

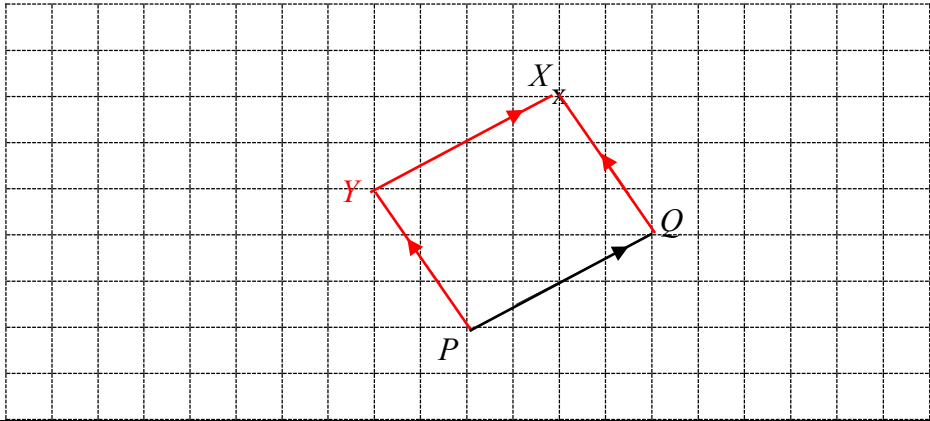
Mark scheme for Sec 4 Math WA2 2026

1	(a)	Show that $3 \sqrt[4]{x^{-4}y^{-8}}^3 = 3x^{-3}y^{-6}$.					[1]
		Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria		
		$3 \sqrt[4]{x^{-4}y^{-8}}^3$ $= 3 x^{-4}y^{-8}^{\frac{3}{4}}$ $= 3x^{-3}y^{-6}$		For “show” questions, working must show use of Laws of Indices explicitly.	Apply Law & Definition of Indices: $\sqrt[n]{a^m} = \sqrt[n]{a^m} = a^{\frac{m}{n}}$ $(ab)^n = a^n b^n$		
	(b)	Hence, simplify $3 \sqrt[4]{x^{-4}y^{-8}}^3 \times \left(\frac{81x^6}{y^8}\right)^{\frac{1}{2}}$ and leave your answer in positive index form.					[2]
		$3 \sqrt[4]{x^{-4}y^{-8}}^3 \times \left(\frac{81x^6}{y^8}\right)^{\frac{1}{2}}$ $= 3x^{-3}y^{-6} \times \left(\frac{9x^3}{y^4}\right)$ $= \frac{27}{y^{10}}$			Apply Law & Definition of Indices: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $a^{-n} = \frac{1}{a^n}, a \neq 0$ $a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $a^0 = 1, a \neq 0$		
		Overall Item Demand	Content	Complexity	Context	Response	
		A01	Low	Medium	Low	Low	

2	Mandy deposited \$150 000 in a 5-year investment plan. For the first two years, it offers a simple interest rate of 4% per annum. After two years, the accumulated sum will enjoy a compound interest of 5% per annum, compounded quarterly for 3 years. Show that the amount of interest she will receive at the end of 5 years is about \$38 000 (rounded to the nearest thousand).						[4]
Solutions/Alternative Methods			Remarks		Skills/Concept/Success Criteria		
Simple interest earned after 2 years $I = \frac{\$150000 \times 4 \times 2}{100}$ $= \$12000$ Accumulated sum after 2 years = \$162 000 Total amount at the end of 5 years					Apply simple interest formula $I = \frac{PRT}{100}$ with correct P, R and T to find simple interest. Adjust r and n according to frequency of calculating compound interest.		

$= \$162000 \left(1 + \frac{5 \div 4}{100} \right)^{3 \times 4}$ $= \$188042.2319$ Total interest earned at the end of 5 years $= \$188042.2319 - \$150\,000$ $= \$38042.23$ $= \$38000 \text{ (nearest thousand)}$ Alternatively, Total interest earned at the end of 5 years $= (\$188042.2319 - \$162000) + \$12000$ $= \$38042.23$ $= \$38000 \text{ (nearest thousand)}$	For “show” question, students should show value \$38042 before rounding off to required accuracy.	Apply compound interest formula $A = P \left(1 + \frac{r}{100} \right)^n$ to find total amount with compound interest. Find total interest by taking the difference of total amount and initial principal.
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Overall Item Demand	Content	Complexity	Context	Response
A02	Medium	Medium	Medium	Somewhat unfamiliar

3	The diagram shows the positions of the points X, P and Q where $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ is represented in the grid below. 	
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(a)	Y is the point such that $PQXY$ is a parallelogram.	
(i)	Mark and label the point Y on the grid above.	[1]
(ii)	Express $\overrightarrow{PY}$ as a column vector.	[1]

Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria	
(i) Mark Y using a cross. (ii) Since $PQXY$ is a parallelogram, $\overrightarrow{PY} = \overrightarrow{QX}$. $\overrightarrow{PY} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$			Apply concept that vectors on opposite sides of a parallelogram are equal. Write down a vector in column vector notation. Draw point Y correctly to form parallelogram $PQXY$.	
Overall Item Demand	Content	Complexity	Context	Response

	A01	Low	Low	Low	Low
(c)	Find $ \overrightarrow{QX} $. Leave your answer in 2 significant figures.				[1]
Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria		
$ \overrightarrow{QX} = \sqrt{(-2)^2 + 3^2}$ $= \sqrt{13}$ $= 3.6 \text{ units (2 s.f.)}$		Take note of accuracy required is 2 s.f.	Find magnitude of a vector using $\left \begin{pmatrix} x \\ y \end{pmatrix} \right = \sqrt{x^2 + y^2}$		
Overall Item Demand		Content	Complexity	Context	Response
A01		Low	Low	Low	Low

4	(a) Two fair dice with sides numbered 1 to 6 are thrown at the same time. Find the probability that both dice show the same number.					[1]																																																										
Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria																																																													
P(both dice show same number) $= \frac{6}{6} \times \frac{1}{6} \quad \text{or} \quad \frac{1}{6} \times \frac{1}{6} \times 6$ $= \frac{1}{6}$ or P(both dice show same number) $= \frac{\text{No. of combined outcomes with same no.}}{\text{Total no. of combined outcomes}}$ $= \frac{6}{6 \times 6}$ $= \frac{1}{6}$ Or use Possibility Diagram to visualize how to get answer. <table><tr><td></td><td></td><td colspan="6">Die 1</td></tr><tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td rowspan="6">Die 2</td><td>1</td><td>1, 1</td><td>2, 1</td><td>3, 1</td><td>4, 1</td><td>5, 1</td><td>6, 1</td></tr><tr><td>2</td><td>1, 2</td><td>2, 2</td><td>3, 2</td><td>4, 2</td><td>5, 2</td><td>6, 2</td></tr><tr><td>3</td><td>1, 3</td><td>2, 3</td><td>3, 3</td><td>4, 3</td><td>5, 3</td><td>6, 3</td></tr><tr><td>4</td><td>1, 4</td><td>2, 4</td><td>3, 4</td><td>4, 4</td><td>5, 4</td><td>6, 4</td></tr><tr><td>5</td><td>1, 5</td><td>2, 5</td><td>3, 5</td><td>4, 5</td><td>5, 5</td><td>6, 5</td></tr><tr><td>6</td><td>1, 6</td><td>2, 6</td><td>3, 6</td><td>4, 6</td><td>5, 6</td><td>6, 6</td></tr></table>				Die 1								1	2	3	4	5	6	Die 2	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4	5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5	6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6	Can visualize using Tree Diagram but no need to show Tree Diagram No need to show Possibility Diagram for 1 mark question	Apply Multiplication Law of Independent Events in Probability: $P((1,1)) = \frac{1}{6} \times \frac{1}{6}$ Apply Addition Law of Mutually Exclusive Events in Probability: $P((1,1) \text{ or } (2,2) \text{ or } (3,3) \text{ or } \dots (6,6))$ $=$ $P((1,1)) + P((2,2)) + P((3,3)) + P((6,6))$ $= \frac{1}{6} \times \frac{1}{6} \times 6$ or Since each combined outcome is equally likely (has same probability) to occur, we can use $P(E) = \frac{n(\text{favourable outcomes in } E)}{n(\text{Sample space})}$ or use Possibility Diagram		
		Die 1																																																														
		1	2	3	4	5	6																																																									
Die 2	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1																																																									
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Overall Item Demand		Content	Complexity	Context	Response																																																											

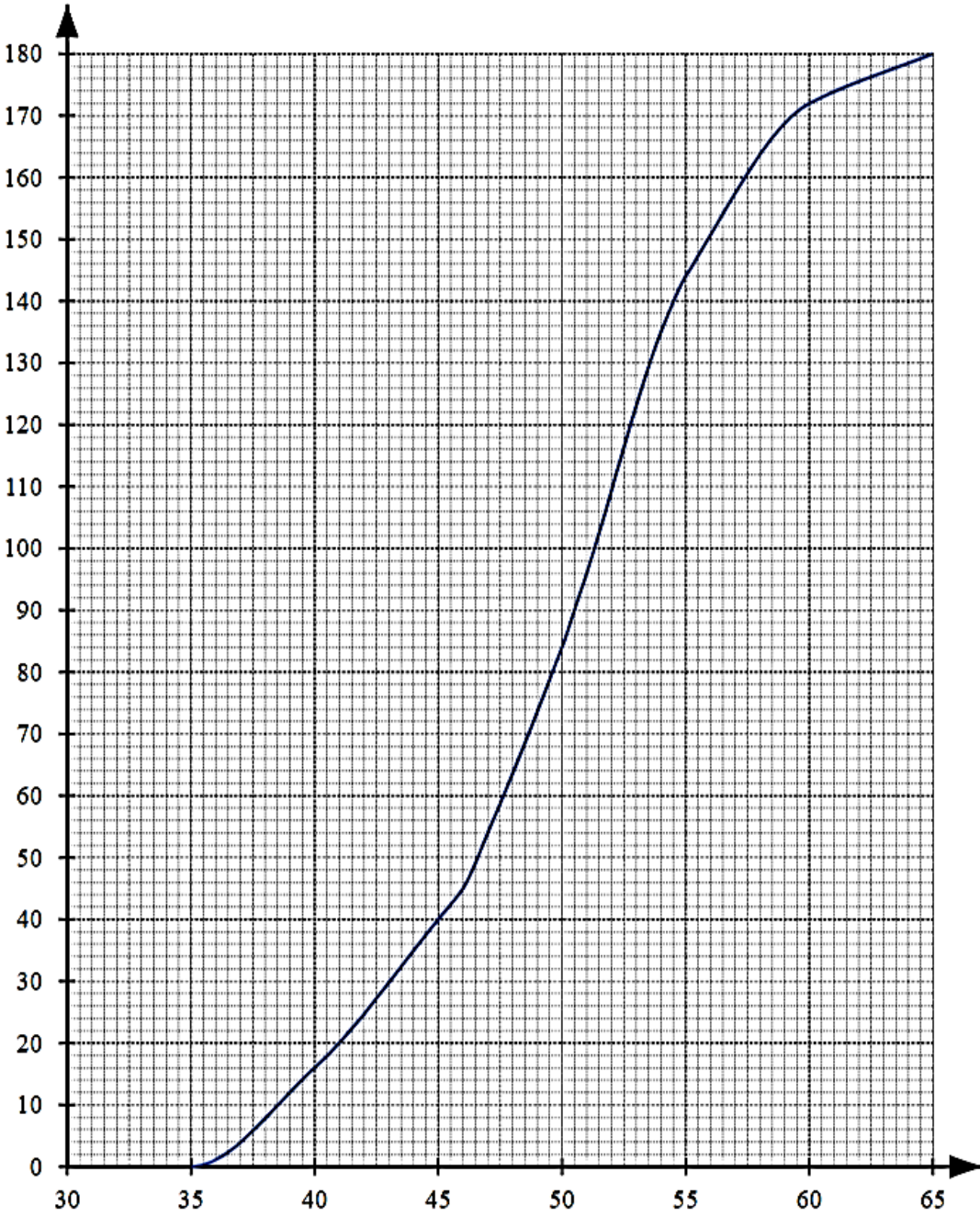
A02		Low	Medium	Low	Medium																			
4	(b) The table below shows the results of a survey in which 60 students were asked whether they played video games every day during the school holidays and whether they had trouble sleeping at night.																							
	<table><tr><td></td><td colspan="2">Played video games</td><td></td></tr><tr><td></td><td>Daily</td><td>Not daily</td><td>Total</td></tr><tr><td>Trouble sleeping</td><td>46</td><td>2</td><td>48</td></tr><tr><td>No trouble sleeping</td><td>8</td><td>4</td><td>12</td></tr><tr><td>Total</td><td>54</td><td>6</td><td>60</td></tr></table>					Played video games				Daily	Not daily	Total	Trouble sleeping	46	2	48	No trouble sleeping	8	4	12	Total	54	6	60
		Played video games																						
		Daily	Not daily	Total																				
	Trouble sleeping	46	2	48																				
	No trouble sleeping	8	4	12																				
Total	54	6	60																					
Two students were selected at random from the whole group of students. Find the probability that																								
(i) at least one student had trouble sleeping at night.																								
[2]																								
Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria																					
$P(\text{at least one student had trouble sleeping})$ $= 1 - P(\text{none had trouble sleeping})$ $= 1 - \frac{12}{60} \times \frac{11}{59}$ $= \frac{284}{295}$ OR $P(\text{at least one student had trouble sleeping})$ $= \frac{12}{60} \times \frac{48}{59} + \frac{48}{60} \times \frac{12}{59} + \frac{48}{60} \times \frac{47}{59}$ $= \frac{284}{295}$ Accept 0.963 (3 s.f.) for Real - World - Context question.		Note there are 3 possible cases here: $P(S'S)$ $+P(SS')$ $+P(SS)$ where S rep the event that the student has trouble sleeping	Apply $P(E) = 1 - P(E')$ and concepts of multiplying probabilities of dependent events (similar to multiplying probabilities along branches of Tree Diagram): $P(AB) = P(A) \times P(A \text{ given } B)$																					
Overall Item Demand		Content	Complexity	Context	Response																			
A02		Medium	Medium	Medium	Medium																			
	(ii) neither of them played video games daily nor had trouble sleeping.				[1]																			
Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria																					
Let A and B rep event that a student played games daily and have trouble sleeping respectively.			Apply $P(AB) = P(A) \times P(A \text{ given } B)$																					

$P(\text{neither played games daily nor had trouble sleeping})$ $= \frac{4}{60} \times \frac{3}{59}$ both from $(A' \cap B')$ $= \frac{1}{295}$ Accept 0.00339 (3 s.f.) for Real - World - Context question.				
Overall Item Demand	Content	Complexity	Context	Response
A02	Medium	Medium	Medium	Taught

5	A little girl has 9 coins in her pocket. She has one 10-cent coin, two 50-cent coins and six \$1 coins. To pay for a drink which costs \$1.50, she randomly takes out a coin from her pocket. If the coin is a 10-cent coin, it is put back into the pocket. Then, she takes out the next coin.			
	(a) Complete the Tree Diagram. <u>1st coin</u> <u>2nd coin</u>			

Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria	
			Compute probability and fill up Tree Diagram	
Overall Item Demand	Content	Complexity	Context	Response
A02	Low	Low	Medium	Low

	(b) Given that the first coin is a 50-cent coin, find the probability that the second coin is a \$1-coin.				[1]
Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria		
$P(\text{2nd coin is \$1 given that the 1st coin is 50-cent})$ $= \frac{3}{4}$ reading from the branch aft "1st coin is 50-cent"		For given event, no need to include probability in calculation.	Identify the conditional probability in the branches of a Tree Diagram.		
Overall Item Demand	Content	Complexity	Context	Response	
A02	Low	Low	Medium	Taught	
	(c) Find the probability that she will have enough to pay for her drink after randomly taking out a coin twice.				[2]
Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria		
$P(\text{enough to pay after two draws})$ $= P(\text{one 50-cent and one \$1 or two \$1})$ $= P(50\text{-cent then \$1}) + P(\$1 \text{ then } 50\text{-cent})$ $\quad\quad\quad + P(\text{two \$1})$ $= \frac{2}{9} \times \frac{3}{4} + \frac{2}{3} \times \frac{1}{4} + \frac{2}{3} \times \frac{5}{8}$ $= \frac{3}{4}$		Some students forgot to consider the case of picking two \$1	Calculate probabilities of combined events using Tree Diagram.		
Overall Item Demand	Content	Complexity	Context	Response	
A02	Medium	Medium	Medium	Taught	

6	On a weekday , 180 cars travelled passed a certain point on a street. The speed of each car was recorded. The cumulative frequency curve below shows the speed, x km/h, and the number of cars whose speed is less than or equal to x km/h.						
<i>Cumulative Frequency</i> 							
(a) Given the following frequency table, calculate an estimate of the mean and standard deviation. [2]							
	Speed (x km/h)	$35 < x \leq 40$	$40 < x \leq 45$	$45 < x \leq 50$	$50 < x \leq 55$	$55 < x \leq 60$	$60 < x \leq 65$
	Frequency	16	24	44	60	28	8

Solutions/Alternative Methods				Re ma rks	Skills/Concept/Succes s Criteria		
Estimated mean $\frac{37.5 \times 16 + 42.5 \times 24 + 47.5 \times 44 + 52.5 \times 60 + 57.5 \times 28 + 62.5 \times 8}{180}$ = 49.833 (5 s.f.) =49.8 km/h (3 s.f.) Standard deviation $\sqrt{\frac{(37.5^2 \times 16 + 42.5^2 \times 24 + 47.5^2 \times 44 + 52.5^2 \times 60 + 57.5^2 \times 28 + 62.5^2 \times 8)}{180} - 49.833^2}$ =6.38 km/h (3 s.f.)					Apply mean formula or calculator to compute mean. (for 1 mark, working is not necessary) Apply S.D formula or calculator to compute S.D. (for 1 mark, working is not necessary)		
Overall Item Demand		Content		Complexity		Context	Response
A01		Low		Low		Low	Low

	(b)	Use the curve to estimate							
		(i)	the median speed,						[1]

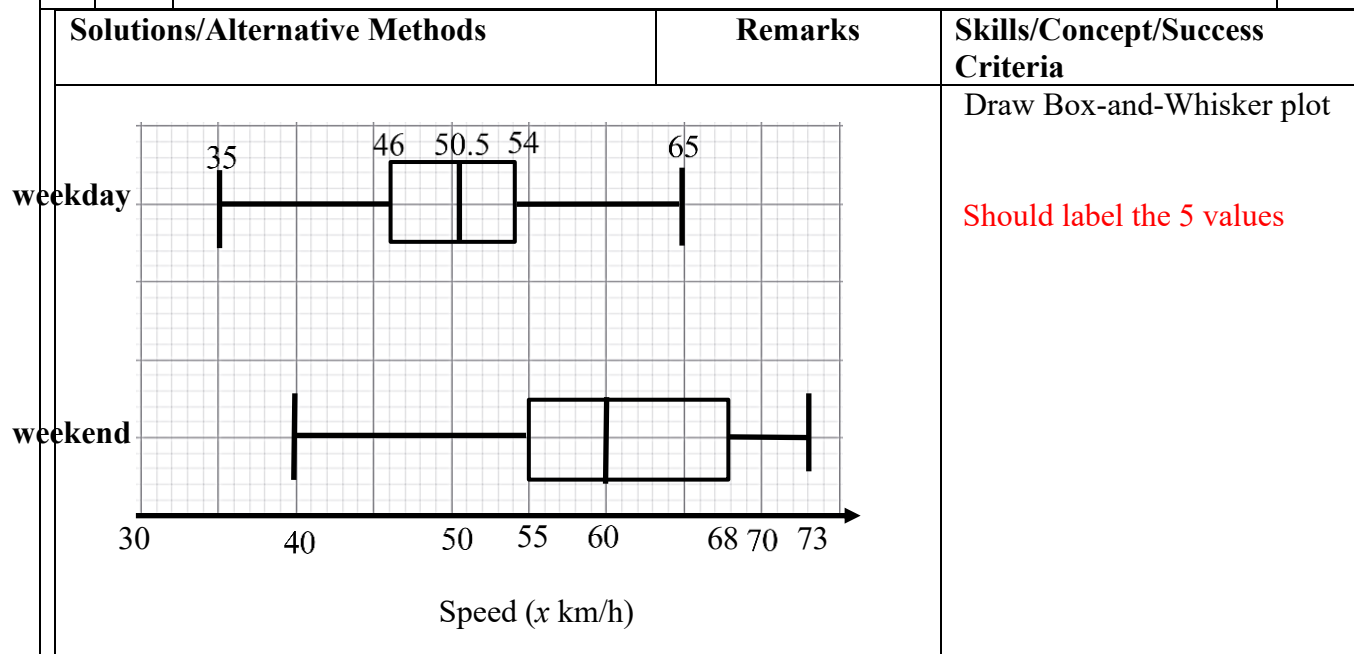
		Solutions/Alternative Methods		Remarks	Skills/Concept/Success Criteria		
		$\frac{n}{2} = \frac{180}{2} = 90$ From the graph, median = 50.5 km/h			Reading median from the CFC		
		Overall Item Demand		Content	Complexity	Context	Response
		A01		Low	Low	Low	Low

		(ii)	the interquartile range.						[1]
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Solutions/Alternative Methods			Remarks		Skills/Concept/Success Criteria		
$\frac{n}{4} = \frac{180}{4} = 45, \quad \frac{3n}{4} = \frac{3 \times 180}{4} = 135$ From the graph, interquartile range = Upper Quartile – Lower Quartile = 54 – 46 = 8 km/h					Finding IQR from the CFC		
Overall Item Demand		Content	Complexity	Context	Response		
A01		Low	Low	Low	Low		

	(c)	The k^{th} percentile speed is 47 km/h. Write down the value of k .				[1]
Solutions/Alternative Methods		Remarks		Skills/Concept/Success Criteria		
From the graph, for speed ≤ 47 km/h, cumulative frequency = 54, i.e., 30% of cars or 54 cars have speed ≤ 47 km/h. $k = \frac{54}{180} \times 100$ $= 30$				Reading 30 th percentile from the CFC		
Overall Item Demand		Content	Complexity	Context	Response	
A02		Medium	Medium	Medium	Medim	
	(d)	A car is picked at random from the 180 cars. The probability that the car travels more than y km/h is $\frac{7}{15}$. Find y .				[2]
Solutions/Alternative Methods		Remarks		Skills/Concept/Success Criteria		
$\frac{7}{15} \times 180 = 84$ cars travelled at $> y$ km / h $180 - 84 = 96$ cars travelled at $\leq y$ km / h From the graph, $y = 51$				Use probability given to find number of cars with speed $> y$ km/h. Deduce number of cars with speed $\leq y$ km/h. Read CFC to find y .		
Overall Item Demand		Content	Complexity	Context	Response	
A02		Medium	Medium	Medium	Taught	

	(c)	On a weekend , another 180 cars travelled passed the same point on the street. The speed of each car was recorded. The box-and-whisker plot below shows the speed of the cars on the weekend . (i) Draw the box-and-whisker plot on the same grid below to illustrate the speeds of 180 cars on the weekday .		[2]



Overall Item Demand	Content	Complexity	Context	Response
A01	Low	Low	Low	Taught

	(ii) Make one comment comparing the speeds of the cars on the weekday and the weekend.	
	<i>Answer</i>	[1]

Solutions/Alternative Methods	Remarks	Skills/Concept/Success Criteria
Medium speed on the weekend = 60 km/h The median speed, 60 km/h, of cars on the weekend is 9.5km/h faster than median speed of cars on the weekday at 50.5 km/h. On average, the speeds of cars are faster on the weekend than on the weekday.	Compare: State tools (median) and numbers Context: Story (who/subject – cars on weekday/weekend, what - speed) Conclusion: a general statement on which group of cars are faster	Compare median and conclude which group has faster speed. Compare: State tools (mean/median/SD/IQR) and numbers Context: Story (who/subject, what) Conclusion: a general statement (without mentioning the tool and numbers) which group performed better (if comparing mean/median) or

				has smaller spread (if comparing SD/IQR)	
Overall Item Demand	Content	Complexity	Context	Response	
A03	Medium	Medium/High	Medium	Medium	

		(iii) If the speed of one car was incorrectly recorded and its speed was updated to 90 km/h in the dataset collected on the weekday , which of the value(s) in the five-number summary of the box-and-whisker plot will change? Based on the change(s), describe briefly what will happen to the lengths of the whiskers and the length of the box.	
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		<i>Answer</i>	[1]
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Solutions/Alternative Methods		Remarks		Skills/Concept/Success Criteria
The maximum value will be changed to 90. Depending on the position of the incorrectly recorded data, the minimum value, lower quartile, medium and upper quartile may or may not change. The upper whisker will become longer while the lower whisker and length of the box may change slightly or remain the same depending on the original ordered position of the incorrectly recorded data.				Able to apply the understanding that LQ, medium and UQ are calculated based on their ordered position, and deduce how an upper outlier affects the components of a five-number summary (min, LQ, medium, UQ, max) and alters the resulting box-and-whisker plot.
Overall Item Demand	Content	Complexity	Context	Response
A03	Medium	Medium	Medium	Unfamiliar

		(iv) Sally says, “On the weekend , the number of cars travelling at up to 55 km/h is greater than the number travelling between 55 and 68 km/h.”	
		Is she correct? Give a reason for your answer.	

		<i>Answer</i>	[1]
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Solutions/Alternative Methods	Remarks	Skills/Concept/Success Criteria
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Sally is wrong. There are about 25% of 180 cars travelling at up to 55km/h. This is less than the number of cars travelling between 55 and 68 km/h, which is about 50% of 180 cars. Accept 45 cars (instead of 25%) vs 90 cars (instead of 50%).		5-number summary in the Box and Whisker plot divide the dataset into four parts, each part consists of 25% of total frequency		Able to apply the understanding that the five-number summary (min, LQ, medium, UQ, max) in the box-and-whisker plot divide the dataset into four parts of 25% each.	
Overall Item Demand	Content	Complexity	Context	Response	
A03	Medium	Medium	Medium	Unfamiliar	

	Q1	Q2	Q3	Q4	Q5	Q6	Actual		Expected	
A01	3		3			6	12	40%	13.5	45%
A02		4		4	4	3	15	50%	12	40%
A03						3	3	10%	4.5	15%