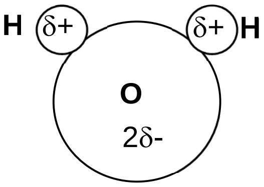


4.2 Practical examples of forced oscillations and resonance *LO (l)*

Resonance is an important phenomenon in many areas of physics and engineering. Resonance is **useful** when oscillation is intended. For example:

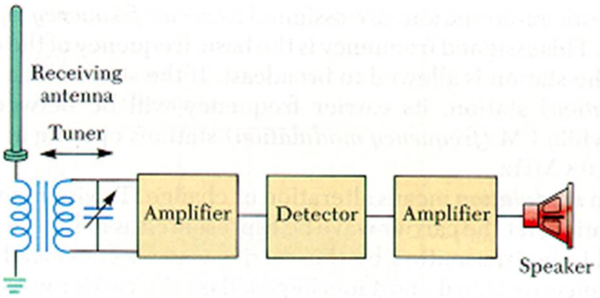
1. Microwave oven

The frequency of microwaves corresponds to the natural frequency of water molecules. When food is placed in the microwave oven, the water molecules in the food resonate when subjected to microwaves. They absorb energy and vibrate with large amplitude that result in heat that cook the food.



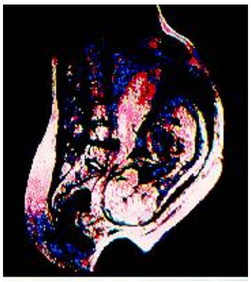
2. Tuning a radio

To tune in to a particular radio station, one adjusts the natural frequency of oscillation of the electrical circuit in the radio to match the frequency of the radio wave emitted by the broadcasting station. Electrical resonance is thus used to isolate and amplify the desired frequency.



3. Magnetic resonance imaging MRI

MRI is a medical imaging technique to create detailed images of the organs and tissues in a body. Any given molecule in the organs and tissues has many resonant frequencies. MRI scanners use strong magnetic fields and radio waves to cause the nuclei of atoms to oscillate. Energy is absorbed whenever resonance appears. The pattern of energy absorption is used to detect the presence of particular molecules within any specimen.

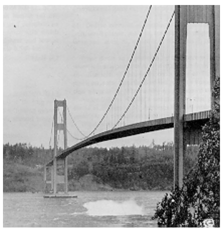


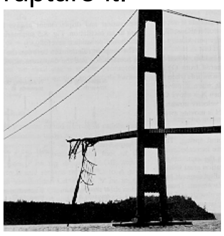
The false colour magnetic resonance image in the picture shows a lateral view of a woman's abdomen during the eighth month of pregnancy. The foetus is in the normal position with head downwards. Magnetic Resonance is replacing X-ray as an imaging system as it has the advantage of not involving ionizing radiation.

In some circumstances, resonance is a nuisance or even a cause of engineering problems. In such instances, resonance should be **avoided**. For example:

1. Vibrations in structures

All mechanical structures have one or more natural frequencies. Care must be taken not to subject a structure to strong external driving force of frequency that matches one of these frequencies, or the resulting oscillations of the structure may rupture it.
For example, the Tacoma Narrows suspension bridge disaster in 1940 caused by wind producing an oscillating resultant force with a driving frequency in resonance with a natural frequency of oscillation of the bridge. An oscillation of large amplitude built up and destroyed the structure.





2. Rattling of aircraft wings

Aircraft designers make sure that none of the natural frequencies at which a wing can vibrate matches the angular frequency of the engines at cruising speed. This is to prevent resonance that can cause the aircraft wings to flap violently.

3 Shattering a wineglass

High-pitched sound waves can shatter fragile objects. For example, an opera singer hitting a top note may shatter a wineglass! This is because of resonance, which occurs when the high-pitched sound wave from the singer matches the high frequency of oscillations of the wineglass.

Practice of science:
Resonance in real life

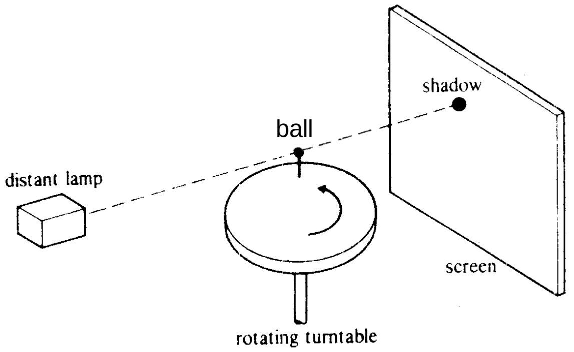
Ways to reduce unwanted resonance in structures or machinery include:

1. increase the degree of damping
2. change the mass or mass distribution of the oscillator or the stiffness of its support to shift its natural frequency away from the driving frequency
3. if possible, change the driving frequency of the driver

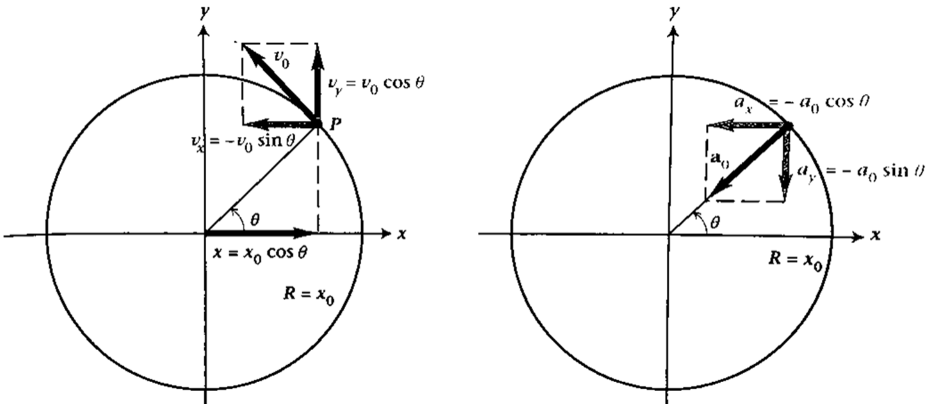
Example 8: 2012 P1Q16
In some circumstances, resonance is useful and in others it is a nuisance.
What is a useful example of resonance?

A The air column in a clarinet resonates when air is blown over the reed.
B The rear-view mirror of a car resonates at a particular engine speed.
C A suspension bridge resonates in a gusting wind.
D An ornament placed on a loudspeaker cabinet resonates when a particular note sounds.

Appendix A: Relationship between SHM and uniform circular motion



In the arrangement shown, a ball attached to a turntable moves with uniform horizontal circular motion. Its shadow is projected on a screen using a distant lamp. The shadow is seen to oscillate back and forth horizontally. It can be shown that the motion of the shadow is SHM!



If P is a point moving with constant tangential speed v_o and radial acceleration a_o in a circle of radius x_o, then the projection or the shadow of P on the x-axis oscillates back and forth between + x_o and $-x_o$.

The projection of the position, velocity and the acceleration on the x-axis will be $x = x_o cos\theta$, $v_x = -v_o sin\theta$ and $a_x = -a_o cos\theta$ respectively.

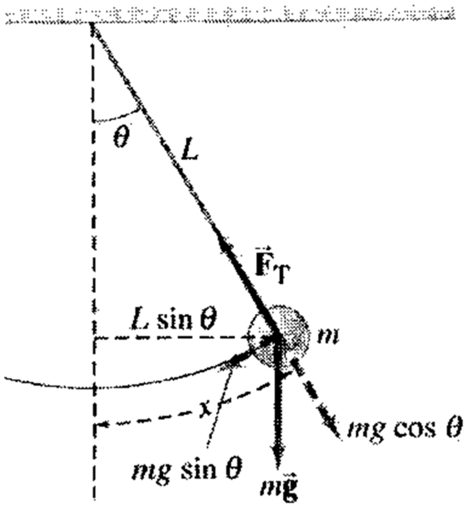
For circular motion of uniform angular velocity ω, θ increases steadily with time t.

I.e. $\theta = \omega t$. Then $x = x_o cos\omega t$, $v_x = -v_o sin\omega t$ and $a_x = -a_o cos\omega t$ respectively.

i.e. $a_x = -a_o\left(\dfrac{x}{x_o}\right) \propto -x$, thus motion along x is SHM!

Appendix B: Simple pendulum as an approximate SHM

Certain oscillatory motion are approximately SHM under specific conditions. For example, the simple pendulum.



Consider a mass *m* attached to the free end of a light piece of string of length *L*.

If the mass is given a small horizontal displacement *x* to the right and then released, the net force (which is the restoring force) on the mass would be

$F = -mg\sin\theta$ or $mg\sin\theta$ to the left.

For small angle θ, $\sin\theta \approx \theta \approx \dfrac{x}{L} \Rightarrow F \approx -mg\dfrac{x}{L}$

From Newton's Second law, $F = ma = -mg\dfrac{x}{L}$

$=> a = -g\dfrac{x}{L}$ and hence $a \propto -x$

Thus motion is simple harmonic provided θ is small.

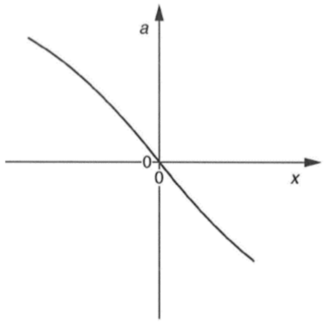
Compare with defining equation of SHM $a = -\omega^2 x$,

$\omega^2 = \dfrac{g}{L}$ and $T = \dfrac{2\pi}{\omega} => T = 2\pi\sqrt{\dfrac{L}{g}}$

Tutorial questions: (Solutions to Qs 1, 7, 9, 13, 16 are provided)

SHM

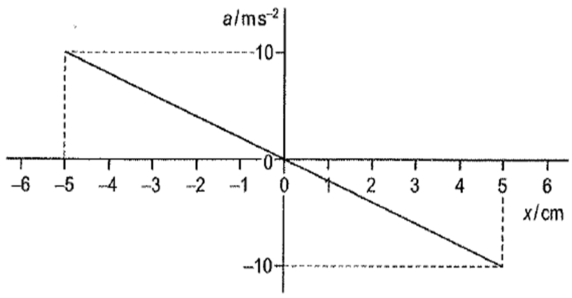
1. 2015P3Q8b
A ball is hanging on an extensible cord. The ball is displaced vertically and then released. The variation with displacement x of the acceleration a of the ball is shown in the figure.



Use the figure to state and explain how it can be deduced that (i) the ball is oscillating and (ii) the oscillations are not simple harmonic.

(i) The line passes through the origin and has a negative slope. This implies the acceleration is always opposite in direction to the displacement from the equilibrium position, indicating the presence of a restoring force for oscillations.
(ii) The line is not straight especially at higher displacement x. This implies the acceleration is not proportional to the displacement, violating a condition for simple harmonic motion.

2. 2008P1Q15.
The defining equation for a particle moving in simple harmonic motion is $a = -\omega^2 x$
where a is the acceleration of the particle, x is the displacement and ω is the angular frequency.



The graph shows how a varies with x for a particle moving in simple harmonic motion. What is the amplitude and period of the motion?

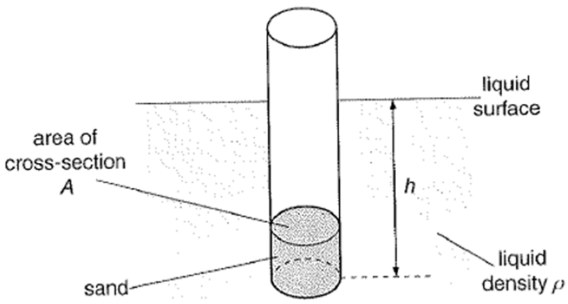
	x_0/cm	period/s
A	5.0	0.44
B	5.0	14
C	10	0.44
D	10	14

3. A particle moves with simple harmonic motion in a straight line with amplitude 0.050 m and period 12 s. Find
   (a) the maximum speed and
   (b) the maximum acceleration of the particle.
   (c) Write down the values of the constants P and Q in the equation x/m = P sin [Q (t/s)] which describes its motion. [0.026 m s^{-1}, 0.014 m s^{-2}, 0.050 m, 0.52 rad s^{-1}]

4. A mass on the end of a light helical spring is given a vertical displacement of 3.0 cm from its rest position and then released. If the subsequent motion is simple harmonic with a period of 2.0 s, through what distance will the bob move in (a) the first 1.0 s, (b) the first 0.75 s. [6.0 cm, 5.1 cm]

5. In order to check the shutter speed of a camera, a photograph of a simple pendulum of period 2.0 s and amplitude 0.030 m is taken. Examination of the photograph shows that the shutter opened as the pendulum passed through equilibrium position and closed after it had moved 0.018 m. Calculate the time for which the shutter was open. [0.20 s]

6. 2013P3Q7 (part)
   (a) A tube, sealed at one end, has a uniform area of cross-section *A*. Some sand is placed in the tube so that it floats upright in a liquid of density ρ as shown in the figure.



   The total mass of the tube and the sand is *m*.
   The tube floats with its base a distance *h* below the surface of the liquid.

   Draw and label arrows on the figure above to show the forces acting on the tube.

   (b) The tube is displaced vertically and then released.
   For a displacement *x*, the acceleration *a* of the tube is given by the expression

   $a = -\left(\frac{\rho A g}{m}\right)x$ where *g* is the acceleration of the free fall.

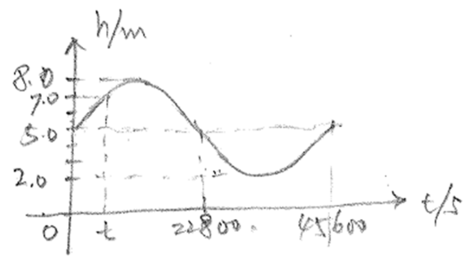
   (i) Explain why the expression leads to the conclusion that the tube is performing simple harmonic motion.
   (ii) The tube has a total mass *m* of 32 g and the area *A* of its cross-section 4.2 cm^2. It is floating in liquid of density ρ of 1.0 x 10^3 kg m^{-3}.
   Show that the frequency of oscillation of the tube is 1.8 Hz. [1.8 Hz]

7. In one harbour, the equation for the depth of water is $h = 5.0 + 3.0\sin\left(\frac{2\pi t}{45600}\right)$ where *h* is given in metres and *t* is the time in seconds.

For this harbour, calculate
(a) the maximum depth of water,
(b) the minimum depth of water,
(c) the time interval between high and low water,
(d) two values of t at which the water is 5.0 m deep,
(e) the length of time for each tide during which the water depth is more than 7.0 m.
[8.0 m, 2.0 m, 22800 s, 0 s and 22800 s, 12200s]

(a) 5.0 + 3.0 = 8.0 m
(b) 5.0 – 3.0 = 2.0 m
(c) $\frac{T}{2} = 22800$ s
(d) 0 s, 22800 s
(e) $2.0 = 3.0\sin\left(\frac{2\pi}{45600}t\right) \Rightarrow t = 5296$ s

Required time = 22800 – 2t = 12200 s (to 3 s.f.)



8. A particle rests on a horizontal platform which is moving vertically in simple harmonic motion with amplitude 50 mm. Above a certain frequency, the particle ceases to be in contact with the platform.
(a) For this minimum frequency, at which point does contact cease?
(b) Find the lowest frequency at which this occurs.
(c) If, instead, the frequency is kept fixed at 1.2 Hz and the amplitude is increased steadily, what is the minimum amplitude above which the object loses contact with the platform? [2.2 Hz, 0.17 m]

Energy in SHM

9. A vibrating mass of 0.060 kg has amplitude of 8.0 cm and a period of 4.0 s. Find the kinetic energy and potential energy at
(i) the middle of the motion
(ii) the maximum displacement. [4.7×10^{-4} J, 0 J, 4.7×10^{-4} J]

(i) At middle of the motion,
KE is maximum, PE is zero.

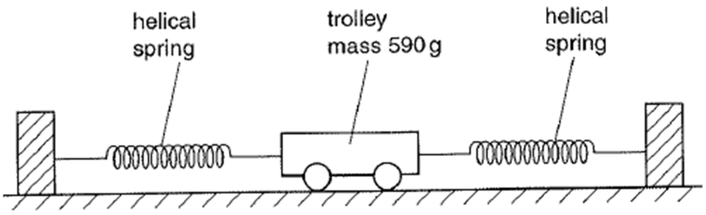
$$KE_{\max} = \frac{1}{2}mv_{\max}{}^2 = \frac{1}{2}m(\omega x_o)^2 = \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2 x_o^2$$

$$= \frac{1}{2}(0.060)\left(\frac{2\pi}{4.0}\right)^2 0.080^2 = 4.7\times10^{-4} \text{ J}$$

(ii) At maximum displacement,
KE is zero, and maximum PE = maximum KE = 4.7 x 10 $^{-4}$ *J.*

10. 2012P3Q3

Two helical springs are attached to a trolley of mass 590 g, as shown in the figure. The trolley is displaced along the axis of the springs and then released. The trolley undergoes simple harmonic motion.

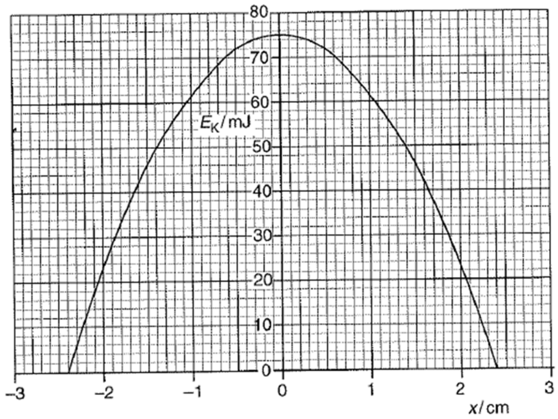


The variation of the kinetic energy E_K of the trolley with displacement x from its equilibrium position is shown in the figure.

(a) Use the data from the figure to calculate the frequency of the oscillation of the trolley.

(b) The trolley loses energy so that its maximum kinetic energy is reduced by 40 mJ. Use the figure, without any further calculation, to determine the amplitude of the oscillations. Show your construction on the figure.

[3.34 Hz, 1.65 cm]

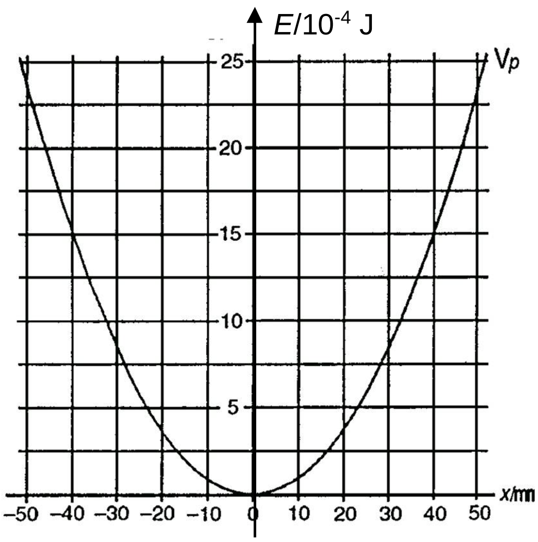


11. N1990P2Q3

(a) Calculate the gain in potential energy when a mass of 150 g is raised through 1.0 mm.

(b) A simple pendulum consists of a light inextensible string to which is attached a bob mass 150 g. The variation with horizontal displacement x of the potential energy E of the bob, is shown in the figure.

In order to set the pendulum into oscillation, the bob is displaced sideways (keeping the string taut) until its centre of mass is raised vertically through 1.0 mm and then released. Using the axes of the figure provided, sketch labelled graphs to show the variation with x of (i) the total energy, (ii) the kinetic energy as the pendulum oscillates.



(c) By reference to the figure, or otherwise, write down the amplitude of oscillation of the pendulum.

[1.5×10^{-3} J, 40 mm]

12. N2008P3Q6

(a) Distinguish between *frequency* and *angular frequency* for a body undergoing simple harmonic motion.

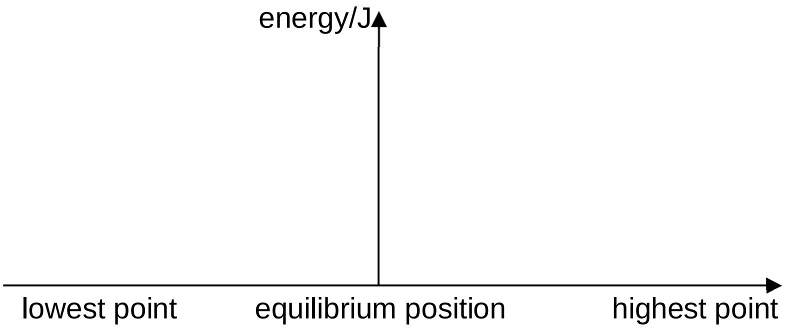
(b) A spring that has an unstretched length of 0.650 m is attached to a fixed point. A mass of 0.400 kg is attached to the spring and gently lowered until equilibrium is reached. The spring has then stretched elastically by a distance of 0.200 m. Calculate for the stretched string (i) the loss in gravitational potential energy of the mass, (ii) the elastic potential energy gained by the mass. Explain why the two answers are different.

(c) The load on the spring is now set into simple harmonic motion of amplitude 0.200 m. calculate (i) the resultant force on the load at the lowest point of its motion, (ii) the angular frequency of the oscillation, (iii) the maximum speed of the mass.

(d) The table below shows the energies of the simple harmonic motion. Complete the table.

	gravitational potential energy/J	elastic potential energy/J	kinetic energy/J	total energy/J
lowest point	0			
equilibrium position				
highest point				

(e) On the axes of diagram below, sketch four graphs to show the shape of the variation with position of the four energies. Label each graph.



[0.785 J, 0.392 J, 3.92 N, 7.00 rad s^{-1}, 1.40 m s^{-1}, 1.57 J]

<u>Damped oscillations, forced oscillations and resonance</u>

13. The suspension of a car may be considered to be a spring under compression combined with a shock absorber which damps the vertical oscillations of the car.

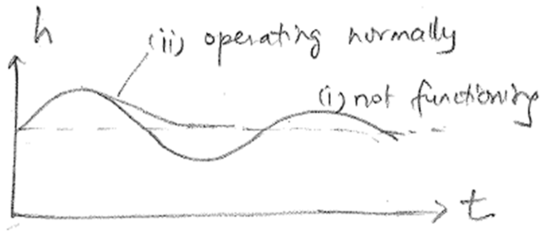
(a) Draw sketch graphs, one in each case, to illustrate how the vertical height of the car above the road will vary with time after the car has passed over a bump if the shock absorber is

(i) not functioning, i.e. slides without resistance,

(ii) operating normally.

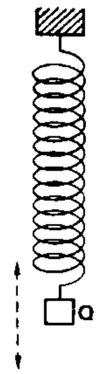
(b) For case (a) (i) above, the amplitude of vertical oscillation of the car becomes very large when the car is driven over a series of bumps at a certain speed. Explain why this happens.

(a)



(b) *The bump on the road acts like a driving oscillator that compresses the springs of the car with a driving frequency that is proportional to the speed of the car. At certain speed, this driving frequency equals the natural frequency of the car. Resonance occurs with maximum energy transfer, resulting in maximum amplitude of vertical oscillations of the car.*

14. The figure shows a mass, suspended from a spring, performing vertical oscillations.



(a) Describe how the direction of acceleration varies with
   (i) the direction of velocity,
   (ii) the direction of the resultant force.

(b) The period of oscillation T is given by the expression $2\pi\sqrt{\frac{m}{k}}$
   where m is the mass and k is the spring constant.
   (i) Calculate the frequency of the oscillation when the mass is 63 g and the spring constant is 1540 N m^{-1}.
   (ii) Calculate the percentage change in the frequency of oscillation when a 5.0 g aluminium foil is wrapped round the mass.

(c) The mass is then immersed in water while still performing vertical oscillations. The resulting motion is said to be a damped simple harmonic motion.
   (i) Explain what is meant by damped simple harmonic motion.
   (ii) State two changes which may be made to the set-up in order to alter the degree of damping.
   (iii) What effects does the degree of damping have on the movement of the mass? [25 Hz, – 4 %]

15. J1997P2Q2
Fig 1 shows a mass which can be made to oscillate vertically between two springs.

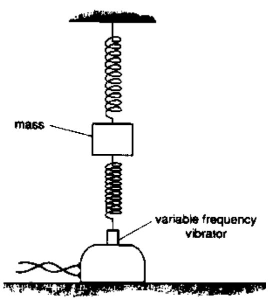


Fig 1

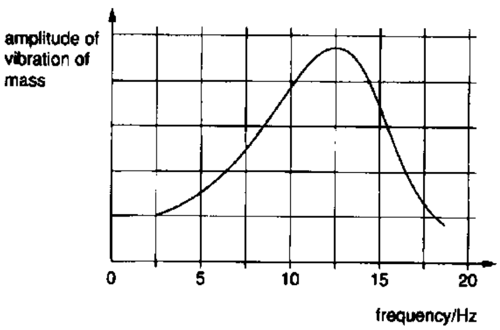


Fig 2

The vibrator itself has constant amplitude. As the frequency is varied, the amplitude of vibration of the mass is seen to change as shown in the graph in Fig 2.

(a) Name the phenomenon which is illustrated in the graph in Fig 2.

(b) Briefly explain this phenomenon for the above case.

(c) For the mass vibrating at maximum amplitude, calculate

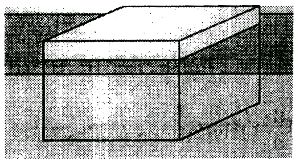
(i) the angular frequency,

(ii) the period.

(d) A light piece of card is fixed to the mass with its plane horizontal. Draw a line in Fig 2 to show the variation with frequency of the amplitude of vibration of the mass.

(e) State and explain one situation in which the phenomenon illustrated here is used to advantage. [78.5 rad s^{-1}, 0.080 s]

16. A block of wood of mass *m* floats in still water, as shown in the picture. When the block is pushed down into the water, without totally submerging it, and is then released, it bobs up and down in the water with a frequency *f* given by the expression $f = \frac{1}{2\pi}\sqrt{\frac{28}{m}}$



where *f* is measured in Hz and *m* in kg.
Surface water waves of speed 0.90 m s^{-1} and wavelength 0.30 m are then incident on the block. These cause resonance in the up-and-down motion of the block.

(a) Explain what is meant by the term *resonance*.

(b) Calculate

(i) the frequency of the water waves,

(ii) the mass of the block.

(c) Describe and explain what happens to the amplitude of the vertical oscillations of the block after the following changes are made independently:

(i) water waves of larger amplitude are incident on the block,

(ii) the distance between the wave crests increases,

(iii) the block has absorbed some water. [3.0 Hz, 78.8 g]

(a) Resonance occurs when the driving frequency of the external periodic force equals the natural frequency of the oscillating system. There is maximum energy transfer and the system oscillates with maximum amplitude.

(b) (i) $f_d = \frac{v}{\lambda} = \frac{0.90}{0.30} = 3.0\,\text{Hz}$

(ii) At resonance, $f_d = f_o = \frac{1}{2\pi}\sqrt{\frac{28}{m}} \Rightarrow 3.0 = \frac{1}{2\pi}\sqrt{\frac{28}{m}} \Rightarrow m = 78.8\text{ g}$

(c) (i) Water wave of larger amplitude has greater energy. Thus there is greater energy transfer at resonance. Hence the amplitude of oscillation increases.

(ii) Increased distance between the wave crests means increased wavelength. Driving frequency $f_d = \frac{v}{\lambda}$ *decreases and no longer equal the natural frequency. Then no resonance occurs and the amplitude of oscillation decreases.*

(iii) The block that has absorbed some water has increased mass. Natural frequency $f_o \propto \frac{1}{\sqrt{m}}$ *decreases and no longer equal the driving frequency. Then no resonance occurs and the amplitude of oscillation decreases.*