Student's copy

Unit 10: Wave Motion

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms displacement, amplitude, period, frequency, phase difference, wavelength and speed

(b) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$

(c) recall and use the equation $v = f\lambda$

(d) show an understanding that energy is transferred due to a progressive wave

(e) recall and use the relationship, $intensity \propto (amplitude)^2$

(f) show an understanding of and apply the concept that wave from a point source and travelling without loss of energy obeys an inverse square law to solve problems

(g) analyse and interpret graphical representations of transverse and longitudinal waves

(h) show an understanding that polarisation is a phenomenon associated with transverse waves

(i) recall and use Malus' law ($intensity \propto cos^2\theta$) to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter

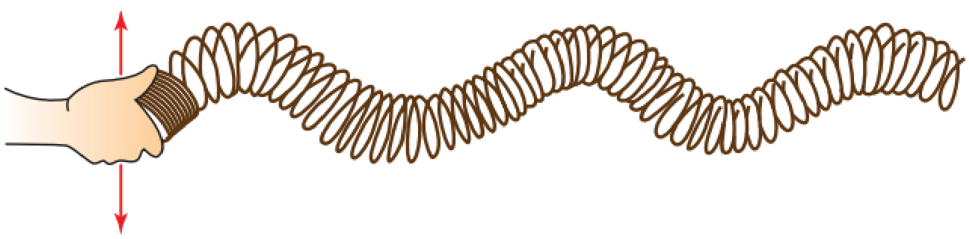
(j) determine the frequency of sound using a calibrated oscilloscope

(k) determine the wavelength of sound using stationary waves.

*(l) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes

*(m) show an understanding of experiments which demonstrate stationary waves using microwaves, stretched strings and air columns

*learning outcomes of topic of Superposition



1 INTRODUCTION

A wave is initiated by a vibrating object (the source) and it travels away from the object.

The particles of the medium vibrate about their rest position at the same frequency as the source.

The wave **transfers energy** between two points in a medium without any net transfer of the matter or material in the medium.

2 PROGRESSIVE WAVES *LO(d)*

A *progressive* wave transfers energy from one place to another without the transfer of medium.

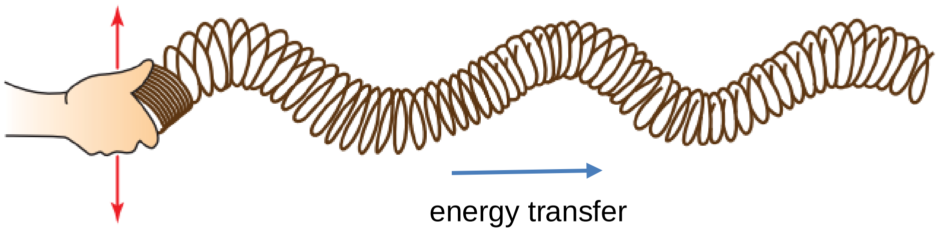
Sound waves, waves on a string, electromagnetic waves etc are progressive waves by nature. Mechanical waves require a medium, such as string or air, to travel; whereas electromagnetic waves can travel through a vacuum, without any medium. (Read Appendix A for EM waves).

Progressive waves can be further classified into **transverse** and **longitudinal** waves.

2.1 Transverse Waves

Transverse waves are waves in which the **oscillation** of the particles of the medium is **perpendicular** to the **direction of energy transfer**.

The passage of a wave through a medium can be demonstrated using a 'slinky' spring.



If the spring is subjected to a repeated up and down motion, a transverse wave is set up. The individual parts of the spring oscillate in the vertical plane while the wave transfer energy horizontally along the spring as shown in the figure above.

Other examples of transverse waves are waves on strings, seismic waves, water waves and electromagnetic(EM) waves.

2.2 Longitudinal Waves

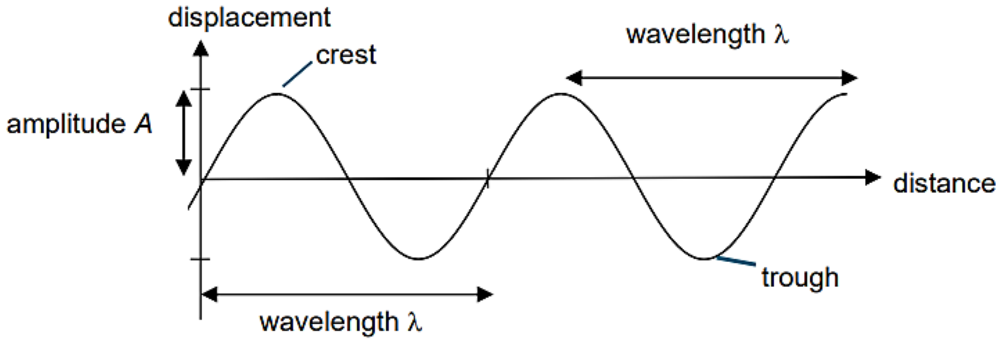
Longitudinal waves are waves in which the **oscillation** of the particles of the medium is **parallel** to the **direction of energy transfer**.

If the slinky spring is repeatedly given a push and a pull, a longitudinal wave is created. The individual parts of the spring move back and forth about their equilibrium positions, causing a series of **compressions** and **rarefactions**.

Compressions occur where the loops of the spring are closer together than at equilibrium while rarefactions appear where the loops are farther apart. All oscillations are parallel to the direction of the energy transfer in a longitudinal wave.

Examples of longitudinal waves are sound waves and longitudinal waves on a slinky spring.

3 WAVE TERMINOLOGY *LO(a),(b),(c)*



Term	Symbol	Unit	Definition
displacement	*x or y*	m	The distance in a specified direction from the equilibrium position
amplitude	*A*	m	the maximum displacement of any particle/point on the wave from its equilibrium position
wavelength	λ	m	The distance between any two successive particles/points on the wave that are in phase (e.g. the distance between successive crests or troughs)
period	*T*	s	The time taken for one complete oscillation of a particle/point in a wave (or the time taken for the wave to travel through one wavelength)
frequency	*f*	Hz or s^{-1}	The number of oscillations(or cycles) per unit time of a point in a wave
wave speed	*v or c*	$m\ s^{-1}$	The distance travelled by the wave per unit time

<u>Worked Example 1</u>: Derive the wave equation $v = f\lambda$

Solution:

By defintition, speed =

By definition, in one period *T*, the distance travelled by the wave is one wavelength λ.

Hence speed v =

Since frequency f = 1/T

$\Rightarrow v = f\lambda$

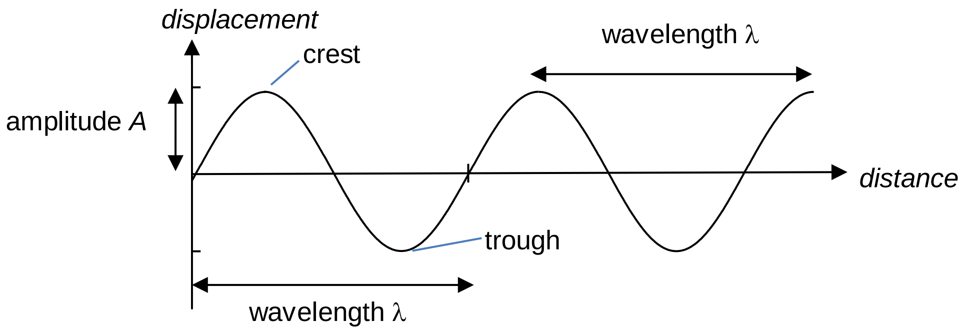
4 GRAPHICAL REPRESENTATION OF WAVES *LO(g)*

There are two types of graphs that are generally used when describing waves. They are displacement–distance and displacement–time graphs. The graphs are sinusoidal (meaning sine of cosine graphs) because the particles in the wave are vibrating in s.h.m..

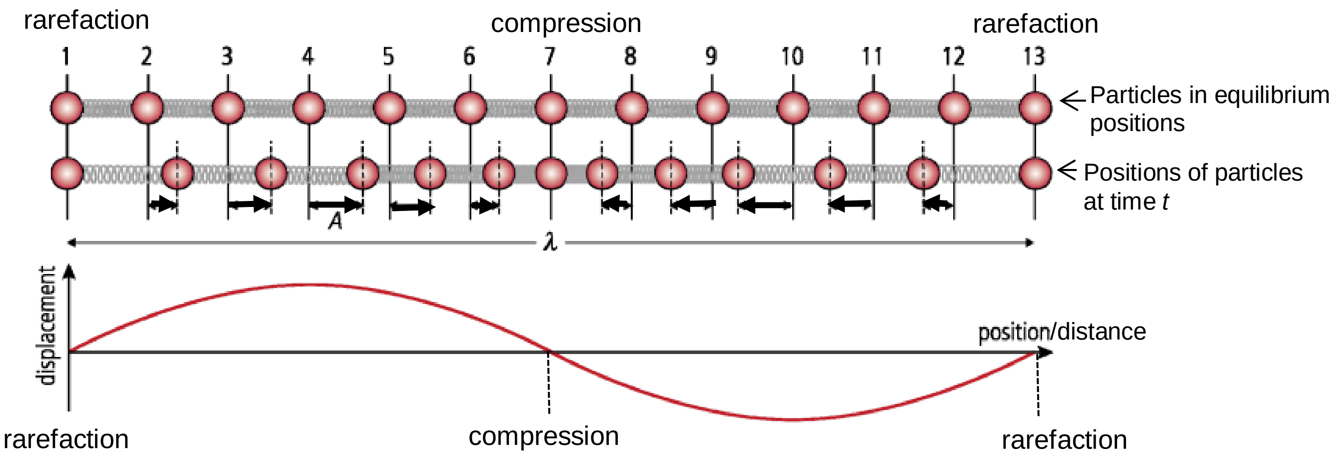
4.1 Displacement-distance graph

This shows how the displacements of the *particles (or points)* in the wave vary with the distance from the source *at a particular instant*. Such a graph is like a **snapshot** of the wave.

This graph can be used to determine the **wavelength** (distance between successive crests or successive troughs).



For **longitudinal waves** such as sound waves, the displacement of the particles is parallel to the motion, so a wave profile is more difficult to draw. But a displacement-distance graph identical to that for transverse waves can be drawn with positive displacements to the right of the equilibrium position and negative displacements to the left.



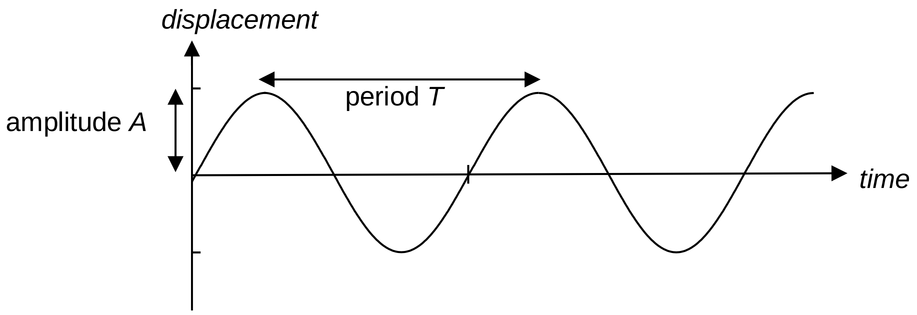
For longitudinal sound waves, the regions of high pressure/density are called compressions and the regions of lower pressure/density are called rarefactions.

The wavelength of a longitudinal wave is the distance between successive compressions, or the distance between successive rarefactions.

4.2 Displacement-time graph

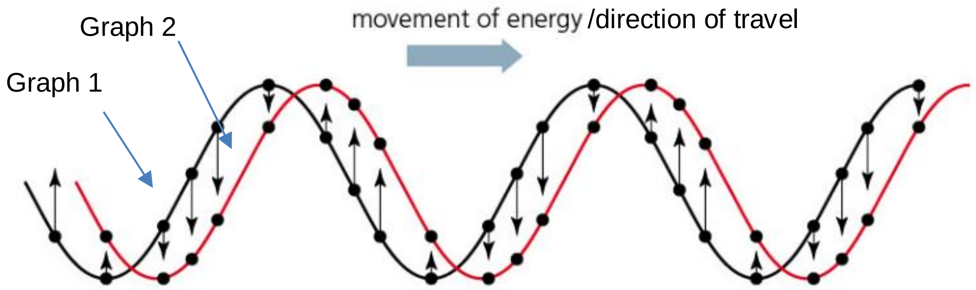
This shows how the displacement of a ***single particle (or point)*** in the wave varies with time. It is the graph of a particle performing s.h.m. with the same frequency as the wave.

From this graph, the **period** of the wave motion can be determined (interval between successive crests or successive troughs on the graph).

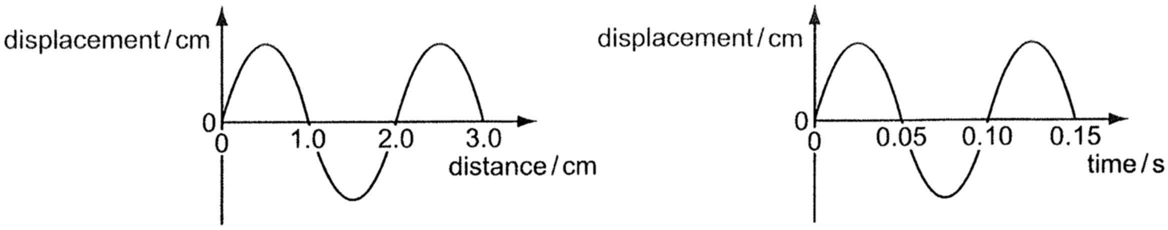


4.3 Graphical representation of a progressive wave

Using a sequence of displacement–distance graphs can provide a good understanding of how the position of an individual particle/point changes with time in a progressive wave. In the figure below, graph 1 represents a progressive wave moving to the right, the arrows shows the direction of movement of particles at that instant. The second graph 2, represents the position of the same wave **a short time later**. Notice that the crest in Graph 2 has shifted to the right, showing that the wave is travelling to the right.



<u>Worked Example 2</u>
The displacement-distance and displacement-time graphs are for a water wave produced in a ripple tank.



What is the speed of the water wave?

A 0.1 m s^{-1} **<u>B</u>** 0.2 m s^{-1} **C** 10 m s^{-1} **D** 20 m s^{-1}

5 PHASE DIFFERENCE *LO(a)*

Phase difference ($\Delta\phi$) **is a measure, in angular form, of the fraction of a cycle two particles in a wave or two waves are out of step by.**

Two particles are exactly ***in phase*** if they are precisely in the same state of disturbance. For instance, any two crests will be in phase and hence, they have zero phase difference. This is the same for any two troughs. However, a crest and a trough will be exactly *180° or π radians out of phase* (or ***anti-phase***).

Phase difference. due to a separation Δx in distance:

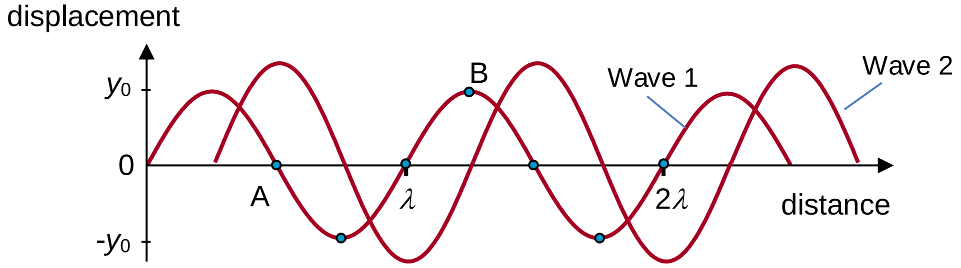
$$\Delta\phi = \frac{\Delta x}{\lambda} 2\pi$$

Phase difference due to a separation Δt in time:

$$\Delta\phi = \frac{\Delta t}{T} 2\pi$$

Worked Example 3

Diagram shows the displacement-distance graphs of Wave 1 and Wave 2 at an instant of time. The waves have the same wavelength but unequal amplitudes.



(a) What is the phase difference between points A and B on Wave 1?
Solution: Phase difference $\Delta\phi = \frac{\Delta x}{\lambda} 2\pi$ = ¾ x 2π = 3/2 π

(b) What is the phase difference between Wave 1 and Wave 2?
Solution: Phase difference $\Delta\phi = \frac{\Delta x}{\lambda} 2\pi$ = ¼ x 2π = ½ π

7 ENERGY AND INTENSITY *LO(d),(e),(f)*

The energy transported by a progressive wave is proportional to the *square* of its amplitude, (similar to energy in shm)

$$E \propto A^2$$

The Intensity of a wave is defined as **the rate of energy flow per unit area perpendicular to the direction of energy transfer.**
Unit: W m^{-2}.

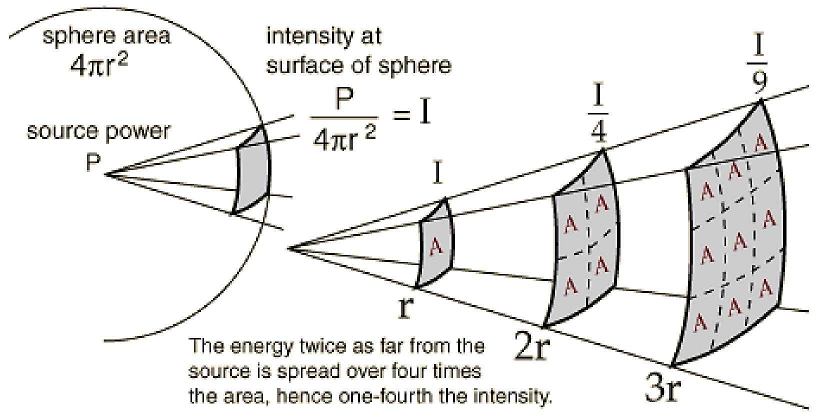
$$\text{Intensity} = \frac{E/t}{\text{Area}} = \frac{\text{Power}}{\text{Area}}$$

Since energy $E \propto A^2$, the intensity of a wave is also proportional to the *square* of its amplitude.

$$\boxed{I \propto A^2}$$

7.1 Intensity of waves from a Point Source

The intensity of a wave is also dependent on the distance it has travelled from its source.
Let *P* be the energy emitted per unit time equally in all directions by a *point* source. Assume no power losses in the medium.



At a distance *r* from the source, the waves will pass through the surface(spherical) area $4\pi r^2$.

Thus intensity of the wave at a distance *r*,

$$I = \frac{Powe}{Area} = \frac{P}{4\pi r^2}$$

If the source has a constant power output, then intensity

$$\boxed{I \propto \frac{1}{r^2}}$$

Thus intensity of waves from a point source follows an **inverse square law** with distance.

Combining the equations above, the amplitude *A* is inversely proportional to the distance *r* from the point source, i.e.

$$\boxed{A \propto \frac{1}{r}}$$

Worked Example 4

A 20 W loudspeaker is emitting sound at its full power in all directions. Calculate
(a) the intensity of sound at a distance of 10 m away.
(b) the power received by a microphone of cross-section area 4.0 cm^2 placed at a distance 10 m away from the loudspeaker.
(c) the amplitude of the vibrations at 10 m given that at 2.0 m, the amplitude of the vibrations is 4.0 cm.

Solution:

(a) $I = \frac{P}{4\pi r^2} = \frac{20}{4\pi(10)^2} \quad =$

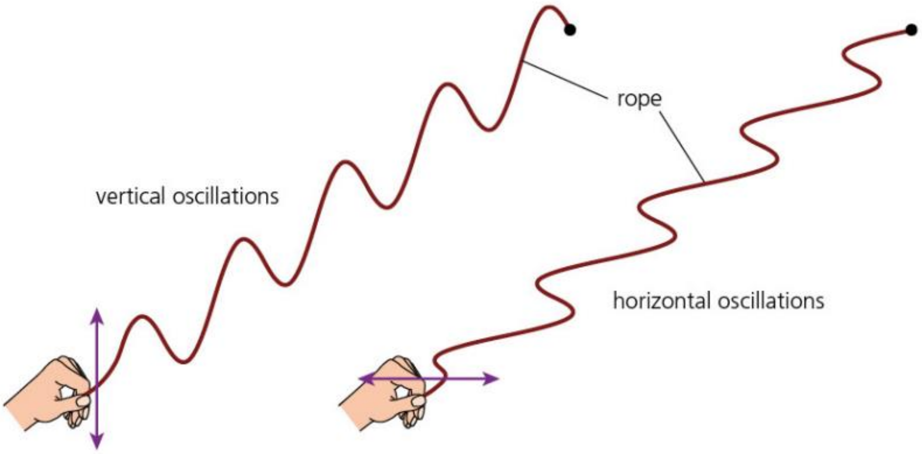
(b) P' = Intensity x Area = $I\,S$ = 0.016 × 4.0 × (10^{-2} x 10^{-2}) =

(c) $A \,\alpha\, \frac{1}{r} \rightarrow \frac{A'}{4.0} = \frac{2.0}{10} \rightarrow A' =$

8 POLARISATION

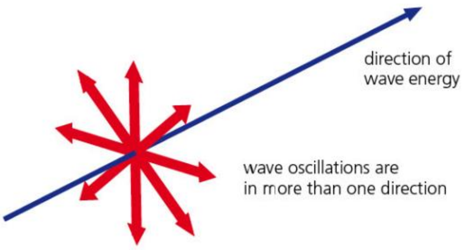
LO(h)

Consider sending transverse waves along a rope. If your hand only oscillates vertically, the rope will oscillate vertically. The wave is said to be plane polarised because it is only oscillating in one plane. If your hand oscillates horizontally, it will produce a wave polarized in the horizontal plane.

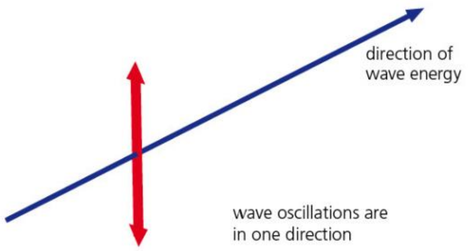


A wave is *plane polarised* if all oscillations transferring the energy of the wave are in the same plane.

An unpolarised wave, on the other hand, has a many randomly possible planes of oscillation. An example of unpolarised waves are EM waves from light sources, which frequently emit transverse waves that oscillate in many planes. If unpolarised EM waves pass through a polarising filter, the transmitted wave will oscillate in one perpendicular plane. The wave is said to be **plane polarised.**



(a) unpolarised electromagnetic wave

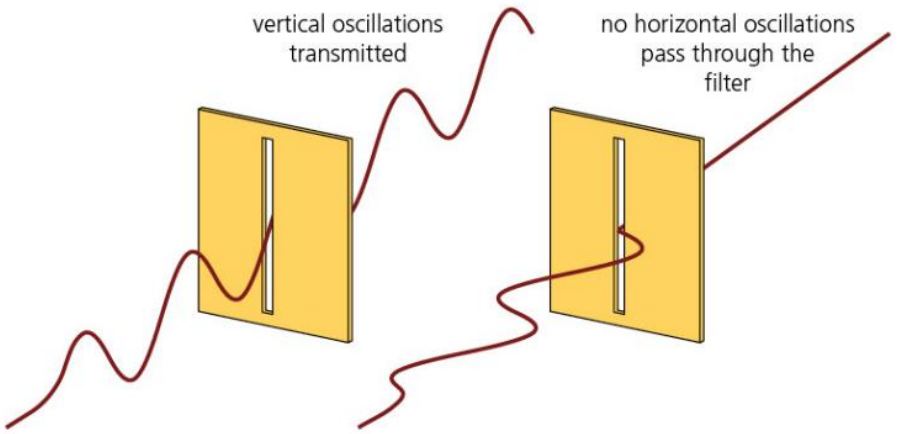


(b) plane polarised electromagnetic wave

8.1 Polarisation of Transverse Rope Waves

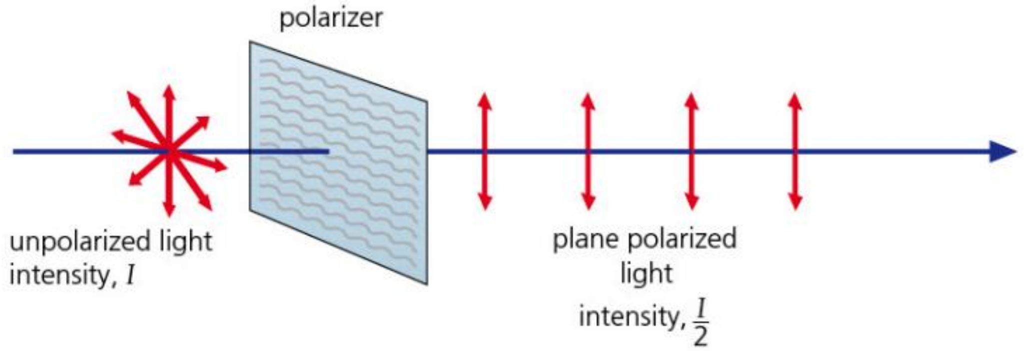
Figure shows a transverse wave on a rope passing through a narrow vertical slit.

Vertically polarised waves, oscillating parallel to the slit, would pass through, but the horizontally polarised wave is blocked.



8.2 Polarisation of EM Waves

Unpolarised light can be converted into plane-polarised light by passing it through a filter called a **polariser**. Polarising filters for light are made of long-chain molecules mostly aligned in one direction. Components of the oscillating field parallel to the long molecules are absorbed, whereas components of the field perpendicular to the molecules are transmitted. The transmitted light is polarised in the vertical plane.



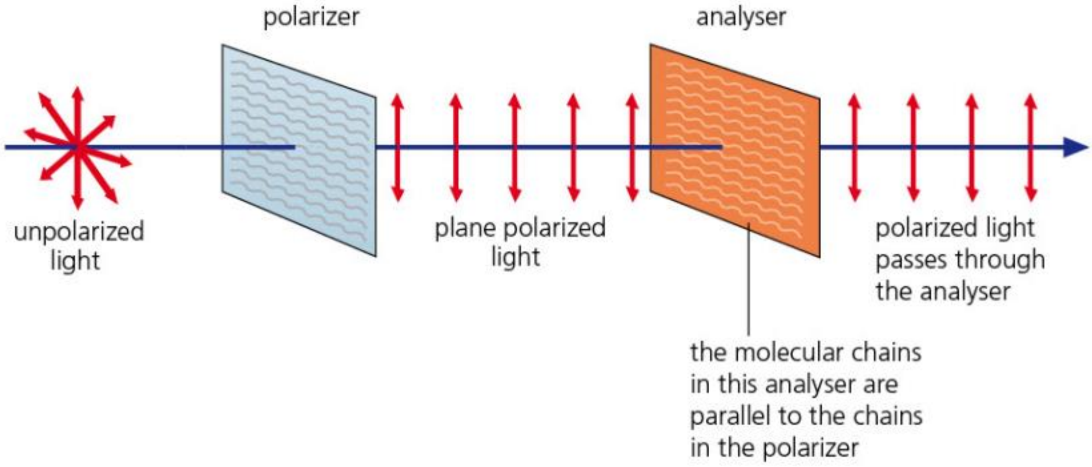
Polarisation is the process of converting an unpolarised wave into a plane polarised wave.

Polarisation can only be achieved with transverse waves.

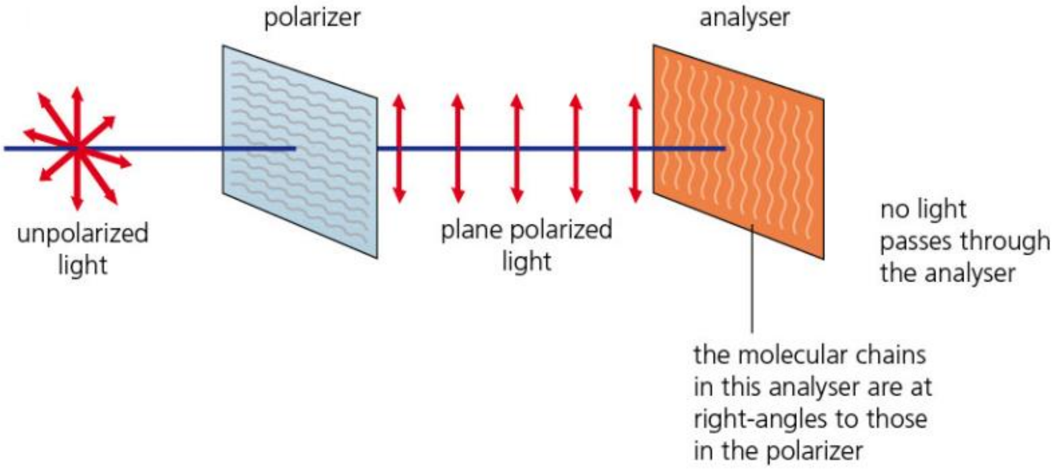
Question" Why is it not possible to polarise longitudinal waves?

Answer: It is not possible longitudinal waves to be polarised because for longitudinal waves, the direction of oscillation is along the direction of energy propagation, so no planes can be cut off.

If a second polarising filter (slso called an analyser) is placed such that it transmits waves in the same plane as the first, the waves will pass through unaffected.

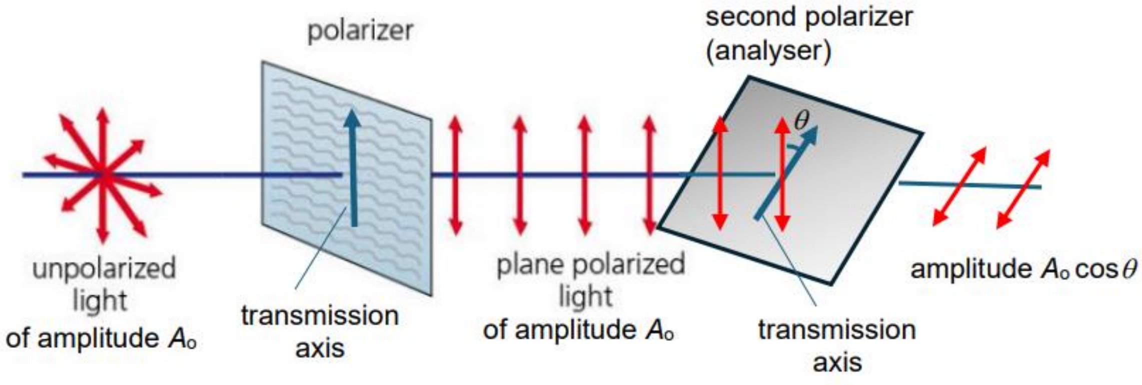


If the analyser is rotated at 90°, then no light will be transmitted by the analyser.

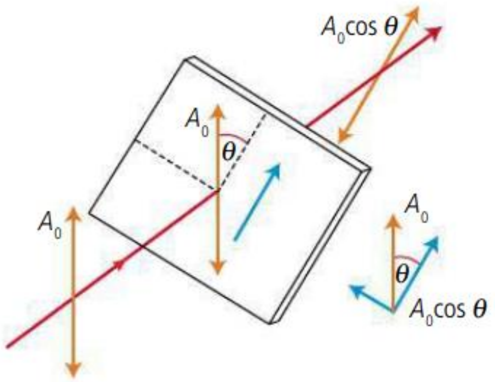


8.3 Malus' Law *LO(i)*

If a beam of plane-polarised light of amplitude A_o strikes a polariser(analyser) whose transmission axis is at an angle θ to the incident polarised light, the transmitted light will have a smaller amplitude equal to the component $A_o \cos\theta$. (by vector resolution)



The beam will still emerge plane polarised, oscillating at an angle θ with respect to the incident light and having an amplitude with amplitude $A_o \cos\theta$.



Since intensity of a light beam is proportional to the square of the amplitude, the intensity of a plane polarised beam transmitted by a polariser is given by

$$\frac{I}{Io} = \left(\frac{Ao\cos\theta}{Ao}\right)^2$$

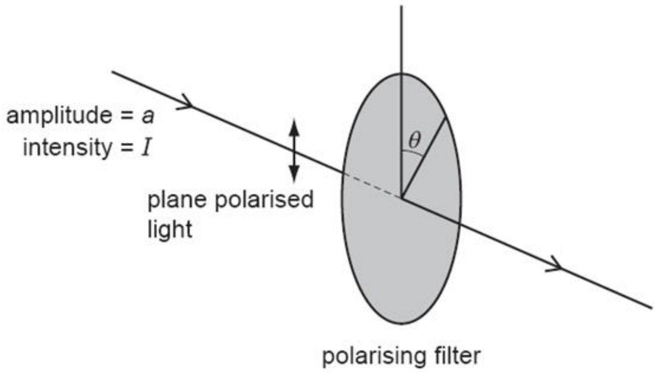
$$\boxed{I = I_0 \cos^2 \theta}$$

where I_0 is the incident intensity and θ is the angle between the polariser transmission axis and the plane of polarisation of the incident beam.

This equation is known as **Malus' Law**.

<u>Worked Example 5</u>

A vertically plane polarised light of amplitude *a* is passed through a polarising filter as shown. The filter is orientated so that the plane of polarisation of the transmitted wave is at an angle of θ to the vertical.

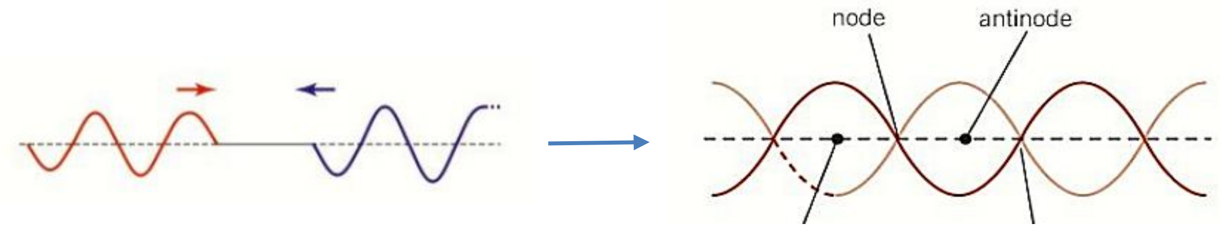


The intensity of the initial beam is I.
What is the intensity of the emerging light when θ is 60.0°?

<u>A</u> 0.250 I **B** 0.500 I **C** 0.750 I **D** 0.866 I

9 STATIONARY WAVES (STANDING WAVES)

When **two progressive waves** of the **same frequency** and **amplitude** moving at the **same speed** in **opposite directions superpose**, a **stationary wave** is formed.

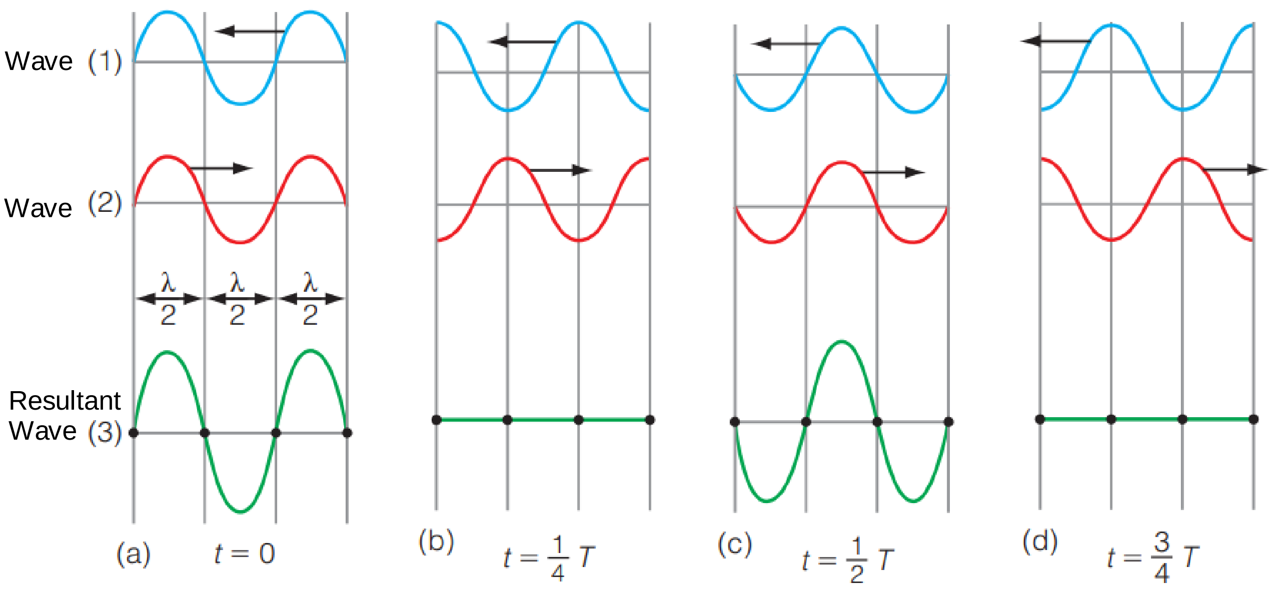


(a) two identical progressive waves superpose

(b) stationary wave pattern

9.1 Formation of stationary waves *LO(l)*

Consider Wave 1 and Wave 2 are two progressive waves travelling in opposite directions with the same frequency and amplitude. The amplitude of the resultant Wave 3 is the resultant amplitude of the two waves superposed. The diagrams show a sequence of snapshots of the wave at different times during one complete cycle.



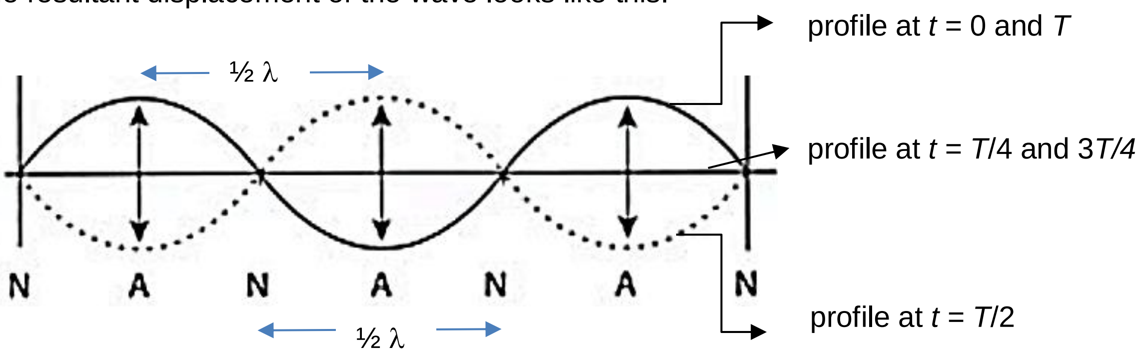
(a) At $t = 0$, Waves 1 and 2 are in phase at the start of the cycle. There is constructive superposition and their displacements add. The combined displacement of both waves is double their original displacement.

(b) At $t = \frac{T}{4}$, both A and B have moved ¼ λ in opposite directions and they are completely antiphase. Their displacements cancel out as the superposition is destructive. Hence the resultant wave has zero resultant displacement.

(c) At $t = \frac{T}{2}$, both A and B are in phase again

(d) At $t = \frac{3T}{4}$, both A and B have moved until they are antiphase to give zero resultant displacement again

Over time, the resultant displacement of the wave looks like this.



There are points along the wave that remain motionless while points on either side are oscillating with the largest amplitude. The points that do not move are called the **nodes** and the points where the wave oscillates with maximum amplitude are called the **antinodes**. The distance between successive nodes or antinodes is ½ λ.

It is clear that the wave profile is not travelling. Hence we call it a **stationary wave** or a standing wave

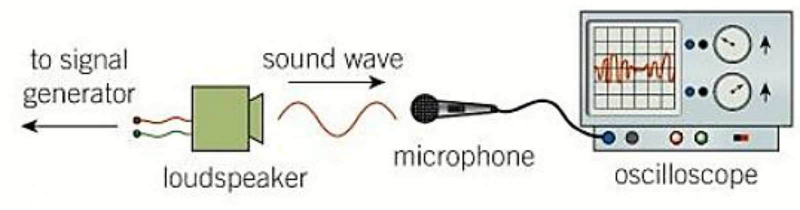
.

9.2 Comparison between stationary wave and progressive wave

	Stationary	**Progressive**
Energy	No net transfer of energy. Energy is confined within the wave and there is interchange of PE and KE.	Energy is transferred in the direction of travel of the wave.
Phase	All particles between two adjacent nodes are in phase. Particles on opposite sides of a node are in anti-phase (i.e. have a phase difference of 180° or π radians).	All particles within one wavelength have different phases.
Amplitude	Varies from zero at the nodes to maximum at the antinodes.	Same for all particles in the wave.
Wavelength	2 × distance between adjacent nodes or antinodes.	Distance between adjacent particles which are in phase. (e.g. between successive crests or successive troughs)
Frequency	All particles vibrate in SHM with same frequency except at the nodes.	All particles vibrate in SHM with same frequency.
Waveform	Does not advance.	Advances with the velocity of the wave.

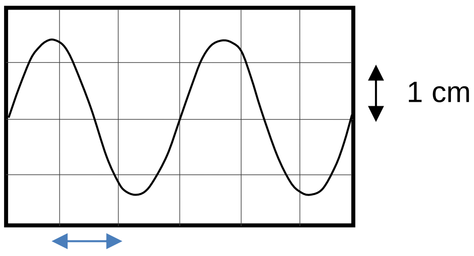
9.3 Determination of Frequency of Sound Using C.R.O. *LO(j)*

The frequency of a sound wave can be measured using a cathode-ray oscilloscope (C.R.O.). The apparatus for this experiment is shown in the diagram below.



The loudspeaker is powered by a signal generator to produce sound waves of a certain frequency.

A microphone placed in front of the loudspeaker picks up the sound and produce a trace on the screen of the oscilloscope. The screen shows a graph of voltage against time for any signal input from the microphone.



The vertical height of the waveform is adjustable using the y-sensitivity in V cm^{1}. The maximum displacement or amplitude of the wave can be determined (in V) from the vertical height.

Each horizontal square of 1 cm on the screen represents a certain time interval which is called the time-base, usually set at μs cm^{-1} or ms cm^{-1}. From the time-base setting, we can determine the period T of the wave. Using this we can calculate the frequency with $f = 1/T$.

For example: in the above diagram,

Let time-base setting : 100 μs cm^{-1}

Period = 3 cm for one wave cycle

$T = 3 \times 100\ \mu s$

$= 300\ \mu s$ or 3.0×10^{-4} s

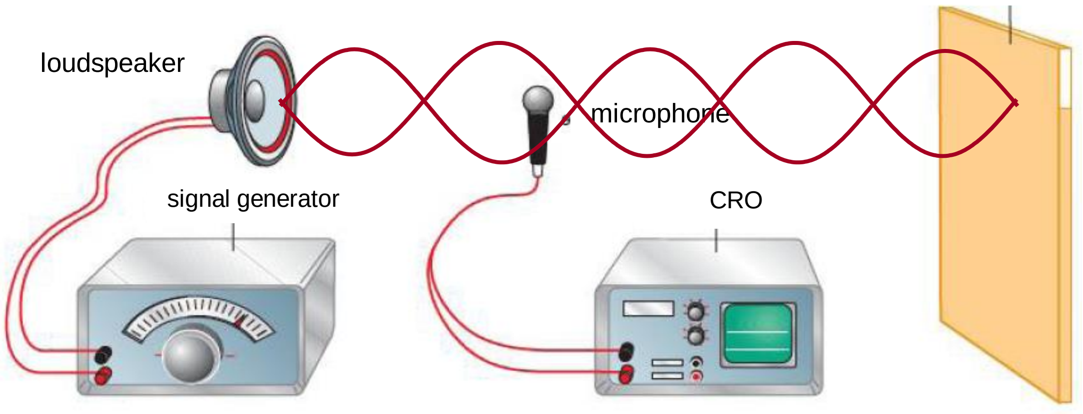
Hence, frequency $f = 1 / T$

$= 1 / (3.0 \times 10^{-4})$

$= 3.3 \times 10^{3}$ Hz

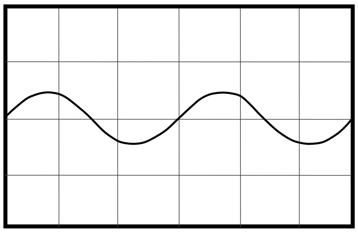
9.4 Determination of Wavelength of Sound *LO(k)*

A setup using a loudspeaker to send a sound wave of known frequency towards the reflector to produce a stationary wave can be used to determine the wavelength of sound.

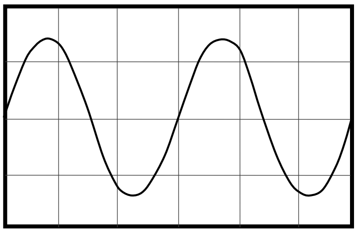


The reflector reflects the wave. The incident and the reflected sound wave superpose to produce a stationary wave pattern consisting of nodes and antinodes.

A microphone connected to the CRO is moved along a straight line between the loudspeaker and the reflector. As the microphone passes by the positions of nodes and antinodes, the amplitude of the display on the CRO will show a corresponding variation. Hence on the screen you will observe:



Near a *pressure* node

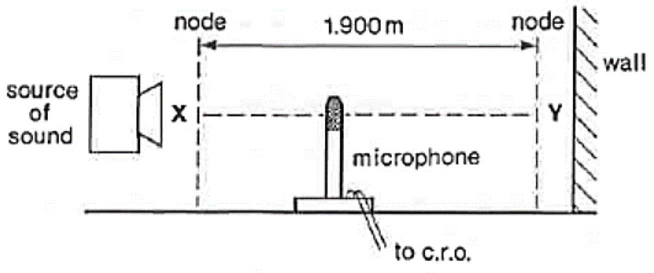


Near a *pressure* anti-node

Therefore when the display on the CRO goes from maximum(antinode) to the next maximum amplitude(antinode), the microphone must have moved by a distance of $\lambda/2$. The wavelength can thus be found.

<u>Worked Example 6</u>

A source of sound of frequency 2500 Hz is placed at a distance from a plane reflecting wall in a large chamber containing a gas. A microphone, connected to a cathode-ray-oscilloscope, is used to detect nodes and antinodes along the line XY between the source and the wall.



The microphone is moved from one node through 20 antinodes to another node, a distance of 1.900 m. What is the speed of sound in the gas?

A 238 m s^{-1} **B** 250 m s^{-1} **C** 330 m s^{-1} **<u>D</u>** 475 m s^{-1}

Solution:

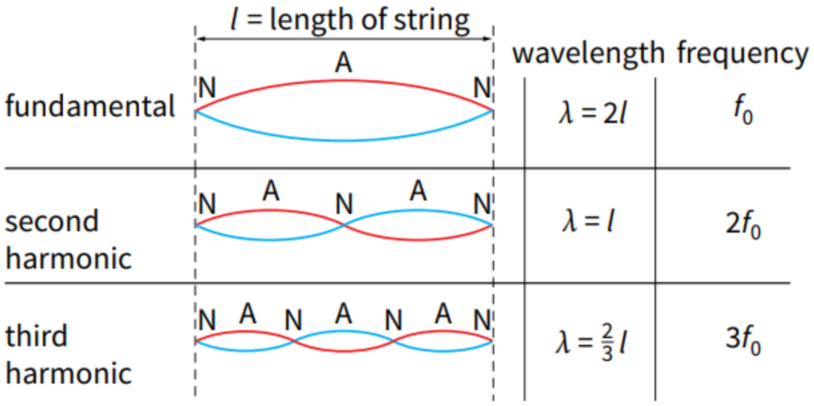
no. of loops $=$ 20

$$\frac{\lambda}{2}=\frac{1.900}{20}\Rightarrow\lambda=0.190\text{ m, v}=f\lambda=2500(0.190)=475\text{ m s}^{-1}$$

9.5 Stationary Waves in Stretched Strings *LO(m)*

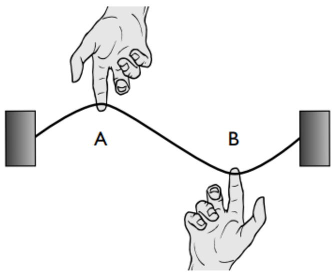
If a string is stretched between two fixed points, these points act as nodes. When the string is plucked, a progressive wave travels along the string and reflected at the fixed ends. This creates two progressive waves travelling in opposite directions that then form a stationary wave.

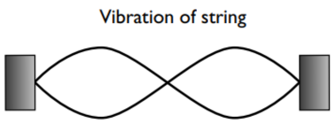
The fundamental frequency f_0 is the minimum or fundamental frequency of a stationary wave for the string. Along with this, the string can form other stationary waves called harmonics at higher frequencies.



Worked Example 7

A stationary wave can be set up by plucking the string at points A and B so that it produces a stationary wave as shown. The length of the string is 0.50 m and the speed of transverse waves on the string is 8.0 m s^{-1}.



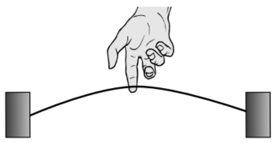


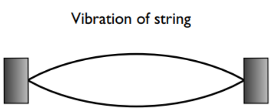
(a) What is the frequency of this stationary wave?

Solution:

(a) 2^{nd} harmonic: wavelength λ = 0.50 m $\Rightarrow$ frequency f = v/λ = 8.0/0.50 = 16 Hz

(b) If the same string is plucked at the centre, what is the frequency of the stationary wave produced?





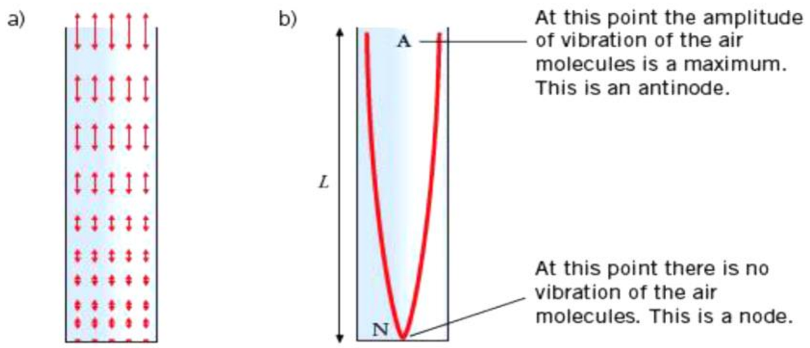
Solution:

(b) 1^{st} harmonic: wavelength λ = 2x0.50 m $\Rightarrow$ frequency f = v/λ = 8.0/1.0 = 8.0 Hz

9.6 Stationary *Sound* Waves in Air Columns (Pipes) *LO(m)*

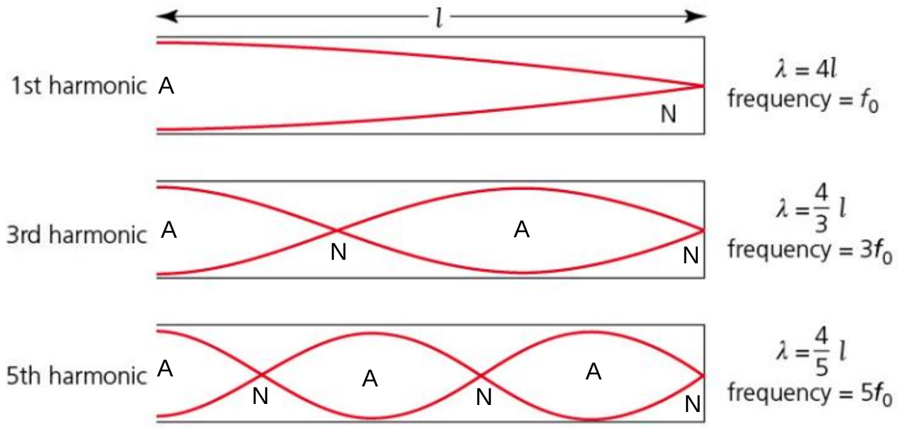
When air is blown across one end of a pipe, progressive sound waves travel to the far end of the pipe where they are reflected and superpose with the incident waves to form stationary *longitudinal* waves.

Recall that sound wave is a longitudinal wave, but the diagram we draw below is more like a transverse wave. Figure (b) shows how we normally represent a standing sound wave, while Figure (a) shows the direction of vibration of the particles along the standing wave.

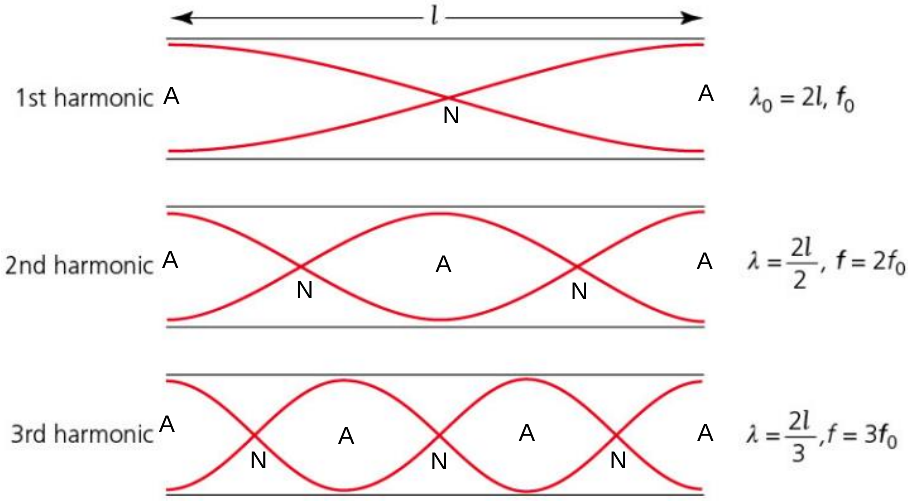


The pipe can be closed at one end (**closed pipe**) or opened at both ends (**open pipe**). In any case, the closed end of a pipe is always a *displacement node* since the air there must be at rest. The open end of a pipe is always a *displacement antinode* because the air there can vibrate freely.

- **Closed pipe**



- **Open pipe**



Worked Example 8

An organ pipe is 0.33 m long, open at one end and closed at the other. The speed of sound is 330 m s^{-1}. Assuming that end corrections are negligible, calculate the frequencies of the fundamental and the 1st overtone(third harmonic),

Solution:

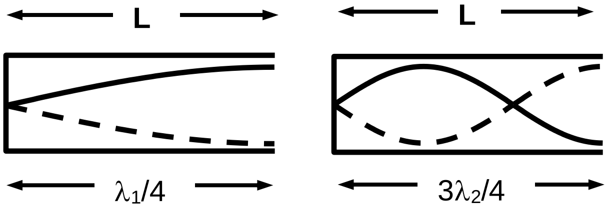
(a) $\lambda_1 = 4L = 1.32$ m

$f_1 = v/\lambda_1 = \mathbf{250\ Hz}$

$\lambda_2 = 4L/3 = 0.44$ m

$f_2 = v/\lambda_2 = \mathbf{750\ Hz}$

or simply $f_2 = 3 f_1 = 750$ Hz

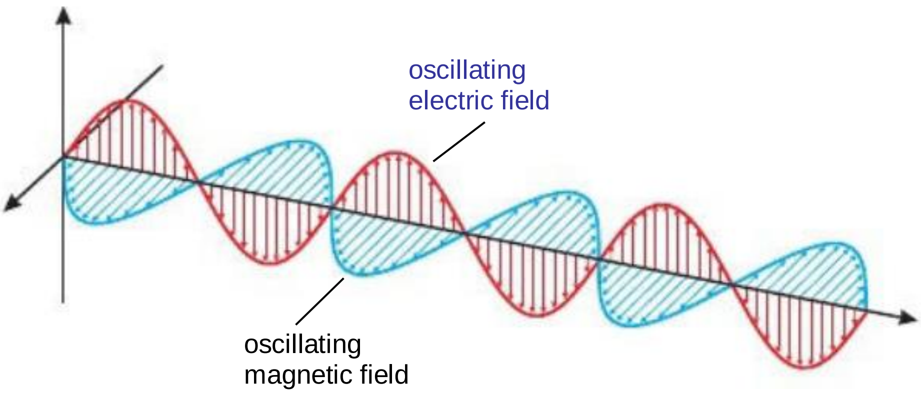


1st harmonic (fundamental)

3rd harmonic (1st overtone)

APPENDIX A: Electromagnetic waves

- Electromagnetic (EM) waves are produced by charged particles when they oscillate. They consist of electric and magnetic fields which oscillate at right angles to each other and to the direction of energy transfer.



Properties of EM waves

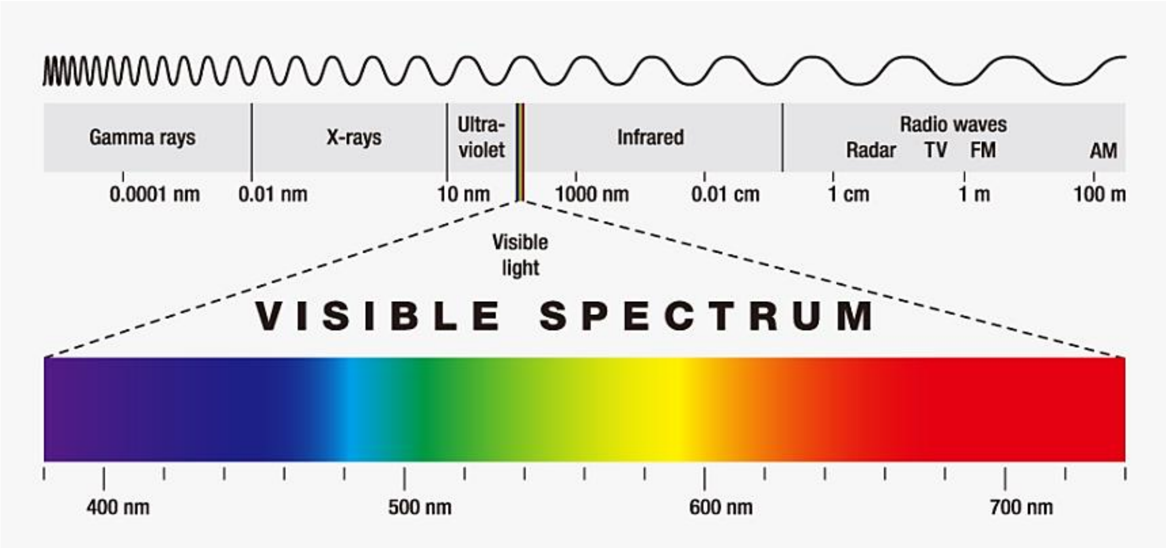
- They are transverse waves.
- They can travel through vacuum (or free space).
- They travel with the same speed, $c = 3.00 \times 10^8$ m s^{-1} in a vacuum.
- They show all the properties common to wave motions: reflection, refraction, interference, diffraction and polarisation.

Electromagnetic Wave Spectrum

The EM spectrum is divided into a series of regions based on the properties of electromagnetic waves in these regions, as illustrated in the diagram below. Note that there is *no clear boundary* between these regions.

Radio waves have the longest wavelength and gamma waves have the shortest wavelengths.

Visible light is in the wavelength range $\mathbf{4.0 \times 10^{-7} - 7.0 \times 10^{-7}}$ **m** or **400 nm – 700 nm** (violet to red region).



Electromagnetic Spectrum

The typical wavelength for each type of EM wave is shown in the table.

EM Wave	*Typical wavelength / m*
X-rays	***10^{-10}***
Ultra-violet	***10^{-8}***
Visible	***10^{-7} (400 – 700 nm)***
Infra-red	***10^{-4} or 10^{-5}***
Radio waves	***0.1 m ~ 10 km***

Appendix B: End Correction in pipes

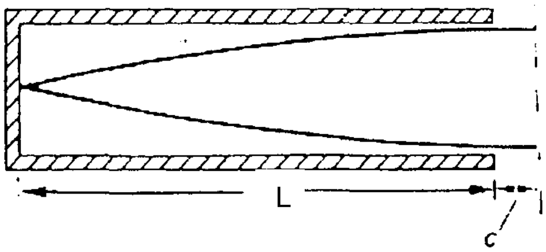
In practice, the vibration of molecules extends outside the pipe at the open end. The excess length at the end is called the ***end correction*** *c.* End correction is a systematic error.

The effective length of the air column at resonance is therefore given by *sum* of the length of the pipe and the end correction.

Consider the fundamental mode of the pipes shown below:

For closed pipe in (a),

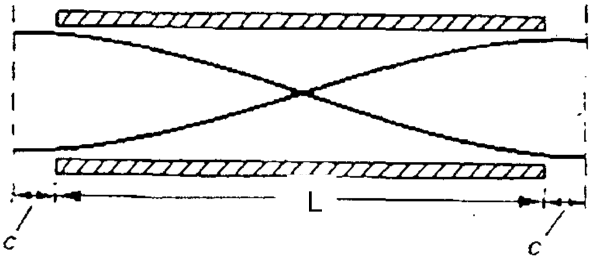
(a)



$$\frac{\lambda}{4} = L + c$$

For open pipe in (b)

(b)



$$\frac{\lambda}{2} = L + 2c$$

<u>**TUTORIAL QUESTIONS ON UNIT 8: WAVE MOTION**</u>
Students are provided with worked solutions for Q5, 8, 12 and 14.

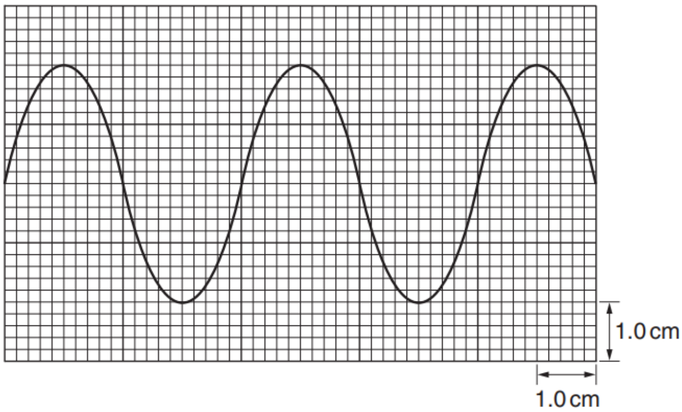
Progressive Waves, Longitudinal and Transverse Waves

1 (a) What is meant by a progressive wave?

(b) Distinguish between transverse and longitudinal waves.

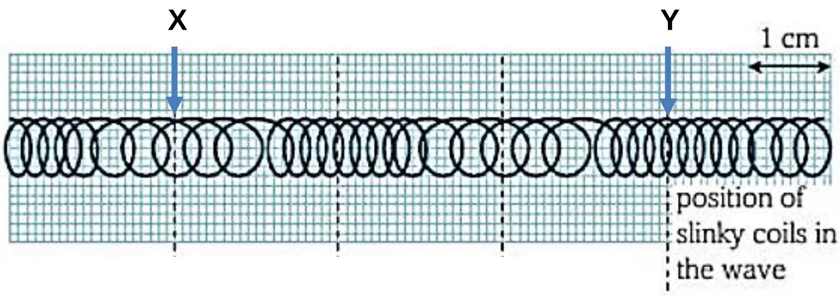
2 (a) A sound wave travels through air. Describe the motion of the air particles relative to the direction of travel of the sound wave.

(b) The sound wave emitted from the horn of a car is detected with a microphone and displayed on a cathode-ray oscilloscope (c.r.o.), as shown in the figure.



The y-axis setting is 5.0 mV cm^{-1}. The time-base setting is 0.50 ms cm^{-1}.
Use the figure to determine the frequency of the sound wave. [500 Hz]

3 The diagram below represents a longitudinal wave travelling in a slinky spring from the left to the right at a frequency of 100 Hz. Two particles in the medium are labelled X and Y.

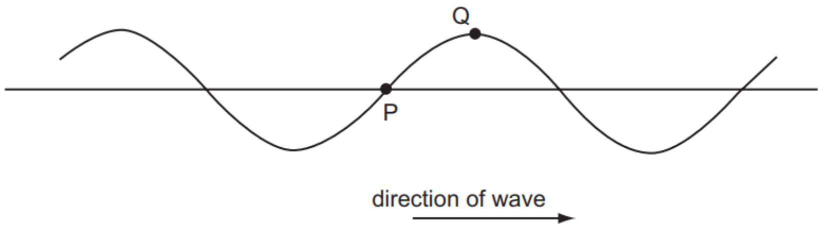


(a) On the figure above, attempt to sketch the displacement-distance graph of the longitudinal wave at that instant. Take displacement to the right as positive.

(b) Identify X and Y, whether it is compression or rarefaction.

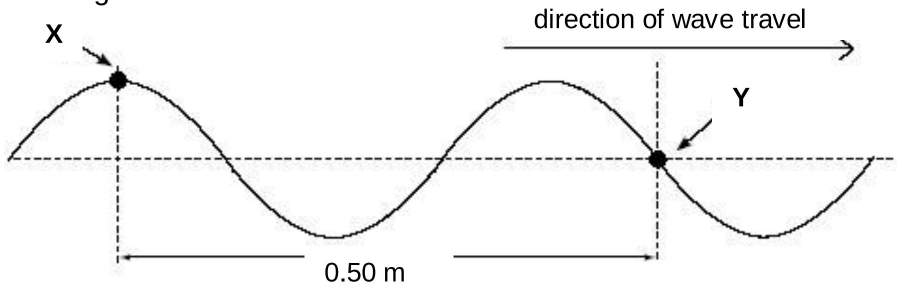
(c) State the distance between X and Y. Hence calculate the speed of the wave in the medium. [4.0 m s^{-1}]

4 The diagram shows a transverse wave on a rope. The wave is travelling from left to right. At the instant shown, the points P and Q on the rope have zero displacement and maximum displacement respectively.



What is the direction of motion, if any, of the points P and Q at this instant? Indicate it on the diagram,

5 The diagram below represents a progressive transverse wave travelling from left to right on a stretched string.



(a) Calculate the wavelength of the wave.
(b) The frequency of the wave is 22 Hz. Calculate the speed of the wave.
(c) State the phase difference between points X and Y on the string
(d) Describe how the displacement of point Y on the string varies in the next half-period.

[0.40 m, 8.8 m s^{-1},, 0.5π or 2.5π rad, 2.4]

Solution:

(a) Wavelength = 4/5 x 0.50 = 0.40 m
(b) speed = frequency × wavelength = 22 × 0.40
= 8.8 (m s^{-1})
(c) 90° or 450° or ½ π , 5/2 π
(d) displacement of Y will be a **positive (or 'up') maximum** at 1/4 T
returns to original position at ½ T

6 A progressive wave moves past two points P and Q, separated by a distance of 0.90 m. graph showing how the displacement y at P varies with time *t* is shown in Fig 1.1. Another graph, Fig 1.2, shows how the displacement of the wave at time *t* =0 varies with distance *x* from point P.

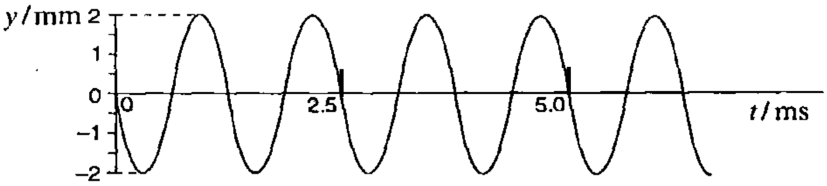


Fig 1.1

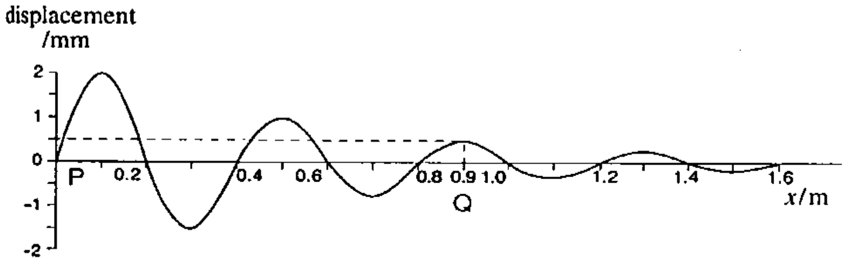


Fig 1.2

Using data from the graphs, deduce for this wave

(i) the wavelength,

(ii) the frequency,

(iii) the speed,

(iv) the phase difference between the oscillations at P and at Q,

(v) the ratio $\dfrac{\text{amplitude at } P}{\text{amplitude at } Q}$

(vi) the ratio $\dfrac{\text{intensity at } P}{\text{intensity at } Q}$

[0.40 m, 800 Hz, 320 m s^{-1}, π/2 rad, 4, 16]

7 A small source of sound radiates energy equally in all directions. At a particular frequency, the intensity of the sound 1.0 m from the source is 1.0 x 10^{-5} W m^{-2} corresponding to an amplitude of oscillation of the air molecules of 70 μm.

Assuming that the sound is propagated without energy loss, what will be

(a) the intensity of the sound,

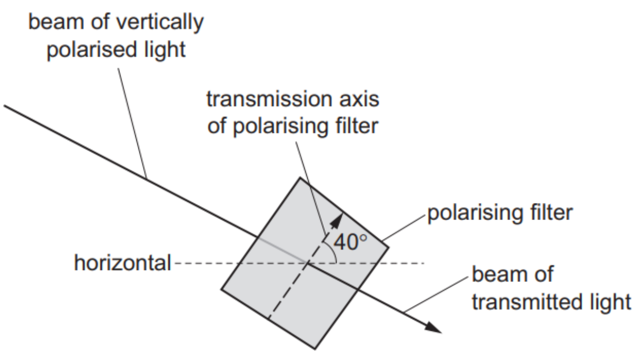
(b) the amplitude of oscillation of the air molecules,

at distance of 5.0 m from the source?

[4.0 x 10^{-7} W m^{-2}, 14 μm]

Polarisation

8 A vertically polarised beam of light is incident normally on a polarising filter. The transmission axis of the filter is at an angle of 40° to the horizontal.



What is the ratio

(a) amplitude of transmitted beam/amplitude of incident beam,

(b) intensity of transmitted beam/intensity of incident beam?

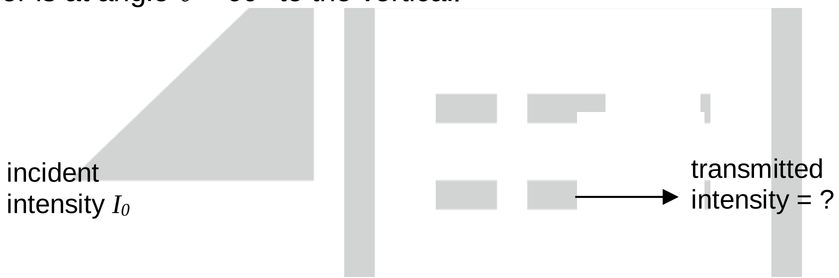
[0.64, 0.41]

Solution:

(a) $A/A_o = \cos 50^o = 0.64$

(b) $I/I_o = \cos^2 50^o = 0.41$

9 Unpolarised light with intensity I_0 passes through 2 polaroids. The axis of one is vertical and that of the other is at angle $\theta = 60°$ to the vertical.

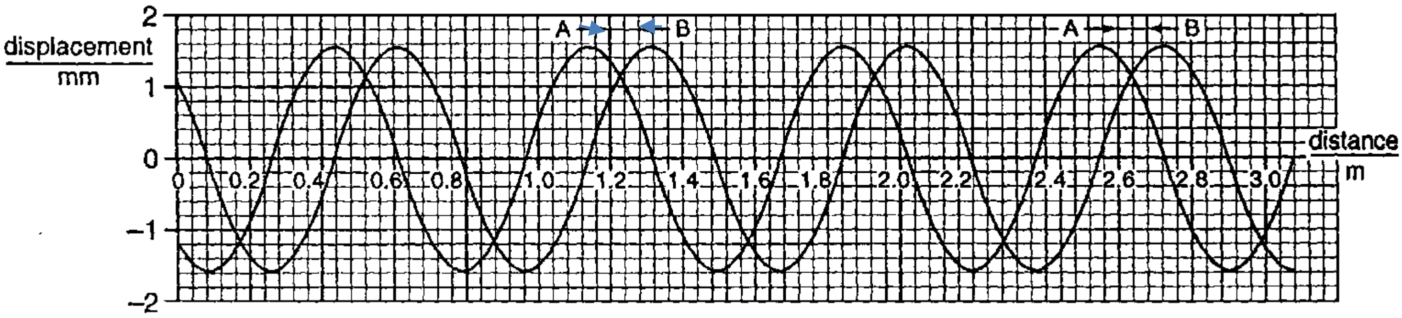


Determine the orientation and intensity of the transmitted light in terms of I_0.

[1/8 I_0]

Stationary Waves

10 The figure below shows a displacement-distance graph for two sound waves **A** and **B**, of the same frequency and amplitude. Wave **A** is traveling to the right and wave **B** to the left.



(a) From the graph, deduce the phase difference between the two waves at the instant shown.

(b) The period of each wave is T. Determine the maximum displacement of the resultant of the two waves

(i) at the instant shown,

(ii) at the instant shown $+\frac{T}{8}$

(iii) at the instant shown $+\frac{3\,T}{8}$

(iv)

(c) The waves have a frequency of 484 Hz. Deduce their speed to 3 significant figures.

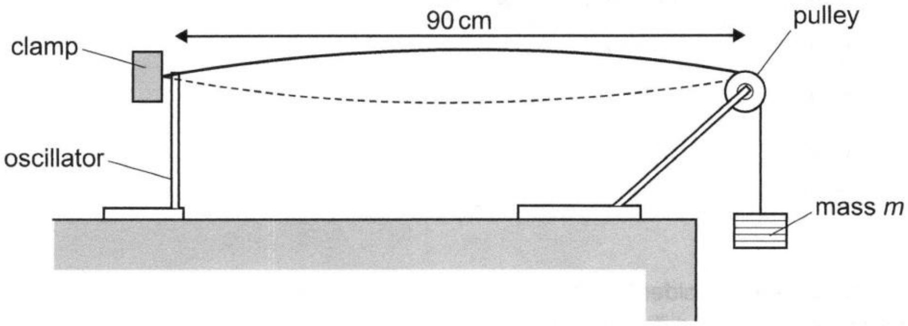
[$\pi/2$ rad, 2.2 mm, 3.2 mm, 0 mm, 342 m s^{-1}]

11 A sound source of frequency 1100 Hz is placed at a distance from a plane reflecting wall. A small microphone runs on a straight track between the source and the wall at a speed of 30 m s^{-1}.

(a) Explain why the sound received by the microphone fluctuates regularly.

(b) If the speed of sound is 330 m s^{-1}, calculate the rate at which nodes are received by the microphone. [200 Hz]

12 The speed of a wave on a stretched string is proportional to the square root of the tension in the string.
The string is held under tension between a clamp and a pulley 90 cm apart by a mass *m* of 400 g hanging from the end of the string.



When the frequency *f* of the mechanical oscillator close to the clamped end is 15 Hz, the stationary wave shown is set up between the clamp and the pulley. The hanging mass *m* and frequency *f* are varied until another stationary wave is formed.

Which combination of *m* and *f* gives a stationary wave?

	m / g	*f* / Hz
A	500	30
B	600	40
C	800	60
<u>D</u>	900	90

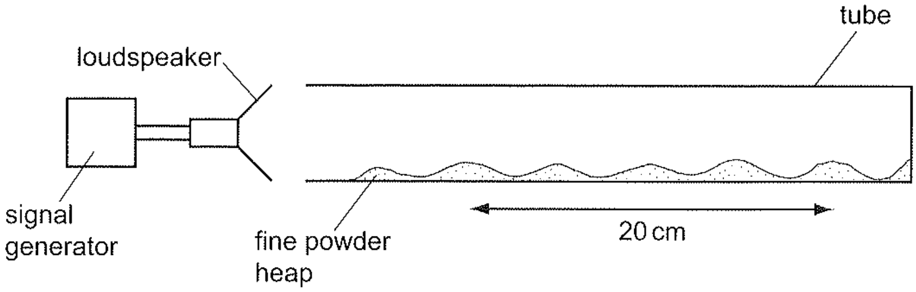
Solution: *Hint: the speed of the wave on a string* α *√tension and Tension T = mg*

$$v = f\lambda \Rightarrow \lambda = \frac{v}{f} \propto \frac{\sqrt{m}}{f}$$

Option D: $\frac{\lambda'}{\lambda} = \sqrt{\frac{m'}{m}} \times \frac{f}{f'}$, such that $\frac{\lambda'}{2(0.90)} = \sqrt{\frac{900}{400}} \times \frac{15}{90}$

$\lambda' = 0.45\ m$ ie = 45 cm $\Rightarrow$ 4th harmonic

13 A horizontal tube, containing a fine powder, is closed at one end. A loudspeaker, conneced to a signal generator, is positioned at the other end.

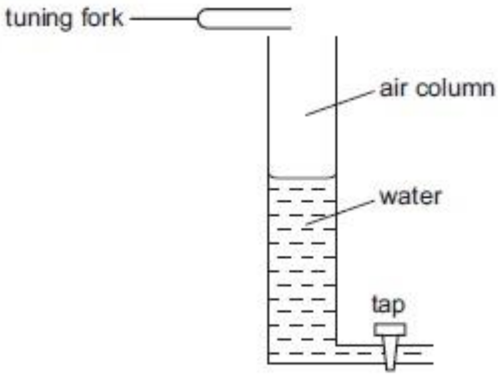


At a particular frequency, a stationary wave is set up inside the tube and the powder forms heaps as shown. The speed of sound is 330 m s^{-1}.

(a) Explain the formation of the heaps in the tube.

(b) Calculate its frequency. [3300 Hz]

14 The diagram shows an experiment to produce a stationary wave in an air column. A tuning fork, placed above the column, vibrates and produces a sound wave. The length of the air column can be varied by altering the volume of the water in the tube.

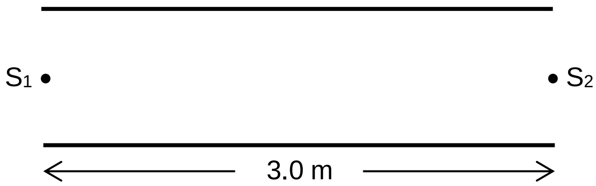


The tube is filled and then water is allowed to run out of it. The first two stationary waves occur when the air column lengths are 0.14 m and 0.42 m.

Calculate the wavelength of the sound wave. [0.56 m]

Solution:
Between 2 consecutive resonant positions = $\lambda/2 = (0.42 - 0.14)$
$\lambda = 0.56$ *m*

15 Two sources of sound waves S_1 and S_2 are placed a distance of 3.0 m apart at either end of a narrow pipe. Both sources are emitting waves of wavelength 1.2 m and of similar amplitude, which travel along the pipe.



(a) Sketch the resultant wave between S_1 and S_2.
(b) In what two ways would the resultant wave in (a) be different if the sources were replaced with sources of microwaves?