TEMASEK JUNIOR COLLEGE

Unit 16: CIRCUITS

Learning Outcomes

Candidates should be able to:

(a) recall and use appropriate circuit symbols.

(b) draw and interpret circuit diagrams containing sources, switches, resistors (fixed and variable), ammeters, voltmeters, lamps, thermistors, light-dependent resistors, diodes, capacitors and any other type of component referred to in the syllabus.

(c) define the resistance of a circuit component as the ratio of the potential difference across the component to the current in it, and solve problems using the equation $V = IR$

(d) recall and solve problems using the equation relating resistance to resistivity, length and cross-sectional area, $R = \dfrac{\rho L}{A}$.

(e) sketch and interpret the $I – V$ characteristics of various electrical components in a d.c. circuit, such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor.

(f) explain the temperature dependance of the resistivity of typical metals (e.g. in a filament lamp) and semiconductors (e.g. in an NTC thermistor) in terms of the drift velocity and number density of charge carriers respectively.

(g) show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.

(h) solve problems using the formula for the combined resistance of two or more resistors in series.

(i) solve problems using the formula for the combined resistance of two or more resistors in parallel.

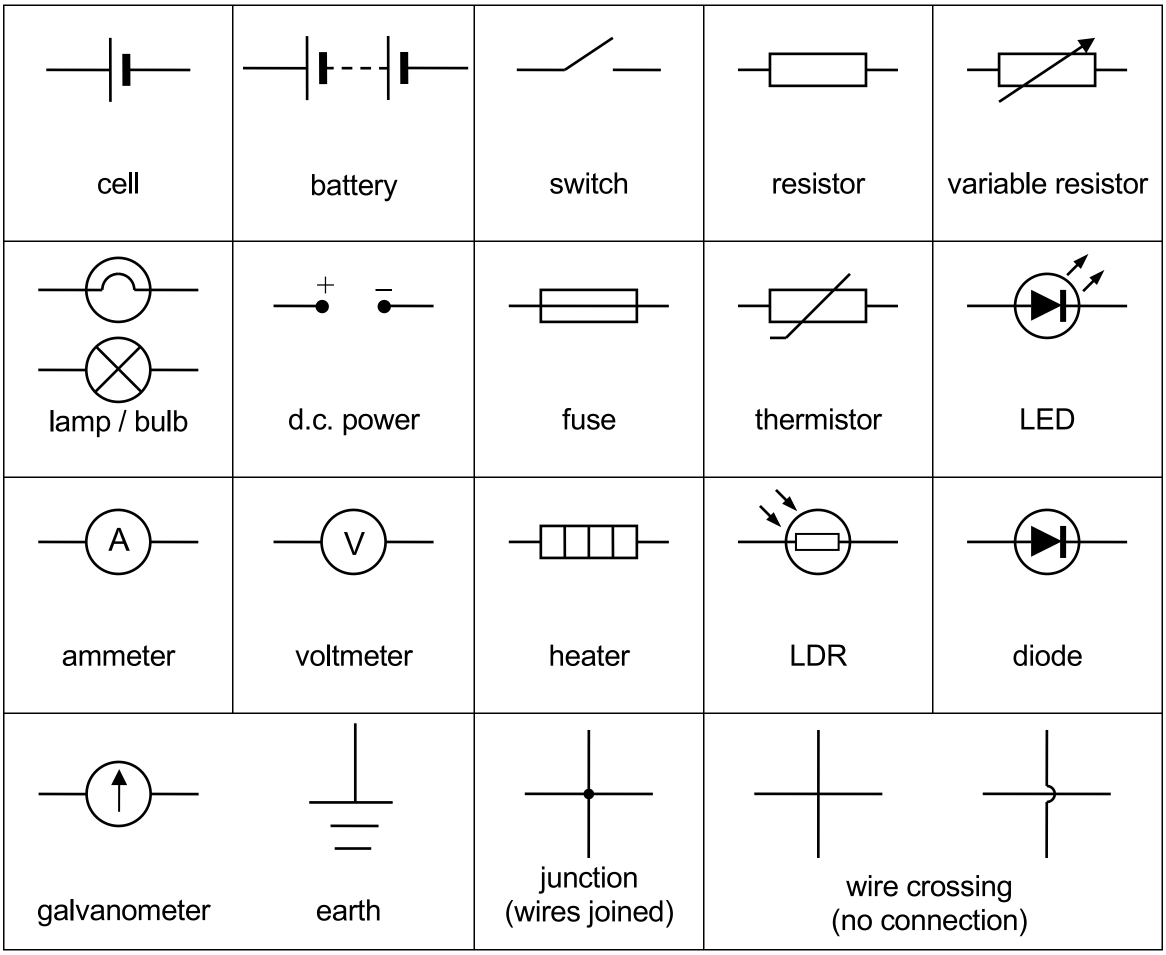
(j) solve problems involving series and parallel arrangements of resistors for one source of e.m.f including potential divider circuits which may involve NTC thermistors and light-dependent resistors.

(k) solve problems using the formulae for the combined capacitance of two or more capacitors in series and in parallel.

(l) describe and represent the variation with time, of quantities like current, charge and potential difference, for a capacitor that is charging or discharging through a resistor, using equations of the form $x = x_0\,e^{-\frac{t}{\tau}}$ or $x = x_0[1 - e^{-\frac{t}{\tau}}]$, where $\tau = RC$ is the time constant.

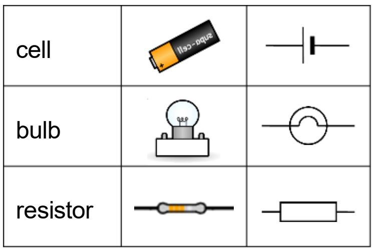
1 Electrical circuit symbols

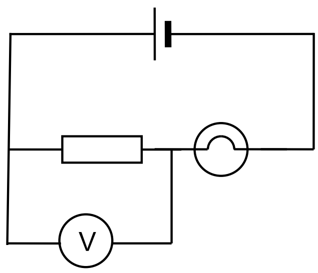
LO(a)(b)



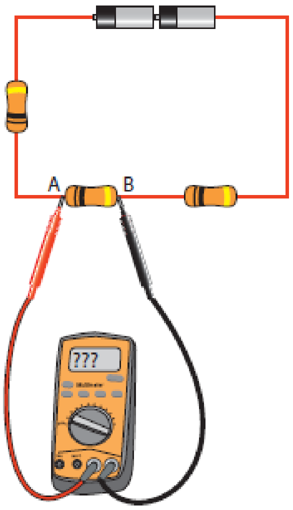
1.1 Practical Circuits

Circuit diagrams are used to show how electrical components are connected together to perform useful tasks. The connecting wires (leads) are drawn as straight lines in the diagram. All circuits need at least one source of energy. A cell is a single component that transfers chemical energy to electrical energy. The longer line represents the positive terminal of a cell. *When cells are connected together they are called a battery.*





1.1.1 Measurement of potential difference



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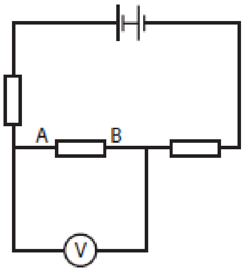
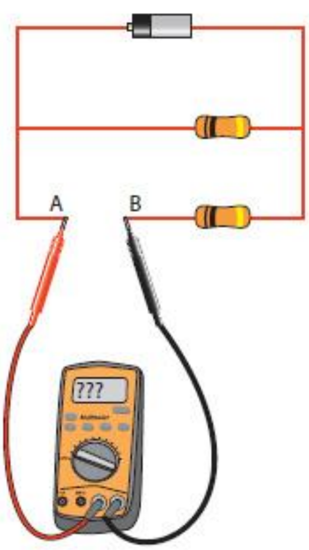


The p.d. is the difference in potential between two points. To measure the p.d. between A and B of a resistor, the voltmeter must be connected in parallel to the resistor, i.e. one end to A, the other to B.

An **ideal voltmeter has infinitely high resistance** so that it does not take any current from the circuit.

1.1.2 Measurement of current



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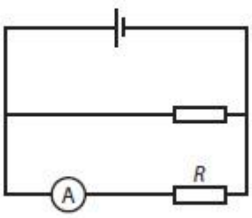


To measure the current flowing through a resistor, the ammeter must be connected so that the same current will flow through the ammeter as flows through the resistor.

An **ideal ammeter has zero resistance** so that it does not change the current in the circuit.

2 Resistance and resistivity

2.1 Resistance *LO (c)*

Microscopically, resistance is due to frequent collisions of the drifting electrons with the vibrating ions in the lattice. When accelerated by an electric field, these free electrons gain kinetic energy but transfer part of this energy to the ions during collisions and hence slow down. This limits the current flow.

Electrical resistance of a conductor is defined as the ratio of the potential difference across the conductor to the current flowing through the conductor.

i.e. $R = \dfrac{V}{I}$

SI unit is ohm (Ω).

One ohm is defined as the resistance of a conductor when the potential difference across the conductor is one volt per ampere of current flowing through it.
(i.e. 1 Ω = 1 V A^{-1})

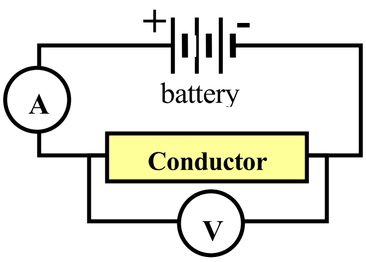
From $R = \dfrac{V}{I} \Rightarrow \boxed{V = IR}$

Conductors which give a constant ratio of *V* to *I* are known as ohmic conductors. They are said to obey Ohm's Law.

Ohm's Law states that the current through a metallic conductor is directly proportional to the potential difference across it, provided the temperature and other physical conditions are kept constant.

Most resistors are not ohmic. Even for a metallic conductor, when current is increased with an increase in p.d., the temperature increases, thus increasing the resistance. The resistance is constant only when the temperature or the other physical conditions are kept constant, e.g. by immersing the specimen in a constant temperature water bath.
Note:

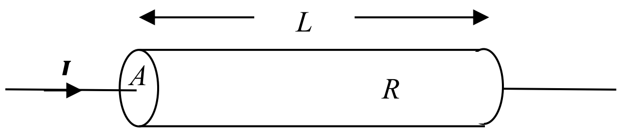
- The diagram below shows the circuit used to determine the resistance of a conductor.



- A low resistance ammeter is connected in series with the conductor to measure the current *I* through it. A high resistance voltmeter is connected across the conductor to measure the potential difference *V* across it.

2.2 Resistivity

LO (d)



The resistance *R* of a uniform conductor of a given material at a given temperature is directly proportional to its length *L* and inversely proportional to its cross-sectional area *A* according to the equation

$$\boxed{R = \frac{\rho L}{A}}$$

where the constant of proportionality ρ is the resistivity and is a characteristic of the material.

SI unit for resistivity is ohm metre (Ω m).

The resistivity of a material is numerically equal to the resistance between opposite faces of a cube of the material, of unit length and unit cross-sectional area.

The table below shows the resistivity for a selection of different materials.

	Substance	Resistivity at 25 °C / Ωm	Uses
Conductors			
Metals	copper gold aluminium tungsten	1.72×10^{-8} 2.42×10^{-8} 2.82×10^{-8} 5.51×10^{-8}	connecting wires microphone contacts power cables light-bulb filaments
Alloys	steel constantan nichrome	20×10^{-8} 49×10^{-8} 100×10^{-8}	 standard resistors heating elements
Semiconductors	carbon germanium silicon	3.5×10^{-5} 0.60 2300	resistors transistors transistors, chips
Insulators	glass polythene	$\sim 10^{13}$ $\sim 10^{14}$	power grid insulators wire insulation

Example 1:

The overhead wire used to supply power to a factory is made of copper of resistivity 1.72×10^{-8} Ωm and has cross-sectional area of 5.00×10^{-5} m^2. Calculate the resistance of one kilometre length of the wire.

Solution:

$$R = \frac{\rho l}{A}$$

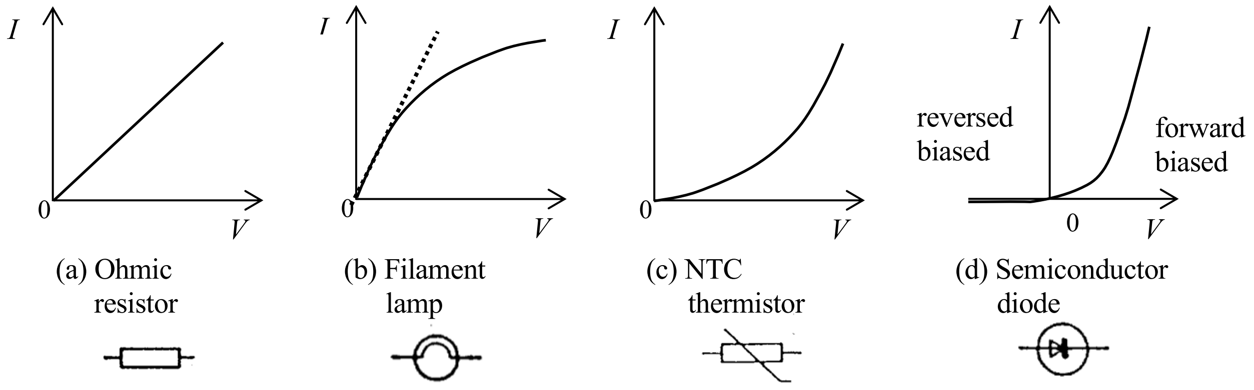
$$= \frac{(1.72\times10^{-8})(1000)}{5.00\times10^{-5}}$$

$$= 0.344\ \Omega$$

2.3 *I – V* characteristics of electrical components *LO (e)*

The variation in resistance can be represented by a plot of current *I* against p.d. *V*, called the *I – V* characteristics. The ratio of *V* to *I* gives the value of resistance *R* at that value of *I*.

The ratio of *V* to *I* on such a curve gives the value of the resistance at that value of p.d.



(a) **Ohmic resistor**: The ohmic resistor is a pure metallic conductor at constant temperature. It obeys Ohm's law and has a constant resistance. Hence the *I – V* relationship is *a straight line through the origin*.

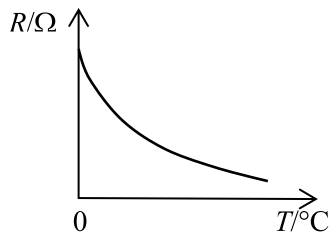
LO (f)

(b) **Filament lamp**: When the current is small, the temperature of the filament lamp is low so the resistance of the filament remains constant. When the current is large, the temperature of the filament wire increases. The lattice ions vibrate more vigorously and the conducting electrons collide more frequently with the lattice ions, hindering the flow of electrons and resulting in a decrease on the drift velocity. Hence the resistance of the filament lamp increases with increased temperature and *the line bends towards larger V to I ratio.*

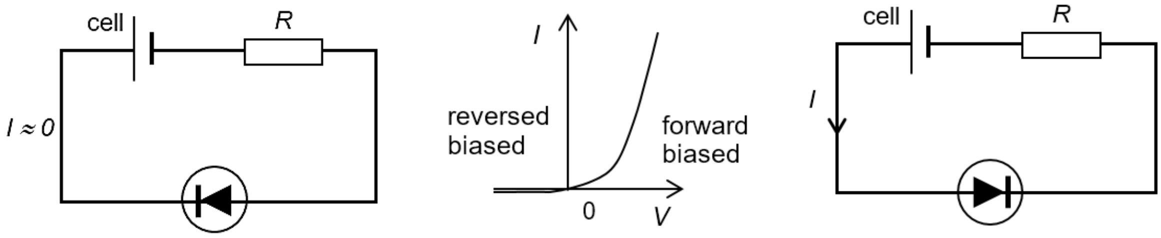
(c) **Negative temperature coefficient (NTC) thermistor**: A thermistor is made of semiconductor materials such as carbon, silicon or germanium.

An increase in voltage and the corresponding increase in current cause a rise in temperature. This would increase the thermal agitation of the atoms and set free more electrons and holes in the crystal lattice. This increase in the number density of charge carriers results in a decrease of the resistance. The effect outweighs the increase in resistance due to the increase in frequency of collisions with the lattice ions. Overall, the resistance decreases and *the line bends towards smaller V to I ratio*.

The **resistance-temperature characteristic** of an NTC thermistor is shown on the right.
The resistance is high at low temperature and resistance is low at high temperature.



(d) **Semiconductor diode**: A diode is a device that passes current in one direction only. When forward-biased, it conducts with negligible resistance when the p.d. exceeds a certain value (for example, about 0.7 V for a p-n junction semiconductor diode). When reverse-biased, only a very small leakage current flows due to a very large resistance.

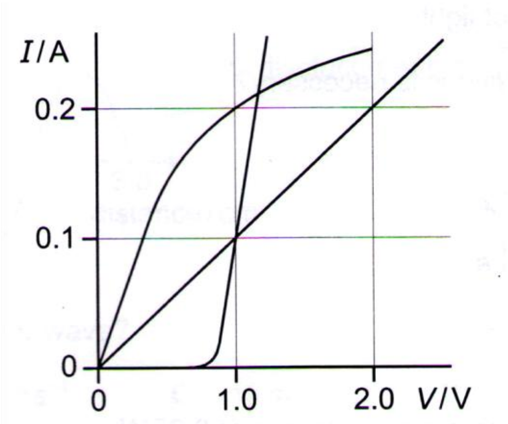


Reverse-biased. Diode does not conduct

Forward-biased. Diode conducts

Example 2:

The graph shows the *I*-*V* characteristics of three electrical components, a diode, a filament lamp and a resistor, plotted on the same axes.



Which statement is correct?

A The resistance of the diode equals that of the filament lamp at about 1.2 V.
B The resistance of the diode is constant above 0.8 V.
C The resistance of the filament lamp is twice that of the resistor at 1.0 V.
D The resistance of the resistor equals that of the filament lamp when V = 0.8 V.

Solution:

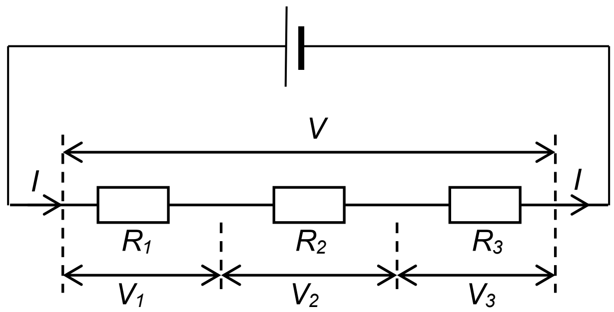
Answer: A
The *I*-*V* characteristics of the diode and the filament lamp intersect at 1.2 V.
Since they have the same potential difference and the same current, both the diode and the filament lamp have the same resistance at 1.2 V.

3 Combinations of resistors (Proof not required)

In practical situations, there are usually more than one resistor in a circuit. The resistors are often joined together in two simple arrangements, series and parallel.

3.1 Resistors in series *LO(h)*

In a series arrangement, the same current flows through each resistor.



Applying the law of conservation of energy, the p.d. across resistor R_1 plus the p.d. across R_2 plus the p.d. across R_3 must be equal to the total p.d. across the combination.

$$V = V_1 + V_2 + V_3$$

The current I through all the resistors is the same.
Applying Ohm's law to each resistor gives

$$IR_{eq} = IR_1 + IR_2 + IR_3$$ where R_{eq} is the equivalent resistance.

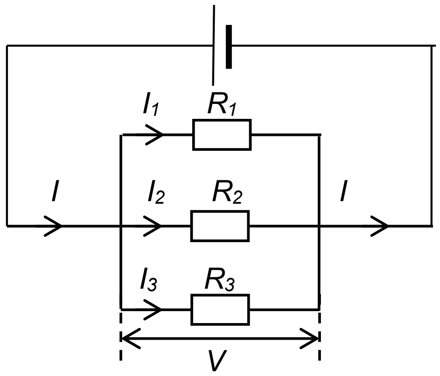
Thus $$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

> The equivalent resistance of resistors in series is the sum of all the individual resistance.

Note:
- For N similar resistors R in series, $R_{eq} = N R$.
- The equivalent resistance R_{eq} must be greater than the greatest resistance.

3.2 Resistors in parallel *LO(i)*

A parallel arrangement is one where the current in each resistor is different but the p.d. is the same.



Applying the law of conservation of charge (or current), the current going into a junction must be equal to the sum of the currents through each resistor.

$$I = I_1 + I_2 + I_3$$

Apply Ohm's law to each resistor gives

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$ where R_{eq} is the equivalent resistance.

Thus $$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

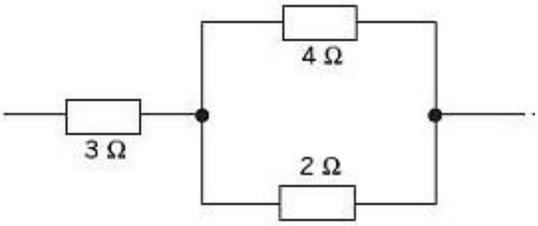
> The reciprocal of the equivalent resistance of resistors in parallel is the sum of the reciprocals of all the individual resistances.

Note:

- For N similar resistors R in parallel, $R_{eq} = \frac{R}{N}$.
- The equivalent resistance R_{eq} must be smaller than the smallest resistance.
- For **two** resistors in parallel, $R_{eq} = \frac{\text{product of two resistance}}{\text{sum of two resistance}}$. i.e. $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$.

Example 3

Three resistors of resistance, 2.0 Ω, 3.0 Ω, and 4.0 Ω are connected. Calculate the total resistance of the three resistors when they are connected
(a) in series
(b) in parallel.
(c) as shown in the diagram



Solution:

(a) In series, the resistances are added together, so

$R = 2.0 + 3.0 + 4.0 = 9.0\ \Omega$

(b) In parallel, the sum of reciprocals are used.

$$\frac{1}{R} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{4.0} = \frac{13}{12}$$

$$\rightarrow \quad R = \frac{12}{13} = 0.92\ \Omega$$

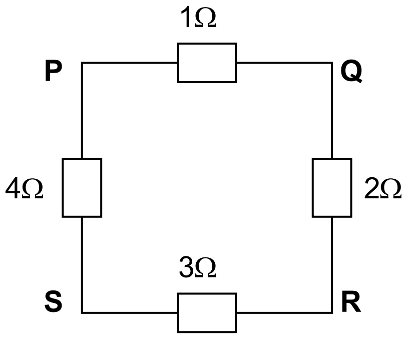
(c) 2.0 Ω and 4.0 Ω in parallel gives combined resistance $\left(\frac{1}{2.0} + \frac{1}{4.0}\right)^{-1} = 1.3\ \Omega$

(Alternatively, using $R = \frac{R_1R_2}{R_1 + R_2} = \frac{2.0 \times 4.0}{2.0 + 4.0} = 1.3\ \Omega$)

Total resistance of 1.3 Ω and 3.0 Ω in series gives (1.3 + 3.0) = 4.3 Ω

Example 4

Four resistors are connected as shown. Across which two points are the resistance of the combination (a) minimum, (b) maximum?



Solution:
(a) Minimum resistance is across **P** and **Q**.

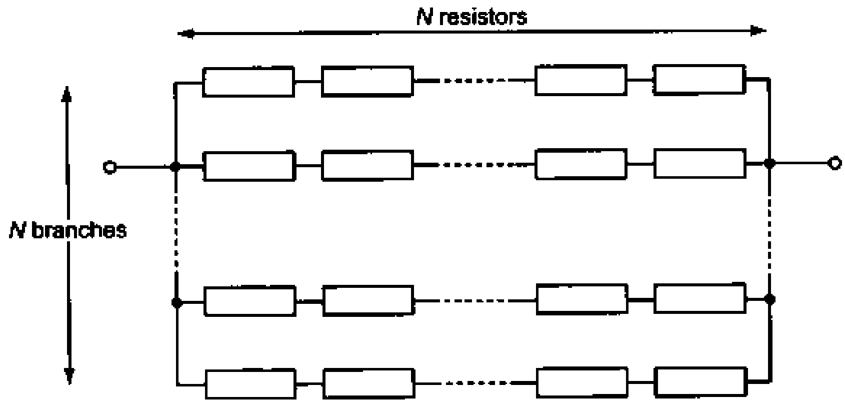
$$R = \frac{(1)(9)}{1+9} = 0.9\ \Omega.$$

(b) Maximum resistance is across **S** and **Q**.

$$R = \frac{5\Omega}{2} = 2.5\ \Omega.$$

Example 5

An array of resistors, each of resistance R, consists of N parallel branches. Each branch contains N resistors connected in series.



What is the total resistance of the array?

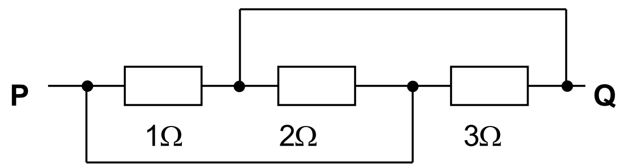
Solution:

In each branch, the total resistance = NR

The combined resistance of N branches in parallel = NR/N = R

Example 6

Three resistors are connected as shown in the diagram using connecting wires of negligible resistance. What is the approximate resistance between points **P** and **Q**?



Solution:

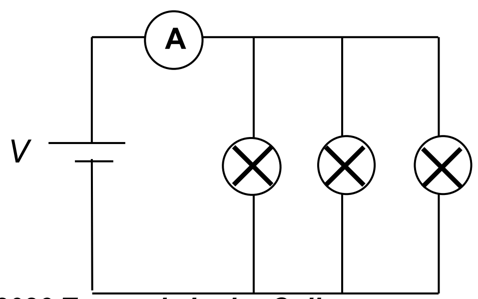
The three resistors are in parallel.

Thus $\dfrac{1}{R_{PQ}} = \dfrac{1}{1} + \dfrac{1}{2} + \dfrac{1}{3}$

$R_{PQ} = 0.5\ \Omega$ (to 1 sf)

Example 7

Three similar light bulbs are connected to a constant-voltage d.c. supply as shown in the diagram. Each bulb operates at normal brightness and the ammeter (of negligible resistance) registers a steady current. The filament of one of the bulbs breaks. What happens to the ammeter reading and to the brightness of the remaining bulbs?

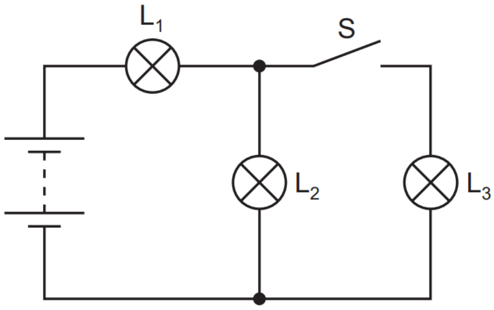


Solution:

- If the filament of one of the bulbs breaks, effective resistance of the circuit increases from ⅓R to ½R.
- Thus **<u>total</u>** current decreases and ammeter reading decreases.
- Brightness of remaining bulbs remains unchanged as $P = \dfrac{V^2}{R}$ remains unchanged.

Example 8

Three identical lamps L_1, L_2 and L_3 are connected to a battery with negligible internal resistance, as shown.



What happens to the brightness of lamps L_1 and L_2 when the switch S is closed?

	lamp L_1	lamp L_2
A	brighter	brighter
B	brighter	dimmer
C	dimmer	brighter
D	dimmer	dimmer

Ans: B

When switch S is closed, effective resistance of L_2 and L_3 decreases. Total resistance of circuit decreases, so total current in circuit increases.

$P = I^2R$ increases $\Rightarrow$ L_1 becomes brighter.

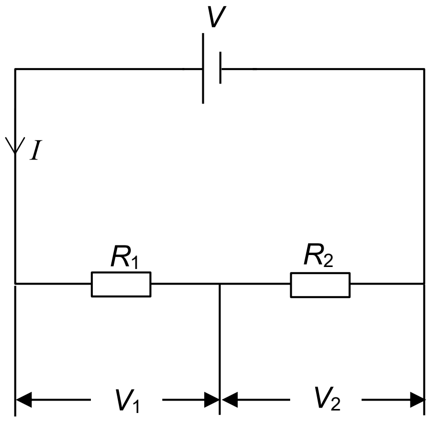
Pd across L_1 increases $\Rightarrow$ pd across L_2 decreases

$P = \dfrac{V^2}{R}$ decreases $\Rightarrow$ L_2 becomes dimmer.

4 Potential divider

LO(j)

The most basic potential divider consists of two resistors with resistances R_1 and R_2 in series with a power supply. The two resistors have the same current in them, and the sum of the p.d.s across the resistors is equal to the source p.d. V.



By Ohm's law, $I = \dfrac{V}{R_1 + R_2}$

$$\boxed{V_1 = R_1 I = \left(\frac{R_1}{R_1 + R_2}\right)V}$$

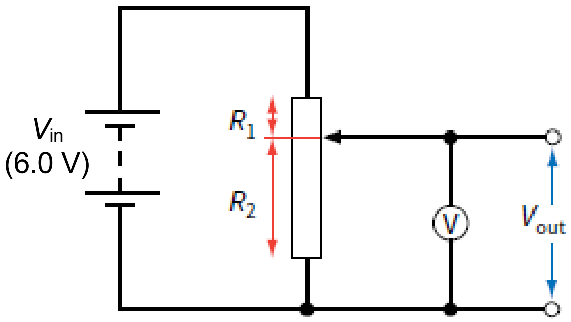
$$\boxed{V_2 = R_2 I = \left(\frac{R_2}{R_1 + R_2}\right)V}$$

Note that $V_1 + V_2 = V$,

I.e. V is shared by R_1 and R_2.

4.1 Using a potential divider to give a variable p.d.

Another more useful potential divider arrangement using a variable resistor, also called a rheostat, allows a range of p.d. to be tapped using a slider.

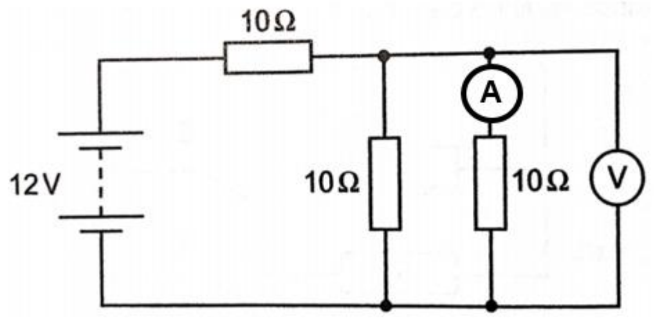


rheostat

By moving the sliding contact, we can achieve any value of V_{out} between 0.0 V (slider at the bottom) and 6.0 V (slider at the top). The output voltage V_{out} depends on the relative values of R_1 and R_2. You can calculate the value of V_{out} using the following potential divider equation:

$$V_{out} = \left(\frac{R_2}{R_1 + R_2}\right)V_{in}$$

Example 9
In the circuit shown, both the voltmeter and ammeter are ideal.
What are the voltmeter and ammeter readings?



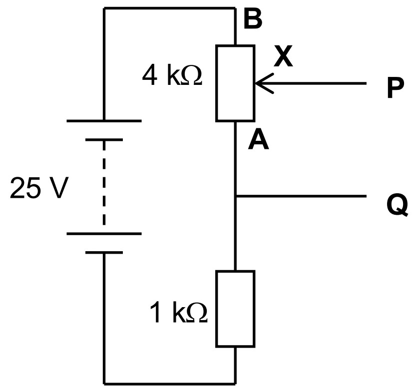
Solution:
The combined resistance of 10 Ω and 10 Ω in parallel is 5 Ω.

By potential divider equation,

$$V = \left(\frac{5}{5+10}\right)12 = 4\text{ V}$$

Current $I = \dfrac{4}{10}$ = 0.4 A

Example 10
The diagram below shows a potential divider circuit which by adjustment of the position of contact **X**, can be used to provide a variable potential difference between the terminals **P** and **Q**.
What are the limits of this potential difference?



Solution:

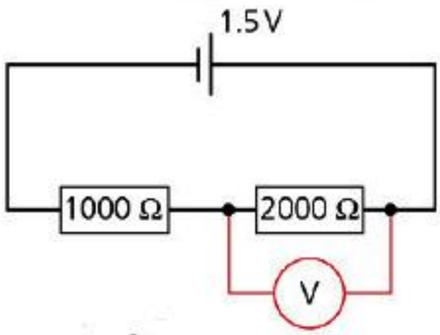
If position of contact **X** is at

A: $V_{PQ} = 0\text{ V}$

B: $V_{PQ} = \left(\frac{4k}{4k+1k}\right)25 = 20\,V$

Thus $0 \le V_{PQ} \le 20\text{ V}$

Example 11
Consider the circuit shown. The cell has an e.m.f of 1.5 V and zero internal resistance. The voltmeter is a non-ideal voltmeter of resistance 5000 Ω



(a) Calculate the p.d. across the 2000 Ω before the voltmeter is connected.
(b) Calculate the p.d. across the 2000 Ω after the voltmeter is connected.

Solution:

(a) By potential divider equation, $V = \dfrac{2000}{1000+2000} x1.5 = 1.0\ V$

(b) Combined resistance of 2000 Ω and 5000 Ω is $(\dfrac{1}{2000}+\dfrac{1}{5000})^{-1} = 1430\ \Omega$

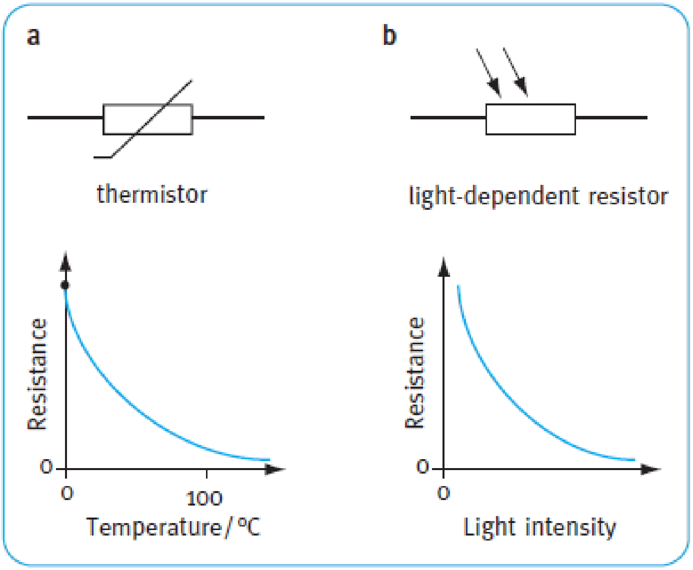
Thus p.d. $V = \dfrac{1430}{1000+1430} x1.5 = 0.88\ V$

Note that the voltmeter reads this p.d. 0.88 V instead of 1.0 V.

4.2 Potential Dividers for temperature and illumination control *LO(g)*

Potential divider circuits are often used in electronic circuits. They are useful when a sensor is connected to a processing circuit. Suitable sensors include **thermistors** and **light-dependent resistors (LDR)**.

Figure below shows the variation with temperature of the resistance of thermistors and LDR.



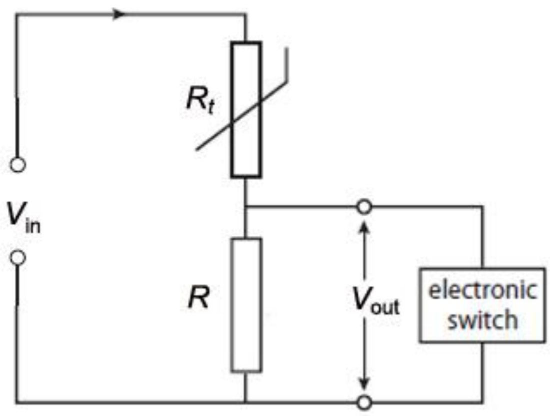
Thermistors and LDRs can be used as sensors because:

- the resistance of a negative temperature coefficient **(NTC)** thermistor **decreases** as its temperature **increases**
- the resistance of a light-dependent resistor (LDR) **decreases** as the incident intensity of light **increases**.

This means that a NTC thermistor can be used in a potential divider circuit to provide an output voltage V_{out} which depends on the temperature and, a light dependent resistor (LDR) can be used in a potential divider circuit to provide an output voltage V_{out} which depends on the intensity of light.

4.2.1 Temperature Control using a thermistor

In the circuit below a thermistor is being used to control temperature, e.g. in a fire alarm.



$$V_{out} = \left(\frac{R}{R+R_t}\right)V_{in}$$

If temperature increases,

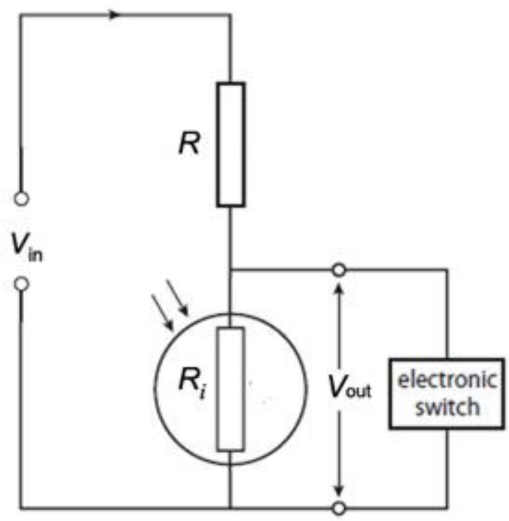
R_t decreases, V_{out} increases.

If the temperature rises, the resistance of the thermistor decreases and the output voltage V_{out} increases. The increase in V_{out} activates an electronic switch that rings the bell.

By changing the setting of the resistor R, one can control the range over which V_{out} varies. This would allow you to set the temperature at which the alarm operates.

4.2.2 Illumination Control using a light dependent resistor (LDR)

In the circuit below, an LDR is connected in series with another fixed resistor R and a source of e.m.f. to form a potential divider controlled by illumination. The potential difference across the resistor R_l is then tapped to control an electronic switch.



$$V_{out} = \left(\frac{R_l}{R_l + R}\right) V_{in}$$

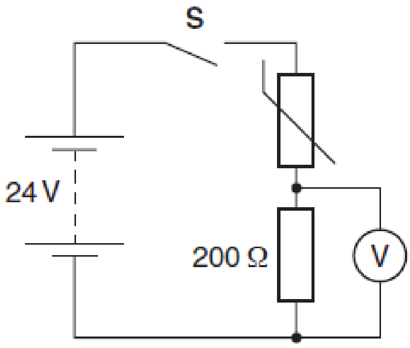
If it is dark,

R_l increases, V_{out} increases.

When light intensity on the LDR dims, its resistance increases, resulting in an increase in V_{out}. The increase in V_{out} in turn activates the electronic switch that puts on the lights in a room. The electronic switch needs a minimum p.d. to activate it, so it does not switch on the lights until V_{out} is big enough.

Example 12

Figure shows a thermistor and fixed resistor of 200 Ω connected through a switch **S** to a 24 V d.c. supply of negligible internal resistance. The voltmeter across the fixed resistor has a very high resistance.



(a) When the switch **S** is closed the voltmeter initially reads 8.0 V. Calculate the resistance R_T of the thermistor.

(b) A few minutes after closing the switch **S** the voltmeter reading has risen to a steady value of 12 V. The value of the fixed resistor remains at 200 Ω. Explain why

(i) the potential difference across the fixed resistor has Increased

(ii) the resistance of the thermistor must now be 200 Ω.

Solution:

(a) $8.0 = \left(\frac{200}{200 + R_t}\right) \text{x } 24$

$R_t = 400\ \Omega$

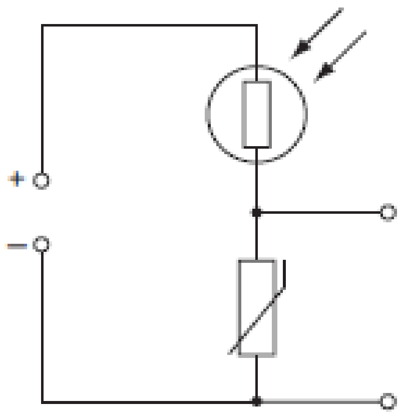
(b)(i) The temperature of the thermistor increases/thermistor is heated up, thus the resistance of thermistor decreases. Hence V increases.

(ii) $12 = \left(\frac{200}{200 + R_t}\right) \text{x } 24$

$\rightarrow R_t = 200\ \Omega$

Example 13

The diagram shows a light-dependent resistor (LDR) and a thermistor forming a potential divider.



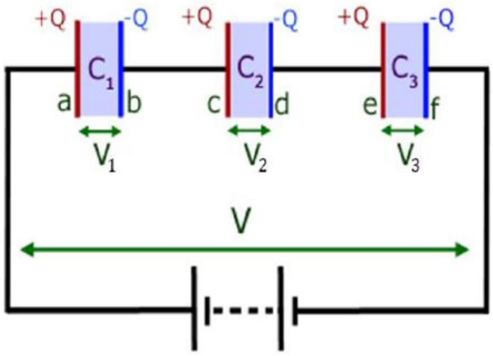
Under which set of conditions will the potential difference across the thermistor have the greatest value?

	illumination	temperature
A	low	low
B	high	low
C	low	high
D	high	high

Solution: $V_{out} = \left(\frac{R_t}{R_t + R_{LDR}}\right) V_{in}$, answer is B

5 Combination of Capacitors *LO(k)*

5.1 Capacitors in Series



Kirchhoff's voltage law states that the sum of the e.m.f.s in any closed loop in a circuit is equal to the sum of the potential differences in the same loop.

$V = V_1 + V_2 + V_3 + \cdots + V_N$

From the equation $C = Q/V$, it is clear that $V = Q/C$ and so substituting this into the expression for Kirchhoff's voltage law gives

$$\frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots + \frac{Q}{C_N}$$

where C_T is the combined capacitance of all the series capacitors. As Q is a constant it can be factorised out to give

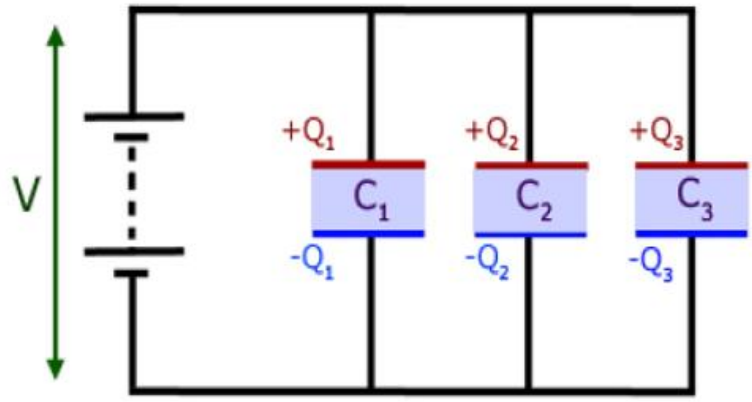
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

Therefore

$$C_T = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}\right)^{-1}$$

Note that this equation is similar to the equation for the total resistance of a number of resistors in parallel.

5.2 Capacitors in Parallel



Kirchhoff's current law states that the total current flowing into a node in a circuit must be equal to the total current flowing out of that node. Therefore, we can state that

$I_T = I_1 + I_2 + I_3 + \cdots + I_N$

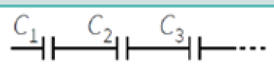
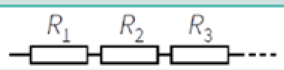
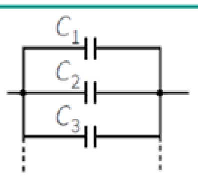
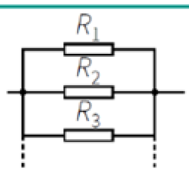
Charge can be stated as $Q = It$, so using the above and factorising out the constant time,

$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_N$

Finally, substituting the equation $C = Q/V$ and that the voltage is the same over each component in parallel we can write

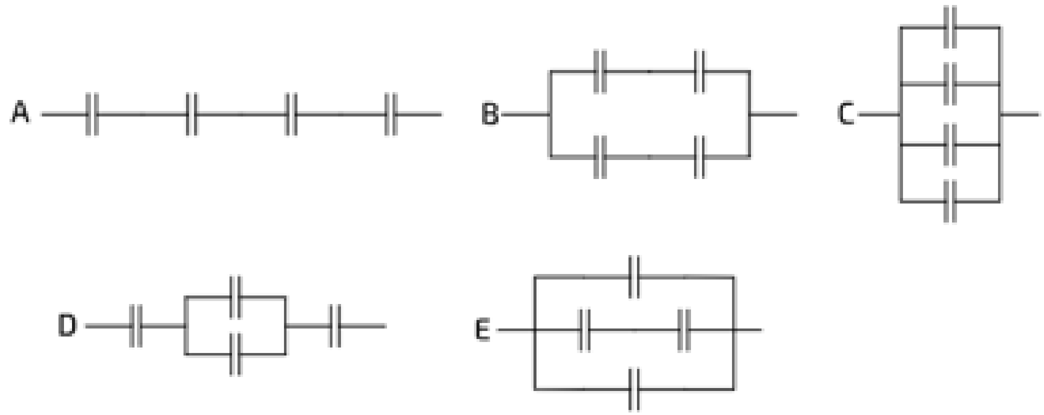
$C_T = C_1 + C_2 + C_3 + \cdots + C_N$

Note that this equation is similar to the equation for the total resistance of a number of resistors in series.

	Capacitors	Resistors
In series		
	store same charge	have same current
	$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$R_{total} = R_1 + R_2 + R_3 + \cdots$
In parallel		
	have same p.d.	have same p.d.
	$C_{total} = C_1 + C_2 + C_3 + \cdots$	$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Example 14

Four capacitors of capacitance 2 μF are connected in five different ways, A – E below.

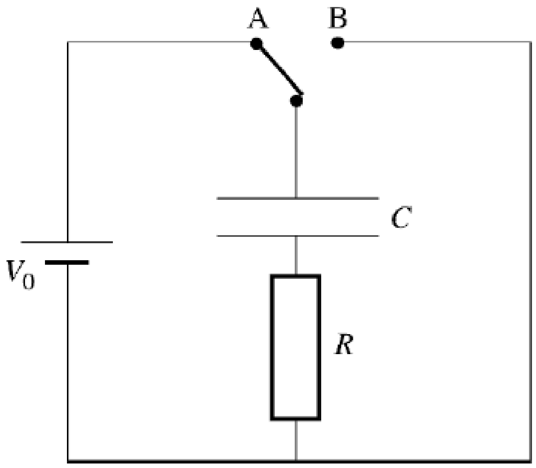


Which of the configurations gives an equivalent capacitance of 2 μF?

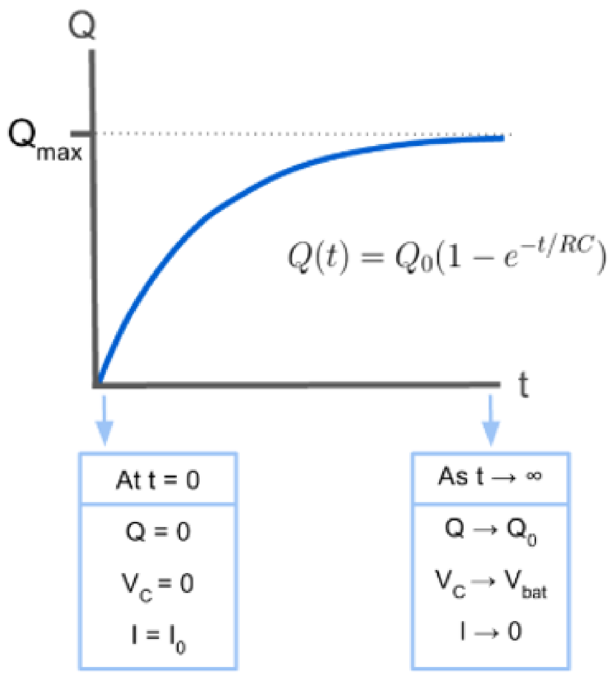
Answer: B

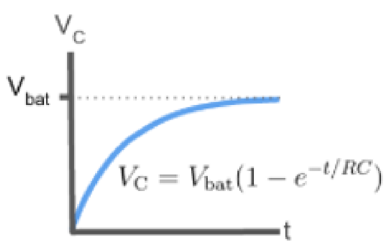
6 Charging and discharging of Capacitors *LO(l)*

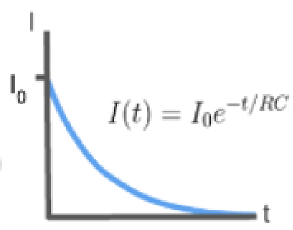
6.1 Charging of Capacitor



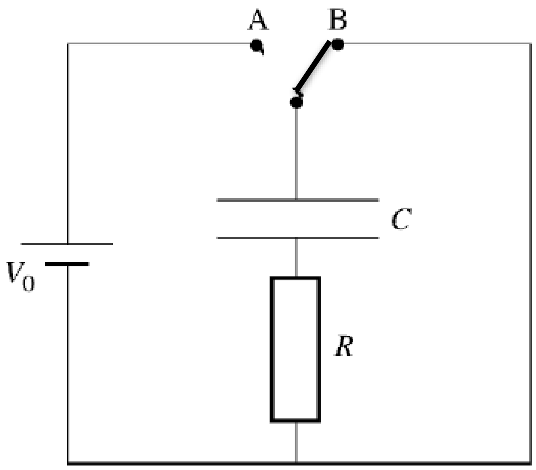
Initially there is no charge on the capacitor. When the switch is flipped to A, the cell will cause a momentary pulse of current to flow and charge will build up on the capacitor. The graph for charge on the capacitor and potential difference across the capacitor all show an exponential increase with time until the capacitor is charged completely ($Q = Q_{max}$) and the potential difference across the capacitor is the same as the e.m.f. of the battery ($V_c = V_o$). The graph for the current, however, shows an exponential decrease with time as the difference in potential between the cell and the capacitor decreases exponentially with time.





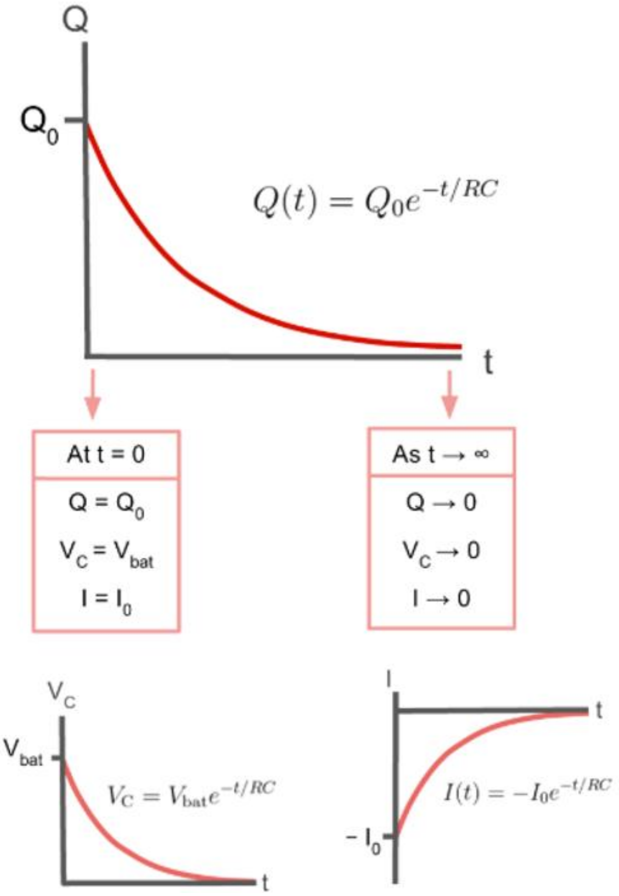


6.2 Discharging of Capacitor



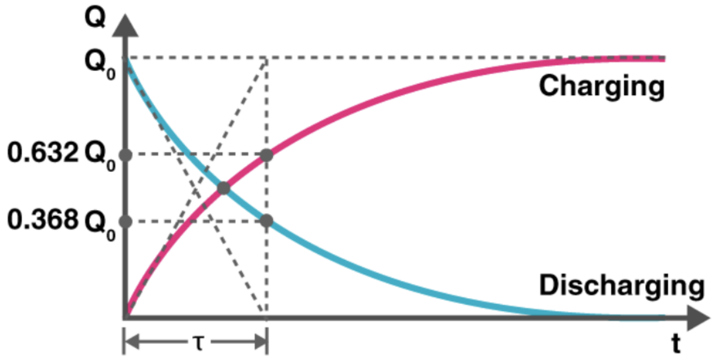
Once a capacitor has been charged, it can then be discharged by disconnecting the power supply and connecting to just the resistor. This can be achieved by flipping the switch in the circuit diagram from A to B.

When the power supply is disconnected and the resistor is connected, the potential difference, V_0, across the plates is at its maximum and given by $V_0 = Q_0/C$ where Q_0 is the initial charge stored on the plates. The electrons packed onto the negative plate are no longer subject to the electric field due to the e.m.f. which held them in such close proximity. They repel one another and so flow round the circuit dissipating electric energy as heat in the resistor. At time t = 0, the resistor circuit is connected and the initial current flowing through the circuit will be V_0/R as given by Ohm's law. As the electrons flow, the charge stored will decrease as the negative plate loses electrons and the positive plate gains electrons until finally there is no longer any potential difference across the capacitor (V = 0 since Q = 0.). Similarly, the current which is momentary will also decrease and eventually reach zero.



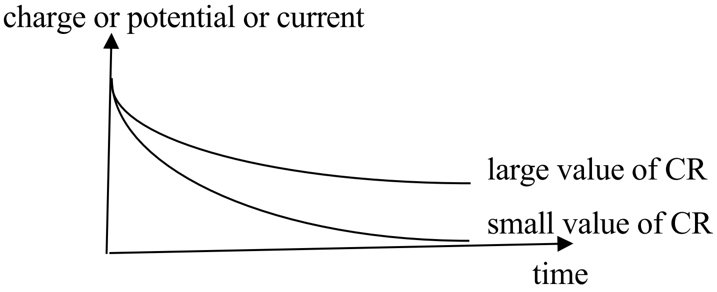
6.3 Time Constant RC

The product RC is often called the time constant and written as: $\tau = RC$.
The time constant τ is the amount of time it takes the charge, current or potential difference to fall to 37 % (i.e. 1/e) of its initial value when discharging or for the charge and potential difference to increase to 63 % of the maximum value when charging.



The magnitude of the time constant dictates how fast the capacitor will charge and discharge. A circuit with a larger resistor and higher capacitance will take longer to charge and discharge. To charge and discharge the capacitor, the direction of the current must be reversed. Turning off the voltage supply will also cause the capacitor to discharge.

The time constant over which this discharging process occurs depends on both the capacitance and also on the magnitude of the resistance in the discharging circuit. The lower the resistance in the discharging circuit, the higher the current can be as current is indirectly proportional to the resistance from Ohm's law ($I \propto 1/R$). If the current is higher, then the charge on the plates will fall to zero in a faster time as $\Delta Q = It$. Equally, the larger the capacitance the larger the charge stored per unit potential difference. This means that the amount of charge that can flow before the voltage drops to zero is higher and so a longer time is needed for the discharge to take place.



Example 15

At time t = 0 s an uncharged capacitor of capacitance 10 μF is connected to a voltage source of potential difference 12 V in series with a resistor of 100 kΩ . What is the charge on the capacitor after 5 s?

Answer:
Using the expression $Q(t) = Q_0(1 - e^{-t/RC})$ and substituting in the values gives an answer of 1.19×10^{-4} F.

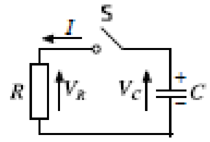
Example 16

At time t = 0 s a charged capacitor of capacitance 5.0 μF is connected to a resistor of 100 Ω. What is the percentage of the original charge still left on the capacitor's plates after 2.0 s?

Answer:
Using the expression $Q(t) = Q_0 e^{-t/RC}$ it can be found that 1.8 % of the original charge is left on the capacitor after 2.0 s.

Appendices (Derivation of charging and discharging formulae – not needed in syllabus)

Discharging capacitors



Here the switch is open and the capacitor is charged. The switch is closed at time $t = 0$ s and the capacitor starts to discharge.

The rate of discharge can be determined by looking at the voltage over the two components. Given that the components are connected together directly, $V_R = V_C$ when the switch is 'on' (closed).

Then using the definition of **resistance** , $V = IR$, and the definition of capacitance, $V = \frac{Q}{C}$, we have $IR = \frac{Q}{C}$. Note that with the direction of current shown, a positive I will discharge the capacitor, and so I is the rate of *decrease* of Q. It follows that $I = -\frac{dQ}{dt}$, and putting these equations together we get:

$$\frac{dQ}{dt}R = -\frac{Q}{C}$$

Separating the variables gives:

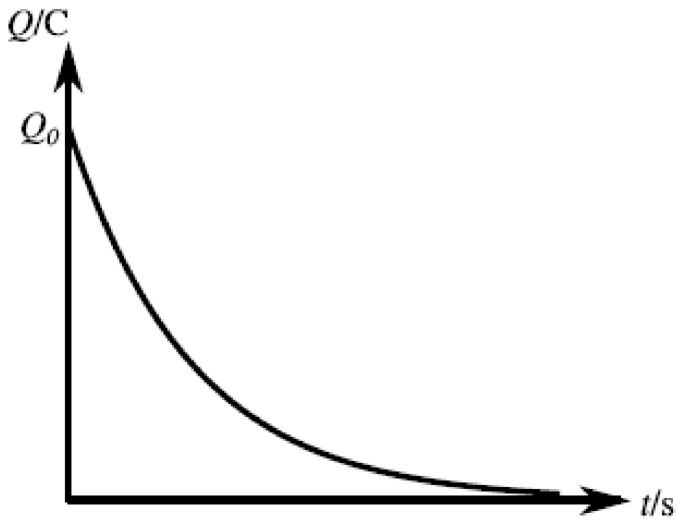
$$\frac{-1}{RC}\int_0^t dt = \int_{Q_0}^{Q(t)} \frac{dQ}{Q},$$

where Q_0 is the charge on the capacitor at $t = 0$ s and $Q(t)$ is the charge on the capacitor at time t. This leads to the final expression:

$$Q(t) = Q_0\, e^{\frac{-t}{RC}}$$

RC is known as the time constant, τ and is the time taken for the capacitor's charge to drop to approximately $37\,\%$ of its original value.

This discharging is an exponential decay, as shown in **Figure 5**:



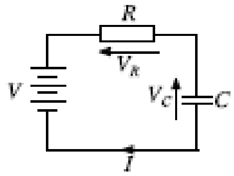
The current flowing in this circuit can be calculated using the definition of current, and the charge on the capacitor. Current is the rate of charge passing past a point, which is the same in this case as minus the rate of charge left on the capacitor - the capacitor losing charge corresponds to a positive current flowing from its positive plate to its negative plate. This gives:

$$I(t) = -\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{Q_0}{RC} e^{\frac{-t}{RC}} = I_0 e^{\frac{-t}{RC}}$$

The potential difference across the capacitor can be calculated even more simply using the definition $Q = CV$. Rearranging this gives $V = \frac{Q}{C}$ and so:

$$V(t) = \frac{Q_0}{C} e^{\frac{-t}{RC}} = V_0 e^{\frac{-t}{RC}}$$

Charging capacitors



Initially there is no charge on the capacitor. The battery will cause a current to flow and charge to build up on the capacitor.

By using **Kirchhoff's Voltage Law** we can write down a relationship between the voltages around the circuit: $V = V_R + V_C$, then by using **Ohm's Law**, $V = IR$, and the definition of capacitance, $V = \frac{Q}{C}$, we have

$$V = IR + \frac{Q}{C}$$

To relate current and charge we use the definition of current which is $I = \frac{dQ}{dt}$. This gives the differential equation:

$$V = R\frac{dQ}{dt} + \frac{Q}{C}$$

Rearranging to solve the equation gives:

$$\int_0^q \frac{dq}{q - CV} = -\frac{1}{RC}\int_0^t dt$$

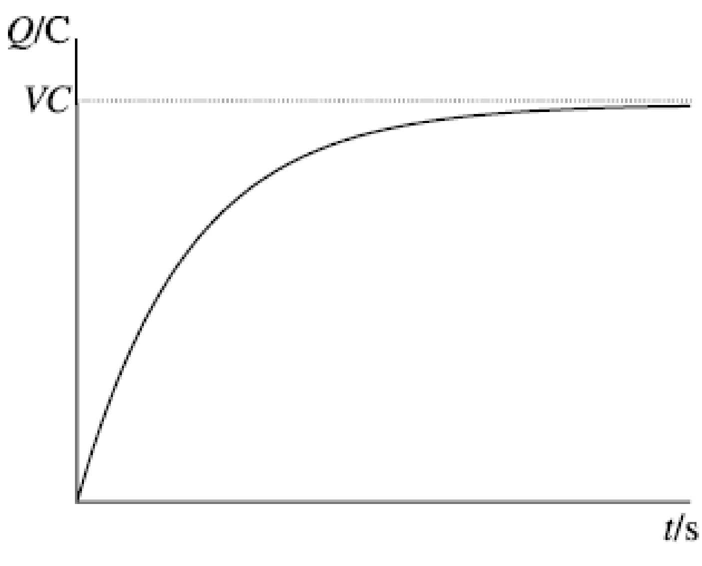
This can be integrated to give:

$$Q(t) = VC(1 - e^{\frac{-t}{RC}})$$

Here VC is the largest charge that will be held on the capacitor and $Q(t)$ is the charge on the capacitor after a time t. Differentiating this expression gives an equation for current.

RC is known as the time constant, τ and represents the time taken for the capacitor to reach approximately $63\,\%$ of its maximum charge.

The graph of charge held on the plates against time is as follows.

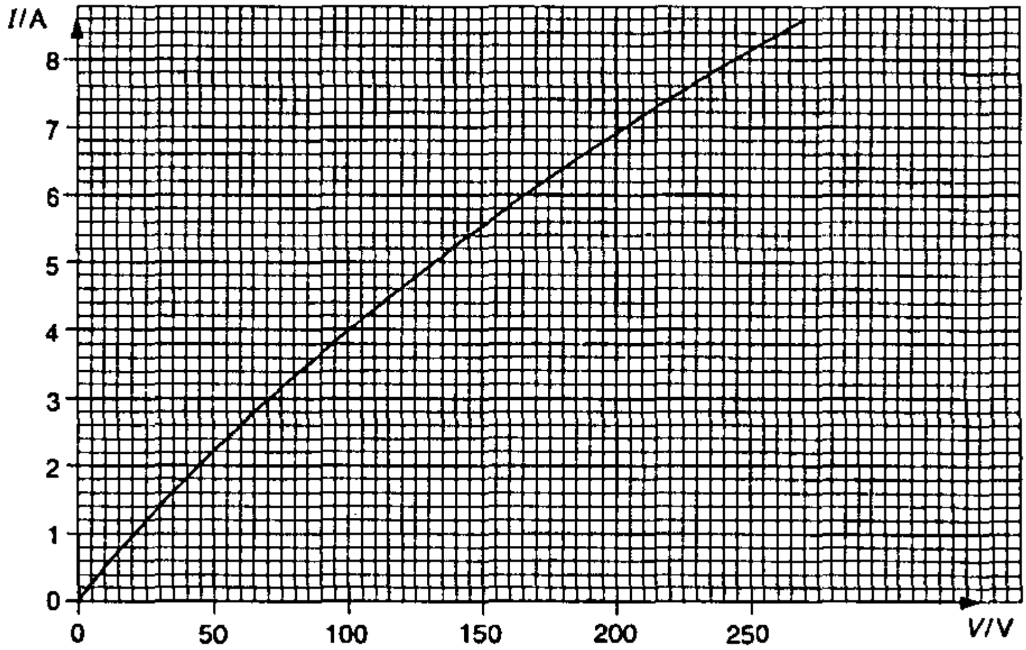


Unit 16: Circuits Tutorial Questions
(Solutions to Q 1, 3, 10, 12 and 18 are provided)

Electrical Resistance and Potential Difference

1. (a) If the equation $V = IR$ is used to define resistance, why is it not then possible to use the same equation to define potential difference?

   (b) One element of an electric cooker has an I - V characteristic as shown in the figure.



   (i) Explain how the characteristic shows that the resistance of the element increases with potential difference.
   (ii) Explain in terms of the movement of charged particles why the resistance increases with potential difference.
   (iii) Use the graph to estimate the potential difference which should be applied to the element if it is to have a resistance of 30 Ω.
   (iv) What will be the current in the element when it has a resistance of 30 Ω?
   (v) What will be the power of the element when it has a resistance of 30 Ω?

[225V, 7.5 A, 1.7 kW]

Solution:

(a) An equation cannot be used to define more than one variable. Since R is defined as the ratio of V to I, the same equation V=RI cannot be used to define V or I.

(b) (i) As V increases, the line bends towards larger V to I ratio. Thus R, which is defined as the ratio of V to I, must have increased.

(ii) As p.d. increases, current increases. Greater joule heating increases the temperature of the metal. Microscopically, the mean kinetic energy of the electrons and lattice ions increases. The lattice ions vibrate more vigorously and the conducting electrons collide more frequently with the lattice ions, hindering the flow of electrons, Hence the resistance of the filament lamp increases.

(iii) If V = 150 V, I = V/R = 150/30 = 5.0 A. Draw a straight line through the origin and (150, 5.0).
This line intersects the curve line at 225 V. Thus for R = 30 Ω, V = 225 V.

(iv) From graph, when V = 225 V, current I = 7.5 A

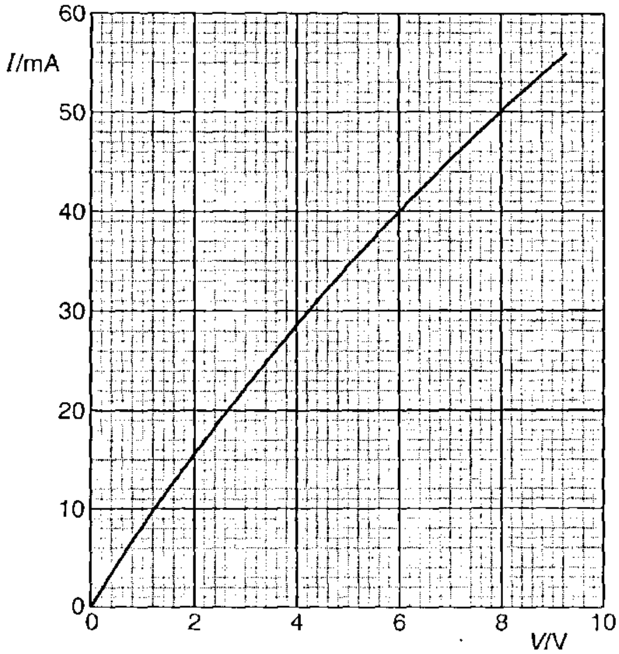
(v) Power P = VI = 225 (7.5) = 1.7 kW

2. A filament lamp operates normally at a potential difference (p.d.) of 6.0 V. The variation with p.d. *V* of the current *I* in the lamp is shown in the graph below.

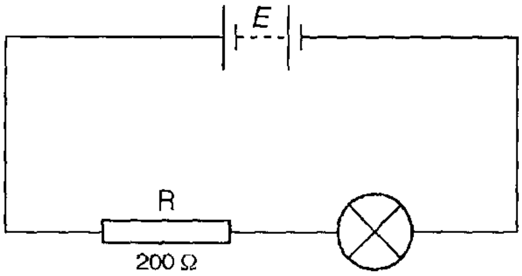
(a) Use the graph to determine, for this lamp,

(i) the resistance when it is operating at a p.d. of 6.0 V.

(ii) the change in resistance when the p.d. increases from 6.0 V to 8.0 V.



(b) The lamp is connected into the circuit as shown.



R is a fixed resistor of resistance 200 Ω. The battery has e.m.f. *E* and negligible internal resistance.

(i) On the given graph, draw a line to show the variation with p.d. *V* of the current *I* in the resistor *R*.

(ii) Determine the e.m.f of the battery for the lamp to operate normally.

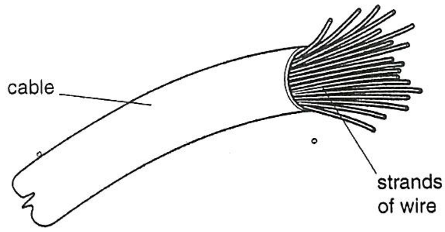
[150 Ω, 10 Ω, 14.0 V]

Solution:

Resistance and Resistivity

3. 2011 P3 Q8

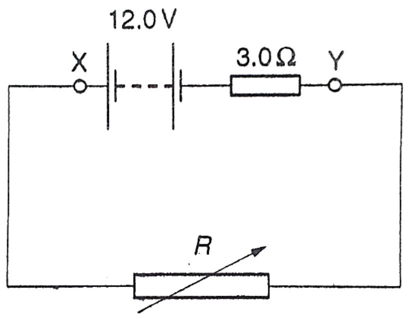
(a) An electric cable is made up of 24 thin strands of copper wire shown.



Each strand has diameter 0.26 mm. Copper has resistivity 1.7×10^{-8} Ω m. Calculate

(i) the resistance of one strand of wire of length 1.0 m,
(ii) the resistance of the cable of length 5.0 m,

(b) A battery has an e.m.f of 12.0 V and negligible internal resistance. A resistor of resistance 3.0 Ω is permanently connected to one of its terminals. The battery and resistor form a power supply with terminals X and Y, as shown.

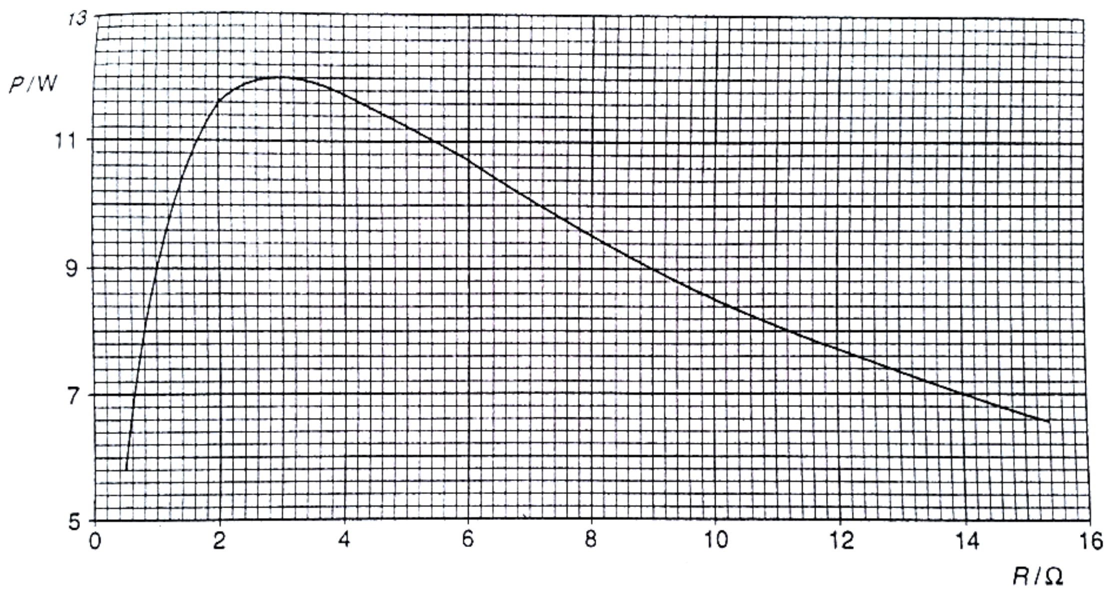


A variable resistor of resistance *R* is connected across terminals X and Y.

(i) Calculate
1. the maximum possible current from the supply,
2. the minimum safe power rating of the 3.0 Ω resistor.

(ii) Suggest the purpose of the 3.0 Ω resistor.

(c) The resistance *R* in the circuit is varied. The variation with *R* of the power *P* dissipated in the variable resistor is shown.



(i) State the two values of *R* at which the power dissipated in the variable resistor is 9.0 W.

(ii) For each of the two values in (i), calculate the efficiency of the transfer of power from the supply to the variable resistor.

(d) A battery is to be designed to provide the power for a component that has a fixed resistance. The internal resistance *r* of the battery is to be chosen to provide maximum efficiency of energy transfer to the component. Use your answers in (c) to state and explain the choice of *r* compared with the component's fixed resistance.

[0.32Ω, 0.067Ω, 4.0A, 48W, 1.0 Ω, 9.0Ω, 25 %, 75 %]

Solution:

(a) (i) $R = \frac{\rho L}{A} = \frac{\rho L}{\frac{\pi}{4}(D)^2} = \frac{1.7x10^{-8}(1.0)}{\frac{\pi}{4}(0.26x10^{-3})^2} = 0.32\,\Omega$

(ii) The strands of wire are arranged in parallel.
Total R = (0.32 x 5.0)/24 = 0.067 Ω

(b) (i) 1. Max I when R = 0.
=> $I_{\max} = \frac{E}{r+R} = \frac{12.0}{3.0+0} = 4.0\text{ A}$
2. Minimum safe power rating of 3.0 Ω resistor is $P = I^2 r = 4.0^2(3.0) = 48W$

(ii) To prevent a large current from being drawn from the battery when R is very low. This way the battery will not be overheated and damaged.

(c) (i) From graph, R = 1.0 Ω and 9.0 Ω
(ii) Efficiency $\eta = \frac{P_o}{P_i} = \frac{I^2R}{I^2(R+r)} = \frac{R}{R+r}$

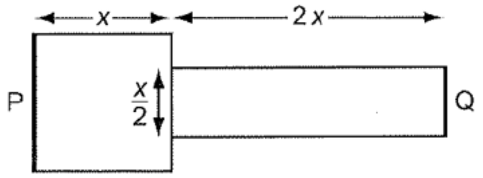
When R = 1.0 Ω, $\eta = \frac{1.0}{1.0+3.0} = 0.25 = 25\%$

When R = 9.0 Ω, $\eta = \frac{9.0}{9.0+3.0} = 0.75 = 75\%$

(d) Efficiency $\eta = \frac{R}{R+r}$*. For max* η*, internal resistance r should be very small compared to the fixed resistance R of the component. From (c), we see that when R is greater than r,* η *is higher.*

4. 2013 P1 Q27

A thin square sheet of metal of uniform thickness and of side *x* has a resistance of 4.0 Ω measured between opposite edges. It is connected to another steel sheet of the same metal of the same thickness but of length 2x and width $\frac{x}{2}$.



What is the resistance between edges P and Q?

A 8 Ω B 12 Ω C 16 Ω D 20

Resistors in series and parallel

5. You are given three resistors of resistance 22 Ω, 47 Ω and 100 Ω. Calculate
   (a) the maximum possible resistance,
   (b) the minimum possible resistance that can be obtained by combining any or all of these resistors.

[169 Ω, 13 Ω]

6. Calculate the combined resistance across A and B in Fig. 2.1 and 2.2.

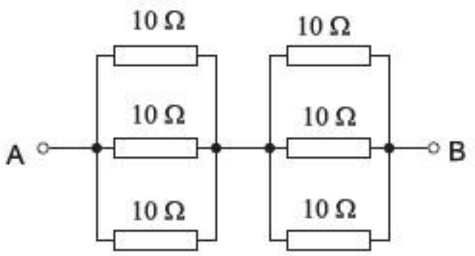


Fig. 2.1

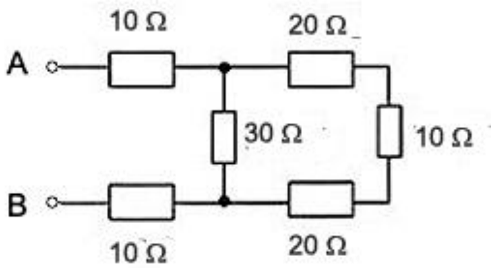
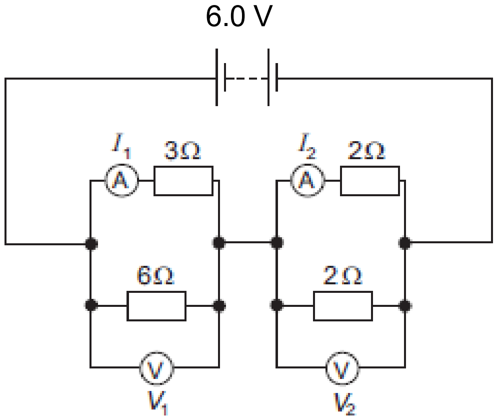


Fig. 2.2

[67 Ω, 39 Ω]

7. You are given some resistors, each of resistance 100 Ω.
   Draw circuit diagrams, one in each case, to show how a number of these resistors may be connected to produce a combined resistance of
   (a) 200 Ω, (b) 50 Ω and (c) 40 Ω.

8. In the circuit shown, the ammeters have negligible resistance and the voltmeters have infinite resistance.

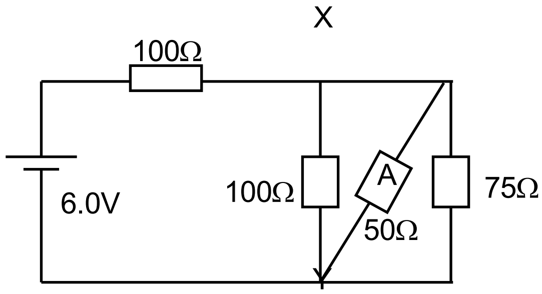


What are the readings on the meters I_1, I_2, V_1 and V_2 on the diagram?

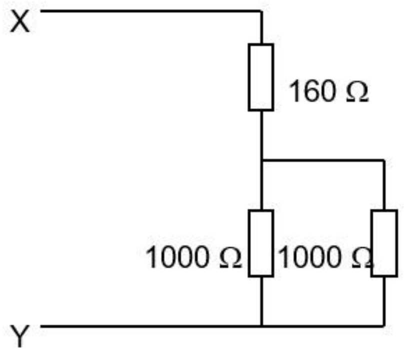
[1.3 A, 1.0 A, 4.0 V, 2.0 V]

9. In the circuit below, calculate
(a) the combined resistance of the circuit
(b) the current in the resistor A

[123 Ω, 0.022 A]



10. The circuit shown is constructed of resistors, each of which has a maximum safe power rating of 0.40 W.



(a) Determine the maximum potential difference that can be applied between X and Y without damage to any of the resistors.

(b) If this potential difference were exceeded, which resistor would be most likely to fail?

[26.4 V]

Solution:

(a)

$$P = I^2R \Rightarrow I = \sqrt{\frac{P}{R}}$$

$$I_{max} \text{ through } 160\,\Omega = \sqrt{\frac{0.40}{160}} = 0.050 \text{ A}$$

$$I_{max} \text{ through } 1000\,\Omega = \sqrt{\frac{0.40}{1000}} = 0.020 \text{ A}$$

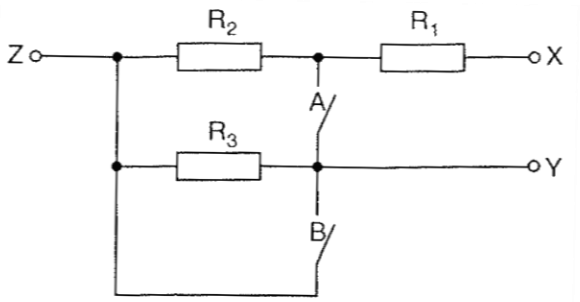
To minimize damage, use the smaller overall current,

$$I_{max} \text{ through circuit } = 2(0.020) = 0.040 \text{ A}$$

$$\Rightarrow V_{max} = \left(160 + \frac{1000}{2}\right)0.040 = 26.4 \text{ V}$$

(b) The 1000 Ω resistors would fail first as they reach the maximum power rating before the 160 Ω resistor.

11. A circuit consists of three resistors R_1, R_2 and R_3, and two switches A and B, as shown in the figure.



The resistances between terminals X and Y is measured for different settings of the switches A and B. The results are shown in the table.

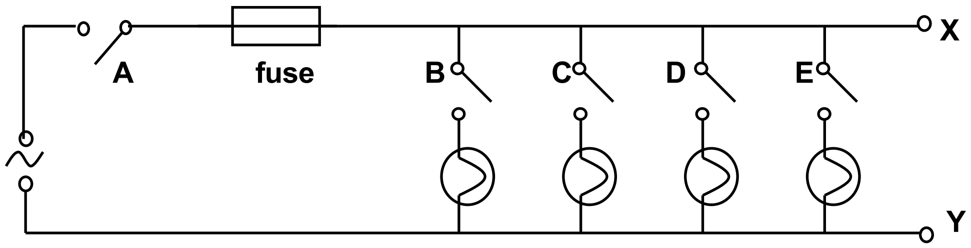
switch A	switch B	resistance between X and Y/Ω
open	open	12
open	closed	10
closed	open	6
closed	closed	6

(a) Determine the resistance of resistor R_1, R_2 and R_3.

(b) Switch A is now closed and switch B is open. Calculate the resistance between terminals X and Z.

[7.3 Ω]

12. A lighting circuit includes four lamps connected as shown below. The resistance of each lamp should be 120 Ω when it is not lit.



A fault is discovered in the circuit, so switch A is turned off and the fuse is removed for safety. A resistance meter is connected between the points X and Y and the following readings are obtained for different switch positions.

switches					resistance meter reading/Ω
A	B	C	D	E	
off	off	off	off	off	14 600 000
off	off	off	off	on	120
off	off	off	on	on	60
off	off	on	on	on	40
off	on	on	on	on	0.2

(a) If there were no fault in the circuit, what would the resistance meter read when switches B, C, D and E are on and A is off?

(b) Why does the resistance meter not read infinity when all the switches are off?

(c) Suggest what the fault in the circuit may be. [30 Ω]

Solution:

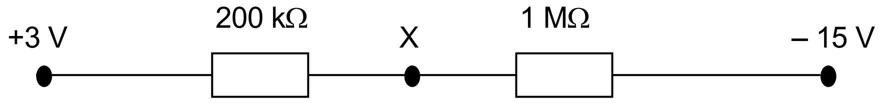
(a) R = ¼ (120) = 30 Ω

(b) The lighting circuit is usually a fixture on the wall. It could be wet due to moisture in the air, humidity, or even rain or snow. This could result in current leakage through any of the switches.

(c) From (a), when switches B, C, D, E are ON, equivalent R should be 30 Ω. Meter reading of 0.2 Ω indicates a short circuit. Bulb B is possibly short circuited due to presence of water in the socket.

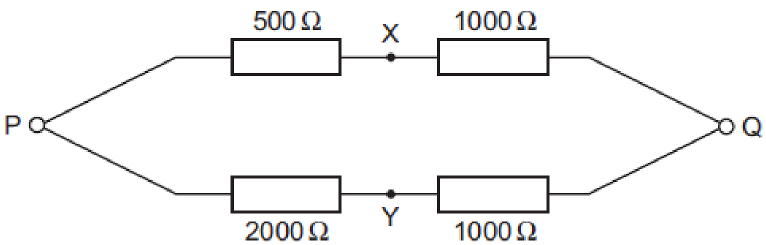
Potential dividers

13. Two resistors of resistance 200 kΩ and 1 MΩ respectively form a potential divider with outer junctions maintained at potentials + 3 V and – 15 V.



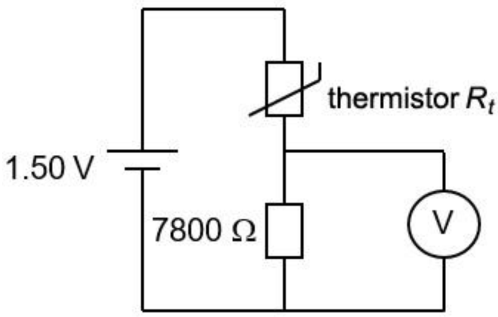
What is the potential at junction X between the resistors? [0 V]

14. A p.d. of 12 V is connected between P and Q.



What is the p.d. between X and Y? [4.0 V]

15. A thermistor has resistance 3900 Ω at 0°C and resistance1250 Ω at 30°C. The thermistor is connected into the circuit below in order to monitor temperature changes. The battery of e.m.f. 1.50 V has negligible resistance and the voltmeter has infinite resistance. And the voltmeter is to read 1.00 V at 0°C.



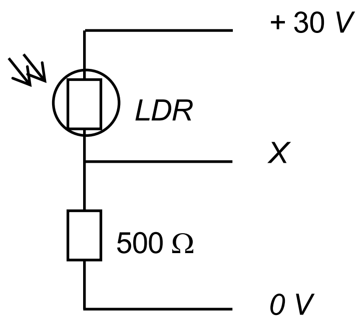
(a) The temperature of the thermistor is increased to 30°C. Determine the reading on the voltmeter.

(b) The voltmeter in the figure is replaced with one having a resistance of 7800 Ω. Calculate the reading on this voltmeter for the thermistor at a temperature of 0°C.

[1.29 V, 0.75 V]

16. The light dependent resistor LDR and a 500 Ω resistor form a potential divider between voltage lines held at +30 V and 0 V as shown in the diagram.
The resistance of the LDR is 1000 Ω in the dark but then drops to 100 Ω in bright light.
What is the corresponding change in the potential at X?

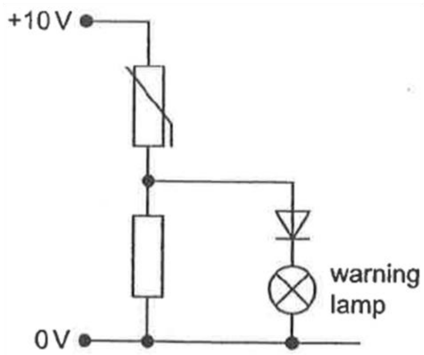
[a rise of 15 V]



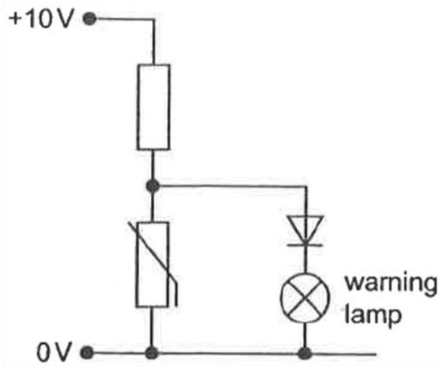
17. A circuit is needed which switches on a warning lamp when the temperature of a thermistor is too high.

Which circuit is suitable?

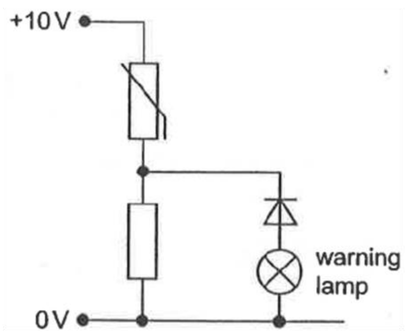
A



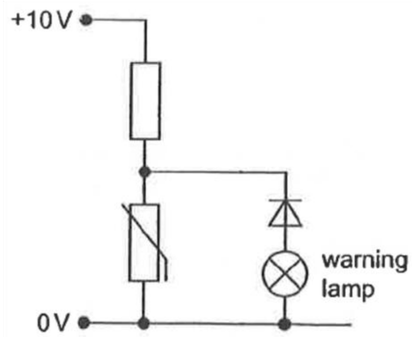
B



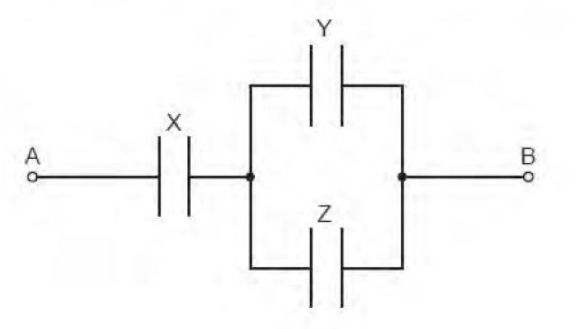
C



D



18. Three capacitors X, Y and Z, each of capacitance 10 μF are connected as shown in the diagram below.

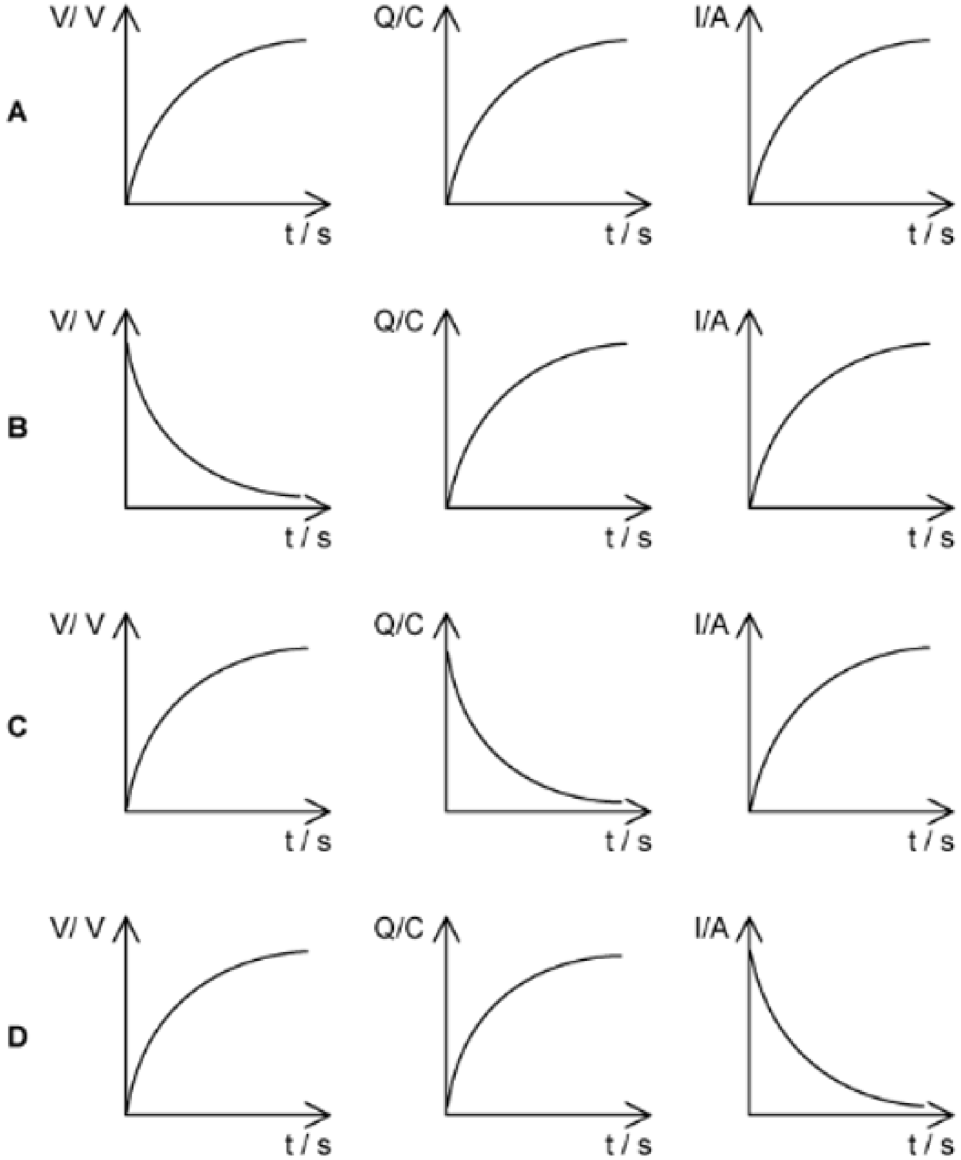


Initially, the capacitors are uncharged. A potential difference of 12 V is applied between point A and B. Determine the magnitude of the charge on one plate of capacitor X.

[80 μC]

Answer:
Effective capacitance of Y and Z = 20 μF
p.d. across capacitor X = 8 V
charge = 10 x 10^{-6} x 8 = 80 μC
Or
Effective capacitance of X, Y and Z = 6.67 μF
p.d. across all capacitors = 12 V
charge = 6.67 x 10^{-6} x 12 = 80 μC

19. A capacitor is initially uncharged. The switch is closed at time t = 0. Which of the following graphs shows how the potential difference *V* across the capacitor, the charge *Q* on the capacitor and the current *I* in the circuit change with time *t*?



20. A 500μF capacitor is connected to a 10 V supply, and is then discharged through a 100kΩ resistor. Calculate:

(a) the initial charge stored by the capacitor,

(b) the initial discharge current,

(c) the value of the time constant,

(d) the charge on the plates after 100s,

(e) the time at which the remaining charge is 2.5×10^{-3} C.

[5.0×10^{-3} C, 1.0×10^{-4} A, 50 s, 6.8×10^{-4} C, 35 s]