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ANDERSON SECONDARY SCHOOL
Weighted Assessment 2 2025
Secondary Four Express/
Five Normal Academic

CANDIDATE NAME:

CLASS: /

INDEX NUMBER:

ADDITIONAL MATHEMATICS 4049

23 April 2025

45 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 30.

Setter: Mr Aaron Wong

1 Differentiate each of the following with respect to x.

(a) $\frac{1}{3}x^6 - x^{-2} + 3$ [1]

(b) $\ln\left(\frac{3x+1}{5-2x}\right)$ [2]

(c) $\frac{3\sin 2x}{\sqrt{5x^2 - 3}}$ [3]

2 The equation of a curve is $y = (2x-3)\sqrt{4x^2+1}$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16x^2 - 12x + 2}{\sqrt{4x^2+1}}$. [2]

(b) Find the range of values of x for which $y = (2x-3)\sqrt{4x^2+1}$ is an increasing function. [4]

3 It is given that $y = P\mathrm{e}^{4x} + Q\mathrm{e}^{-3x}$, and that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} = 12\mathrm{e}^{4x} + 3\mathrm{e}^{-3x}$.

Find the value of each of the constants P and Q. [4]

4 The equation of a curve is $y = x + \dfrac{3x-1}{x-2}$, where $x > 0$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Find the equation of the tangent to the curve at $x = 1$. [1]

(c) Find the equation of the normal to the curve at $x = 1$. [2]

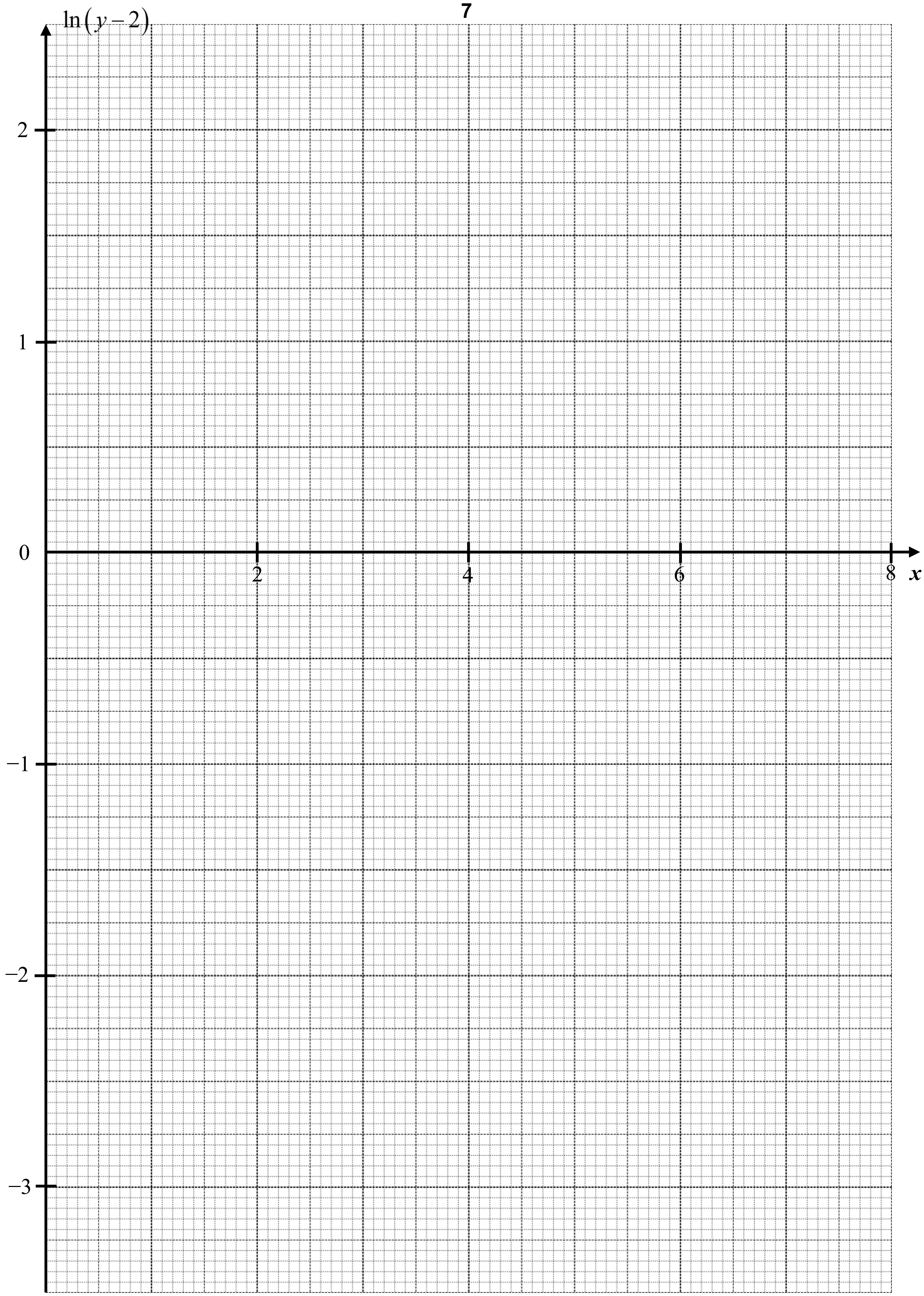
5 The table shows experimental values of two variables, x and y, which are connected by the equation $y = A\mathrm{e}^{\frac{x}{k}} + 2$, where A and k are constants.

x	2	4	6	8
y	2.135	2.368	3	4.718

(a) On the grid provided on the next page, draw a graph of $\ln(y-2)$ plotted against x. [3]

(b) Use your graph to estimate the value of A and of k. [4]

(c) On the same diagram, draw the straight line representing the equation $y - 2 = \mathrm{e}^{-\frac{x}{5}}$. Hence find the value of x for which $\frac{x}{k} + \ln A = -\frac{x}{5}$. [2]



END OF PAPER

2025 Secondary 4E/5N Additional Mathematics WA2
Mark Scheme

1 Differentiate each of the following with respect to x.

(a) $\frac{1}{3}x^6 - x^{-2} + 3$ [1]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{3}x^6 - x^{-2} + 3\right) = \frac{1}{3}(6)x^5 - (-2)x^{-3}$$
$$= 2x^5 + 2x^{-3}$$ **[B1]**

(b) $\ln\left(\frac{3x+1}{5-2x}\right)$ [2]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\frac{3x+1}{5-2x}\right)\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1) - \ln(5-2x)\right)$$ **[M1] – applying logarithmic law**

$$= \frac{3}{3x+1} + \frac{2}{5-2x}$$ **[A1]**

OR

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\frac{3x+1}{5-2x}\right)\right) = \frac{\frac{3(5-2x)-(-2)(3x+1)}{(5-2x)^2}}{\frac{3x+1}{5-2x}}$$ **[M1] – differentiate directly**

$$= \frac{17}{(5-2x)(3x+1)}$$ **[A1]**

(c) $\frac{3\sin 2x}{\sqrt{5x^2-3}}$ [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3\sin 2x}{\sqrt{5x^2-3}}\right) = \frac{\left(\sqrt{5x^2-3}\right)(6\cos 2x) - \left(\frac{10x}{2\sqrt{5x^2-3}}\right)(3\sin 2x)}{\left(\sqrt{5x^2-3}\right)^2}$$

[M2]
1m for $\left(\sqrt{5x^2-3}\right)(6\cos 2x)$
1m for $\left(\frac{10x}{2\sqrt{5x^2-3}}\right)(3\sin 2x)$

$$= \frac{2\left(5x^2-3\right)(6\cos 2x) - 10x(3\sin 2x)}{2\left(\sqrt{5x^2-3}\right)^3}$$

$$= \frac{6\left(5x^2-3\right)\cos 2x - 15x(\sin 2x)}{\left(\sqrt{5x^2-3}\right)^3}$$ **[A1]**

2 The equation of a curve is $y = (2x-3)\sqrt{4x^2+1}$.

(a) Show that $\frac{dy}{dx} = \frac{16x^2-12x+2}{\sqrt{4x^2+1}}$. [2]

$$\frac{dy}{dx} = 2\sqrt{4x^2+1} + (2x-3)\left(\frac{1}{2}\right)(8x)\left(\frac{1}{\sqrt{4x^2+1}}\right)$$ **[M1]**

$$= \frac{2(4x^2+1)+4x(2x-3)}{\sqrt{4x^2+1}}$$

$$= \frac{16x^2-12x+2}{\sqrt{4x^2+1}} \quad \text{[Shown]}$$

[A1] – Making a single fraction, leading to RHS

(b) Find the range of values of x for which $y = (2x-3)\sqrt{4x^2+1}$ is an increasing function. [4]

For y to be an increasing function, $\frac{dy}{dx} > 0$, hence,

$$\frac{16x^2-12x+2}{\sqrt{4x^2+1}} > 0$$ **[M1]**

$$\frac{2(8x^2-6x+1)}{\sqrt{4x^2+1}} > 0$$

$$\frac{2(4x-1)(2x-1)}{\sqrt{4x^2+1}} > 0$$

Since $\sqrt{4x^2+1} > 0$, then

$$16x^2-12x+2 > 0$$ **[M1]**

$$2(4x-1)(2x-1) > 0$$

$$(4x-1)(2x-1) > 0$$ **[M1]**

$$x < \frac{1}{4} \quad \text{or} \quad x > \frac{1}{2}$$ **[A1]**

3 It is given that $y = P\mathrm{e}^{4x} + Q\mathrm{e}^{-3x}$, and that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} = 12\mathrm{e}^{4x} + 3\mathrm{e}^{-3x}$.

Find the value of each of the constants P and Q. [4]

$$y = P\mathrm{e}^{4x} + Q\mathrm{e}^{-3x}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4P\mathrm{e}^{4x} - 3Q\mathrm{e}^{-3x}$$ **[M1]**

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 16P\mathrm{e}^{4x} + 9Q\mathrm{e}^{-3x}$$ **[M1]**

Substituting $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} = 12\mathrm{e}^{4x} + 3\mathrm{e}^{-3x}$

$$16P\mathrm{e}^{4x} + 9Q\mathrm{e}^{-3x} + 2\left(4P\mathrm{e}^{4x} - 3Q\mathrm{e}^{-3x}\right) = 12\mathrm{e}^{4x} + 3\mathrm{e}^{-3x}$$

$$24P\mathrm{e}^{4x} + 3Q\mathrm{e}^{-3x} = 12\mathrm{e}^{4x} + 3\mathrm{e}^{-3x}$$

[M1] – correct substitution or comparison of coefficients

By comparing coefficients,

$24P = 12$

$P = \dfrac{1}{2}$ and $Q = 1$ **[A1]** – Both P and Q

4 The equation of a curve is $y = x + \dfrac{3x-1}{x-2}$.

(a) Find $\dfrac{dy}{dx}$. [2]

$$\frac{dy}{dx} = 1 + \frac{(x-2)(3)-(3x-1)}{(x-2)^2} \qquad \textbf{[M1]}$$

$$= \frac{(x-2)^2 + 3x - 6 - 3x + 1}{(x-2)^2}$$

$$= \frac{x^2 - 4x - 1}{(x-2)^2}$$

[A1] – accept $1 - \dfrac{5}{(x-2)^2}$ as well

(b) Find the equation of the tangent to the curve at $x = 1$. [1]

At $x = 1$,

$$\frac{dy}{dx} = \frac{1^2 - 4(1) - 1}{(1-2)^2}$$

$$= -4$$

$$y = 1 + \frac{3(1)-1}{1-2}$$

$$= -1$$

Equation of tangent:

$$y - (-1) = -4(x-1)$$

$$y = -4x + 3 \qquad \textbf{[B1]}$$

(c) Find the equation of the normal to the curve at $x = 1$. [2]

Gradient of normal $= \dfrac{-1}{-4}$

$$= \frac{1}{4} \qquad \textbf{[M1]}$$

Equation of normal:

$$y - (-1) = \frac{1}{4}(x-1)$$

$$y = \frac{1}{4}x - \frac{5}{4} \qquad \textbf{[A1]}$$

5 The table shows experimental values of two variables, x and y, which are connected by the equation $y = Ae^{\frac{x}{k}} + 2$, where A and k are constants.

x	2	4	6	8
y	2.135	2.368	3	4.718

(a) On the grid provided on the next page, draw a graph of $\ln(y-2)$ plotted against x. [3]

x	2	4	6	8
$\ln(y-2)$	–2.002	–1.000	0	1.000

B1 – correct table of values
B1 – plotting all the points correctly
B1 – best-fit line

(b) Use your graph to estimate the value of A and of k. [4]

$$y = Ae^{\frac{x}{k}} + 2$$

$$y - 2 = Ae^{\frac{x}{k}}$$

$$\ln(y-2) = \ln A + \ln e^{\frac{x}{k}}$$

$$\ln(y-2) = \frac{x}{k} + \ln A \qquad \textbf{[M1]}$$

$$\text{gradient} = \frac{1-(-1)}{8-4}$$

$$= 0.5\ (\pm 0.1) \quad \textbf{[M1]}$$

$$\frac{1}{k} = 0.5$$

$$k = 2(\pm 0.5) \quad \textbf{[A1]}$$

$$y\text{-intercept} = -3$$

$$\ln A = -3$$

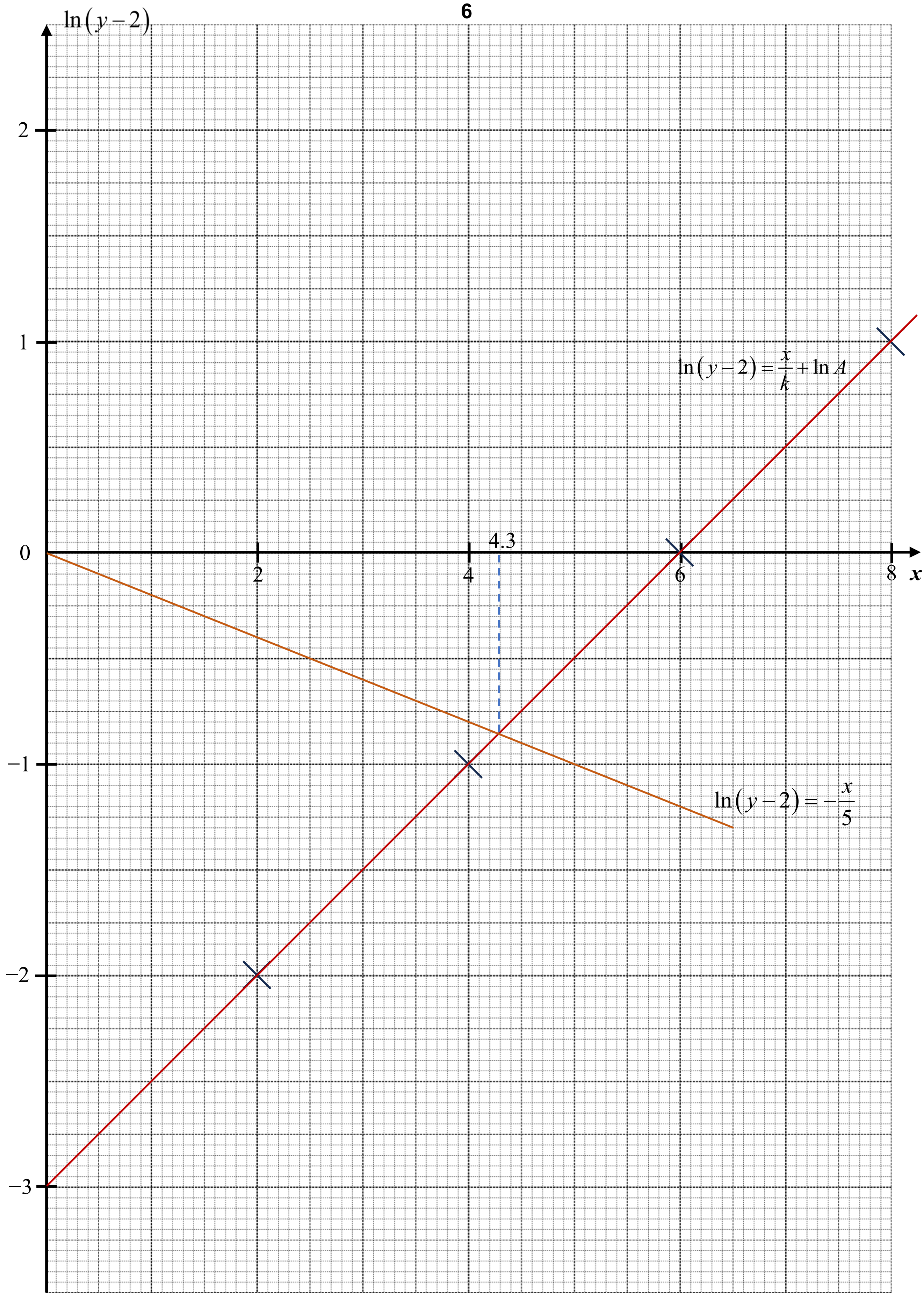
$$A = e^{-3}$$

$$= 0.0498 \ \text{(accept 0.0474 to 0.0523) (3s.f.)} \quad \textbf{[A1]}$$

(c) On the same diagram, draw the straight line representing the equation $y - 2 = e^{-\frac{x}{5}}$. Hence find the value of x for which $\frac{x}{k} + \ln A = -\frac{x}{5}$. [2]

$y - 2 = e^{-\frac{x}{5}} \Rightarrow \ln(y-2) = -\frac{x}{5}$ Draw straight line $\ln(y-2) = -\frac{x}{5}$ **[M1]**

$x = 4.3(\pm 0.1)$ **[A1]**


ln(y − 2)
2
1
0
−1
−2
−3
2
4
4.3
6
8
x
ln(y − 2) = x/k + ln A
ln(y − 2) = −x/5