SUPERPOSITION

Learning Outcomes

Candidates should be able to:

(a)* explain and use the principle of superposition in simple applications

(b)* show an understanding of experiments which demonstrate standing (stationary) waves using microwaves, stretched strings and air columns

(c)* explain the formation of a standing (stationary) wave using a graphical method, and identify nodes and antinodes, differentiating between pressure and displacement nodes and antinodes for sound waves.

(d)* determine the wavelength of sound using standing (stationary) waves

(e) show an understanding of the terms diffraction, interference, coherence, phase difference and path difference

(f) show an understanding of phenomena which demonstrate two-source interference using water waves, sound waves, light and microwaves

(g) show an understanding of the conditions required for two-source interference fringes to be observed

(h) recall and use the equation $\lambda = ax/D$ to solve problems for double-slit interference, where a is the slit separation and x is the fringe seperation

(i) recall and use the equation $a\sin\theta = n\lambda$ to solve problems involving the principal maxima of a diffraction grating, where a is the slit seperation

(j) describe the use of a diffraction grating to determine the wavelength of light (knowledge of the structure and use of a spectrometer are not required).

(k) show an understanding of phenomena which demonstrate diffraction through a single slit or aperture, or across an edge, such as the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap, or the diffraction of sound waves from loudspeakers or around corners

(l) recall and use the equation $b\sin\theta = \lambda$ to solve problems involving the positions of the first minima for diffraction through a single slit of width b

(m) recall and use the Rayleigh criterion $\theta \approx \lambda/b$ for the resolving power of a single aperture, where b is the width of the aperture

** Covered in Unit 10: Waves*

Temasek Junior College

1 Principle of superposition *LO (a)*

Principle of superposition states that when two or more waves of the same kind meet at a point, the resultant displacement of the waves at any point is the vector sum of the displacement due to each wave acting independently.

1.1 Important terms *LO (e)*

(a) Phase difference

Phase difference is an angular measure of the fraction of a cycle two particles in a wave or two waves are out of step.

- When the phase difference between two waves is zero, or an integer multiple of a cycle, the waves are said to be **in phase**.
- When the phase difference between two waves is an odd integer multiple of half cycle, the waves are said to be **anti-phase**.

(b) Coherence

Two sources are said to be **coherent** if waves from each source have a constant phase difference between them (and therefore they must have the same frequency).

Two coherent waves need not be in phase. For example,

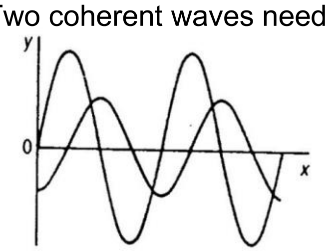


Fig (a) two waves from coherent sources

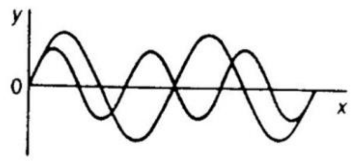
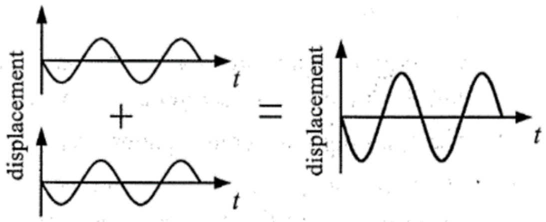


Fig (b) two waves from incoherent sources

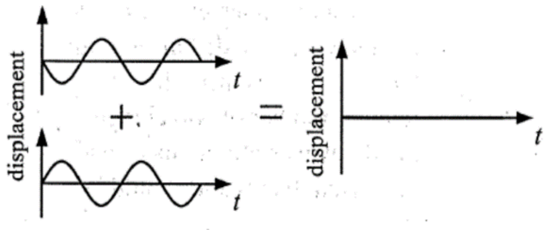
(c) Interference

Interference refers to the results of the superposition of two or more waves of the same kind. The displacement of the resultant wave at any point is given by the principle of superposition.

- **Constructive interference** occurs when two or more waves meet in phase and superpose to produce a resultant wave of maximum amplitude (where the amplitudes add up).



- **Destructive interference** occurs when two or more waves meet anti-phase (or 180° out of phase) and superpose to produce a resultant wave of zero or minimum amplitude (where the amplitudes cancel).



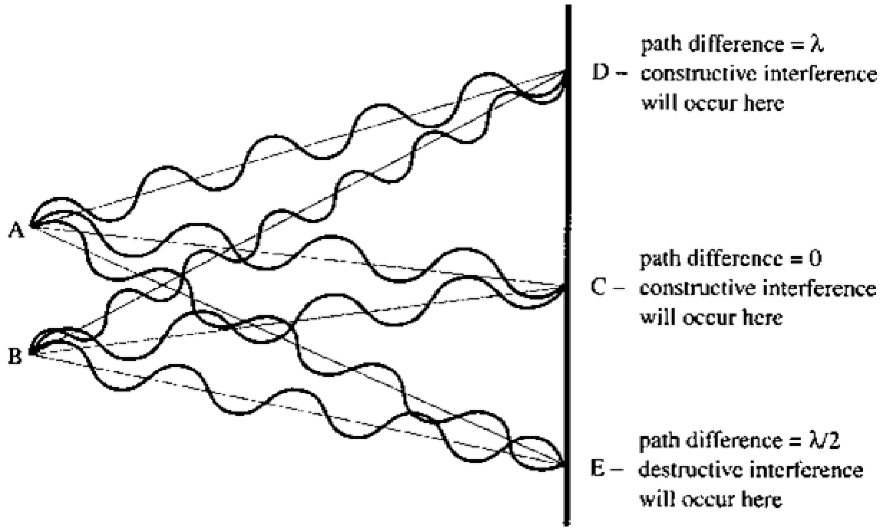
(d) Path difference

Path length of a wave is the distance travelled by a wave. Path difference is the difference in the two path lengths. i.e.

Path difference is the difference in the distance travelled by two waves from their respective coherent wave sources to a point.

Path difference is often expressed in terms of a number of wavelengths.

For example, consider waves originating from two coherent sources that oscillate **<u>in phase,</u>**



- if the path difference is zero or an integer number of wavelengths, the waves from the two sources would meet in phase and interfere constructively. i.e.

Condition for **constructive interference** is **path difference = $n\lambda$, where n = 0,1,2,3,...**

- if the path difference is an odd integer number of half wavelengths, the waves from the two sources would meet anti-phase and interfere destructively. i.e.

Condition for **destructive interference** is **path difference = $(n + ½)\lambda$, where n = 0,1,2,3,...**

Note:

1. Since path difference of one wavelength λ corresponds to a phase difference ϕ of 2π radians (or 360°), we can write the proportion

$$\boxed{\frac{\phi}{2\pi} = \frac{\Delta L}{\lambda}}$$

Thus, path difference ΔL of $n\lambda$ is equivalent to phase difference ϕ of **$(n)2\pi$ rad**, and path difference ΔL of **$(n + ½)\lambda$** is equivalent to phase difference ϕ of **$(n + ½)2\pi$ rad**, where n = 0,1,2,3,...

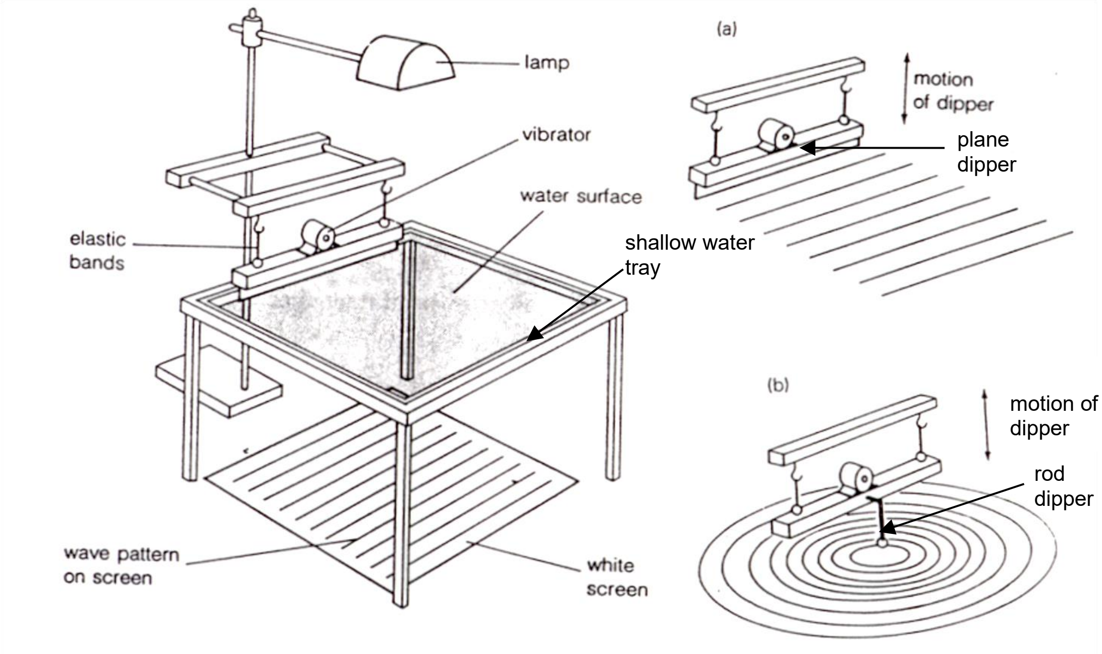
2. For two coherent sources that oscillate **<u>anti-phase</u>**, the conditions above above would be reversed.

2 Diffraction *LO (e)*

Diffraction is the spreading of waves around an obstacle or through a gap, into its geometrical shadow.

Geometrical shadow refers to region which the waves would not have reached if they had travelled in a straight line.

2.1 Diffraction experiment with a ripple tank *LO (k)*



Diffraction of waves can be demonstrated using a ripple tank as shown. Plane or circular water waves can be generated using a plane dipper or a rod dipper respectively. Light from a lamp shines upon the water from above and illuminates a white screen placed directly below the tank.

When plane water waves produced by the ripple tank pass through obstacle, gaps (or slits) of varying widths, the following diffraction effect could be observed.

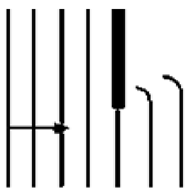


Fig (a) obstacle

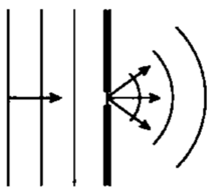


Fig (b) small gap

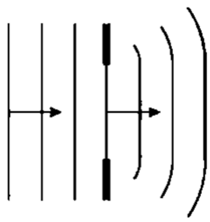
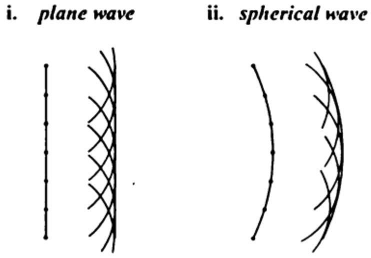


Fig (c) wide gap

Fig (a) shows diffraction of water waves around an obstacle. In Fig (b), the gap width is **comparable to wavelength**, **diffraction is significant** as the waves emerge from the gap with circular wavefronts, as if spreading out from a point source. The waves spread noticeably around the gap into the geometrical shadow. In Fig (c), the gap width is much larger than the wavelength, diffraction is insignificant. The spreading of the waves around the edges of the gap becomes less.

Note:
1. Diffraction does NOT change the wavelength of the wave; i.e. the space between adjacent wavefronts remains constant.

2. Diffraction can be explained with **Huygen's principle**, which states that every point on a wavefront can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The new wavefront is the envelope of all the wavelets as illustrated in the diagram.



3. Two-source interference

3.1 Conditions for observable interference pattern *LO (g)*

For interference pattern to be **observable**,

1. The waves must be of the **same kind and superpose at a point**.
2. The waves must be **coherent**. i.e. the waves have a constant phase difference between them. (Otherwise the pattern will change with time due to changing phase difference and quick shifting patterns are not observable).
3. The waves must have **about the same amplitude**. (Otherwise the pattern has poor contrast due to incomplete cancellation of wave amplitudes at points of destructive interference).
4. For transverse waves, they must be **unpolarised or have the same plane of polarisation**. (Interference can only occur for wave oscillations in the same plane).

3.2 Two-source interference experiments *LO (a, f)*

Let's look at experiments which demonstrate two-source interference of water waves, sound waves, microwaves and light waves.

(a) Water waves

The interference of surface water waves can be demonstrated using a ripple tank with two vibrating rod dippers. The two sets of circular waves pass over one another and superpose. The two dippers are **coherent** sources producing waves that are in phase because they are **fixed to the same vibrator**.

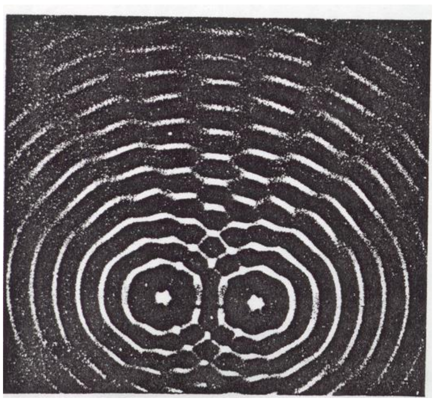


Fig (a)

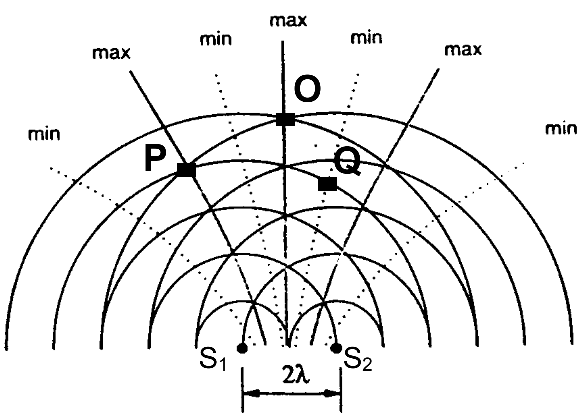


Fig (b)

Fig (a) shows the resulting interference pattern. In Fig (b), S_1 and S_2 represent the two sources of water waves. The solid circular lines represent the wavefronts, which are the crests of the waves. **Stationary** waves is formed between the two sources where the two waves propagate in opposite directions. But everywhere else, an **interference pattern** consisting of maxima and minima are seen.

At points where the path difference of the waves from the two sources is an integer number of wavelengths, the waves meet in phase (a wave crest from one source always meet a wave crest of the other) and interfere constructively. The resultant amplitude is maximum due to the sum of the amplitudes of the waves. These points form lines known as **Maxima.**

At points where the path difference of the waves from the two sources is is an odd integer number of half wavelengths, the waves meet anti-phase (a wave crest from one source always meet a wave trough of the other) and interfere destructively. The resultant amplitude is zero or minimum due the difference of the amplitudes of the waves. These points form lines known as **Minima.**

For example,

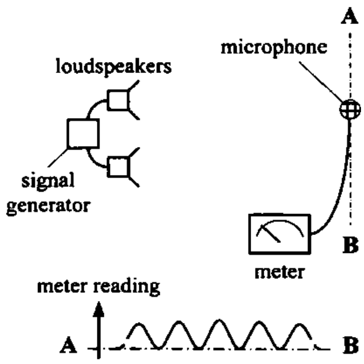
- At O, path difference = $S_2O - S_1O = 5\lambda - 5\lambda = 0\lambda$,
  constructive interference occurs and a central maximum is obtained.

- At P, path difference = $S_2P - S_1P = 5\lambda - 4\lambda = \lambda$,
  constructive interference occurs and a first order maximum is obtained.

- At Q, path difference = $S_1Q - S_2Q = 4\lambda - 3\tfrac{1}{2}\lambda = \tfrac{1}{2}\lambda$,
  destructive interference occurs and a first order minimum is obtained.

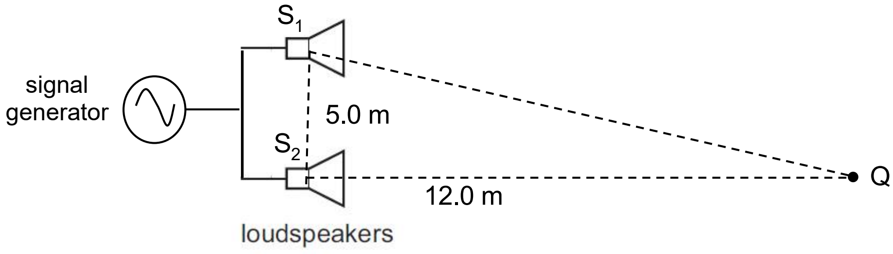
(b) Sound waves *LO (f)*

Two loudspeakers connected to a **single signal generator** produces two **coherent** sets of sound waves as shown. Sound waves from the two loudspeakers superpose along line AB with alternate constructive and destructive interference, resulting in strong and weak signals along line AB.



If a microphone connected to a galvanometer is moved along the line AB, the metre reading would increase and decrease regularly.

Example 1:
A signal generator is connected to two identical loudspeakers S_1 and S_2 separated by 5.0 m. The loudspeakers emit sound of the same wavelength 2.0 m and in phase. A person is standing at point Q, 12.0 m away from S_2. Identify whether he will hear a loud or soft sound.



$S_1Q = \sqrt{(5.0^2 + 12.0^2)} = 13.0$ m

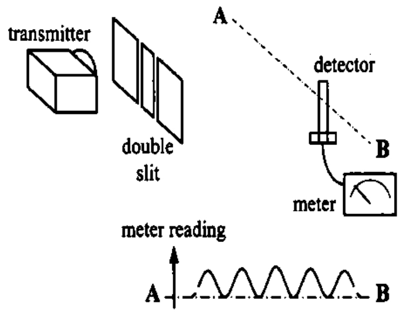
path difference $= S_1Q - S_2Q = 13.0 - 12.0 = 1.0$ m
$= \frac{1}{2}\lambda$ since λ is 2.0 m

Since path difference = ______, waves meet ______________ and interfere _____________.

The resultant amplitude is ___________, and thus he will hear a ________, sound.

(c) Microwaves *LO (f)*

Microwaves from the transmitter is passed through a double slit provided by aluminium plates as shown. Diffraction at the double slit produces two **coherent** sources. Microwaves from the double slit superpose along line AB with alternate constructive and destructive interference, resulting in strong and weak signals along line AB.

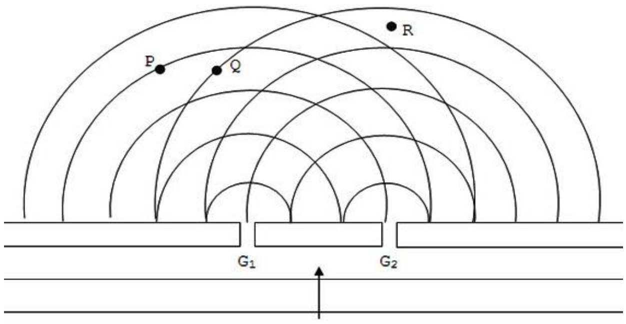


If a detector connected to a galvanometer is moved along the line AB, the metre reading would increase and decrease regularly.

Example 2:
The figure shows an interference pattern produced by microwaves of frequency 1.0 x 10^{10} Hz. The waves are directed towards two gaps G_1 and G_2. A detector is placed at the points P, Q and R. Each arc that is drawn in the figure represents a wavefront.

(i) Calculate the wavelength λ of the microwaves.

(ii) State, with explanations, what is detected at P, Q and R.



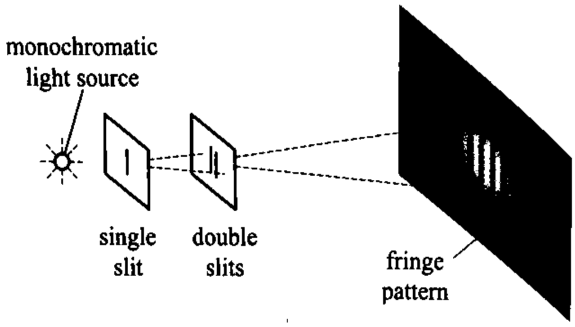
(i) Wavelength $\lambda = c / f = 3.0 \times 10^8 / 1.0 \times 10^{10} = 0.030$ m or 3.0 cm.

(ii) At P, path difference = $6\lambda - 4\lambda = 2\lambda$. The waves meet in phase and interfere constructively. Resultant amplitude is sum of the wave amplitudes. Thus a maximum is detected.
At Q, path difference = $5\lambda - 3.5\lambda = 1.5\lambda$. The waves meet anti-phase and interfere destructively. Resultant amplitude is difference of the wave amplitudes. Thus a minimum is detected.
At R, path difference = $5.5\lambda - 4.5\lambda = 1\lambda$. … a maximum is detected.

(d) Light waves *LO (f, h)*

In 1801, Thomas Young conducted an experiment that demonstrated the interference of light.

The experiment, well known as the **Young's double slit experiment**, provided strong experimental evidence that light and other electromagnetic radiation show wave-like nature.



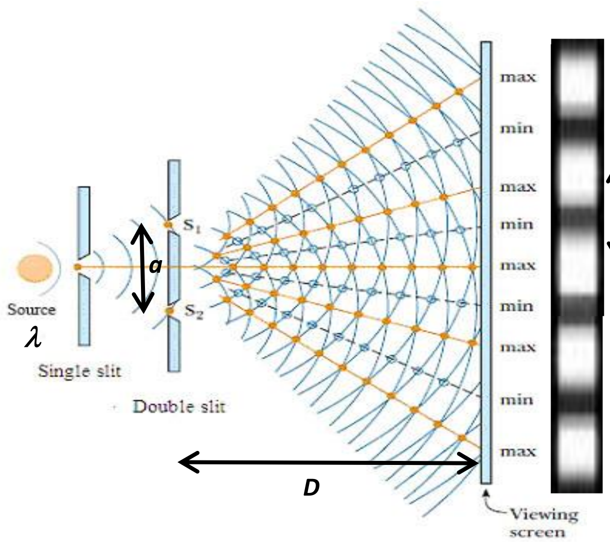


Fig (a)

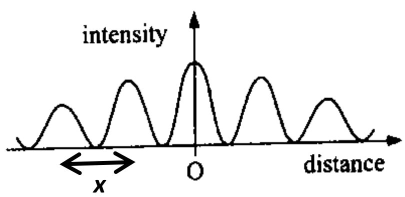


Fig (b)

In Fig (a), a monochromatic light source is placed in front of a single slit followed by a double slit of slit separation *a*. The screen is placed a distance *D* from the double slit.

Diffraction at the single slit produces a point light source that emits semi-circular wavefronts. Waves from this point source are further split into two coherent point sources at the double slit S_1 and S_2. These two slits serve as a pair of **coherent** light sources because waves emerging from them **originate from the same wavefront** and therefore maintain a constant phase difference. A stable interference pattern can then be obtained.

The lights from the double slit superposed on the screen with alternate constructive and destructive interference, resulting in equally spaced bright fringes (maxima) and dark fringes (minima) near the centre on the screen as shown in Fig (a).

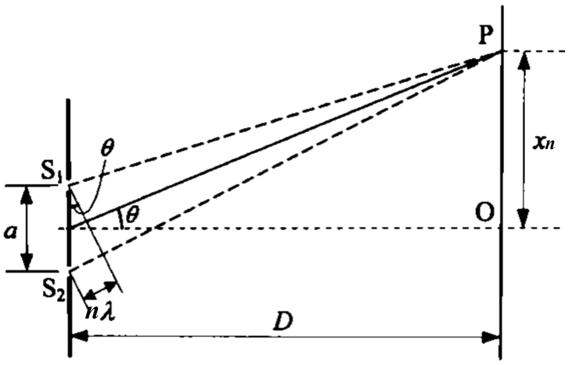
Fig (b) shows the typical variation of light intensity with distance from the centre O on the screen. The intensity of the bright fringes is greatest for the central fringe and decreases for higher orders. (See Appendix A for explanations).

Fringe separation *x* is related to the wavelength λ of the source, screen distance *D* and double-slit separation *a* by the expression

$\boxed{x = \dfrac{\lambda D}{a}}$ valid only if $a \ll D$ and for fringes close to central maximum

<u>Proof (not required):</u>

The figure shows the interference of light from a double slit.



At **O**, path difference between the waves is zero. They meet in phase and interfere constructively to produce the central bright fringe (or central maximum).

For n^{th} bright fringe to occur at **P**,
path difference $S_2P - S_1P = n\lambda$

For $a \ll D$, S_2P and S_1P approximately parallel,

path difference $S_2P - S_1P \approx a\sin\theta = n\lambda$ => $\sin\theta = \dfrac{n\lambda}{a}$

But at P, $\tan\theta = \dfrac{x_n}{D}$, where x_n is distance of n^{th} fringe from the central axis.

For small angle, $\tan\theta \approx \sin\theta$ => $\dfrac{x_n}{D} = \dfrac{n\lambda}{a}$ => $x_n = \dfrac{n\lambda D}{a}$

Likewise for the adjacent (n + 1)th fringe, $x_{n+1} = \dfrac{(n+1)\lambda D}{a}$

Thus fringe separation $x = x_{n+1} - x_n$ => $\boxed{x = \dfrac{\lambda D}{a}}$ valid only if $a \ll D$

Note:

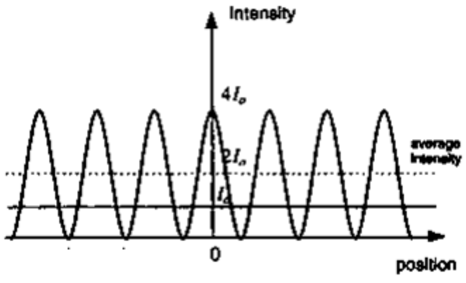
1. Since the interference fringes are produced by waves emerging from S_1 and S_2, these two slits must be very close to each other in order that waves are able to overlap over a significantly large region. Typical slit separation ***a* ~ 10^{-4} m.** The screen distance from the double slit is much greater than *a*, typically ***D* ~ 1 m**. And typical fringe separation ***x* ~ 10^{-3} m**.

2. Since $x = \dfrac{\lambda D}{a}$, fringe separation *x* increases with increasing wavelength λ, increasing screen distance *D* from the double slit but decreasing slit separation *a*. And fringes are **equally spaced (near the centre)** since fringe separation x is constant for the same λ, *D* and *a*.

3. The **intensity** of the bright fringes is:
   (i) **inversely proportional to D^2** (light intensity decreases with distance from the source)
   (ii) **proportional to (resultant amplitude)^2 and in turn proportional to (slit width)^2.**

   Let amplitude of wave from each slit be *A* and intensity of each wave be I_o.
   At the bright fringes, $A_R = (A + A) = 2A$. since intensity $I \propto A^2$, then $I_R = 4I_o$.
   At the dark fringes, A_R = zero if there is complete cancellation. Hence I_R is also zero.

   Note that interference cannot create or destroy energy but merely **redistributes** it over the screen. The average intensity on the screen would be $2I_o$ regardless of whether the sources are coherent. If waves from the two sources were incoherent, there would be no fringe pattern and the intensity would have the uniform value $2I_o$ for all points on the screen.



4. The additional single slit is necessary in Young's double slit experiment is to produce a point light source. This is because two separate light sources are incoherent sources. Light pulses called photons are emitted spontaneously and randomly resulting in abrupt change in the phase of the light waves (Quantum Physics). But **laser** beam is a coherent source of light. Why?

5. A **monochromatic** light source is used to ensure the fringes are distinct. White light source would produce coloured fringes. As fringe separation varies linearly with wavelength, each fringe will be tinged with violet on the inner side towards the centre and red on the outer side (except for the central white fringe). Why?

Example 3:
In a double-slit interference experiment using light of wavelength 540 nm, the separation of the slits is 0.700 mm. The fringes are viewed on a screen at a distance of 2.75 m from the double slit. Calculate the separation of the fringes observed on the screen.

Fringe separation $x = \dfrac{\lambda D}{a}$

$$= \frac{540\times10^{-9}(2.75)}{0.700\times10^{-3}} = 2.12 \text{ mm}$$

Online simulations:
http://phet.colorado.edu/en/simulation/wave-interference

http://vsg.quasihome.com/interfer.htm

4 Single slit diffraction

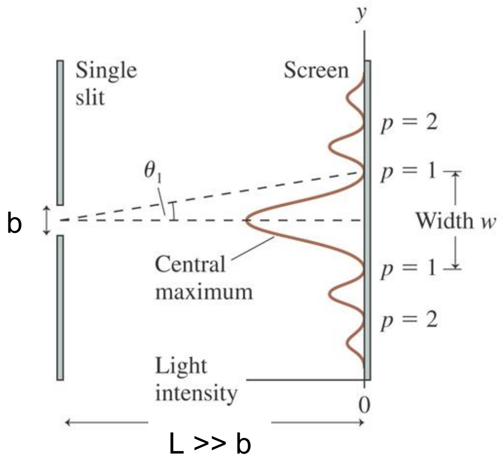


Fig (a)

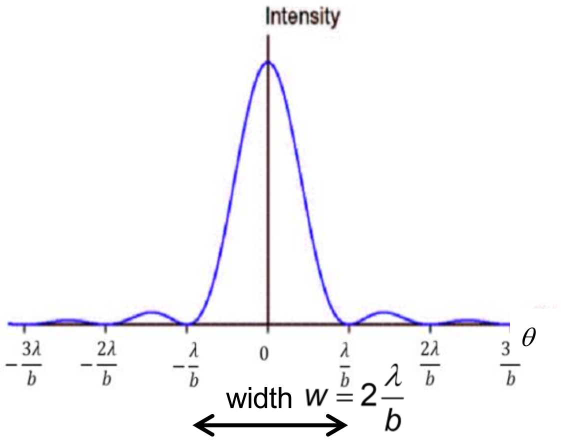


Fig (b)

Fig (a) shows the diffraction pattern formed on a screen when a monochromatic light is incident normally onto a single slit. The pattern is the result of interference between light waves diffracted by the slit.

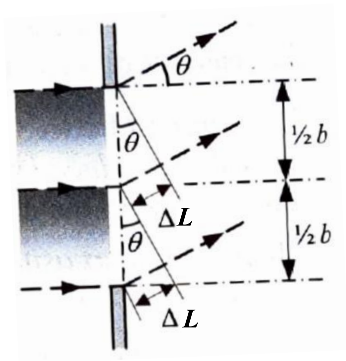
Fig (b) shows the variation of intensity with $\sin\theta$ for the diffraction pattern. It consists of an intense broad central maximum (or central bright fringe) with subsidiary maxima which are narrower and of much less intensity on both sides. The width of the central bright fringe is twice that of the subsidiary maxima.

The angle θ of the **first minima** is related to light wavelength λ and slit width b by the expression

$$\boxed{\sin\theta = \frac{\lambda}{b}}$$

Proof (not required):

The figure shows the diffraction of light from a single slit.



Consider the diffracted beam from the single slit as two halves. At any angle θ, path difference between uppermost waves of the two halves is $\Delta L = \frac{1}{2}b\sin\theta$.

Path difference between all other matching waves of the two halves is the same.

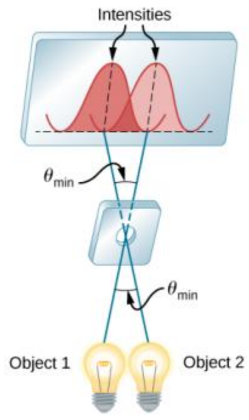
Condition for first minima is path difference $\Delta L = \frac{1}{2}\lambda$, where waves of the upper and lower halves meet anti-phase and interfere destructively.

Thus $\frac{1}{2}b\sin\theta = \frac{1}{2}\lambda$ => $\boxed{\sin\theta = \frac{\lambda}{b}}$ for the first minima

4.1 Resolving power

LO (m)

The resolving power of an imaging system is its ability to produce two separate distinguishable images of two separate objects. And the resolving power depends on the wavelength coming from the objects, how close they are to each other and how far away they are from the observer.



When light passes through an aperture, a diffraction pattern is formed. For an optical system, the aperture could be the **pupil of the observer's eye** or the **objective lens of a telescope**. When there are two sources of light, two diffraction patterns will be formed by the system. How close can these patterns be for us to still recognize that there are two sources?

In the late 19th century, English physicist, John William Strutt, 3rd Baron Rayleigh, proposed what is now known as the **Rayleigh criterion for resolution of two patterns:**

When the central maximum of the diffraction pattern from one source is directly over the first minimum of the diffraction pattern from the second source, the patterns are just distinguishable.

Diffracted patterns further apart than this limit will be distinguishable (resolved) and those closer will not.

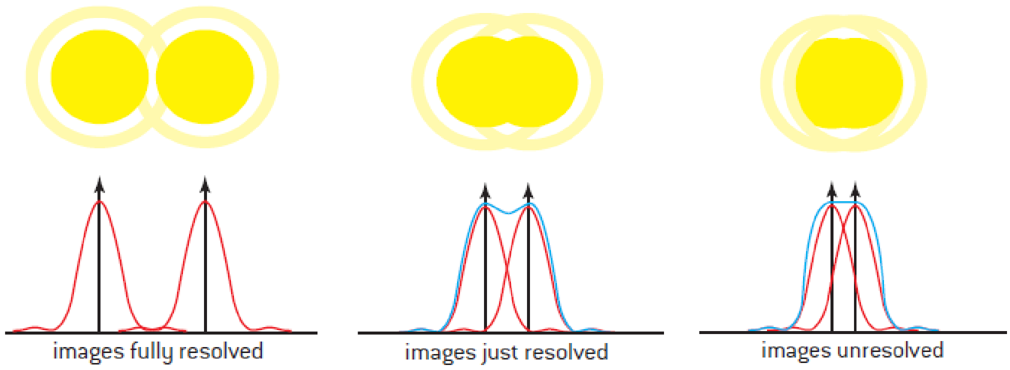


Fig (a) Fig (b) Fig (c)

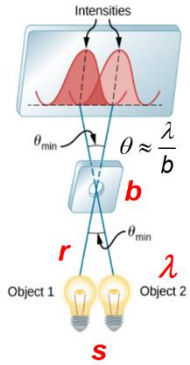
Fig (a) to (c) show the possible diffraction intensity patterns produced by two separate point sources when viewed through a circular aperture at some distance away. The variation of intensity with angle for each of the diffraction patterns is shown below the image.

For single slit diffraction, the first minimum occurs at angle θ with the straight-through position given by $\sin\theta = \frac{\lambda}{b}$.

Thus for small θ, **Rayleigh criterion** for resolving power of a single aperture of diameter b is given by

$$\boxed{\theta \approx \frac{\lambda}{b}}$$

Example 4:
A double star is at a distance of 20 light years from the Earth. A telescope with a diameter of 3.0 m is used to view the star. (A light year is the distance light travels in a vacuum in one year = 9.46×10^{15} m).
What is the approximate minimum separation between the two stars of the double star that can be detected by the telescope?



By Rayleigh criterion, minimum angle for resolution $\theta_{min} \approx \frac{\lambda}{b}$ for small θ

Angle subtended by double star $\theta \approx \frac{s}{r}$

=> $\frac{s}{r} \approx \frac{\lambda}{b}$ => $\frac{s}{20(9.46 \times 10^{15})} \approx \frac{5 \times 10^{-7}}{3.0}$ (assume λ of 500 nm)

=> $s \approx 3 \times 10^{10}$ m

Online resources

To understand the Rayleigh criterion better:

https://www.edinst.com/de/news/the-rayleigh-criterion-for-microscope-resolution/

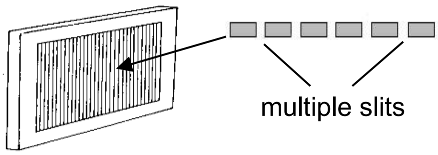
To facilitate visualization of Rayleigh criterion (3 slides):

5 Multi-slit interference

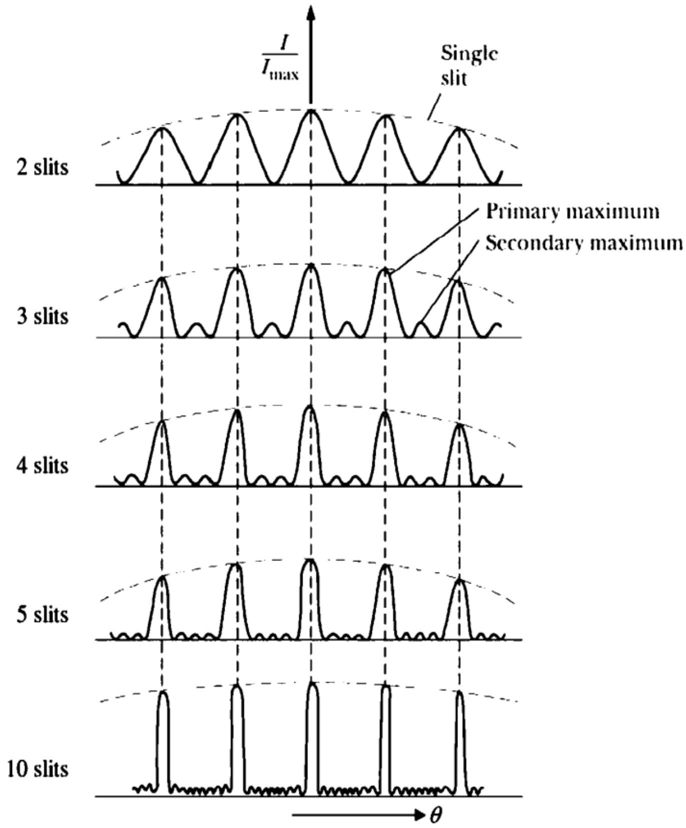
5.1 Diffraction grating

LO (*i*)

A diffraction grating consists of a large number of **parallel, closely spaced** slits as shown.



A diffraction grating is actually a **multiple-slit** plate. The figure on the right shows the interference patterns for grating with varying number of slits.



For same slit separation, the maxima (bright fringes) of its interference pattern occur at exactly the same angular positions as that of a double-slit interference.

But these maxima are **brighter** since the pattern is formed by light diffracted from many slits instead of two. They are also **sharper** (with numerous secondary maxima in between them - not in syllabus).

A typical diffraction grating would have **1000 slits per mm**. Thus, the slit separation *d* is much smaller than that used in the double slit experiment. This means the maxima of diffraction grating occur at **very large diffraction angle θ** and so usually **only a few orders of maxima can be seen**.

5.2 Determining the wavelength of light

LO (m)

Diffraction grating is used to determine the wavelength of light. By measuring the angle at which a particular diffracted image appears, the wavelength of the light producing that image may be determined.

A spectrometer is an apparatus used to investigate the spectra.

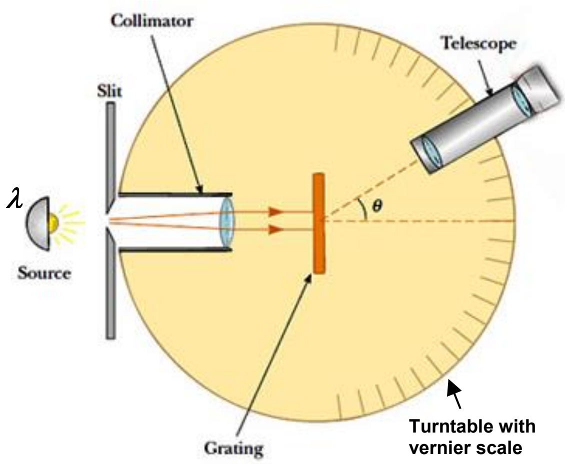


Fig (a)

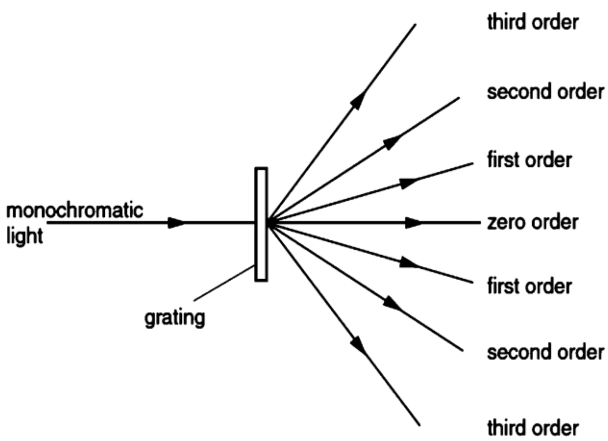


Fig (b)

Fig (a) shows a spectrometer with a grating placed on its turntable. When a collimated beam of **monochromatic** light is incident **normally** on a diffraction grating placed on the round table of the spectrometer, interference maxima can be seen through the telescope at different angle θ.

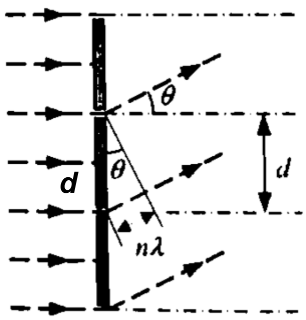
Fig (b) shows a schematic diagram of the interference pattern due to a diffraction grating. The incident light wave diffracts at every slit of the grating, forming **coherent** light sources. The diffracted light waves then interfere with one another to produce the interference pattern that can be viewed through the telescope.

The angle of diffraction θ for ***n*^th^** →

The angle of diffraction θ for $\boldsymbol{n^{th}}$ **order bright fringe (maxima)** is related to the light wavelength λ and the slit separation d by the expression

$\boxed{d \sin\theta = n\lambda}$ where n = 0,1,2,3…

<u>Proof (not required):</u>



Consider a coherent monochromatic light beam incident normally on a diffraction grating.

For waves from any two adjacent slits, path difference is $d \sin\theta$, where θ is the angle of deviation of the n^{th} maxima from the zero order maximum.

Condition for n^{th} bright fringe (maxima) is path difference = $n\lambda$ where the waves meet in phase and interfere constructively.

Thus $\boxed{d \sin\theta = n\lambda}$ where n = 0, 1, 2, 3,…

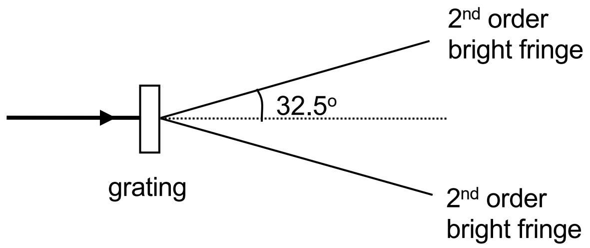
Note:

1. Diffraction grating is often specified with the number of lines per metre, N. The required slit separation d is then the reciprocal of N.

2. To find the maximum number of orders, set angle of diffraction θ to 90° and determine the maximum possible order n. i.e.

$$d \sin 90^o \geq n\lambda => n \leq \frac{d}{\lambda}$$

Example 5:
Monochromatic light is incident on a grating having 600 lines per mm. The second order maximum is observed at an angle of 32.5° to the central maximum. Determine
(i) the wavelength of the incident light,
(ii) the total number of fringes seen on the screen.



(i) $d\sin\theta = n\lambda$

$$\left(\frac{10^{-3}}{600}\right)\sin 32.5^{o} = 2\lambda \Rightarrow \lambda = 4.5\times10^{-7}\text{ m}$$

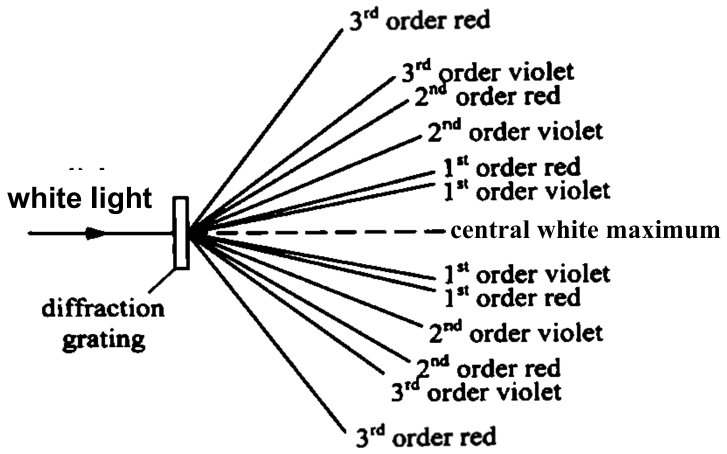
(ii) $d\sin 90^{o} \geq n\lambda$

$$\Rightarrow \left(\frac{10^{-3}}{600}\right)\sin 90^{o} \geq n\left(4.5\times10^{-7}\right) \Rightarrow n \leq 3.7$$

Thus maximum order $n = 3$

Total number of fringes = 3+3+1 = 7

5.3 White light spectrum



When a collimated beam of white light is incident normally on a diffraction grating, **sets of coloured spectra** are observed on both sides of a **central white maximum** as shown above.

The zeroth order spectrum (central maximum) is white since all colours overlap here. Each other order of spectrum consists of violet colour on the inner side and red colour on the outer side. This is because diffraction angle θ increases with wavelength λ according to $d\sin\theta = n\lambda$. Red light with the longest wavelength is diffracted through the largest angle while violet light with the shortest wavelength is diffracted the least.
Two colours of different orders may **overlap** if the diffraction angles θ are the same. For example, a red maximum may overlap with a higher adjacent order of violet maximum to form a purple maximum.

This can be deduced from the equation $d\sin\theta = n\lambda$. For same θ (overlap) and d (same grating), $n\lambda$ is a constant. Thus the condition for overlapping of spectra of two different colours is

$$\boxed{n_1\lambda_1 = n_2\lambda_2}$$

Example 6:
A parallel beam of white light is shone normally on a diffraction grating having 6500 lines in 1 cm. Assuming the wavelengths of yellow and blue light in air are 600 nm and 400 nm respectively, show that there is overlapping between the second order yellow spectrum and the third order blue spectrum.

$d \sin\theta = n\lambda$

$\Rightarrow n_Y \lambda_Y = n_B \lambda_B$ for same θ and d

$\Rightarrow \dfrac{n_Y}{n_B} = \dfrac{\lambda_B}{\lambda_Y} = \dfrac{400}{600} = \dfrac{2}{3}$

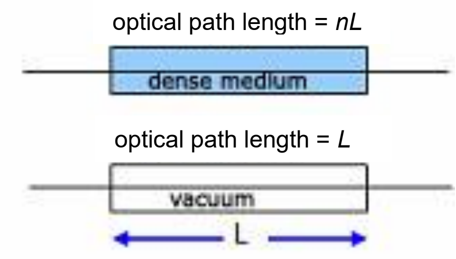
=>*Second order yellow spectrum overlaps third order blue spectrum).*

Note:
1. For experiment with diffraction grating, one could determine the wavelength using **higher order** maxima. An advantage is the larger angle involved would lead to smaller percentage error. A disadvantage would be the beams are dimmer.

2. The incident beam must be **normal** to the diffraction grating. Otherwise there will be a phase difference between light waves arriving at the slits. Fringe pattern may still be observed, but the 0^{th} order maxima (central maximum) will not be symmetrical about the grating normal and the same order maxima on opposite sides of the 0^{th} order maxima would emerge at different angle. (2019P3Q8)

3. Light slows down when it passes through a dense material of refractive index n where
   $n = \dfrac{\text{velocity of light in vacuum}}{\text{velocity of light in material}} = \dfrac{c}{v}$

   Thus light takes n times longer to pass through a dense medium of refractive index n compared to air for the same length. Thus the optical path length in a material of length L and refractive index n is nL. If one part of a light beam travels a distance L in air and the other a distance L in the material of refractive index n, then an **optical path difference** will exist between them where

   optical path difference = $(nL - L) = (n - 1)L$

   If the two beams overlap, an interference pattern will result.

Example 7:
Calculate the path difference between two beams of yellow-green light (wavelength 550 nm), one of which passes through 60 μm of glass of refractive index 1.5 and the other passes through the same distance in free space (a vacuum). The two beams are initially in phase.

Path difference = (n – 1)L = (1.5 – 1)60 x 10^{-6} = 3.0 x 10^{-6} m

$$= \frac{3.0 \times 10^{-5}}{550 \times 10^{-9}} = 54\frac{1}{2} \text{ wavelengths}$$

If the waves were to meet, they would meet anti-phase and interfere destructively

6 Real-life applications of interference

6.1 LIGO interferometer to measure gravitational waves (2019P2Q6 Data Analysis)

In 1916, Albert Einstein predicted the existence of gravitational waves. The Laser Interferometer Gravitational-wave Observatory (LIGO) have recently confirmed the existence of gravitational waves generated by the merger of two massive black holes in 2015 and colliding neutron stars in 2017.

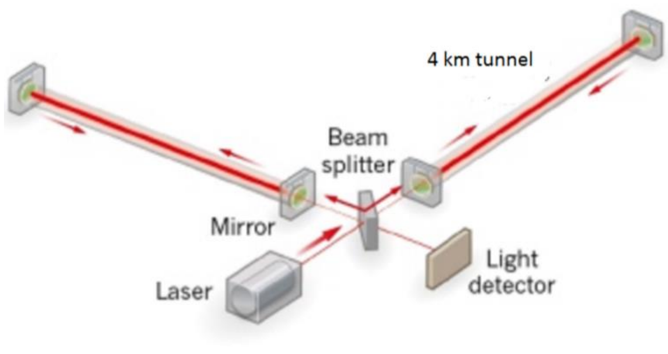


Fig (a)

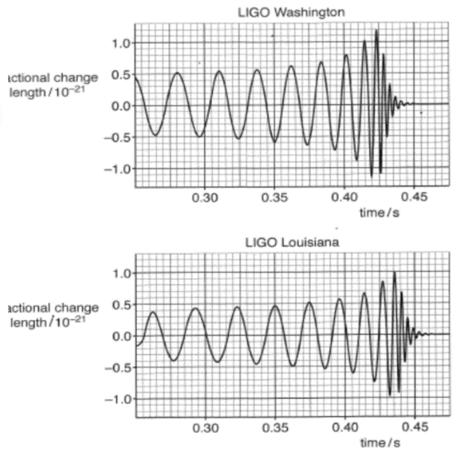
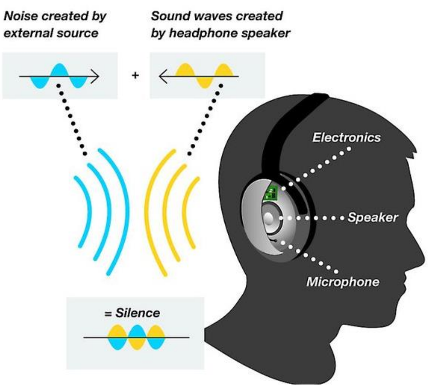


Fig (b)

LIGO comprises two enormous laser interferometers located 3000 km apart in Louisiana and Washington States of America. Fig (a) shows a schematic diagram of a laser interferometer. Interferometers work by merging two or more sources of light to create an interference pattern that provides information about the object or phenomenon being studied. In the absence of gravitational waves , the split laser beams in the two arm lengths are designed to meet anti-phase to give a zero resultant wave.

Gravitational waves cause space itself to stretch in one direction and simultaneously compress in a perpendicular direction. In LIGO, this causes one arm of the interferometer to get longer while the other gets shorter, then vice versa, back and forth as long as the wave is passing. This affect the distance traveled by each laser beam causing them to shift in and out of "phase" as they merge while the wave is causing the arm lengths to oscillate. Fig (b) shows the fractional change in the two arm lengths due to passing gravitational waves.

6.2 Noise-cancelling headphones



A small microphone mounted inside the headphones of an airplane pilot detects outside noise. The headphone's circuitry processes the electronic signal, reproduces the noise in a form that is anti-phase with the original noise, and plays it back through the headphones.

Because of destructive interference, a quieter background is produced.

6.3 X-Ray crystallography (X-ray diffraction)

Significant diffraction occurs when waves pass through slit of separation comparable to the wavelength of the incident wave. In X-ray diffraction, a crystal provides a suitable grating as the spacing between atoms or ions in the crystal lattice is comparable to the wavelength of X-rays (~ 10^{-10} m).

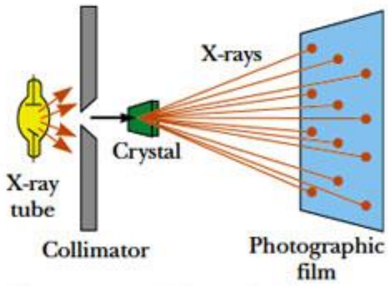


Fig (a)

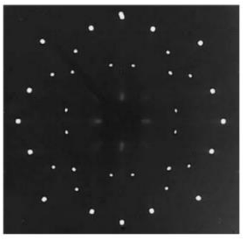


Fig (b)

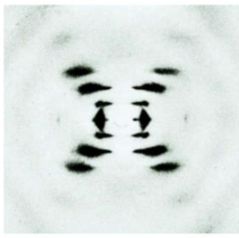
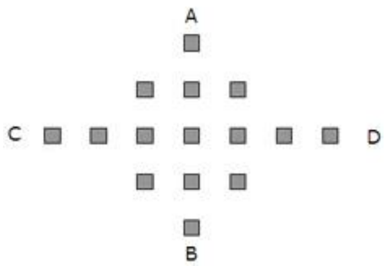


Fig (c)

Fig (a) illustrates the diffraction of X-rays when it is directed onto a crystal. The pattern of spots (maxima) as shown in Fig (b) is due to the complex three-dimensional structure of the crystal. Patterns such as these help in determination of spacing between the atoms and the nature of the crystal structure.

X-ray diffraction has also been applied with great success toward understanding the structure of biologically important molecules, such as nucleic acids. A famous result was the X-ray diffraction patterns of DNA as shown in Fig (c), obtained by Rosalind Franklin in 1952. It played the pivotal role in the discovery in 1953 by James Watson and Francis Crick of the double helix structure of the nucleic acid DNA.

Example 8:
A distant street light, which may be taken as a point source of light of wavelength 5.90 x 10^{-7} m, is viewed through a nylon curtain and the pattern of light seen is as shown.
Explain why the pattern appears.



The pattern appears as the curtain acts like two diffraction gratings perpendicular to each other. They produce an interference pattern that is the result of superposition from two perpendicular patterns. The brightest dot at the centre is the superposition of the two central maxima.

Online resources:

Sound interference:

http://www.sound-physics.com/Sound/Interference/

How mufflers work:

http://auto.howstuffworks.com/muffler.htm

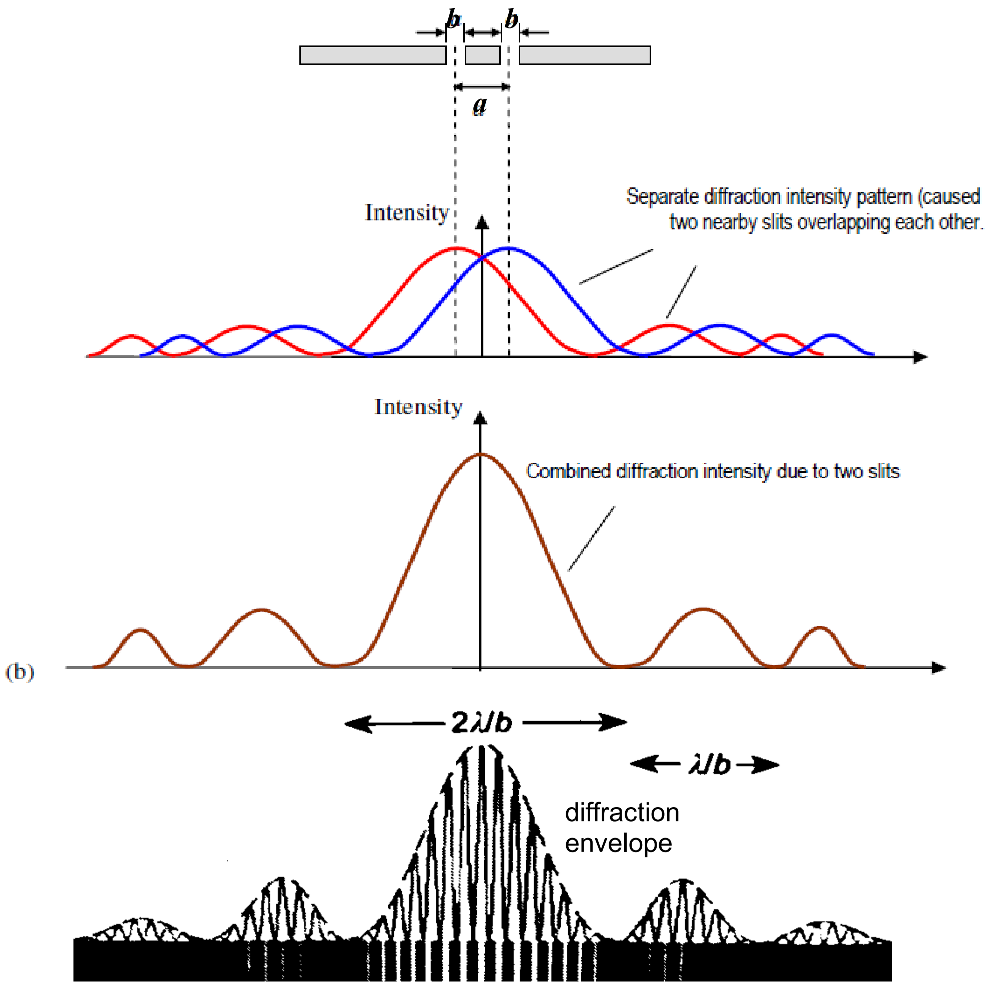
Noise-cancelling headphones:

http://electronics.howstuffworks.com/gadgets/audio-music/noise-canceling-headphone3.htm

Active noise cancellation:

http://www.school-for-champions.com/science/noise_cancellation.htm

Appendix A: Intensity pattern of double-slit experiment



For two finite slits, each of slit width *b*, separated by slit separation *a*, the intensity pattern due to both slits is a combination of both the single slit pattern and the double slit pattern. The intensity of the bright fringes is greatest for the central fringe and decreases for higher orders on the two sides. For the same slit separation *d*, the fringe separation *x* remains the same.

Tutorial questions: Solutions to Questions 1, 2, 7, 9, 11 and 12 are provided.

Principle of superposition and Diffractions

1. Explain the following terms: (i) *coherent* sources, (ii) *diffraction*, (iii) *interference,* (iv) constructive interference and (v) destructive interference.

Solution:

(i) coherent sources produce waves which have a constant phase difference between them.

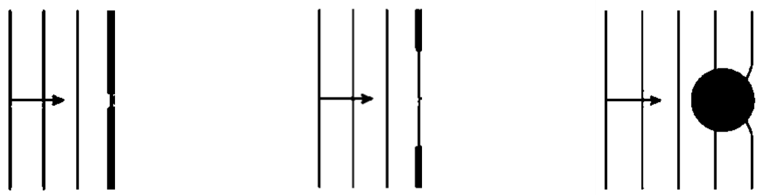
(ii) diffraction is the spreading of waves around an obstacle or through a gap, into its geometrical shadow.

(iii) interference occurs when two or more waves (of the same kind) meet and superpose to produce a resultant wave.

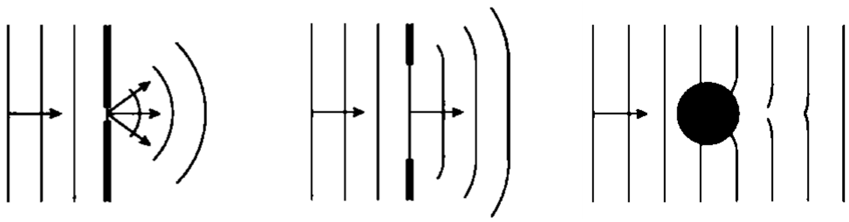
(iv) constructive interference occurs when two or more waves meet in phase and superpose to produce a resultant wave of maximum amplitude.

(v) destructive interference occurs when two or more waves meet anti-phase and superpose to produce a resultant wave of zero or minimum amplitude.

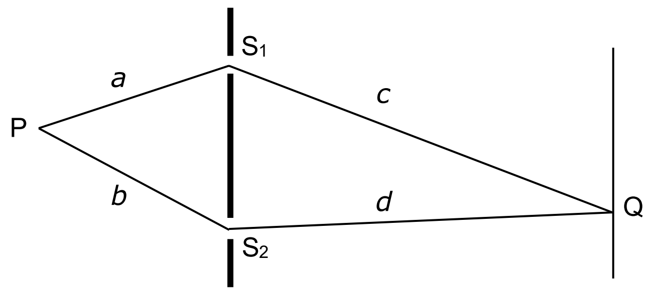
2. The diagrams represent plane wavefronts approaching obstacles(which are shaded) respectively. Draw on each diagram, lines illustrating diffraction, to represent the wavefronts after passing through the gaps.



Solution:



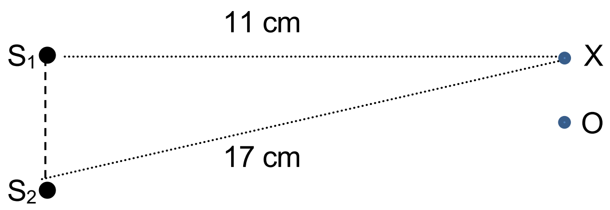
3. Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point source P, which is not equidistant from slits S_1 and S_2.



Write an expression expressing the *condition* for *destructive* interference at Q in terms of *a, b, c, d* and λ.

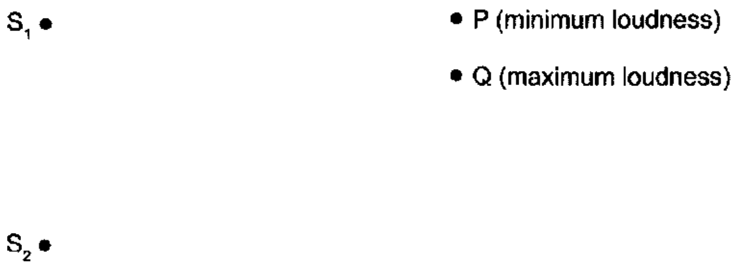
Two-source interference

4. Two point sources produce water waves at the same phase as shown below. Point O is equidistant from S_1 and S_2. X and O are points of constructive interference. S_1 X = 11 cm and S_2 X= 17 cm. There are two more points of constructive interference between O and X.



(a) Express the path difference at X in terms of the wavelength λ.
(b) Find the value of λ.
(c) If the frequency of the source is 5 Hz, what is the wave speed? [2 cm, 10 cm s^{-1}]

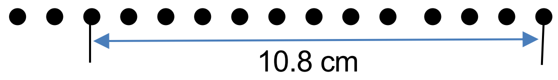
5. (2013P1Q22) Two coherent sources of sound S_1 and S_2 give rise to a minimum loudness at point P, and maximum loudness at point Q.



The amplitude of the sound produced by S_2 is reduced, the frequency of the sound produced by both sources is unchanged.
What happens to the loudness of the sound at P and at Q?

	loudness at P	loudness at Q
A	decreases	decreases
B	increases	decreases
C	increases	no change
D	no change	no change

6. Red light of wavelength 633 nm from a laser is passed through a double slit. The diagram shows the pattern of dots (maxima) produced on the screen. The distance from the slits to the screen is 3.02 m.



(a) What is the fringe separation?
(b) What is the slit separation? [0.900 cm, 2.12×10^{-4} m]

7. Coherent light is incident normally on a double slit. Interference fringes are observed on the screen placed parallel to the plane of the double slit and 2.3 m from it. Point P on the screen is equidistant from the two slits A and B.
Assume that, for the fringes near point P on the screen, the light reaching the screen from slit A alone has intensity I and that from slit B alone has intensity $\frac{1}{3}I$.
Determine the intensity, in terms of I, of a dark fringe near P. [0.18 I]

Solution:
$I = kA^2$
$\Rightarrow \frac{1}{3}I = k\left(\frac{1}{\sqrt{3}}A\right)^2$
Destructive interference occurs at a dark fringe.
Resultant amplitude $= A - \frac{1}{\sqrt{3}}A = 0.42\,A$.
Intensity $= k(0.42\,A)^2 = 0.18\,kA^2 = 0.18\,I$.

8. In a Young's double slit experiment, the fringe separation observed using yellow light was found to be 0.275 mm. The yellow lamp, giving a wavelength of 550 nm is replaced with a purple one giving wavelength of 400 nm in the violet and 600 nm in the red. The remainder of the apparatus is undisturbed. Calculate:
(a) the distance between the fringes formed by the violet light,
(b) the distance between the fringes formed by the red light and
(c) the distance from the purple fringe on the axis to the next purple fringe observed. [0.20 mm, 0.30 mm, 0.60 mm]

9. In the Young double slit experiment, describe the effects on the (a) intensity and (b) fringe separation of the fringes when:
   (i) both slits are made narrower whilst keeping the slit separation constant;
   (ii) width of one slit is reduced to half that from the other slit;
   (iii) the slits separation is decreased;
   (iv) a thin sheet of transparent plastic is inserted in front of one slit;
   (v) both slits are covered with a polaroid and one polaroid is rotated.

Solution:

(i) Narrower slits means the amplitude of each wave is reduced. So the resultant amplitude and hence intensity decrease (intensity $\propto$ amplitude^2)
No change in fringe separation x since no change in a, λ and D.

(ii) Halving width of one slit means amplitude of one source is half of the other. Resultant intensity of bright fringes decreases but resultant intensity of dark fringes increases due to incomplete cancelation of the two amplitudes.
Hence, fringes will not be well defined / contrast will be reduced.
No change in fringe separation.

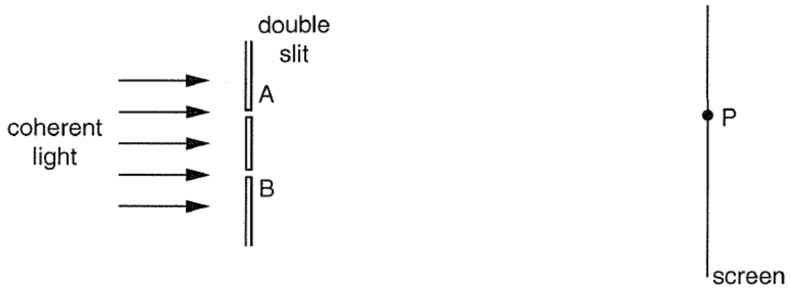
(iii) No change in intensity since slit width is unchanged.
Since $x = \frac{\lambda D}{a}$, decreasing slits separation a will increase fringe separation x

(iv)

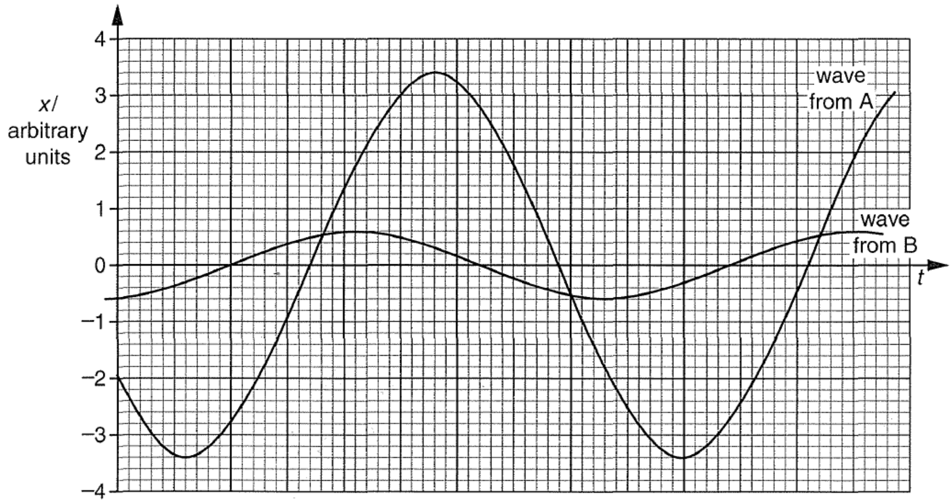
Plastic sheet increases the optical path length of the light waves and introduces an extra path difference of (nL - L) where L is the thickness of the plastic sheet. Fringe pattern will be shifted along the screen in the direction where the plastic sheet is.
No change in intensity and no change in fringe separation.

(v) When axes of polarisation of both polaroids are aligned, well-defined interference fringes are seen. When one polaroid is rotated, the pattern become less and less defined as the contrast between the maxima and minima decreases. When the axes of polaroids are perpendicular to each other, the fringe pattern disappears to give uniform illumination.
No change in fringe separation.

10. (2014P3Q4b) Coherent light is incident normally on a double slit, as shown in the figure (not to scale).



Light passes through the two slits A and B and is incident on a screen. The variation with time t of the displacement x of the light arriving at point P on the screen is shown in the graph.



(i) Use the graph to determine the phase difference between the waves from slit A and from slit B that arrive at point P. [1]

(ii) Dark fringes and bright fringes are both formed on the screen. Determine, for the bright fringe and the dark fringe closest to point P, the ratio

$$\frac{\text{intensity of light at the dark fringe}}{\text{intensity of light at the bright fringe}}$$ [3]

[57°, 0.49]

Single slit diffraction and Rayleigh criterion

11. 2018P2Q3 part

(b) Laser light is incident normally on a single slit of width 12 μm. The light diffracts and a diffraction pattern is observed on the screen 2.7 m away as Illustrated in Fig. 3.1.

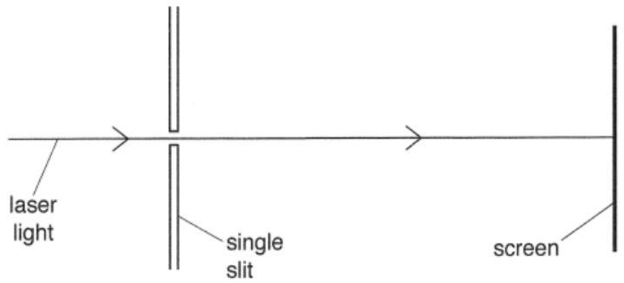


Fig. 3.1

The screen is parallel to the plane of the single slit.
A diffraction pattern is formed on the screen. Part of the diffraction pattern showing the variation with displacement from the centre of the pattern of the intensity of the light is as shown in Fig. 3.2.

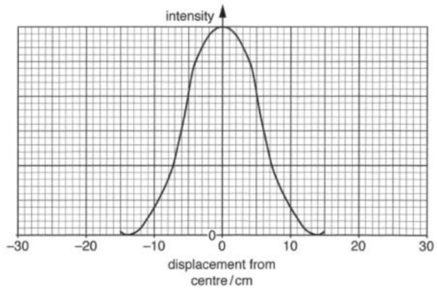


Fig. 3.2

Calculate the wavelength λ of the laser light. [3]

(c) The single slit in (b) is now replaced by a narrower slit. On Fig. 3.2. Sketch the new diffraction pattern. [2]

(d) (i) State and explain the changes to the observed diffraction pattern in Fig. 3.2 when visible light of longer wavelength is used. [2]

(ii) When white light is incident on a single slit, the central fringe is coloured at the edges and has a white central region. Explain this observation. [2]

[6.22 x 10^{-7} m]

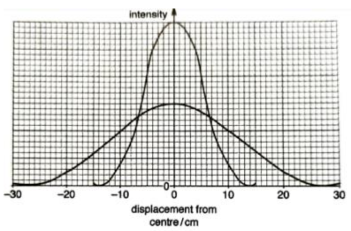
Solution:

(b) Condition for first minima $\sin\theta = \frac{\lambda}{b}$. But $\tan\theta = \frac{x}{D}$.

For small angle, $\sin\theta \approx \tan\theta$

$$=> \frac{\lambda}{b} \approx \frac{x}{D} => \frac{\lambda}{12\times10^{-6}} \approx \frac{0.14}{2.7} => \lambda = 6.22\times10^{-7} \text{ m}$$

(c)

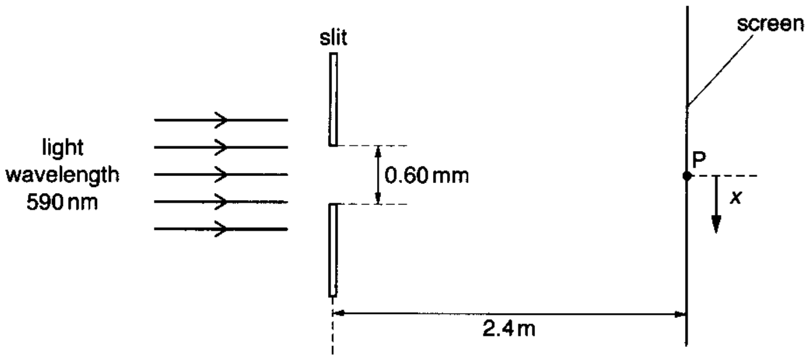


sinθ = λ/b. Narrower slit has smaller b, thus θ increases, causing the diffraction to spread out. Also, narrower slit allows less light to pass through and hence intensity decreases.

(d) (i) sinθ = λ/b. Longer wavelength are diffracted by larger angle. Thus first minima occurs further from the centre and hence the diffraction pattern is broader.

(ii) Different wavelengths in the white light diffract and interfere constructively at different angular position, with the longer wavelength further away from the straight through position resulting in coloured edges. The central region is white as all wavelengths interfere constructively here.

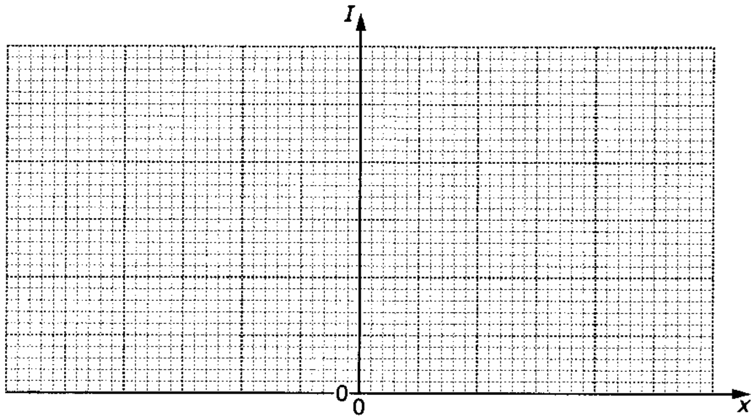
12. (2017P3Q5) Parallel light of wavelength 590 nm is incident on a rectangular slit of width 0.60 mm, as shown in the figure (not to scale).



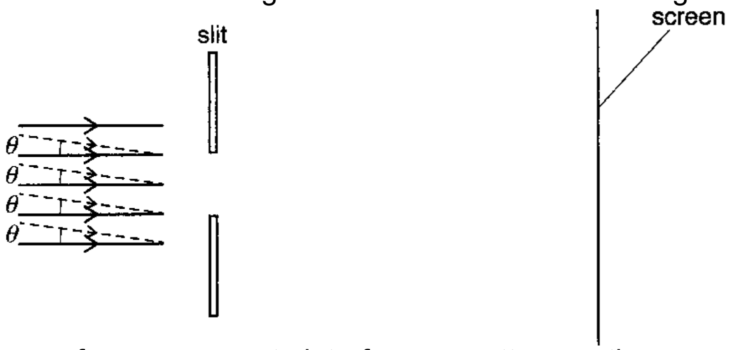
Light passing through the slit is incident on a screen that is 2.4 m from the plane of the slit. The centre of the interference pattern formed on the screen is at P.

(a) (i) Calculate the width of the central fringe, as observed on the screen. [3]

(ii) Sketch a graph to show the variation with distance *x* from point P of the intensity *I* of the light on the screen. [3]



(b) Parallel light from a second source and of the same wavelength 590 nm is also incident on the slit. The angle between the two beams of light is θ as shown.



Each beam forms a separate interference pattern on the screen.

(i) Explain what is meant by the *Rayleigh criterion* for the resolution of the two patterns. [2]

(ii) Calculate the angle θ, in rad, such that the two interference patterns are just resolved. [2]

[4.72 mm, 9.83×10^{-4} rad]

Solution:

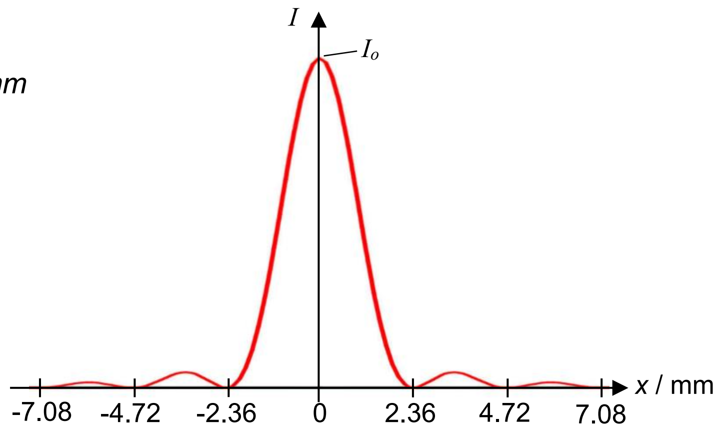
(a)(i) $\sin\theta = \dfrac{\lambda}{b}$ for first minima, $\tan\theta = \dfrac{x}{D}$

For small angle, $\sin\theta \approx \tan\theta$

$$=> \frac{590\times10^{-9}}{0.60\times10^{-3}} = \frac{x}{2.4} => x = 2.36 \text{ mm}$$

$\therefore$ *Width of central fringe = 2x = 4.72 mm*

(ii)
- Central maxima is twice as wide as each of the subsidiary maxima on both sides
- Its peak intensity >> that of subsidiary maxima, whose peak intensity decreases with distance from the centre



(b) (i) Rayleigh criterion specifies that when the central maximum of the diffraction pattern from one source is directly over the first minimum of the diffraction pattern from the second source, the patterns are just distinguishable.

(ii) $\theta = \dfrac{\lambda}{b} = \dfrac{590\times10^{-9}}{0.60\times10^{-3}} = 9.83\times10^{-4}$ rad

13. (2021P3Q5) A double slit consists of two parallel slits, each of slit width 0.100 mm. The separation of the slits is 1.40 mm. Parallel light of wavelength 590 nm is incident normally on the double slit. A screen is placed parallel to the plane of the double slit at a distance of 2.60 m from the slits.
(i) Initially, one of the two slits is covered.
Calculate the width of the central fringe of the single-slit diffraction pattern seen on the screen. Give your answers to three significant figures.
(ii) Both slits are now uncovered.
Estimate the number of fringes resulting from double-slit interference that are seen within the central maximum produced by the single-slit diffraction.

[0.0307 m, 28]

Diffraction grating

14. Monochromatic light of wavelength 5.0×10^{-7} m falls normally on a diffraction grating having 600 lines per mm.
    (a) Calculate the angular position of the first order spectrum on one side of the normal.
    (b) What is the highest order of spectrum obtained on this side of the normal?
    [17.5°, 3]

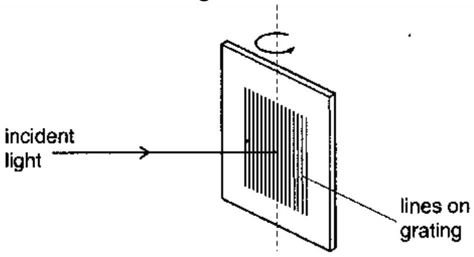
15. A diffraction grating has 400 lines per mm and is illuminated normally by monochromatic light of wavelengths 4.5×10^{-7} m and 6.0×10^{-7} m.
    In what respective order of spectrum does each colour overlap and what is the corresponding angle of diffraction? [4th and 3rd order; 46°]

16. Light from a mercury lamp is incident normally on a plane diffraction grating ruled with 6000 lines per cm. The spectrum contains two strong yellow lines of wavelength 577 nm and 579 nm. What is the angular separation of the second-order diffracted beams corresponding to these two wavelengths? [0.19°]

17. (2019P3Q8c) Light of wavelength 5.90×10^{-7} m is incident normally on a diffraction grating having 750 lines per millimetre.

(i) Determine the angles of diffraction at which maxima of diffracted light are seen. [3]

(ii)



The diffraction grating is now rotated through a small angle so that the incident light is no longer normal to the grating as shown. The resulting diffraction pattern is no longer symmetrical.

Suggest qualitatively, any other effects on the appearance of the diffraction pattern produced on the screen. [2]

[0°, 26.3°, 62.3°]