Secondary 2 Express Notes

definitions

1. Rational Numbers

- numbers that can be expressed as fractions

i.e. $2 = \frac{2}{1}$, $0.5 = \frac{1}{2}$, $0.\dot{3} = \frac{1}{3}$

2. Irrational Numbers

- numbers that cannot be expressed as fractions

i.e. $\sqrt{2}$, π, ...

3. Prime Numbers

- numbers that can be divided only by itself and 1.

i.e. 2, 3, 5, 7, ...

4. Integers

- whole numbers

i.e. −1, 0, 2, ...

NUMBERS AND OPERATIONS

prime factorisation

Highest Common Factor (HCF)

1. Look for common factors.
2. Take the number with the smallest power

Lowest Common Multiple (LCM)

1. Look for common factors.
2. Take the number with the highest power.
3. Multiply them together with the other uncommon numbers.

E.g. Find HCF and LCM of 36 and 64.

$36 = 2^2 \times 3^2$

$64 = 2^6$

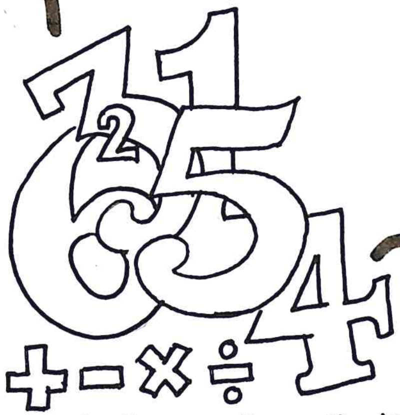
Thus, HCF $= 2^2 = 4$

LCM $= 2^6 \times 3^2 = 576$

SIGNIFICANT FIGURES

* zeroes in between non-zero digits are significant
  i.e. 5.003 = 4 significant figures
* zeroes in front of decimals are not significant
  i.e. 0.00305 = 3 significant figures
* zeroes behind decimal point is significant.
  ie. 4.000 = 4 significant figures
* zeroes behind in whole numbers may or may not be significant.
  i.e. 200 = 1/2/3 significant figures.

NUMBERS AND OPERATIONS



indices

LAWS OF INDICES

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. $a^0 = 1$
7. $a^{-m} = \frac{1}{a^m}$
8. $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
9. $\sqrt[n]{a} = a^{\frac{1}{n}}$
10. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

prime factorisation

HIGHEST COMMON FACTOR (HCF)

1. Look for common numbers.
2. Take the number with the smallest power.

LOWEST COMMON MULTIPLE (LCM)

1. Look for common numbers.
2. Take the number with the highest power.
3. Multiply them together with the other uncommon numbers.

E.g. Find HCF and LCM of 36 and 64.

$36 = 2^2 \times 3^2$

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LCM $= 2^6 \times 3^2 = 576$

definitions

1. Rational Numbers
   - numbers that can be expressed as fractions
   - i.e. $2 = \frac{2}{1}$; $0.5 = \frac{1}{2}$, $0.\dot{3} = \frac{1}{3}$
2. Irrational Numbers
   - numbers that cannot be expressed as fractions.
   - i.e. $\sqrt{2}$, π, ...
3. Integers → whole numbers
   → i.e. 2, −2, ...

standarD FoRM

$\boxed{A \times 10^n}$

where $1 \le A < 10$ and n is an integer.

E.g.

Express as standard form:

(a) $3573 = 3.573 \times 10^3$

(b) $0.00892 = 8.92 \times 10^{-3}$

SIGNIFICANT FIGURES

* zeroes in between non-zero digit is significant
* zeroes in front of decimal are not significant
* zeroes behind decimal is significant
* zeroes behind whole numbers may or may not be significant.

E.g. Round off 568 $\approx$ 600 (1sf)

$\approx$ 570 (2sf)

- must be in the same units
- cannot be in fraction nor decimals

properties

map scales

DISTANCE SCALE

$\boxed{1 : n}$

(both units are in cm)

AREA SCALE

$\boxed{1 : n^2}$

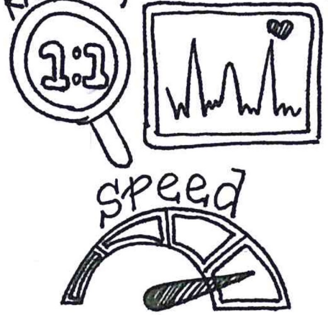
E.g. (a) Given that 2cm represents 5km, find the scale in the form $1 : n$.

M : A
2cm : 5km
1cm : 2.5km
1 cm : 250000cm
1 : 250 000 #

(b) If the area on the map is $5cm^2$, find the actual area, in km^2.

M : A
1cm : 2.5km (map scale)
$(1cm)^2 : (2.5km)^2$
$1cm^2 : 6.25km^2$ (area scale)
$5cm^2 : 31.25km^2$ #

RATIO, Rate, Proportion



DIRECT PROPORTION

If y is directly proportional to x, then

$\boxed{y = kx}$

INVERSE PROPORTION

If y is inversely proportional to x, then

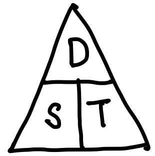
$\boxed{y = \frac{k}{x}}$

i.e. Given that y is directly proportional to x^2, and $y = 12$ when $x = 2$, find an equation connecting y and x.

Step 1 : Form equation $\Rightarrow y = kx^2$

step 2 : Find constant value, k. $\Rightarrow 12 = k(2^2)$
$\therefore k = 3$

step 3 : substitute k into equation. $\Rightarrow y = 3x^2$ #



- distance = speed x time
- speed = $\frac{\text{distance}}{\text{time}}$
- time = $\frac{\text{distance}}{\text{speed}}$
- AVERAGE SPEED

  = $\frac{\text{total distance taken}}{\text{total time taken.}}$

Ratio, Rate, Proportion

Speed

Properties

* must be in the same units
* cannot be in fraction nor decimal

MAP SCALES...

Distance Scale

$\boxed{1 : n}$

(both are in cm)

Area Scale

- square of the distance scale

$\boxed{1 : n^2}$

E.g.

(a) Given that 2cm represents 5km, find the scale in the form $1 : n$.

2cm : 5km
1cm : 2.5km
1cm : 2500m
1cm : 250000cm

Thus $1 : 250000$ *

(b) If the area on the map is 5cm^2, find the actual area in km^2.

1cm : 2.5km

Area scale: $1\text{cm}^2 : 6.25\text{km}^2$

$5\text{cm}^2 : 31.25\text{km}^2$ *

Direct

- If y is directly proportional to x, then

$\boxed{y = kx}$

Indirect

- If y is indirectly proportional to x, then

$\boxed{y = \frac{k}{x}}$

% Percentage

- a fraction with 100 as denominator

(1) Percentage Change

$$\frac{\text{New value} - \text{original value}}{\text{original value}} \times 100\%$$

- Distance = speed x time
- $\text{speed} = \frac{\text{distance}}{\text{time}}$
- $\text{Time} = \frac{\text{distance}}{\text{speed}}$
- $\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$

E.g. Given that SGD\$1 = US \$1.30, find how much Aaron will get, if he wants to exchange US \$100.

US \$1.30 ⟶ SGD\$1

US \$100 ⟶ SGD \$ $\left(\frac{100}{1.30}\right)$

= \$76.92 #

money exchange

PROFIT OR LOSS

PROFIT

- gain money from the deal

$\boxed{\text{Profit} = \text{Selling price} - \text{cost price}}$

LOSS

- lose money from the deal.

$\boxed{\text{Loss} = \text{cost price} - \text{selling price}}$

TAXATION

Financial transactions

INTEREST

SIMPLE INTEREST

$\boxed{\text{INTEREST (I)} = \frac{P \times R \times T}{100}}$

where P = principal (amt of \$\$ loan/invest)

R = rate

T = time (units must be in years)

HIRE PURCHASE

E.g.

Dan bought a TV set on hire purchase. The TV costs \$3500. He paid a downpayment of 10% and paid the balance over a period of 2 years at a rate of 2% per annum. Calculate his monthly instalment.

SOLUTION

Balance = 90% of \$3500 = \$3 150

Interest = $\frac{\$3150 \times 2 \times 2}{100}$ = \$126

Total he has to pay = \$3 150 + \$126

= \$3276

∴ monthly instalments = \$3276 ÷ 24

= \$136.50 #

① INCOME TAX

$\boxed{\text{Chargeable Income} = \text{Annual Gross Income} - \text{Total Reliefs}}$

E.g.

Alan earns \$36,000 in a year. He gets wife relief of \$1000, child relief of \$500 each and NS relief of \$200. He has three children. Calculate the amount of tax he has to pay.

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable (\$)
First \$20 000 Next \$20 000	0 2	0 200
First \$30 000 Next \$10 000	- 3.5	200 350

LOOK

Chargeable Income = \$36000 − (\$1000 + \$1500 + \$200)

= \$33 300

First \$30 000 ⇒ Tax to pay = \$200

Balance \$3 300 ⇒ Tax to pay = 3.5% of \$3300

= \$115.50

Total tax = \$200 + \$115.50

= \$315.50

GST & SERVICE CHARGE

* GST is always calculated after service charge is added to the bill.

Financial Transactions

Money exchange

E.g. Given that SGD\$1 = US\$1.30, find how much Aaron will get if he wants to exchange US\$100?

US\$1.30 → S\$1

US\$100 → $\frac{100}{1.3} \times$ S\$1 = S\$76.92

TAXATION

TAX INCOME

Chargeable income = Annual gross income − Total amount of reliefs

E.g.

Alan earns \$36 000 in a year. He gets wife relief of \$1000, child relief of \$500 each and NS relief of \$200. He has three children. Calculate amount of tax he has to pay.

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable (\$)
First \$20 000	0	0
Next \$10 000	2	200
First \$30 000	–	200
Next \$10 000	3.5	350

Chargeable income = \$36 000 − \$1000 − \$1500 − \$200
= \$33 300

LOOK First \$30 000 ⇒ Tax to pay = \$200

Balance \$ 3 300 ⇒ Tax to pay = 3.5% of \$3300
= \$ 115.50

Total tax = \$200 + \$115.50
= \$315.50 ⌗

GST & service charge

- GST always calculated after the service charge is added to the bill

PROFIT OR LOSS

① Profit = Selling price − Cost price

② Loss = Cost price − Selling Price

INTEREST

Simple interest

$$\boxed{\text{Interest (I)} = \frac{P \times R \times T}{100}}$$

where P = principal (amt of \$ loan/invest)
R = rate
T = time (years)

Compound Interest

$$\boxed{\text{Total amount} = P\left(1+\frac{r}{100}\right)^n}$$

where P = principal
r = rate
n = no. of periods.

Hire Purchase

E.g. Dan bought a TV set on hire purchase. The TV costs \$3500. He paid a downpayment of 10% and paid the balance over a period of 2 years at a rate of 2% per annum. Calculate his monthly instalment.

Solution

Balance = 90% of \$3500 = \$3150
Interest = \$3150 × 2% × 2 = \$126
Total amount = \$ 3150 + \$126 = \$3276
Monthly = \$3276 ÷ 24 mths = \$136.50/month ⌗.

NUMBER PATTERN

constant | same difference (+ or −)

n^{th} term

$$T_n = \underbrace{a}_{\text{first term}} + (n-1)\underbrace{d}_{\text{constant difference}}$$

Eg 1.

Find the n^{th} term of

$$3,\ 9,\ 15,\ 21,\ \ldots$$

$+6 \quad +6 \quad +6$

$$\begin{aligned} \therefore\ T_n &= 3 + (n-1)(6) \\ &= 3 + 6n - 6 \\ &= 6n - 3 \ \# \end{aligned}$$

Eg 2.

Find the n^{th} term of

$$12,\ 9,\ 6,\ 3,\ 0,\ \ldots$$

$-3 \quad -3 \quad -3$

$$\begin{aligned} \therefore\ T_n &= 12 + (n-1)(-3) \\ &= 12 - 3n + 3 \\ &= 15 - 3n \ \# \end{aligned}$$

NUMBER PATTERN

constant / same difference (+ or −)

n^{th} term : $a + (n-1)d$

the first term

constant difference

E.g.1

Find the n^{th} term of

3, 9, 15, 21, ...

$\boxed{\text{Solution}}$

$$T_n = 3 + (n-1)(6)$$
$$= 3 + 6n - 6$$
$$= 6n - 3 \text{ \#}$$

E.g.2

Find the n^{th} term of

12, 9, 6, 3, 0, ...

$\boxed{\text{Solution}}$

$$T_n = 12 + (n-1)(-3)$$
$$= 12 - 3n + 3$$
$$= -3n + 15 \text{ \#}$$

ALGEBRAIC expressions

EXPANSION

"RAINBOW" METHOD

1. $a(b \pm d) = ab \pm ad$
2. $(a+b)(c+d)$

   $= ac + ad + bc + bd$

NOTATIONS (SIMPLIFICATION)

1. $ab = a \times b$
2. $\frac{a}{b} = a \div b$
3. $d^2 = d \times d$

   $\therefore d^n = \underbrace{d \times d \times \dots \times d}_{n \text{ times}}$
4. $\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$
5. $\frac{ab}{c} = \frac{a}{c} \times b$ OR $a \times \frac{b}{c}$

E.g.

Simplify $\frac{4}{x-1} + \frac{3}{x+2}$

* always check whether there are any expressions that can be factorised. If yes, factorise first.

Find common denominator.

$$\frac{4(x+2)}{(x-1)(x+2)} + \frac{3(x-1)}{(x+2)(x-1)}$$

$$= \frac{4(x+2)+3(x-1)}{(x-1)(x+2)}$$

$$= \frac{4x+8+3x-3}{(x-1)(x-2)}$$

$$= \frac{7x-5}{(x-1)(x-2)} \#$$

FACTORISATION

1. Taking out common factors

Factorise $2xy + 6y^2$.

$\boxed{\text{solution}}$

$2xy + 6y^2 = 2y(x+3y)\#$

2. By grouping

Factorise (a) $xd + yc + xc + yd$

(b) $2x + ax - 2y - ay$

$\boxed{\text{solution}}$

(a) $(xd + xc) + (yc + yd)$ * grouping common terms together.

$= x(d+c) + y(c+d)$ * taking out common factors from each group

$= (d+c)(x+y)\#$

(b) $(2x + ax) - (2y + ay)$

$= x(2+a) - y(2+a)$

$= (2+a)(x-y)$

3. By cross-factorisation / multiplication frame

Factorise $x^2 - 5x + 6$

$\boxed{\text{solution}}$

CROSS FACTORISATION

x	-3	$-3x$
x	-2	$-2x$
x^2	$+6$	$-5x$

MULTIPLICATION FRAME

$\times$	x	-2
x	x^2	$-2x$
-3	$-3x$	$+6$

$\therefore x^2 - 5x + 6$

$= (x-3)(x-2)$

ALGEBRAIC IDENTITIES

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a-b)(a+b)$

ALGEBRAIC expressions

expansion

"Rainbow" Method

① $a(b \pm d) = ab \pm ad$

② $(a+b)(d+k)$

$= ad + ak + bd + bk$

notations

① $ab = a \times b$

② $\frac{a}{b} = a \div b$

③ $d^2 = d \times d$

$d^5 = d \times d \times d \times d \times d$

$d^n = \underbrace{d \times d \times d \ldots\ldots \times d}_{n \text{ times}}$

④ $\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$

⑤ $\frac{ab}{c} = \frac{a}{c} \times b$

Example 1

Simplify $\frac{4}{x-1} + \frac{3}{x+2}$.

Solution

$\frac{4(x+2)}{(x-1)(x+2)} + \frac{3(x-1)}{(x+2)(x-1)}$ → Find common denominator.

$= \frac{4(x+2) + 3(x-1)}{(x-1)(x+2)}$

$= \frac{4x + 8 + 3x - 3}{(x-1)(x+2)}$

$= \frac{7x+5}{(x-1)(x+2)}$ #

factorisation

① Taking out common factors

$3x + 6x^2 = 3x(1+2x)$

② By grouping

eg.

$xd + yc + xc + yd$

$= (xd + yd) + (yc + xc)$

$= d(x+y) + c(y+x)$

$= (x+y)(c+d)$ #

OR

$2x + ax - 2y - ay$

$= x(2+a) - y(2+a)$

$= (x-y)(2+a)$ #

③ By cross-factorisation

Factorise $x^2 - 5x + 6$.

Solution

x	-3	$-3x$
x	-2	$-2x$
x^2	$+6$	$-5x$

Thus, $x^2 - 5x + 6 = (x-3)(x-2)$ #.

special products

① $(a+b)^2 = a^2 + 2ab + b^2$

② $(a-b)^2 = a^2 - 2ab + b^2$

③ $(a+b)(a-b) = a^2 - b^2$

SOLVING algebraic equations

two unknowns (SIMULTANEOUS EQNS)

e.g. Solve $x+y=5$
$2x+3y=13$.

solution

By ELIMINATION

$x+y=5$ — Eqn1
$2x+3y=13$ — Eqn2

step 1 Make one variable to be the same.

Eqn1 ×2 $\quad 2x+2y=10$ — Eqn3

Eqn2 − Eqn3 $\quad y=3$ #

step 2 substitute the value found into any of the equations to find the other value.

sub $y=3$ in Eqn1
$x+3=5$
$x=2$ #

BY SUBSTITUTION

$x+y=5$ — Eqn1
$2x+3y=13$ — Eqn2

step 1 Make one variable to be the subject of formula using any of the equations.

From Eqn1, $x=5-y$ — Eqn3

step 2 substitute Eqn3 into Eqn2

$2(5-y)+3y=13$
$10-2y+3y=13$
$10+y=13$
$10-10+y=13-10$
$y=3$

step 3 substitute value found into any equations to find the other value.

sub $y=3$ in Eqn3
$\therefore x=2$ #

one unknown

LINEAR

eg. solve $3x-5=7$

solution

$3x-5=7$
$3x-5+5=7+5$
$\frac{3x}{3}=\frac{12}{3}$
$x=4$ #

eg. solve $\frac{5}{x+1}=\frac{2}{x-8}$

solution

*we cross-multiply.

$5(x-8)=2(x+1)$
$5x-40=2x+2$
$5x-40+40=2x+2+40$
$5x=2x+42$
$5x-2x=2x-2x+40$
$3x=40$
$x=\frac{40}{3}=13\frac{1}{3}$ #

QUADRATIC

① By cross-factorisation/multiplication frame

e.g. Solve $x^2-x-6=\underline{0}$

solution

*ensure that LHS or RHS is equal to ZERO.

$x^2-x-6=0$
$(x-3)(x+2)=0$
$\therefore x-3=0$ OR $x+2=0$
$x=3$ OR $x=-2$ #

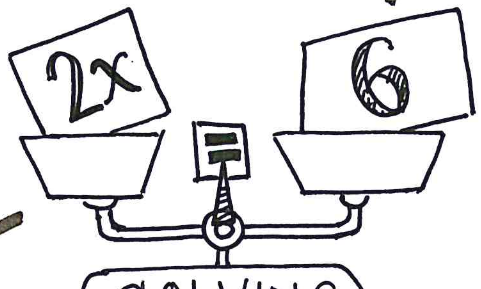
INEQUALITIES

* treat the inequalities like it represents the equal signs
* you "flipped" the sign when divided/multiply by a negative value.

eg. solve $-3x\geq 6$

solution

$\frac{-3x}{-3}\geq\frac{6}{-3}$
$x\leq -2$



SOLVING algebraic equations

two unknowns

Eg. Solve $x+y=5$
$2x+3y=13$

① ELIMINATION

$x+y=5$ —①
$2x+3y=13$ —②

step 1: make one unknown common.

①×2 $\quad 2x+2y=10$ —③

③-② $\quad -y=-3$
$y=3$ #

step 2: Substitute to find the other unknown.

Sub $y=3$ in ①
$x+3=5$
$x=2$ #

② SUBSTITUTION

$x+y=5$ —①
$2x+3y=13$ —②

From ①,
$x=5-y$ —③

sub ③ in ②
$2(5-y)+3y=13$
$10-2y+3y=13$
$y=3$ #

sub $y=3$ in ③
$x=5-3$
$x=2$ #

one unknown

LINEAR

Eg. Solve $3x-5=7$.

$3x-5=7$
$3x-5+5=7+5$
$3x=12$
$x=\frac{12}{3}=4$ #

Eg. Solve
$$\frac{5}{x+1}=\frac{2}{x-8}$$

* cross-multiply.

$5(x-8)=2(x+1)$
$5x-40=2x+2$
$3x=-38$
$x=-\frac{38}{3}$ #

QUADRATIC

① By cross-factorisation.

Solve $x^2-x-6=0$.

$\boxed{\text{solution}}$

$x^2-x-6=0$
$(x-3)(x+2)=0$
$x-3=0$ or $x+2=0$
$x=3$ # $\quad x=-2$ #

② By quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\boxed{\text{Solution}}$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-6)}}{2(1)}=\frac{1\pm\sqrt{25}}{2}$$
$$=\frac{1+5}{2} \text{ or } \frac{1-5}{2}$$
$$=3 \text{ or } -2 \text{ \#}$$

③ completing the square

x^2+ax+b
$=(x+\frac{a}{2})^2+b-(\frac{a}{2})^2$

ie.
x^2+4x+8
$=(x+2)^2+8-(2)^2$
$=(x+2)^2+4$ #

INEQUALITIES

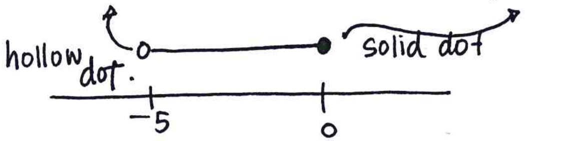
* treat the inequalities like an equal sign
* "flipped" the sign when divided/multiply by a negative value.

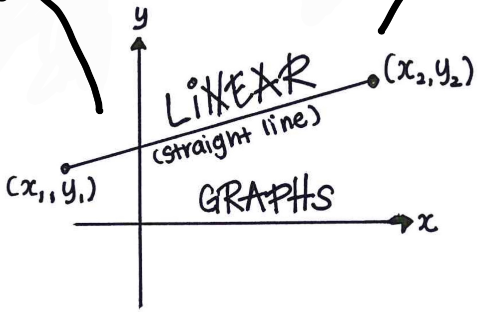
Eg.
$-3x\geqslant 6$
$x\leq 6\div(-3)$
$x\leq -2$ #

SOLVING SIMULTANEOUS INEQUALITIES

Eg. Solve $3x-1<5x+9\leq -7x+9$

$3x-1<5x+9$ OR $5x+9\leq -7x+9$
$-2x<10 \qquad 12x\leq 0$
$x>-5 \qquad x\leq 0$



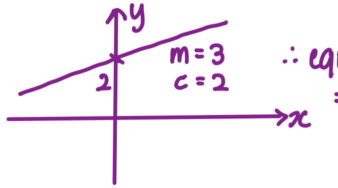


Varying GRADIENTS

co-ordinate geometry

Positive Gradient ($m>0$)

eg.

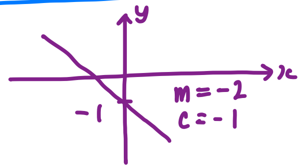


$\therefore$ equation
$\Rightarrow y = 3x+2$.

From left to right, the line slopes upwards.

Negative Gradient ($m<0$)

eg.

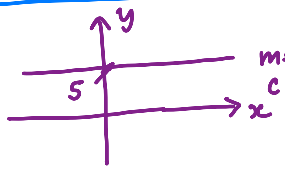


$\therefore$ equation
$\Rightarrow y = -2x-1$

From left to right, the line slopes downwards.

Zero Gradient ($m=0$)

eg.



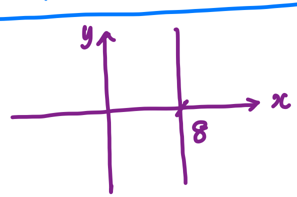
m=0
c=5

$\therefore$ equation
$\Rightarrow y = 5$

The line is horizontal and parallel to x-axis.

Undefined Gradient

eg.

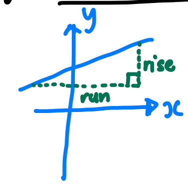


m= undefined
c=0

$\therefore$ equation
$\Rightarrow x = 8$

The line is vertical and parallel to y-axis

Gradient (Steepness)



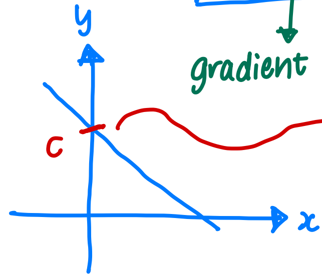
$\boxed{\text{gradient} = \frac{\text{rise}}{\text{run}}}$

Equation of Line

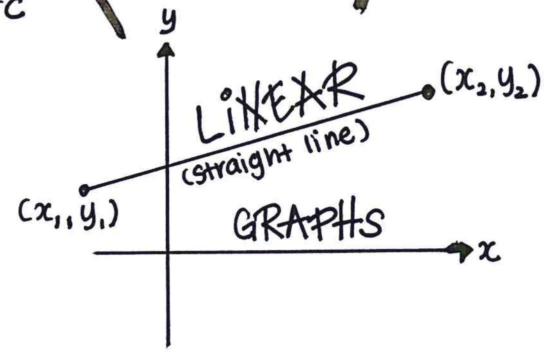
$$\boxed{y = mx + c}$$

gradient

y-intercept (the point in which the line cuts the y-axis)



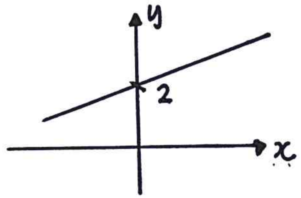
LINEAR (straight line) GRAPHS



Types of Graphs

ie.
$y = mx + c$

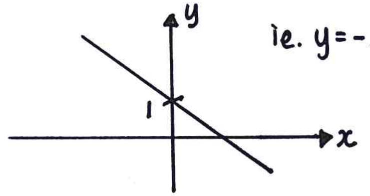
① $m > 0$ [positive gradient]



ie. $y = 3x + 2$

From left to right, the line slopes upwards.

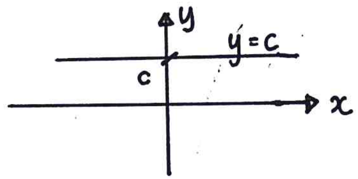
② $m < 0$ [negative gradient]



ie. $y = -2x + 1$

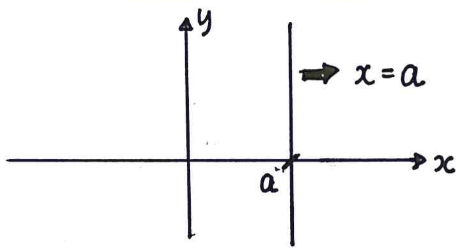
From left to right, the line slopes DOWNWARDS

③ $m = 0$ [zero gradient]



The line is HORIZONTAL.

④ $m = \infty$ [unlimited gradient]



The line is VERTICAL.

co-ordinate geometry

① Length (Distance) of Line

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

② Gradient (Steepness)

$$\frac{y_1 - y_2}{x_1 - x_2}$$

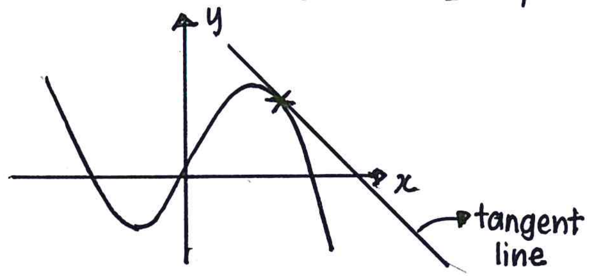
③ Equation of Line

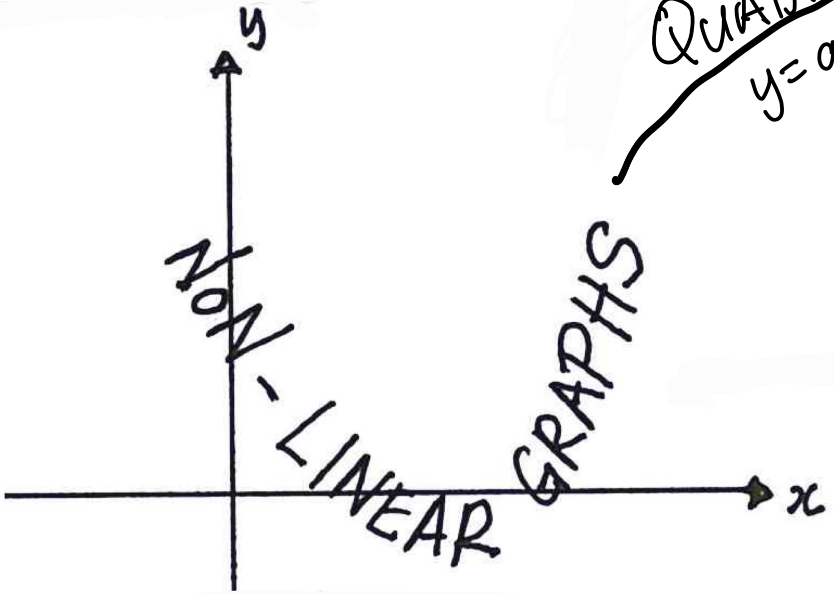
$$y = mx + c$$

where m = gradient
c = y-intercept
(the point in which it cuts y-axis).

④ Tangent

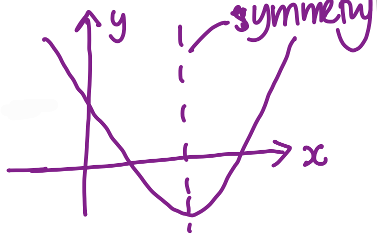
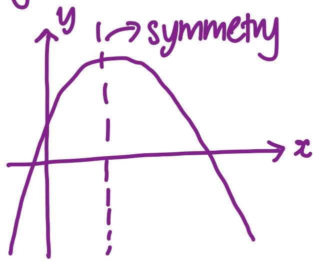
- a line that cuts the graph at ONE point.





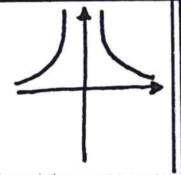
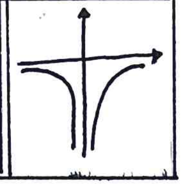
QUADRATIC GRAPHS

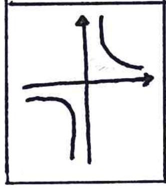
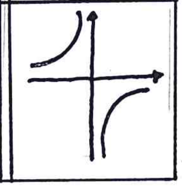
$y = ax^2 + bx + c$

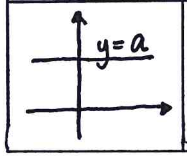
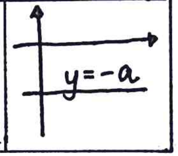
$y = ax^2 + bx + c$	
$a > 0$	$a < 0$
e.g. $y = \underline{\underline{2}}x^2 + 5x + 7$ 	e.g $y = -x^2 + 6x + 3$ 
• minimum point • symmetry line passes through minimum point	• maximum point • symmetry line passes through maximum point.

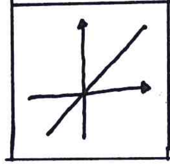
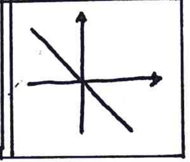
NON-LINEAR GRAPHS

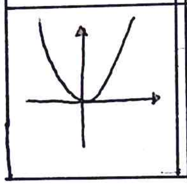
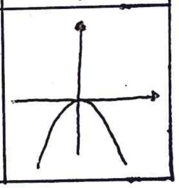
$y = ax^n$ where $-2 \le n \le 3$

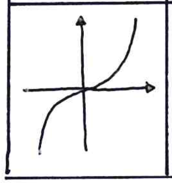
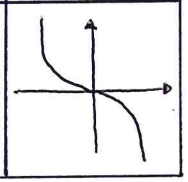
$y = \frac{a}{x^2}$	
$a > 0$	$a < 0$
i.e. $y = \frac{1}{x^2}$	i.e. $y = -\frac{1}{x^2}$
	

$y = \frac{a}{x}$	
$a > 0$	$a < 0$
i.e. $y = \frac{1}{x}$	i.e. $y = -\frac{1}{x}$
	

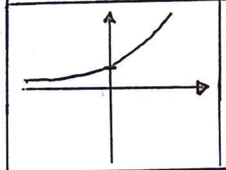
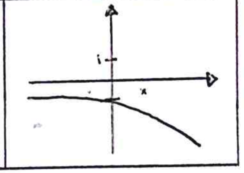
$y = ax^0$	
$a > 0$	$a < 0$
i.e. $y = 1$	i.e. $y = -1$
	

$y = ax$	
$a > 0$	$a < 0$
i.e. $y = 2x$	$y = -2x$
	

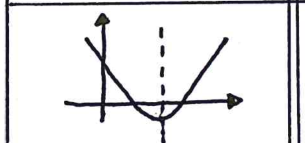
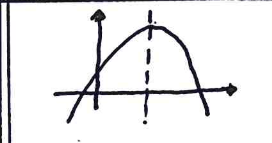
$y = ax^2$	
$a > 0$	$a < 0$
i.e. $y = x^2$	i.e. $y = -x^2$
	

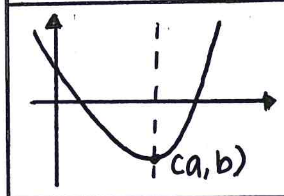
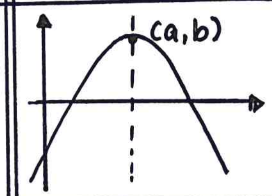
$y = ax^3$	
$a > 0$	$a < 0$
i.e. $y = x^3$	i.e. $y = -x^3$
	

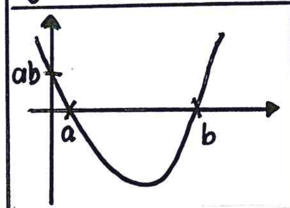
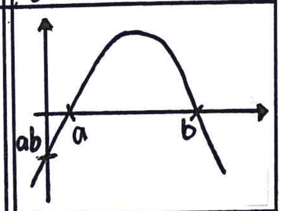
POWER GRAPHS "Strong"

$y = ka^x$	
$k > 0$	$k < 0$
i.e. $y = 2^x$	i.e. $y = -(2^x)$
	

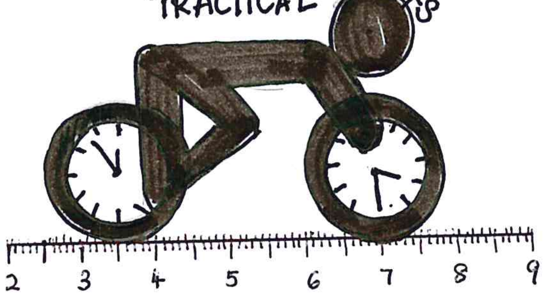
Quadratic graphs $y = ax^2 + bx + c$

$y = ax^2 + bx + c$	
$a > 0$	$a < 0$
	
• minimum pt • symmetry line passes through minimum pt	• maximum pt • symmetry line passes through maximum pt.

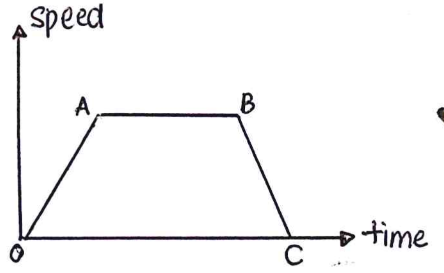
$y = (x-a)^2 + b$	$y = -(x-a)^2 + b$
	
• min point = (a, b) • symmetry line: $x = a$	• max point = (a, b) • symmetry line $x = a$

$y = (x-a)(x-b)$	$y = -(x-a)(x-b)$
	
• minimum pt • x-intercepts $\Rightarrow x = a, x = b$ • y-intercept $\Rightarrow c = ab$ • symmetry line $\Rightarrow x = \frac{a+b}{2}$	• maximum pt • x-intercepts $\Rightarrow x = a, x = b$ • y-intercepts $\Rightarrow c = ab$ • symmetry line $\Rightarrow x = \frac{a+b}{2}$

PRACTICAL GRAPHS



Speed-time graphs



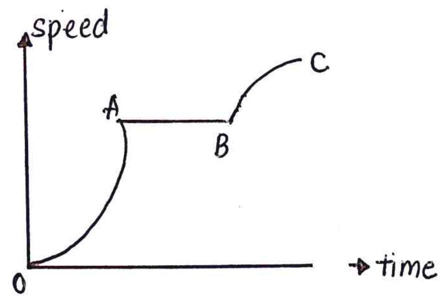
$\boxed{\text{Gradient = Acceleration}}$

$\boxed{\text{Area under Graph = Distance travelled}}$

OA ⇛ constant acceleration
⇛ increasing speed

AB ⇛ zero acceleration
⇛ constant speed

BC ⇛ constant deceleration
⇛ decreasing speed

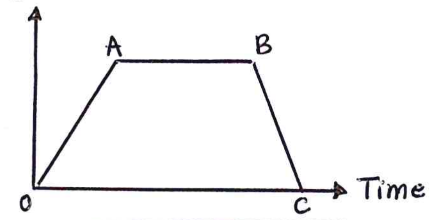


OA ⇛ increasing acceleration
⇛ increasing speed.

AB ⇛ zero acceleration
⇛ constant speed

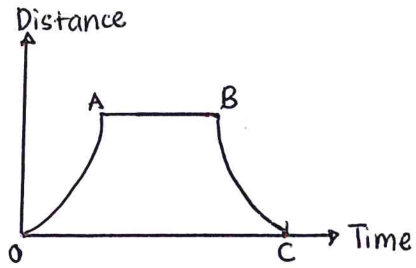
BC ⇛ decreasing acceleration.

Distance-Time Graphs

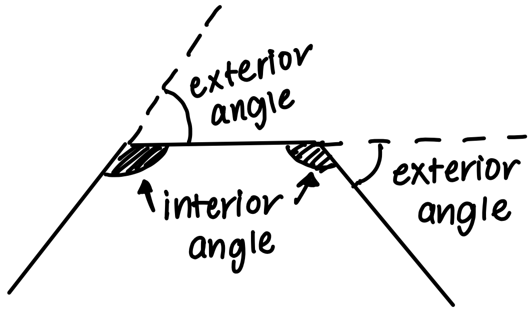


$\boxed{\text{Gradient = speed}}$

OA ⇛ constant speed
AB ⇛ zero speed (at rest)
BC ⇛ constant speed.

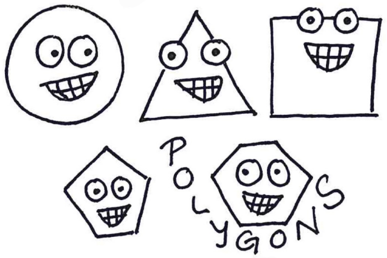


OA ⇛ increasing speed
AB ⇛ at rest
BC ⇛ decreasing speed



FORMULAES

1. interior + exterior = 180°
2. Total sum of interior angles of n-sided
   $\boxed{(n-2)\times 180^\circ}$
3. Each INTERIOR angle of n-sided regular
   $\boxed{\frac{(n-2)\times 180^\circ}{n}}$
4. Total sum of EXTERIOR angles of n-sided
   $\boxed{\text{total} = 360^\circ}$
5. Each Exterior angle of n-sided regular
   $\boxed{\frac{360^\circ}{n}}$



definition

plane figure that has 3 or more straight edges

REGULAR POLYGON

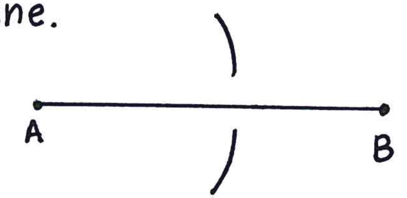
* all sides are equal in length
* each interior angles are equal
* each exterior angle are equal

types of polygons

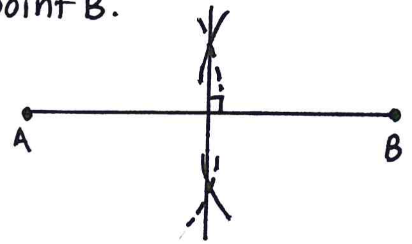
no. of sides	NAME
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

CONSTRUCTION OF PERPENDICULAR BISECTOR

Step 1: Put compass at point A. Measure out the width of compass to be more than half the length of the line. Mark an arc above and below line.



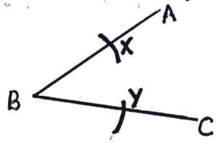
Step 2: using the same extension, do the same on point B.



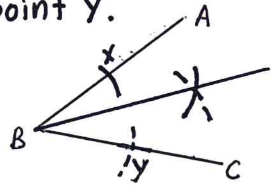
Step 3: Draw a line joining the two intersections.

CONSTRUCTION OF ANGLE BISECTOR

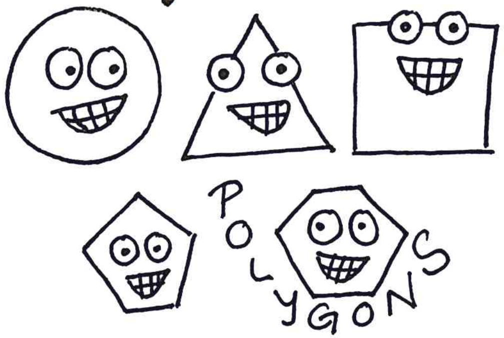
Step 1: Place compass at point B. Draw arcs to cut line AB at X and BC at Y.



Step 2: Place compass at X. Draw arc between lines AB and BC. Repeat the process at point Y.



Step 3: Draw a line from point B to the point of intersection of the two arcs.



Geometrical Construction

definition → plane figure that has 3 or more straight edges

REGULAR
- * all sides are equal
- * interior angles are equal
- * exterior angles are equal

Types

Number of Sides	NAME
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nanagon
10	Decagon

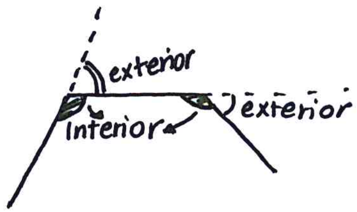
Formulaes

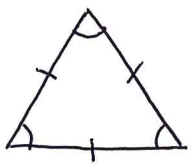
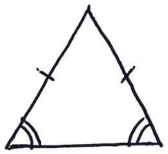
① Total sum of interior angles of n-sided: $\boxed{(n-2)\times 180^\circ}$

② Each interior angle of a n-regular sided: $\boxed{\dfrac{(n-2)\times 180^\circ}{n}}$

③ Total sum of exterior angles of n-sided $= 360^\circ$

④ Each exterior angle of n-regular sided: $\boxed{\dfrac{360^\circ}{n}}$



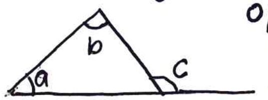
EQUILATERAL	ISOSCELES
	

TRIANGLES

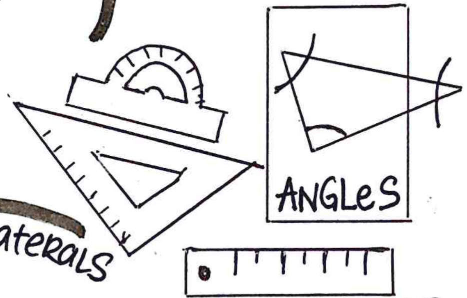
PROPERTIES

① sum of interior angles = 180°

② Exterior angle = sum of 2 interior opposite angles



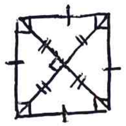
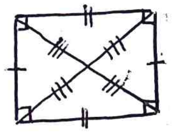
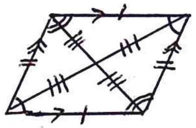
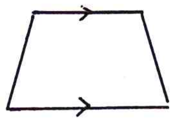
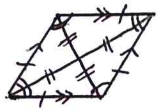
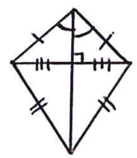
$\boxed{\angle a + \angle b = \angle c}$



& PLANE FIGURES

QUADRILATERALS

① Four sided figures.

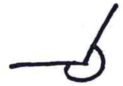
SQUARE	RECTANGLE	PARALLELOGRAM	TRAPEZIUM	RHOMBUS	KITE
					
• 4 equal sides • 4 right angles • opposite sides parallel • diagonals bisect at right angles	• Opposite sides equal • Opposite sides parallel • 4 right angles	• Opposite sides equal • Opposite sides parallel • Diagonally opposite angles are equal	• one opposite side parallel	• all sides equal • opposite sides parallel • diagonally opposite angles are **EQUAL**	• adjacent sides are equal • diagonals bisect at right angles

TYPES OF ANGLES

① Acute (less than 90°) 

② Right (90°) 

③ obtuse (more than 90° but less than 180°) 

④ reflex (more than 180° but less than 360°) 

⑤ complementary (angles add up to 90°)

⑥ supplementary (add up to 180°)

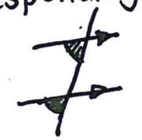
⑦ vertically opposite = EQUAL

⑧ adjacent angles on a straight line adds up to 180°

⑨ angles at a point (360°)

⑩ alternate angles **EQUAL**

⑪ corresponding angles **EQUAL**

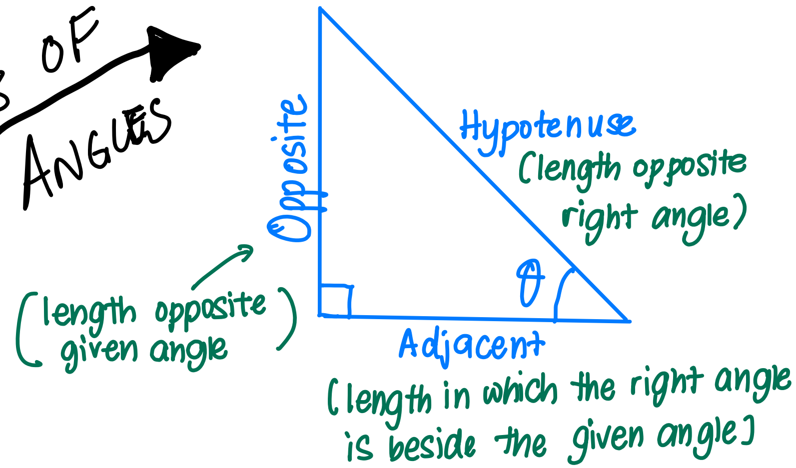


⑫ interior angles add up to 180°

TRIGONOMETRY

WE COULD USE A SINE FROM ABOVE

RATIOS OF ACUTE ANGLES



$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ (SOH)

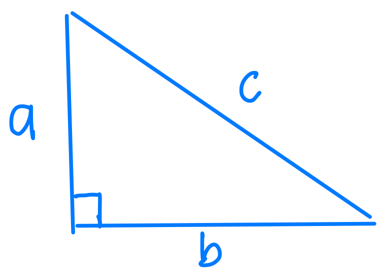
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ (CAH)

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ (TOA)

* to find value of angle (θ), just need to
1. $\theta = \sin^{-1}$
2. $\theta = \cos^{-1}$
3. $\theta = \tan^{-1}$

PYTHAGORAS' THEOREM

- can only be used for right-angled triangles.
- used to calculate length



$$\boxed{a^2 + b^2 = c^2}$$

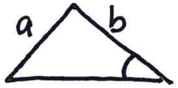
TRIGONOMETRY

SINE RULE

WHEN TO USE????

① given 2 angles and 1 side opposite given angle

② given 2 sides and 1 angle opposite given side



$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

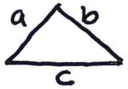
where A, B, C are angles

a, b, c are lengths

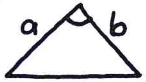
COSINE RULE

WHEN to USE??

① all three sides given

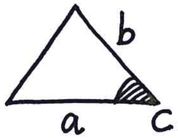


② given 2 sides and 1 angle in between.



$$\boxed{\begin{aligned} a^2 &= b^2+c^2-2bc\cos A \\ b^2 &= a^2+c^2-2ac\cos B \\ c^2 &= a^2+b^2-2ab\cos C \end{aligned}}$$

AREA OF TRIANGLE

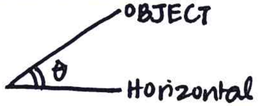


$$\boxed{\text{Area} = \frac{1}{2}ab\sin C}$$

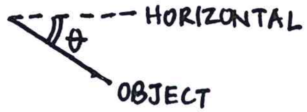
Bearings

* always start from North direction
* goes in clockwise direction
* displayed as three-digit form ie. 000°

Angle of elevation

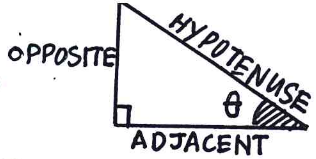


Angle of depression



WE COULD USE A SINE FROM ABOVE

Ratios of acute angles

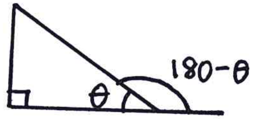


① $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ (SOH)

② $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ (CAH)

③ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ (TOA)

Ratios of obtuse angles

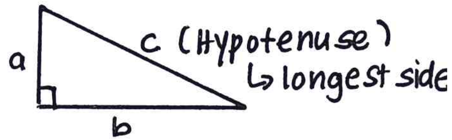


① $\sin(180-\theta)^\circ = \sin\theta$

② $\cos(180-\theta)^\circ = -\cos\theta$

Pythagoras' Theorem

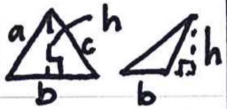
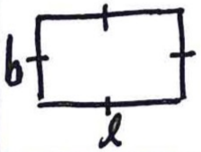
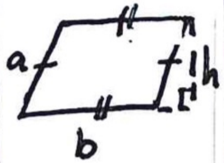
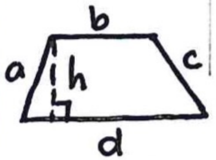
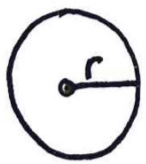
* to calculate length
* used only for right-angled Δs



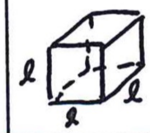
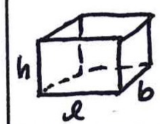
$$\boxed{c^2 = a^2+b^2}$$

2D& 3d-fig (MENSURATION)

Plane Figures

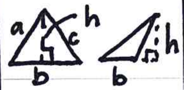
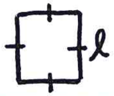
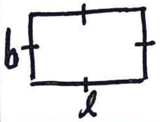
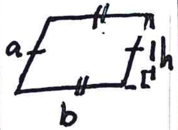
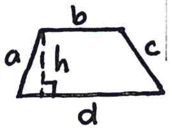
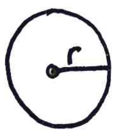
Type	Diagram	Perimeter	Area
Triangle		$a+b+c$	$\frac{1}{2}bh$
Square		$4l$	l^2
Rectangle		$2b+2l$	bl
//gram		$2a+2b$	bh
Trapezium		$a+b+c+d$	$\frac{1}{2}(b+d)(h)$
Circle		πd or $2\pi r$	πr^2

Solids

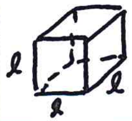
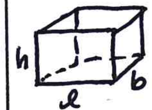
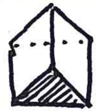
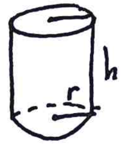
Type	Diagram	Surface Area	Volume
Pyramid		—	$\frac{1}{3} \times$ base area $\times h$
Cube		$6l^2$	l^3
Cuboid		$2lh+2lb+2bl$	blh
Prism		—	Cross sectional Area $\times h$
Cone		curved SA $= \pi r l$ Total SA $= \pi r l + \pi r^2$	$\frac{1}{3}\pi r^2 h$
Cylinder		curved SA $2\pi r h$ Total SA $2\pi r h + 2\pi r^2$	$\pi r^2 h$
Sphere		$4\pi r^2$	$\frac{4}{3}\pi r^3$

2D & 3d-fig (MENSURATION)

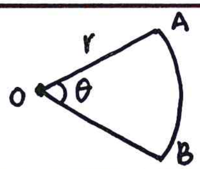
PLANE FIGURES

TYPE	DIAGRAM	PERIMETER	AREA
Triangle		$a+b+c$	$\frac{1}{2}bh$
Square		$4l$	l^2
Rectangle		$2b+2l$	bl
//gram		$2a+2b$	bh
Trapezium		$a+b+c+d$	$\frac{1}{2}(b+d)(h)$
Circle		πd or $2\pi r$	πr^2

SOLIDS

Type	DiagraM	SurFace Area	VoLume
Pyramid		—	$\frac{1}{3} \times \text{base area} \times h$
Cube		$6l^2$	l^3
Cuboid		$2lh+2lb+2bl$	blh
Prism		—	Cross sectional Area $\times h$
Cone		curved SA $=\pi rl$ Total SA $=\pi rl+\pi r^2$	$\frac{1}{3}\pi r^2 h$
Cylinder		curved SA $2\pi rh$ Total SA $2\pi rh+2\pi r^2$	$\pi r^2 h$
Sphere		$4\pi r^2$	$\frac{4}{3}\pi r^3$

sectors/segments



Degrees	Radians
Arc length (AB) $\frac{\theta}{360} \times 2\pi r$	Arc length (AB) $s = r\theta$
Area of sector $\frac{\theta}{360} \times \pi r^2$	Area of sector $A = \frac{1}{2}r^2\theta$

CONVERSION

$\pi \text{ rad} = 180°$

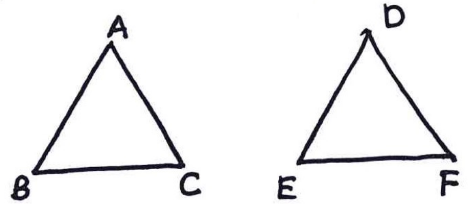
$1 \text{ rad} = \frac{180°}{\pi}$

$1° = \left(\frac{\pi}{180}\right) \text{rad.}$

Congruence & Similarity

definition

* exact same shape and size.



Triangle ABC $\equiv$ Triangle DEF

properties

① corresponding lengths are **EQUAL**

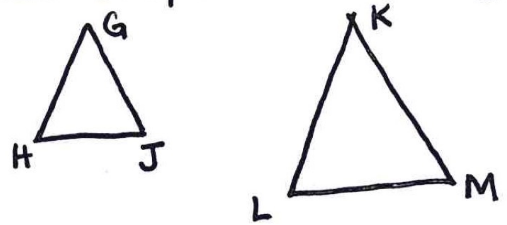
i.e. $AB = DE$, $BC = EF$, $AC = DF$

② corresponding angles are **EQUAL**

i.e. $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

definition

* same shape different size



properties

① corresponding angles are **EQUAL**

$\angle G = \angle K$, $\angle H = \angle L$, $\angle J = \angle M$

② ratio of corresponding sides are **EQUAL**

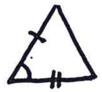
$$\boxed{\frac{GH}{KL} = \frac{HJ}{LM} = \frac{GJ}{KM}}$$

Congruence & Similarity

Congruence

Tests

1. side-side-side (SSS)
   * 3 equal corresponding sides.
2. side-angle-side (SAS)

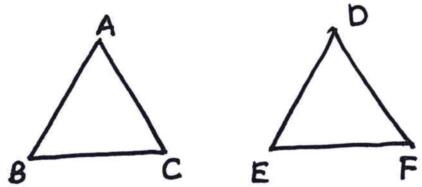


3. angle-side-angle (ASA)

4. right angle-hypotenuse-side (RHS)

definition

* exact same shape and size.



Triangle ABC ≡ Triangle DEF

properties

1. corresponding lengths are EQUAL
   i.e. AB = DE, BC = EF, AC = DF
2. corresponding angles are EQUAL
   i.e. $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

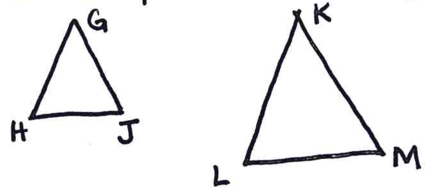
Similarity

Tests.

1. Angle-Angle (AA)
   - 2 corresponding angles equal
2. side-side-side (SSS)
   - 3 corresponding sides in the same ratio
3. side-side-angle (SSA)
   - 2 corresponding sides in same ratio and 1 included angle equal.

definition

* same shape different size



properties

1. corresponding angles are EQUAL
   $\angle G = \angle K$, $\angle H = \angle L$, $\angle J = \angle M$
2. ratio of corresponding sides are EQUAL

$$\boxed{\frac{GH}{KL} = \frac{HJ}{LM} = \frac{GJ}{KM}}$$

scalefactor

$$k = \frac{\text{Final image length}}{\text{Original image length}}$$

- $k < 1 \Rightarrow$ Reduction
- $k > 1 \Rightarrow$ enlargement

Area

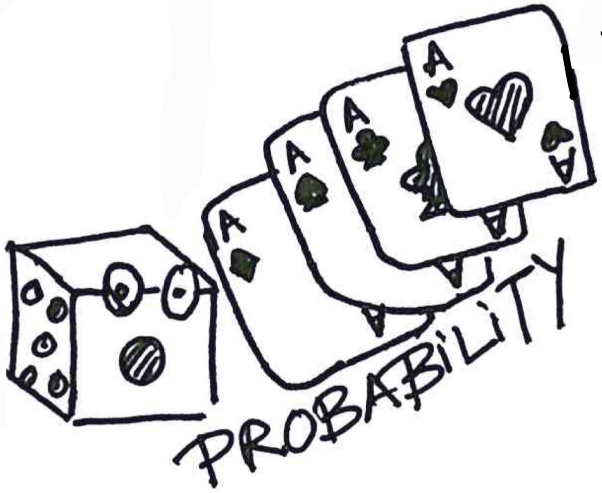
Area of Similar Figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

volume

Vol of Similar Fig

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$



definition

P(E) = probability of an event happening

$$P(E) = \frac{\text{No. of possible outcomes for event}}{\text{Total number of outcomes}}$$

properties

- $P(E) = 0 \Rightarrow$ impossible event to happen
- $P(E) = 1 \Rightarrow$ event will definitely happen.

e.g. Given that we roll a die numbered from 1 to 6, find the probability of getting

(i) a number 3,

(ii) a prime number.

SOLUTION

(i) $P(\text{getting a '3'}) = \frac{1}{6}$

(ii) $P(\text{getting a prime no.}) = \frac{3}{6} = \frac{1}{2}$ #

① POSSIBILITY diagrams

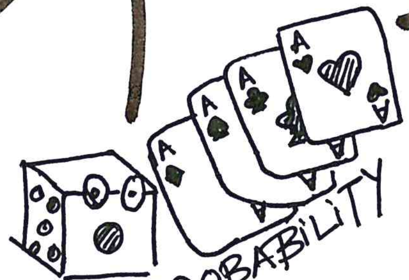
– usually meant for 2 events

Eg. Draw a possibility diagram for the event when a coin and a die is thrown

Coin

Die		H	T
	1		
	2		
	3		
	4		
	5		
	6		

DIAGRAMS



PROBABILITY

definition

$P(A)$ = probability of an event happening

$$\boxed{P(A) = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}}$$

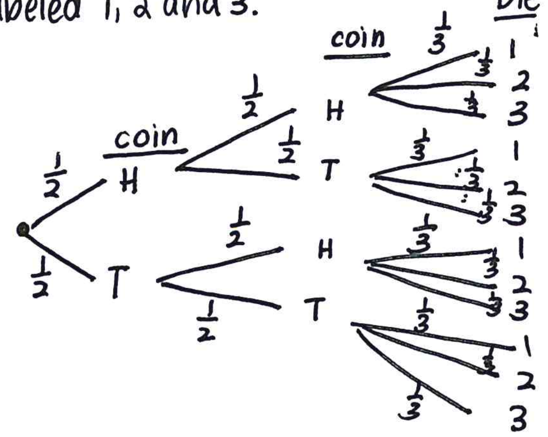
properties

* $P(A) = 0 \Rightarrow$ not possible
* $P(A) = 1 \Rightarrow$ surely possible
* $0 \leq P(A) \leq 1$

② TREE DIAGRAMS

– meant for 2 events or more.

Eg. Draw a tree diagram to show the possible outcome of throwing 2 coins and a three-sided die labeled 1, 2 and 3.



Outcome

H, H, 1 $\quad P = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

H, H, 2

⋮

T, T, 2

T, T, 3 $\quad P = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

MUTUALLY EXCLUSIVE

* Both events cannot happen at the same time.

ie. A = {throwing a coin and head appears}
B = {throwing a coin and tail appears}
thus A and B are mutually exclusive.

$$\boxed{P(A \text{ OR } B) = P(A) + P(B)}$$

INDEPENDENT

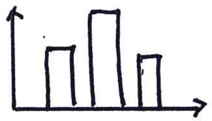
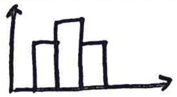
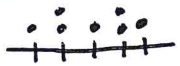
* If A happens, B can also happen at the same time.

ie. A = {throwing a coin and gets head}
B = {throwing a dice and even number appears}

$$\boxed{P(A \text{ and } B) = P(A) \times P(B)}$$

STATISTICAL DIAGRAMS	REASONS FOR MISINTERPRETATION OF DATA
BAR GRAPH	* vertical scale does not start from zero
PIE CHART	* distorted sense of proportion
PICTOGRAM	* the size of the pictures may not be UNIFORM
LINE GRAPH	* vertical scale does not start from zero * horizontal scale is not uniform
STEM-AND-LEAF	* key is not provided
HISTOGRAM	* the scale does not start from zero.

data representation

TYPE	GRAPH	PROPERTIES
Pictogram	4A ★★ 4B ★	- easy to present - inaccurate
Pie Chart		- compares size of individual to whole - doesn't show proportion
Line Chart		- shows trends - tracks changes with respect to time
Bar Graph		- accurate - useful in comparison among categories - does not show proportion in relation to whole
Histogram		- accurate - useful for comparison - does not show proportion
Dot diagram		- useful for highlighting clusters - not suitable for large set.
stem-and-leaf	stem | leaf 1 | 1 2 | 3 3 4 3 | 4	- good for comparing data back to back - not suitable for large data.

Measures Of Central Tendency

MEAN

ungrouped

$$\text{Mean} = \frac{\text{total sum of data}}{\text{no. of data}}$$

e.g.
2, 3, 5, 7

$$\text{Mean} = \frac{2+3+5+7}{4} = 6\frac{3}{4}$$

grouped

e.g. * Find mid-class value first.

CLASS	FREQ	MID-CLASS
$0 \le x < 2$	5	1
$2 \le x < 4$	15	3

$$\text{Mean} = \frac{1(5) + 3(15)}{5+15} = \frac{50}{20} = 2.5$$

mode/modal

value with the highest frequency OR simply the value that occurs the most often.

e.g.

stem	leaf
1	0 0 2
2	3 4 5 5 5 7
3	1 2

Key: $1|0 = 10$

mode = 15 #

median

position = $\frac{11+1}{2} = 6^{th}$

∴ median = 25 #

e.g.

CLASS	FREQ	MID-CLASS
$0 \le x < 2$	5	1
$2 \le x < 4$	15	3

mode = $2 \le x < 4$ #

median

position = $\frac{20+1}{2} = 10.5^{th}$ position (between 10^{th} & 11^{th})

∴ median class = $2 \le x < 4$ #

median

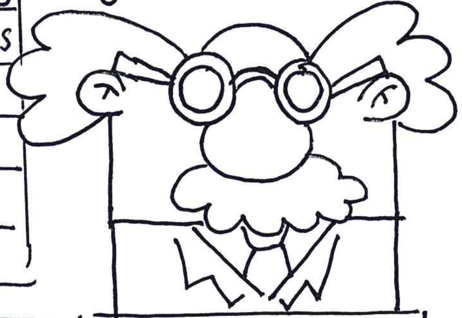
– the middle value of data.

step ① Find the total no. of values (n).

step ② Find the middle position. $\left[\frac{n+1}{2}\right]^{th}$ position

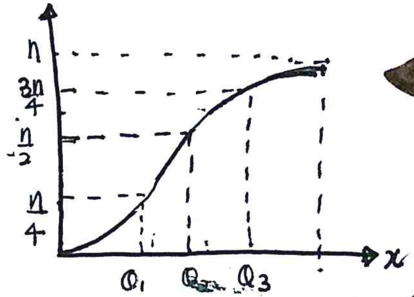
step ③ state the median value/class.

DATA ANALYSIS



CUMULATIVE FREQUENCY

graph



table

Frequency	
Marks	No. of pupils
$20<x\leq 30$	5
$30<x\leq 40$	11
$40<x\leq 50$	4

⇒

Cumulative Frequency	
Marks	No. of pupils
$x\leq 20$	0
$x\leq 30$	5
$x\leq 40$	16
$x\leq 50$	20

1. Range = Largest value − Smallest value
2. Lower Quartile (Q_1) = 25%
   Median (Q_2) = 50%
   Upper Quartile (Q_3) = 75%
3. Interquartile Range (IQR) = $Q_3 - Q_1$
4. n^{th} percentile = n% of TOTAL frequency.

MEASURE OF CENTRAL TENDENCY

MEAN

AVERAGE = $\dfrac{\text{SUM OF VALUES}}{\text{TOTAL NUMBER OF VALUES}}$

ungrouped

E.g	Average
2, 3, 5, 7	$= \dfrac{2+3+5+7}{4} = 6\frac{3}{4}$

grouped

E.g.	Average
(table below)	$\dfrac{1(5)+3(15)}{20} = 2\frac{1}{2}$

CLASS	FREQ
$0<x\leq 2$	5
$2<x\leq 4$	15

MEDIAN

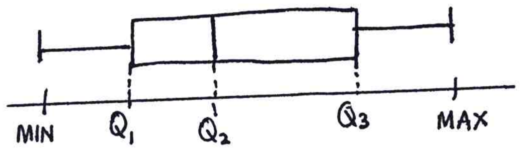
the middle value

ODD	EVEN
$\left(\frac{n+1}{2}\right)^{th}$ value	mean of 2 middle values: $\dfrac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$

MODE / MODAL

value with the highest frequency OR simply the value that occurs most often.

BOX-AND-WHISKER



MEAN & STANDARD DEVIATION

* standard deviation → measures spread of data
→ the lower the value, the MORE consistent it is.

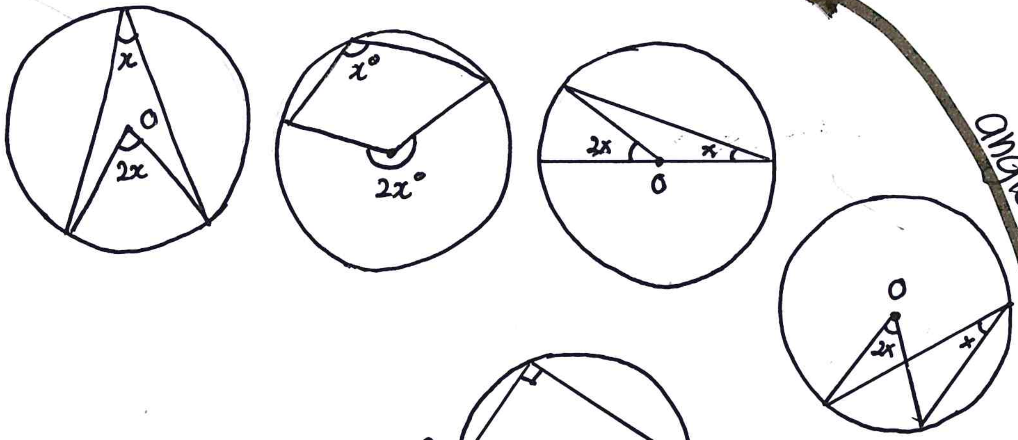
UNGROUPED	GROUPED
Mean, $\bar{x} = \dfrac{\Sigma x}{n}$	Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$
SD, $\sigma = \sqrt{\dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2}$	SD, $\sigma = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \left(\dfrac{\Sigma fx}{\Sigma f}\right)^2}$

GEOMETRY

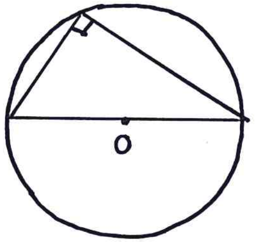
circle properties • circle properties • circle properties • circle properties •

angle

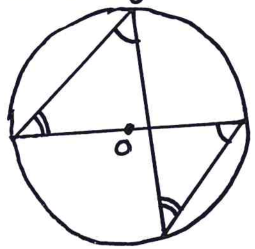
① Angle at centre is equal to twice angle at circumference



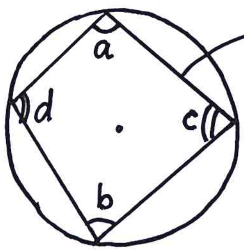
② Angle in a semicircle = 90°



③ Angles in the same segment are <u>EQUAL</u>.



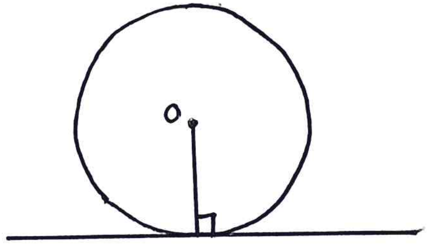
④ Angles in opposite segments add up to 180°.



cyclic quadrilateral (all points on circumference of circle)

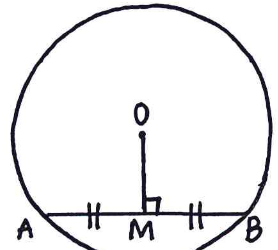
$(\angle a + \angle b = 180°)$ OR $(\angle c + \angle d = 180°)$

⑤ Tangent to circle is perpendicular to radius.



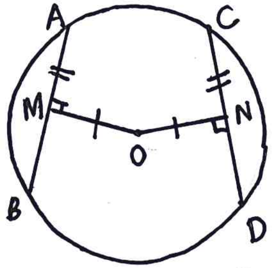
symmetrical

①



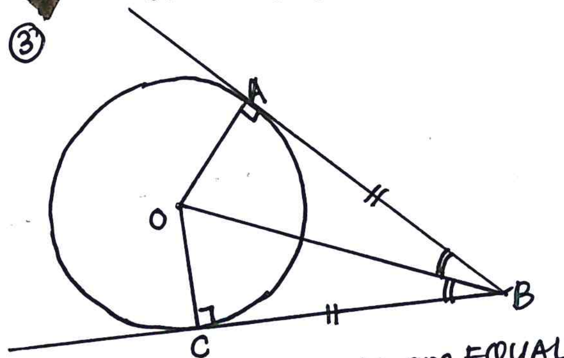
* perpendicular bisector of a chord passes through the centre.
* line joining midpoint AB to centre of circle is perpendicular to chord

②

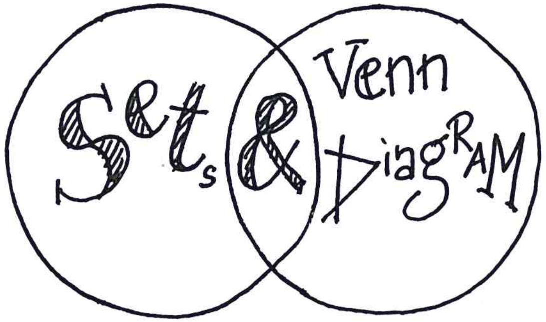


* equal chords equidistant from centre. (If AB = CD, then OM = ON)
* chords are equidistant from centre are equal in length. (If OM = ON, then AB = CD)

③



* tangents from external points are <u>EQUAL</u> (AB = CB).
* lines from external point to centre of circle bisects angle.



VENN diagrams

UNIVERSAL SET

→ represented by a box.

Examples

If $n(\mathcal{E}) = 100$, $n(A) = 20$, $n(B) = 95$

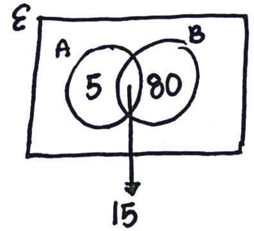
Find the smallest / largest possible value

for (i) $(A \cup B)'$

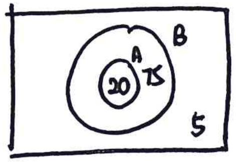
(ii) $(A \cap B)$

Solution:

(i)



thus smallest possible = 0



thus largest possible = 5

(ii) smallest possible value of $(A \cap B) = 15$

Largest possible value of $(A \cap B) = 20$

Language & Notation

① $\in$ = element of

② $\notin$ = not an element of

③ ϕ OR $\{\}$ = empty/null set

④ $\mathcal{E}$ = universal set

⑤ $\subseteq$ = subset

ie. $A = \{1,2,3\}$; $B = \{3,1,2\}$

thus $A \subseteq B$

⑥ $\subset$ = proper subset

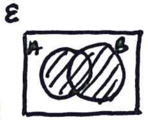
ie. $A = \{1,2\}$; $B = \{1,2,3,4\}$

thus $A \subset B$

⑦ $\cup$ = union

ie. $A = \{1,2\}$; $B = \{3,4\}$

thus $A \cup B = \{1,2,3,4\}$

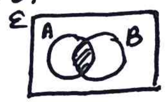


⑧ $\cap$ = intersect

= common elements

ie. $A = \{3,4,8\}$; $B = \{5,8,10\}$

thus $A \cap B = \{8\}$



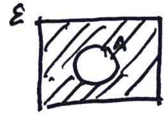
⑨ A' = complement of A

= elements out of set A.

ie. $\mathcal{E} = \{1,7,11,12\}$

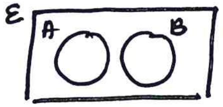
$A = \{7,12\}$

thus $A' = \{1,11\}$



⑩ Disjoint sets = NO common elements.

$A \cap B = \phi$



MATRICES

MULTIPLICATION

CONDITION

number of columns of 1st matrix = number of rows of 2nd matrix

$$\boxed{(m \times \underline{n})(\underline{n} \times p)}$$

SAME

E.g. If $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 10 \\ 3 \end{pmatrix}$,

thus AB

$$= \underbrace{\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}}_{2\times2}\underbrace{\begin{pmatrix} 10 \\ 3 \end{pmatrix}}_{2\times1}$$

$$= \frac{\text{FINAL PRODUCT}}{2\times1}$$

$$= \begin{pmatrix} R_1C_1 \\ R_2C_1 \end{pmatrix}$$

$$= \begin{pmatrix} (1\times10)+(3\times3) \\ (2\times10)+(5\times3) \end{pmatrix}$$

$$= \begin{pmatrix} 19 \\ 35 \end{pmatrix}$$

E.g. If $A = (3 \ \ 3)$ and $B = (4 \ \ 4 \ \ 5)$

$$AB = \underbrace{(3 \ \ 3)}_{1\times2}\underbrace{(4 \ \ 4 \ \ 5)}_{1\times3}$$

= Not possible.

NOTE

$$\boxed{AB \neq BA}$$

ORDER

$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ — element, rows, columns

$m \times n$ — number of rows (m), number of columns (n)

TYPES

① row matrix ⇒ ONLY one row
⇒ $(1 \ \ 2 \ \ 3)$

② column matrix ⇒ ONLY one column
⇒ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

③ IDENTITY MATRIX : $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ → diagonals are 1 and the rest are zeroes

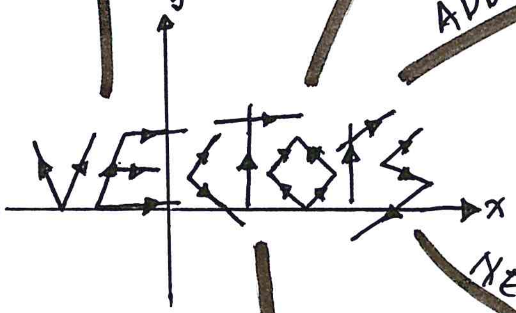
④ ZERO/NULL MATRIX : (0)

⑤ EQUAL MATRICES : $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

ADDITION SUBTRACTION

* can only ADD and SUBTRACT matrices of the SAME ORDER.

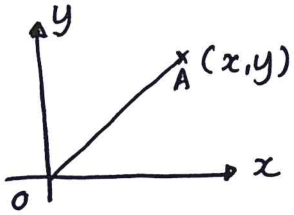
VECTORS



POSITION VECTOR

* a vector that starts from origin

ie.



Thus $\overrightarrow{OA} = \begin{pmatrix} x \\ y \end{pmatrix}$

ie. $\overrightarrow{PA} = \overrightarrow{OA} - \overrightarrow{OP}$

OR $\overrightarrow{PA} = \overrightarrow{PO} + \overrightarrow{OA}$

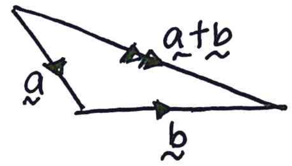
① COLUMN VECTOR $= \begin{pmatrix} x \\ y \end{pmatrix}$

② magnitude of vectors = length.

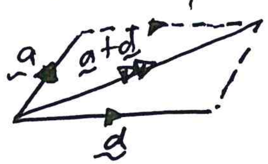
ie. $AB = \begin{pmatrix} x \\ y \end{pmatrix}$; $|AB| = \sqrt{x^2 + y^2}$

ADDITION

① TRIANGLE LAW



② PARALLELOGRAM LAW



NEGATIVE

* equal magnitude but opposite direction.

* $\boxed{\text{equal vectors}}$

↳ equal magnitude and same direction.

* $\boxed{\text{SCALAR VECTOR}}$

ie.

case 1 : Parallel lines

$\overrightarrow{CD} = k\overrightarrow{FG}$

since there are NO common points, thus CD // FG.

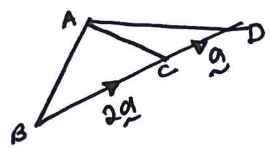
CASE 2 : collinear

$\overrightarrow{AB} = k\overrightarrow{BC}$

Since B is a common point, thus A, B, C are collinear.

Finding ratio of area in a vector question:

① CASE 1 : SHARING of common height.

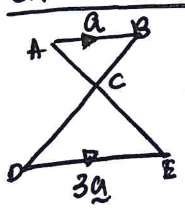


Find ratio of triangle ABC to triangle ACD.

Sol^n: $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ACD} = \dfrac{\cancel{\frac{1}{2}} \times BC \times \cancel{h}}{\cancel{\frac{1}{2}} \times CD \times \cancel{h}}$

$= \dfrac{2}{1} = 2$ //

② case 2 : SIMILAR TRIANGLES



Find ratio of △ABC : △CDE.

Sol^n: $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle CDE} = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$ //