20 NUCLEAR PHYSICS

H2 Physics 9478

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Learning Outcomes

Candidates should be able to:

(a) infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the atomic nucleus.

(b) distinguish between nucleon number (mass number) and proton number (atomic number).

(c) show an understanding that an element can exist in various isotopic forms, each with a different number of neutrons in the nucleus, and use the notation ${}^{A}_{Z}X$ for the representation of nuclides.

(d) show an understanding of the spontaneous and random nature of nuclear decay.

(e) infer the random nature of radioactive decay from the fluctuations in count rate.

(f) show an understanding of the origin and significance of background radiation.

(g) show an understanding of the nature and properties of α, β and γ radiations (knowledge of positron emission is not required).

(h) define the terms activity and decay constant, and recall and solve problems using the equation $A = \lambda N$.

(i) infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_0 \exp(-\lambda t)$ where x could represent activity, number of undecayed particles or received count rate.

(j) define and use half-life as the time taken for a quantity x to reduce to half its initial value.

(k) solve problems using the equation $\lambda = \ln 2 / t_{\frac{1}{2}}$.

(l) discuss qualitatively the applications (e.g. medical and industrial uses) and hazards of radioactivity based on:

- (i) half-life of radioactive materials,
- (ii) penetrating abilities and ionising effects of radioactive emissions.

(m) represent simple nuclear reactions by nuclear equations of the form ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$.

(n) state and apply to problem solving the concept that nucleon number, charge and mass-energy are all conserved in nuclear processes.

(o) show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the (anti)neutrino [knowledge of the antineutrino and the zoo of particles is not required].

(p) show an understanding of the concept of mass defect.

(q) recall and apply the equivalence between energy and mass as represented by $E=mc^2$ to solve problems.

(r) show an understanding of the concept of nuclear binding energy and its relation to mass defect.

(s) sketch the variation of binding energy per nucleon with nucleon number.

(t) explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission.

20.1 The Nuclear Atom

❖ Introduction

Just as the chemical energy liberated in fire can be used for good and evil, the energy in nuclear "fire" can also be put to benevolent or malevolent uses. A slow and controlled release of nuclear energy can be harnessed to sustainably power our megacities safely and efficiently, but the reckless deployment of weapons of mass destruction can obliterate the surface of the Earth many times over and effectively turn the clock back on progress made in civilisation. This topic provides opportunities to discuss a whole host of existential questions.

There is also a strong argument to learn about nuclear physics as part of scientific literacy. We should appreciate that while the fallout from nuclear disaster can be extremely hazardous, life on Earth has coped with an environmental level of background radiation. There are many creative applications that we have found to use nuclear radiation productively. Discussing such socio-scientific issues provides opportunities to situate scientific and technological understanding in the context of humanistic and economic concerns.

In terms of the physics concepts, this topic builds from Electric Fields when discussing Rutherford scattering of α-particles from the atomic nucleus, as well as for understanding the characteristics of nuclear radiation. The rest of the topic is otherwise descriptive or involves data handling and simple mathematical modelling, such as the exponential curve in simple examples of radioactive decay, as well as applying conservation laws (including the famous mass-energy equivalence) in nuclear reactions and processes.

❖ Rutherford α-scattering Experiment

Atoms, the basic building blocks of matter, were once thought to be the smallest indivisible particle. However, with his discovery of the electrons in 1897, J.J. Thomson concluded that electrons are part of an atom because his experiments consistently produced electrons regardless of the metal used. He further postulated that these very light and negatively charged electrons were distributed throughout a uniform sea of positive charges in an atom, much like the way that plums are evenly distributed in a pudding. He thus named this model as the 'plum-pudding' model.

In 1909, under the direction of Ernest Rutherford, Hans Geiger and Ernest Marsden investigated the structure of an atom. As shown in Fig. 20.1, a beam of α-particles (helium nuclei) having an energy of 7.7 MeV emitted by the decay of radium is directed towards a thin gold foil. Deflected α-particles were detected as flashes on the fluorescent screen. To minimise the scattering of the α-particles by air molecules, their experimental set up was enclosed in vacuum.

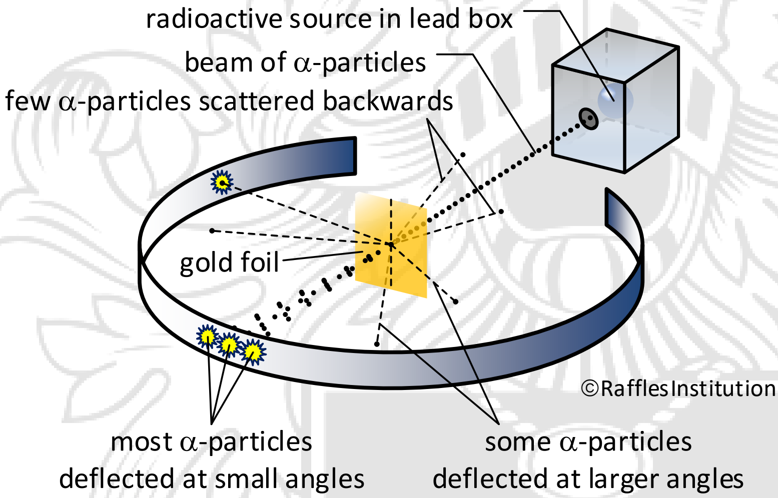


Fig. 20.1 A schematic of the Rutherford α-particle scattering experiment. Almost all of the α-particle are deflected at small angles, but a small number are deflected at large angles.

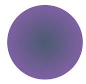
Based on the 'plum-pudding' model of the atom, they had expected the α-particles to be deflected by only a very small angle ($\sim 1°$ at most). Much to their surprise, a **very small number** of α-particles (1 in about 8000) were deflected at large angles ($> 90°$) and a few were even scattered backwards ($\sim 180°$).

To explain the large deflections of some of the α-particles, Rutherford theorized in 1911 that all the **positive charge** of the atom and **most of the mass** was concentrated in a tiny space inside the atom called the **nucleus** and the electrons orbit around the nucleus (similar to planets revolving around the sun).

When an α-particle (having a charge of $+2e$) comes close to a gold nucleus (having a charge of $+79e$), it experiences a strong Coulombic repulsion and suffers a large deflection due to the large mass of the nucleus.

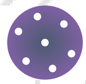
[Modern variant of Rutherford's experiment]

Models of the Atom



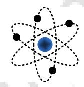
Pre-1897

Indivisible Atom
An atom was thought to be the smallest component of matter and could not be further broken down into smaller constituents.



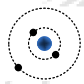
1897

Plum Pudding Model
Atoms compose of the negatively charged electrons evenly distributed within a cloud of positive charges.



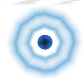
1911

Planetary Model
Atoms consist of negatively charged electrons orbiting around a very small, dense, positively charged nucleus.



1913

Bohr Model
Similar to Rutherford's model but electrons are in stationary states and hence do not radiate EM energy.



Post-1926

Quantum Model
Electrons are distributed in region described by the electron density cloud around a positively charged nucleus. The greatest probability of locating the electron is at the densest region.

The amount of deflection depends on how close the path of the incoming α-particle is to the gold nucleus. This distance is called the impact parameter *b* (Fig. 20.2).

If the impact parameter is large, (for instance, α-particle 5) then the repulsion between the α-particle and the gold nucleus is weaker and the α-particle experiences only a **small deflection**. For small *b* (particles 1, 2 & 3), the repulsion gets stronger and the α-particle experiences a **larger deflection**, as shown in Fig. 20.2.

The impact parameter *b* (not in syllabus) is the perpendicular distance between the path of the projectile and the centre of the field (the target).

An α-particle colliding head-on (zero impact parameter) is deflected by 180°.

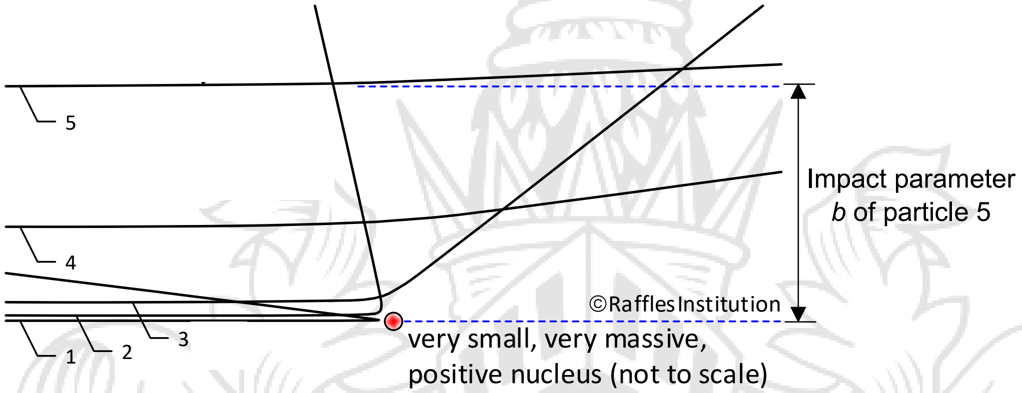


Fig. 20.2 (not to scale) The amount of deflection experienced by α-particles depends on the impact parameter. The smaller the impact parameter, the greater the deflection angle.

When a large number of α-particles is incident on a gold foil, most of the α-particles pass through the foil with little deflection (Fig. 20.3). On the other hand, a very small proportion of the α-particles are deflected at large angles, suggesting that the gold nucleus is not just **extremely small** but also **very massive**. Rutherford estimated the size of the nucleus to be in the order of a few femto-metre.

α-particles passing through an atom based on the plum-pudding model are also expected to experience minimal deflection.

1 femto-metre equals 10^{-15} m.

The gold foil used in Rutherford α-scattering experiment has a thickness of 8.6×10^{-6} cm.

Given that the separation between gold atoms is 2.88 Å, the gold foil is about 300 atoms thick.

For comparison, a strand of human hair is up to 10000 times thicker.

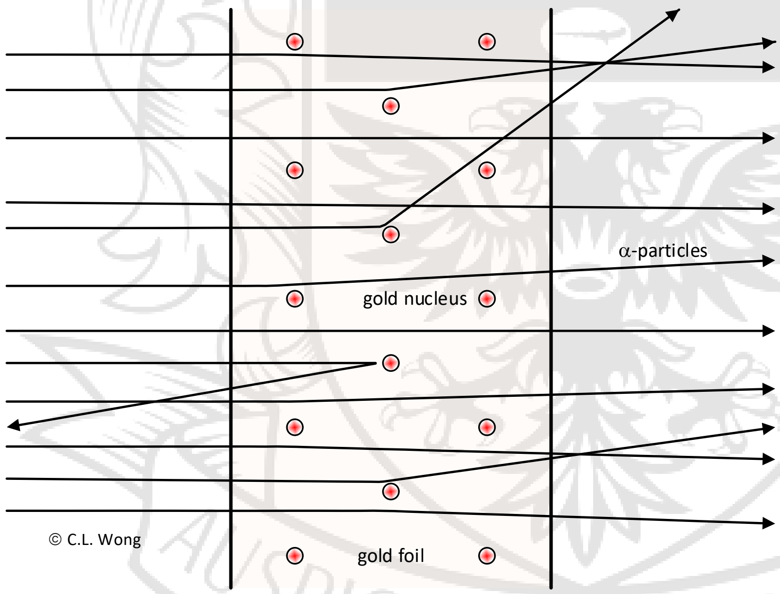


Fig. 20.3 (not to scale) Most α-particles did not pass close to the gold nuclei and hence experienced little deflection.

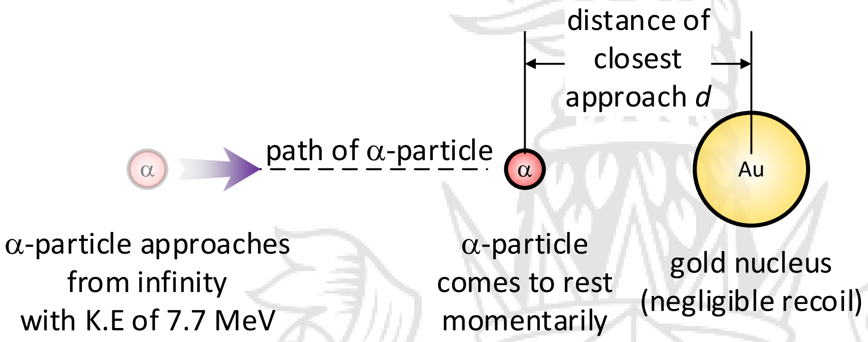
Example 1: Distance of Closest Approach

An α-particle ($+2e$ charge) of energy 7.7 MeV is directed head-on (zero impact parameter) towards a gold nucleus ($+79e$ charge).

$1\text{ MeV} = 1.60 \times 10^{-13}$ J

Calculate the distance of closest approach for the α-particle.

[Solution]



Since the gold nucleus is much more massive than the α-particle, the recoil of the gold nucleus is negligible (approximate masses of α-particle and gold nucleus are 4 *u* and 197 *u*, respectively).

As the α-particle approaches the nucleus, its kinetic energy is gradually converted to electrostatic potential energy. At distance of closest approach *d*, the α-particle is momentarily at rest.

The size of the nucleus ranges from 1.8 fm to 15 fm while the size of an atom ranges from 30 pm to 300 pm.

An atom is approximately 20 000 times larger than its nucleus. If the nucleus is the size of a marble (1 cm), then the atom will be the size of two football fields (200 m).

❖ The Nucleus

After the discovery of the nucleus of an atom, Rutherford turned his attention to the scattering of $\alpha-$particles on lighter nuclei. By 1917, Rutherford had succeeded in producing the hydrogen nuclei by bombarding hydrogen gas with $\alpha-$particles. When he repeated the experiment with pure nitrogen gas, hydrogen nuclei were also detected. He concluded that the hydrogen nucleus (hydrogen being the simplest and lightest element), is the building block of the nucleus. Rutherford named this particle the **proton**.

Discovery of the neutron

However, the disparity between the atomic (proton) number of an atom and its atomic mass led Rutherford to hypothesize the existence of a neutral particle in the nucleus. In 1931, physicists detected an extremely penetrating radiation produced when very energetic α-particles bombarded the nuclei of beryllium, boron and lithium. This radiation was initially thought to be gamma radiation even though it was more penetrating than any gamma radiation ever known.

In 1932, James Chadwick, a former student of Rutherford, performed a series of experiments that conclusively dismissed the gamma radiation hypothesis. Instead, he found that the new radiation consisted of neutral particles which had approximately the same mass as the protons. Chadwick named this particle the **neutron**.

It is now an established fact that the nucleus of an atom consists of protons and neutrons, collectively known as **nucleons**. Table 20.1 shows the properties of the particles that make up an atom.

Properties	**proton**	**neutron**	**electron**
Charge / 10^{-19} C	1.602177	0	–1.602177
Rest mass / kg	1.672622×10^{-27}	1.674927×10^{-27}	9.109383×10^{-31}
Rest energy / MeV	938.2720	939.5654	0.5109989

Table 20.1 Properties of proton, neutron and electron

In nuclear physics, the masses of subatomic particles are typically given to higher precision.

❖ Nuclear Force

How is it possible to pack protons into such a tiny space as the nucleus when they repel each other so strongly when they are near? This Coulombic repulsion would tear the nucleus apart. So, physicists hypothesized that there must be an even stronger attractive force between the nucleons. Indeed, it is this force, called the **nuclear force,** that binds all the protons and neutrons together in the nucleus. Note that the nuclear force is:

- **the same** between a proton and a proton, a proton and a neutron and a neutron and another neutron.
- **extremely short range**, and acts over 1 to 3 fm, or, not beyond the closest nucleon.

The nuclear force is a residual effect of the strong interaction (not in syllabus). The strong interaction is one of the four fundamental interactions of nature, the others being electromagnetism, the weak interaction and gravitation.

Example 2: Repulsive Force between Protons

Calculate the Coulomb repulsive force between two protons separated by a distance of 1 fm.

[Solution]

The magnitude of the strong nuclear force is approximately 100 times that of the Coulomb repulsive force.

Example 3: Density of Nucleus

Studies have shown that the nucleus of an atom is approximately spherical with constant density. The nuclear radius is given by

$$R = R_0 A^{1/3}$$

where R_0 has a value ranging from 1.2 to 1.3 fm.

Estimate the density of the nucleus.

[Solution]

Consider a nucleus of mass number *A*. The mass of this nucleus is approximately equal to $m_p A$, where m_p is the mass of proton.

Thus, the density of nucleus is

❖ Standard Nuclear Notation

$$^{12}_{6}\mathrm{C}$$

mass number *A*
The number of nucleons in the nucleus (a.k.a. *nucleon number*)

The nucleus contains ***Z*** protons and ***N*** neutrons.
$A = N + Z$

atomic number *Z*
The number of protons in the nucleus (a.k.a. *proton number*)

chemical symbol
This is the symbol for the element *carbon*

Mass number *A* is not the same as atomic mass. Mass number is an integer and has no unit. It is simply the number of nucleons in the nucleus. Atomic mass has unit of kg or *u*.

$^{12}_{6}\mathrm{C}$ 98.9% $^{13}_{6}\mathrm{C}$ 1.1% $^{14}_{6}\mathrm{C}$ trace

isotopes
Nuclides of the same element
Same *Z*, different *N*

❖ Nuclides and Isotopes

Definition of Nuclide

A nuclide is the atomic species with a specific mass number *A* and proton number *Z* (e.g., $^{56}_{26}\mathrm{Fe}$)

Definition of Isotope

Isotopes are nuclides that have the same number of protons but different number of neutrons.

E.g. $^{12}_{6}\mathrm{C}$ and $^{14}_{6}\mathrm{C}$ are isotopes of carbon, also referred to as carbon-12 and carbon-14. Isotopes of the same element exhibit identical chemical properties and exist in different proportions in nature, e.g. $^{16}_{8}\mathrm{O}$ (99.8%), $^{17}_{8}\mathrm{O}$ (0.037%) and $^{18}_{8}\mathrm{O}$ (0.163%).

In the case of hydrogen, its isotopes are given specific names: hydrogen ($^{1}_{1}\mathrm{H}$), deuterium ($^{2}_{1}\mathrm{H}$) and tritium ($^{3}_{1}\mathrm{H}$).

A **radioisotope** is an isotope that is radioactive, e.g. Iodine-131. Radioisotopes can be naturally occurring or produced artificially.

❖ Unified Atomic Mass Unit (u)

Definition of unified atomic mass unit

One atomic mass unit (1 *u*) is equal to one-twelfth of the mass of a carbon-12 atom.

The mass of carbon-12 atom in the ground state is exactly 12 *u* while the mass of its nucleus is 11.996706 *u*.

The atomic mass unit *u* is the standard unit used to represent mass on an atomic scale.

1 mole of carbon-12 contains 6.022×10^{23} atoms with a mass of 0.012 kg. So, in terms of kg, the unified atomic mass unit *u* is

$$1\,u = \frac{1}{12} \times \frac{0.012}{6.022 \times 10^{23}} = 1.660539 \times 10^{-27} \text{ kg}$$

$u = 1.66 \times 10^{-27}$ kg can be found in the data page during examinations.

Example 4: "Missing Mass"

Calculate the total mass of 6 protons, 6 neutrons and 6 electrons in terms of *u*.

[Solution]

Mass of proton:

Mass of neutron:

Mass of electron:

Total mass of 6 protons, 6 neutrons and 6 electrons:

Why is the mass of the carbon-12 atom (which is 12 u) less than the total mass of its constituents?

We will answer this when we study the concept of mass defect.

20.2 Radioactive Decay

❖ Stability of Nuclei

There are at least 3300 known nuclides (both natural and artificial), but only about 250 of them are stable (they are indicated as ■ in Fig. 20.4). An atom with an unstable nucleus is known as a **radioactive nuclide (or radionuclide)**.

Nuclide is the atomic species with a specific mass number *A* and proton number *Z*.

Lead (Z = 82) is the heaviest element to have stable isotopes.

Nuclides with even *Z* and even *N* are most stable while nuclides with odd *Z* and odd *N* are least stable due to pairing effects.

Data provided by IAEA

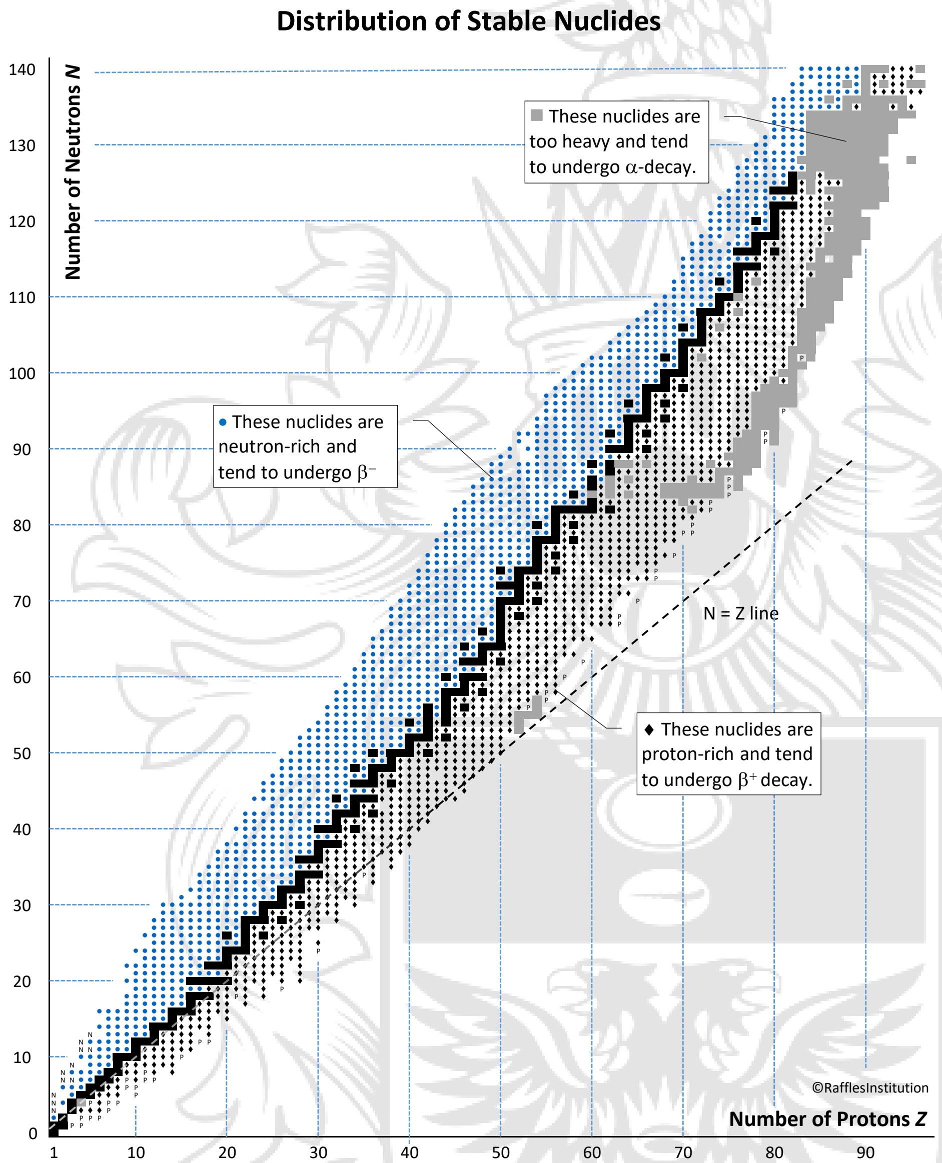


Fig. 20.4 Distribution of nuclides from ${}^{1}_{1}H$ to ${}^{236}_{96}Cm$ and the main decay mode of radionuclides.

❖ Radioactivity

A radioactive nuclide will spontaneously decay to a more stable nuclide by emitting some forms of ionising radiation. This decay is a nuclear process known as **radioactivity** and the atom is said to have undergone **radioactive decay**.

Radioactive decay is both **spontaneous** and **random.** It is spontaneous because the probability of decay is unaffected by any external factors such as temperature, pressure or chemical composition. It is random because it is impossible to predict which nucleus will decay next or when the next decay will occur.

This random nature of radioactive decay results in the **fluctuations in the count rate** measured by a detector (refer to appendix for types of detectors).

However, the probability that a given atom or nucleus will decay is constant over time. Hence a mathematical description of the decay rate of a large number of identical radioactive atoms can be formulated.

Definition of Radioactive Decay

Radioactivity or **radioactive decay** is the process whereby a radionuclide is transformed into a more stable nuclide by emitting some forms of ionising radiation.

Meaning of radioactive decay being spontaneous

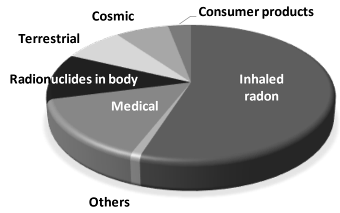
Radioactive decay is **spontaneous** because the probability of decay of a nucleus is unaffected by any external factors such as temperature, pressure or chemical composition.

Meaning of radioactive decay being random

Radioactive decay is **random** as it is impossible to predict which nucleus will decay next or when it will happen. This results in fluctuations in the count rate measured by a detector.

❖ Background Radiation

Background radiation is the ionising radiation in our environment that humans are constantly exposed to. The sources of background radiation can be natural and artificial. A list of sources is given in Table 20.2.



Contributions to background radiation from various sources

Natural sources	Artificial sources
Air, water & food	Medical imaging
Soil & rock (terrestrial)	Consumer products/materials
Cosmic rays (from outer space)	Nuclear accidents
Internal	

Table 20.2 Sources of background radiation

The **Earth** is a **natural** source of background radiation. Radionuclides such as thorium, uranium and radium exist naturally in soil and rock. The decay of these radionuclides produces **radon** – a radioactive gas – which readily seeps out from the ground into underground spaces, buildings and the atmosphere. Radon is by far the biggest source of natural background radiation. Other radioactive sources include underground drinking water, bananas, nuts and many other natural materials.

Some of these radionuclides have extremely long half-life (billions of years) and their activity has largely remained constant in the history of mankind.

Cosmic rays from outer space consist of very energetic positively charged ions ranging from protons to iron and even heavier nuclei originating from outside our solar system.

Food that are radioactive mainly contain potassium and radium.

Radioactive nuclides such as potassium-40 and carbon-14 can also be found within **our body** so we are constantly being exposed to ionising radiations from within. This is known as internal radiation.

Our exposure to the various natural sources of ionising radiations depends on our location, altitude and even the time of the day.

The most significant source of **artificial** background radiation comes from **medical imaging** – x-rays, computed tomography (CT), positron emission tomography (PET) and single-photon emission computed tomography (SPECT). Radioactive nuclides such as fluorine-18, iodine-123 and technetium-99m are often used as a *tracer*.

Consumer products such as tobacco are radioactive. Heavy smokers are constantly dosed with significant levels of radiation to localised spots in the lungs. Construction materials (concrete, stones and granite), petroleum and fuels are also radioactive.

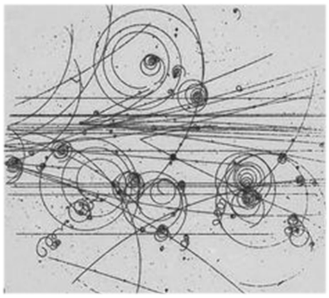
Polonium-210 is found in tobacco products.

The two most infamous **nuclear accidents** occurred in Fukushima, Japan (2011) and Chernobyl, Ukraine (1986) which resulted in significant loss of life and substantial contamination to the immediate vicinity and into the atmosphere. In the Fukushima disaster, millions of litres of contaminated radioactive water were discharged into the Pacific Ocean although the radioactivity level was still deemed to be safe.

Japan is not the only country dumping radioactive waste into the oceans. Many countries with nuclear capabilities have been dumping tons of waste since WW2.

❖ Types of Radiations

Early researchers detected **three** types of **ionising radiation** in the study of radioactive emission. These radiations are given the names **alpha**, **beta** and **gamma** radiations (in increasing order of their **penetrating power**). In a cloud chamber (see Appendix), these radiations leave behind distinguishable tracks that can be photographed and studied.



Tracks left by ionising particles in a cloud chamber.

- Alpha radiations produce tracks that are **short** and **thick** (**strong** ionising power) and of **similar length** (same kinetic energies)
- Beta radiations produce tracks that are **long** and **thin** (**moderate** ionising power) and of **varying lengths** (a range of kinetic energies).
- Gamma radiations (**low** ionising power) leave behind a cloud of wiggly, short and thin tracks due to ionised electrons.

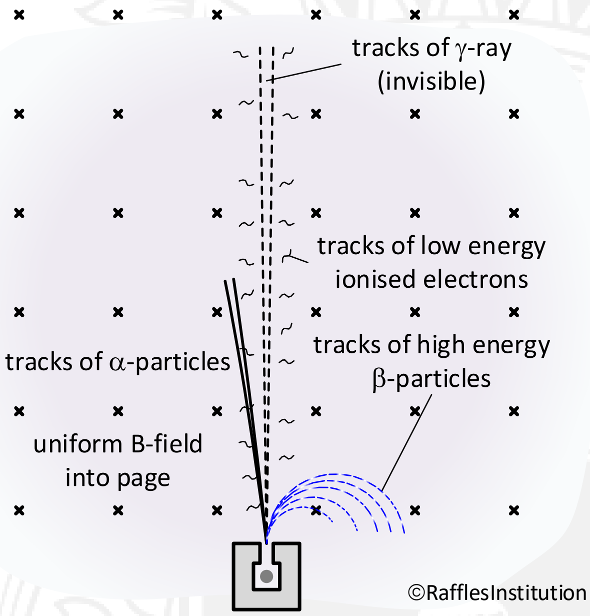


Fig. 20.5 Deflection of α, β and γ radiations in a magnetic field

When a magnetic field is applied, alpha radiations and beta radiations are deflected in opposite directions. Based on the direction of the magnetic field and the deflection of the radiations, we can deduce that alpha radiations are positive while beta radiations are negative. On the other hand, gamma radiations are not deflected so it must be neutral.

By measuring the specific charge (q/m) of alpha and beta radiations, it was determined that alpha radiations are helium nuclei (^4_2He) and beta radiations are electrons ($^{0}_{-1}\text{e}$). Further investigations revealed that gamma radiations are electromagnetic (EM) radiations.

In 1914, Rutherford observed the diffraction of gamma radiations from crystal surfaces and measured their wavelengths, proving that gamma radiations are EM radiations.

❖ Alpha Decay

Alpha decay refers to the spontaneous emission of an α-particle (helium-4 nucleus) by a radioactive nucleus. The nuclear equation is:

$$^{A}_{Z}X \rightarrow \, ^{A-4}_{Z-2}Y + \, ^{4}_{2}He$$

where X and Y are the chemical symbols for parent and daughter nuclides respectively.

The typical kinetic energy of the α-particles ranges from 4-9 MeV. However, **all** α-particles emitted by a particular radionuclide have the **same kinetic energy** (Example 20.12).

As there are only two particles produced after the decay, they will move with equal and opposite momenta if the parent nucleus was at rest initially, as linear momentum is conserved.

Alpha decay is common in nuclei with too many nucleons (mass number $A > 209$). The emission of an α-particle decreases the mass number by 4 and results in a more stable daughter nucleus.

The α-particle eventually acquires two electrons to become a helium atom.

The daughter nucleus may still be radioactive.

❖ Beta Decay

Beta decay refers to the spontaneous emission of a β-particle (electron $^{0}_{-1}e$ for β^- decay or positron $^{0}_{+1}e$ for β^+ decay), by a radioactive nucleus. The nuclear equations are as follows:

$$^{A}_{Z}X \rightarrow \, ^{A}_{Z+1}Y + \, ^{0}_{-1}e + \, ^{0}_{0}\bar{\nu} \qquad (\beta^-)$$

$$^{A}_{Z}X \rightarrow \, ^{A}_{Z-1}Y + \, ^{0}_{1}e + \, ^{0}_{0}\nu \quad (\beta^+)$$

Beta particles are created in the nucleus.

β^- decay is common in nuclei with **too many neutrons**. Electron emission from the nucleus converts one of the neutrons into a proton:

$$^{1}_{0}n \rightarrow \, ^{1}_{1}p + \, ^{0}_{-1}e + \, ^{0}_{0}\bar{\nu}$$

Assume beta decay to be β^- decay, unless stated otherwise.

β^+ decay is common in nuclei with **too many protons**. Positron emission (not required in syllabus) from the nucleus converts one of the protons into a neutron:

$$^{1}_{1}p \rightarrow \, ^{1}_{0}n + \, ^{0}_{1}e + \, ^{0}_{0}\nu$$

β^+ decay decay is not in H2 Physics syllabus.

A free proton does not decay into a neutron. Can you explain why?

❖ **Gamma Decay**

When a nucleus has just undergone an alpha or a beta decay, the daughter nucleus may be left in an excited state (this means that it possesses too much energy and will emit the excess energy subsequently).

$${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

where * indicates that a nucleus is in an excited state.

As with an excited atom that de-excites back to its ground state by emitting a photon, an excited nucleus will also de-excite to its ground state by emitting a photon. However, because the emitted photon in this case is much more energetic, it is a γ-ray photon whereas in the case of excited atoms, the photons emitted are less energetic. They can be x-ray photons, uv light photons or just visible light photons.

Example 5: Radioactive Decay Series

Uranium-238 (${}^{238}_{92}U$) undergoes a series of decays to a final stable nuclide. The particles emitted in successive decays are

$\alpha \quad \beta \quad \beta \quad \alpha \quad \alpha$

Which nuclide is not produced during this series of decay?

A ${}^{228}_{88}Ra$ **B** ${}^{230}_{90}Th$ **C** ${}^{234}_{91}Pa$ **D** ${}^{234}_{92}U$

[Solution]

Knowledge of chemical symbol of the nuclides is not required.

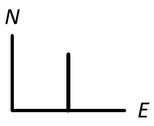
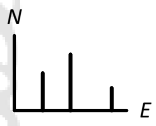
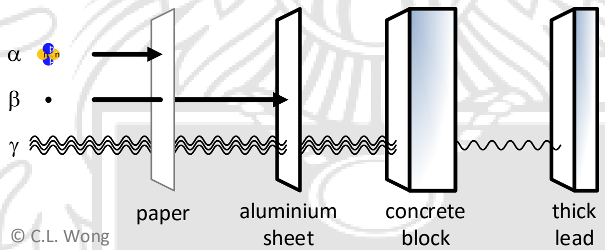
Properties	**alpha**	**beta (β^-)**	**gamma**
Nature	helium nucleus	electron	γ-ray photon
Charge	$+2e$	$-e$	0
Rest Mass/ kg	6.644657×10^{-27}	9.109383×10^{-31}	0
Energy	 well-defined 4 – 9 MeV	 range from 0.01 – 5 MeV	 discrete energies (keV – MeV)
Speed	$0.05c$	up to $0.9c$	c
Ionising power	strong	moderate	very weak
Range in air	few centimetres	few metres	very long
Penetrating power	low	medium	high
Stopped by			

Table 20.3 Properties of Alpha, Beta & Gamma Radiations

Ionising power is the ability of the radiation to knock out electrons from atoms.

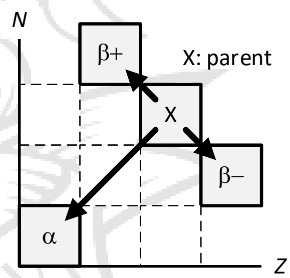
Penetrating power is the ability of the radiation to pass through a material before being completely absorbed. A radiation with high ionising power has low penetration power.

When particles are absorbed by a material, their energies are converted into **internal energy** of the material.

Decay	Transformation	Example	Reason
α-decay	${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$	${}^{238}_{92}U \to {}^{234}_{90}Th + {}^{4}_{2}He$	Nucleus too large
β^- decay	${}^{A}_{Z}X \to {}_{Z+1}{}^{A}Y + {}^{0}_{-1}e + \bar{\nu}$	${}^{14}_{6}C \to {}^{14}_{7}N + {}^{0}_{-1}e + \bar{\nu}$	Nucleus has too many neutrons
β^+ decay	${}^{A}_{Z}X \to {}_{Z-1}{}^{A}Y + {}^{0}_{1}e + \nu$	${}^{64}_{29}Cu \to {}^{64}_{28}Ni + {}^{0}_{1}e +$	Nucleus has too many protons
γ-decay	${}^{A}_{Z}X^* \to {}^{A}_{Z}X + \gamma$	${}^{87}_{38}Sr^* \to {}^{87}_{38}Sr + \gamma$	Nucleus has excess energy after a decay

Table 20.4 Summary of Alpha, Beta & Gamma Decays

Transition diagram



❖ Law of Radioactive Decay

The probabilistic nature of radioactive decay makes it all but impossible to predict which nucleus will decay at a given time. However, since all nuclei of a radioactive nuclide has an equal probability of decaying, it is possible to determine the proportion of the nuclei that will decay in any time interval.

We define the **decay constant** λ as the **probability per unit time** of the decay of a nucleus. According to probability theory, for a sample containing N radioactive nuclei, the **number of radioactive decays per unit time** will be λN. So, the variation with time of the number of undecayed radioactive nuclei dN/dt is given by

$$\boxed{\frac{dN}{dt} = -\lambda N}$$

The negative sign indicates that dN/dt is negative since N, the number of radioactive nuclei, decreases with time.

We can rewrite the equation above in the form

$$\frac{dN}{N} = -\lambda dt$$

which, upon integration, gives

$$\boxed{N = N_0 e^{-\lambda t}}$$

where N_0 represents the number of undecayed radioactive nuclei at time $t = 0$. This is the **law of radioactive decay**.

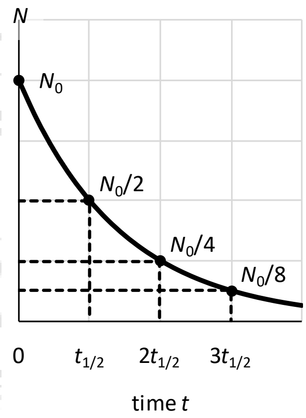


Fig. 20.6 Exponential decrease in the number of radioactive nuclei.

❖ Activity

The **activity** A of a radioactive source is the **number of radioactive decays per unit time** or **rate of radioactive decay** in the source.

$$A = -\frac{dN}{dt} = \lambda N$$

Since $N = N_0 e^{-\lambda t}$,

$$A = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$$

where $A_0 = \lambda N_0$ is the activity at time $t = 0$.

The SI unit of activity is the becquerel (Bq) where

1 becquerel = 1 decay s^{-1}

Activity is high when the sample contains a large number of radioactive nuclei N or the nuclei has a high decay constant λ (high probability of decay).

Both N and A decay exponentially with time.

❖ Corollaries

If a sample contains N number of a radioactive nuclei, then the amount of radioactive substance is

$$n = \frac{N}{N_A}$$

and the mass of this radioactive substance is

$$m = N \times m_{\text{atom}} = n \times M_{\text{molar}}$$

Molar mass M_{molar} is the mass of 1 mole of the radioactive nuclide.

Therefore, the equation $N = N_0 e^{-\lambda t}$ can be written in the following forms:

$$n = n_0 e^{-\lambda t}$$

$$m = m_0 e^{-\lambda t}$$

where n_0 and m_0 are the initial amount and the initial mass of radioactive substance.

According to Learning Outcome **(i)**, we can use the equation

$$\boxed{x = x_0 e^{-\lambda t}}$$

where x can represent the activity A, the number of undecayed nuclei N, the count rate C, the number of moles n of radioactive substance or the mass m of undecayed radioactive substance.

❖ Half-life

The **half-life** $t_{1/2}$ of a radioactive nuclide is the time taken for the **number of undecayed nuclei** to be **reduced to half its original number**. Suppose the initial number of radioactive nuclei is N_0, then after one half-life, only half of the original nuclei remain. So

$$\tfrac{1}{2} N_0 = N_0 e^{-\lambda t_{1/2}}$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

If $t_{1/2}$ is the time for half the number of radioactive nuclei to decay, does it imply that all radioactive nuclei would decay in two half-lives?

The answer is **no**. In first half-life, one-half of the original nuclei will decay and $\tfrac{1}{2} N_0$ undecayed radioactive nuclei are left. After another half-life, half of those remaining will decay and $\tfrac{1}{4} N_0$ are left. So, after n half-lives, the number of radioactive nuclei left is

$$N = N_0 \left(\tfrac{1}{2}\right)^n = N_0 \left(\tfrac{1}{2}\right)^{t/t_{1/2}}$$

where t is the time elapsed and n can be any real number.

Compare this equation with $N = N_0 e^{-\lambda t}$. Can you show that $\left(\tfrac{1}{2}\right)^{t/t_{1/2}} = e^{-\lambda t}$?

Activity

The **activity** of a radioactive source is the number of radioactive decays per unit time in the source.

Decay Constant

The **decay constant** is the probability per unit time of the decay of a nucleus.

Half-life

Half-life $t_{1/2}$ of a radioactive nuclide is the time taken for the number of undecayed nuclei to be reduced to half its original number.

Example 6: Rate of Energy Released

The decay constant of uranium-238 ($^{238}_{92}\text{U}$) is 1.55×10^{-10} yr^{-1}.

Given that the energy released per alpha decay of $^{238}_{92}\text{U}$ is 4.28 MeV, determine the rate at which energy is released in watts when 1.00 g of $^{238}_{92}\text{U}$ undergo radioactive decay.

See Example 20.12.

[Solution]

Use $m_{\text{U-238 atom}} \approx 238\ u$ when no data is given.

The activity of $^{238}_{92}\text{U}$ remains constant for an appreciable amount of time due to its very long half-life.

$1\text{ MeV} = 1.60\times10^{-13}\text{ J}$

Example 7: Half-life

At time $t = 0$, a radioactive sample contains 3.50 μg of pure carbon-11 ($^{11}_{6}C$), which has a half-life of 20.4 min. Determine

(a) N_0, the initial number of nuclei in the sample,

(b) the activity A of the sample after 8.00 h.

[Solution]

$3.50\ \mu g = 3.50 \times 10^{-9}$ kg.

$m_{\text{C-11 atom}} \approx 11\ u$

Recall: $\lambda = \dfrac{\ln 2}{t_{1/2}}$

❖ **Carbon Dating**

Carbon dating is a technique commonly used to determine the age of organic archaeological sample. Nuclear reactions due to cosmic radiations in the upper atmosphere create radioactive carbon-14 ($^{14}_{6}C$) isotope.

The ratio of carbon-14 to carbon-12 in carbon dioxide molecules in our atmosphere has a constant value of 1.30×10^{-12}. The carbon atoms in all living organism have this same ratio because all organisms continuously exchange carbon dioxide with their surroundings. When an organism dies, however, it no longer absorbs carbon-14 from the atmosphere and so this ratio decreases as carbon-14 decays with a half-life of 5730 yr.

This means that there is one carbon-14 atom to 7.69×10^{11} carbon-12 atoms.

Using this technique, scientists have been able to identify samples of wood, charcoal, bone and shell between 1000 to 25000 years old. The limit of carbon dating is approximately 60000 years.

Example 8: Carbon Dating

A living specimen contains one atom of carbon-14 ($^{14}_{6}C$; $t_{1/2} = 5730$ yr) for every 7.69×10^{11} stable carbon-12 ($^{12}_{6}C$). An archaeological sample of wood contains 21.0 mg of carbon. The activity of the sample is measured to be 120 day^{-1}. Determine the age of the sample of wood.

[Solution]

$m_{\text{C-12 atom}} = 12\ u$. We use mass of carbon-12 to compute *N* because ratio of carbon-14 to carbon 12 is extremely minuscule.

We should compare the activity of the same mass of carbon in a living specimen and in an archaeological sample.

❖ **Correcting for Background Count Rate**

In any experiment on radioactivity, the measured count rates should be corrected for the background count rate since background radiation is always present. The count rate due to the radioactive nuclide alone, also known as the true count rate, is the difference between the observed count rate and the background count rate.

$$\text{true count} = \text{observed count} - \text{background count}$$

❖ Applications and Hazards of Radioactivity

Radioactive substances emit radiation that can be used in medical or industrial applications. They may also damage living cells due to their ability to penetrate skin and do ionising damage to cells. The penetrating and ionising ability of alpha, beta and gamma radiation can be found in Table 20.3 on pg. 16.

Radioactive Tracers in Medical Applications

A small amount of radioactive iodine-131 (half-life of 8 days, emits both beta and gamma radiation) can be taken by a patient to evaluate thyroid function. The amount of iodine absorbed by the thyroid gland can be determined from the radiation intensity measured around the neck area. The gamma rays can be detected externally due to its high penetrating power, allowing imaging to confirm the tracer's location.

However, technetium-99m (half-life of 6 hours, emits only gamma radiation) is more commonly used today for diagnostic imaging because iodine-131 can damage tissue due to its beta radiation being more ionising than gamma radiation. In addition, the much shorter half-life of technetium-99m means that the patient will be subjected to much lower radiation dosage.

Radioactive tracers can also be used in emergencies to locate internal haemorrhages. Radioactive chromium-51, which is taken up by red blood cells, accumulates at the site of bleeding together with the leaked blood, and the location of the increased radiation intensity can be detected.

Irradiation of Food and Medical Equipment

Most bacteria, worms and insects are easily destroyed by exposure to gamma radiation from radioactive cobalt. The process is effective in destroying harmful bacteria and worms in food like meat, fruits and vegetables. Medical equipment while in their protective covering can also be sterilised this way. Bone, cartilage and skin used for grafting is also irradiated to reduce the chance of infection.

Cobalt-60, with a half-life of 5.27 years, is typically chosen to emit gamma rays of high penetrating power to penetrate the products and/or packaging to destroy any bacteria or pests thoroughly. Its activity is high enough to emit a constant, powerful stream of gamma rays that can effectively kill microbes but with a long enough half-life to require infrequent replenishment.

Recall: $A_0 = \frac{\ln 2}{t_{1/2}} \times N_0$

Radioactive Tracers and Radiation in Agricultural Use

Radioisotopes like phosphorus-32 are added to fertilizers to track how much is absorbed by the plant, helping to optimize fertilizer use. The half-lives of the radioisotopes used are typically of shorter duration (a couple of weeks) to ensure the crops which are exposed to them are safe for consumption.

Exposing seeds or cuttings to radiation can create genetic variability, leading to desirable traits such as resistance to diseases, pests, or environmental stresses. The radioactive sources chosen emit radiation of high penetrating power (like gamma rays) to penetrate thick tissue walls of seeds or stem cuttings.

Radiation Hazard

Ionising radiation damages living tissue by breaking chemical bonds in atoms and molecules. This can disrupt cellular control processes, leading to abnormal cell growth, cancer, or tissue failure. Acute exposure to high doses can cause burns, radiation sickness, premature ageing, or death.

The **type of radiation** and **route of exposure** determine the extent of harm.

- Since alpha radiation has strong ionising power but very low penetration, it is dangerous only if an alpha emitter is ingested.
- Beta radiation can penetrate skin slightly and may cause internal damage if a beta emitter is ingested.
- Gamma radiation has high penetrating power but weak ionising ability; they can reach internal organs even from external sources.

The **half-life** of a radioactive nuclide also affects hazard level. Those with short half-lives release radiation more intensely over a short time, posing acute risks. Others with long half-lives emit radiation more slowly but remain hazardous for extended periods, particularly if the material accumulates in the body or environment.

Stochastic (random) health effects

Stochastic effects arise from long-term, low-level exposure. The probability of these effects increases with dose. Cancer is the primary health effect from radiation exposure, caused by damage at the cellular or molecular level. This disruption to control processes due to cellular damage leads to uncontrolled growth of cancer cells.

Radiation can also cause genetic mutations (changes in DNA) which can be passed to offspring. There can also be fetal development defects like stunted growth or deformities if exposure occurs during pregnancy.

Non-stochastic (deterministic) health effects

Non-stochastic effects appear in cases of exposure to high levels of radiation and become more severe as the exposure increases. These include skin burns, hair loss, organ failure, and radiation sickness, which can be fatal if exposure is extreme.

Medical patients receiving radiation treatments often experience acute effects because they are receiving relatively high doses of radiation during treatment.

20.3 Nuclear Processes and Conservation Laws

❖ Representing Nuclear Reactions based on Conservation Laws

A **nuclear reaction** is a process during which two nuclei, or a nucleus and a subatomic particle (proton, neutron or energetic electron), **collide** with each other to produce one or more nuclides that is/are different from the original nuclides. Hence, a nuclear reaction must cause **nuclear transformation** of at least one nuclide to another. Nuclear reactions may either be natural or artificial. Examples of nuclear reactions are presented in the table below.

Natural	Artificial
Fusion of light elements, such as hydrogen, powers the star and is responsible for the creation of virtually all elements.	Fusion of deuterium and tritium or fission of ${}^{235}_{92}U$ or ${}^{239}_{94}Pu$ in thermonuclear weapons. These are uncontrolled nuclear chain reactions.
Heavy, naturally occurring nuclides like uranium and thorium can spontaneously undergo fission without any external trigger.	Fission of ${}^{235}_{92}U$ in nuclear power plants generates tremendous amount of energy. These are controlled nuclear chain reactions.

All nuclear reactions may be presented in the form of an equation in the standard form similar to chemical reactions. The notations of common particles involved in nuclear reactions are given in the table below.

Name	Symbol
proton	${}^{1}_{1}H$ or ${}^{1}_{1}p$
neutron	${}^{1}_{0}n$
electron	${}^{0}_{-1}e$
positron	${}^{0}_{1}e$
α-particle	${}^{4}_{2}He$

In any nuclear reaction, the total number of nucleons and the total charge must be conserved. For example, consider the nuclear reaction

$$ {}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H $$

We can see that the total number of nucleons before the reaction ($14+4=18$) is equal to the total number after the reaction ($17+1=18$). Furthermore, the total charge ($Z=9$) is the same before and after the reaction.

❖ Existence of (Anti)Neutrino during Beta Decay

In addition to charge and total number of nucleons, energy and momentum must also be conserved in nuclear reactions.

These conservation laws for energy and momentum in a β-decay were used to predict the existence of the neutrino.

The kinetic energy of β-particles emitted during beta decay ranges from zero up to a maximum value (Fig. 20.7). Initially, this led to concerns that beta decay might violate the principle of energy conservation as the range of kinetic energy showed that they are not consistently carrying away all the energy released in the decay.

Additionally, cloud chamber experiments revealed that the daughter nucleus does not recoil directly opposite to the direction of the emitted β-particle. This unexpected behaviour suggested a violation of the principle of momentum conservation.

In a cloud chamber, ionising radiations form visible tracks.

To preserve the conservation laws of energy and momentum, Wolfgang Pauli hypothesized an undetected neutral particle with a very small mass. This particle was eventually detected in 1956 by Frederick Reines and Clyde Cowan, which won them the Nobel Prize in 1995.

It is now known that most of the energy released during beta decay is carried away by the β-particle and the neutrino. In addition, total momentum is conserved if we include the momentum of the neutrino in our calculation (Fig. 20.8).

(Anti)Neutrinos are neutral and nearly undetectable. It does not leave a track in the cloud chamber and has no biological or chemical effect.

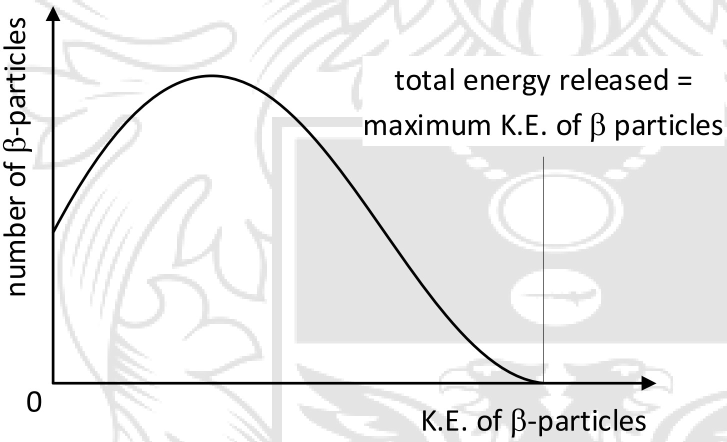


Fig. 20.7 Range of kinetic energy of beta particles in beta decay

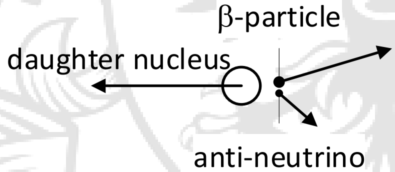


Fig. 20.8 Total momentum is conserved in beta decay

20.4 Mass Defect and Nuclear Binding Energy

❖ Equivalence of Mass and Energy

In 1905, Einstein established that energy and mass are equivalent quantities related by

$$E = mc^2$$

When a particle emits energy, its mass decreases. The decrease in mass Δm would be equal to the amount of energy emitted ΔE divided by the square of the speed of light:

$$\Delta m = \frac{\Delta E}{c^2}.$$

A particle at rest has an enormous amount of energy by virtue of its mass alone. This is known as **rest mass energy**:

$$E_0 = m_0 c^2$$

Probably the clearest experimental proof of the equivalence of mass and energy occurs in nuclear reactions: the release of large amounts of energy is accompanied by a decrease in mass.

In view of the equivalence of mass and energy, the classical principle of conservation of energy must be regarded as a special case of the principle of conservation of mass-energy. However, this more general principle needs to be invoked only when dealing with certain nuclear phenomena or when speeds comparable to the speed of light are involved.

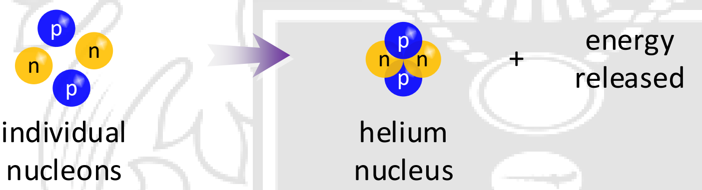


Fig. 20.9 Conservation of mass-energy in nucleus formation

❖ Mass Defect and Nuclear Binding Energy

Mass Defect

The **mass defect** of a nucleus is the difference between the total mass of the separate nucleons and the mass of the nucleus.

The **mass defect of the nucleus** is

$$\Delta m_{\text{nucleus}} = Zm_p + (A - Z)m_n - m_{\text{nucleus}}$$

$\Delta m_{\text{atom}} \approx \Delta m_{\text{nucleus}}$

Nuclear Binding Energy

Nuclear binding energy is the energy equivalent of the mass defect of a nucleus. It is the energy required to separate to infinity all the nucleons of a nucleus.

Atomic binding energy is the energy required to break up an atom into free electrons and individual nucleons.

Electronic binding energy is the energy required to completely remove all electrons from an atom. Electronic binding energies are typically much smaller than nuclear binding energies so they may be neglected.

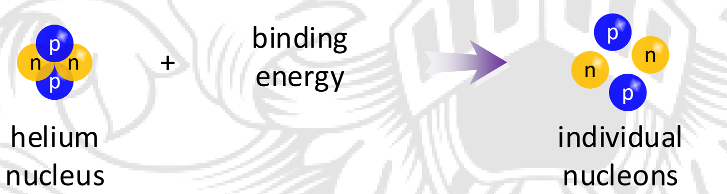


Fig. 20.10 Energy required in the disintegration of a nucleus

This process is accompanied by an **increase** in the rest mass of the system. Thus, the nuclear binding energy (B.E.) and the mass defect Δm of the nucleus are related by the equation:

$$\text{B.E.} = \Delta m_{\text{nucleus}} c^2$$

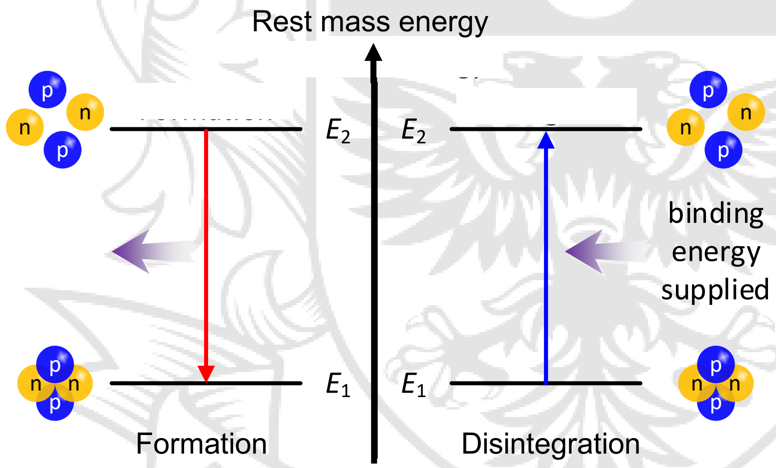


Fig. 20.11 Energy released during nucleus formation vs binding energy

The energy released during the formation of a nucleus is numerically equal to the binding energy of the nucleus – however, it is not the definition of nuclear binding energy.

Example 9: Pair Production

A gamma ray photon of energy E can interact with a nucleus that is at rest to produce an electron and its anti-particle, a positron. This process is known as pair-production. Calculate the minimum energy of this gamma ray photon.

[Solution]

$$\gamma \rightarrow e^{-} + e^{+}$$

By the principle of mass-energy equivalence, the energy of the gamma ray photon must be at least equal to the total rest mass energy of the electron and the positron.

$E_{\text{min}} =$

At this minimum energy, the electron and positron are formed at rest. However, the total momentum of the system must be non-zero as gamma ray photon has momentum. Hence pair production is possible only when the gamma ray photon interacts with a massive nucleus. The momentum of the photon is transferred to the nucleus without appreciable increase in energy of the nucleus.

A positron is the anti-particle of an electron. It has the same mass as an electron but of the opposite charge.

Pair production can also be achieved with two gamma ray photons travelling in opposite directions.

Example 10: Nuclear Binding Energy

Calculate the binding energy in MeV of the following nuclides ${}^{2}_{1}\text{H}$, ${}^{56}_{26}\text{Fe}$ and ${}^{236}_{92}\text{U}$ given that

mass of deuteron $m_{\text{D}} = 3.343583 \times 10^{-27}$ kg

mass of iron 56 nucleus $m_{\text{Fe}} = 92.861315 \times 10^{-27}$ kg

mass of uranium 236 nucleus $m_{\text{U}} = 391.889065 \times 10^{-27}$ kg

mass of proton $m_{\text{p}} = 1.672622 \times 10^{-27}$ kg

mass of neutron $m_{\text{n}} = 1.674927 \times 10^{-27}$ kg

[Solution]

Mass defect for ${}^{2}_{1}\text{H}$

$\Delta m =$

$=$

$=$

Binding energy of ${}^{2}_{1}\text{H}$

B.E. $=$

$=$

$=$

Mass defect for $^{56}_{26}\text{Fe}$

$\Delta m =$

$=$

$=$

Binding energy of $^{56}_{26}\text{Fe}$

B.E. $=$

$=$

$=$

Mass defect for $^{236}_{92}\text{U}$

$\Delta m =$

$=$

$=$

Binding energy of $^{236}_{92}\text{U}$

B.E. $=$

$=$

$=$

❖ Binding Energy Per Nucleon and Nuclear Stability

Since nuclear binding energy is the energy needed to separate the nucleus into its constituent nucleons, this energy should therefore be an indication of the **stability** of the nucleus. While the nuclear binding energy of a heavier nucleus is greater than that of a lighter nucleus because of the greater number of nucleons, this does not mean that a heavier nucleus is more stable. To determine stability, we calculate the **binding energy per nucleon**:

We can interpret binding energy per nucleon as the **average energy** require to remove a nucleon from the nucleus until it is completely disintegrated.

$$\text{binding energy per nucleon} = \frac{\text{B.E.}}{A}$$

The **higher** the value of its binding energy per nucleon, the **more stable** is the nucleus.

Binding Energy Per Nucleon

Binding energy per nucleon is the total energy needed to completely separate all the nucleons in a nucleus to infinity divided by the number of nucleons in the nucleus.

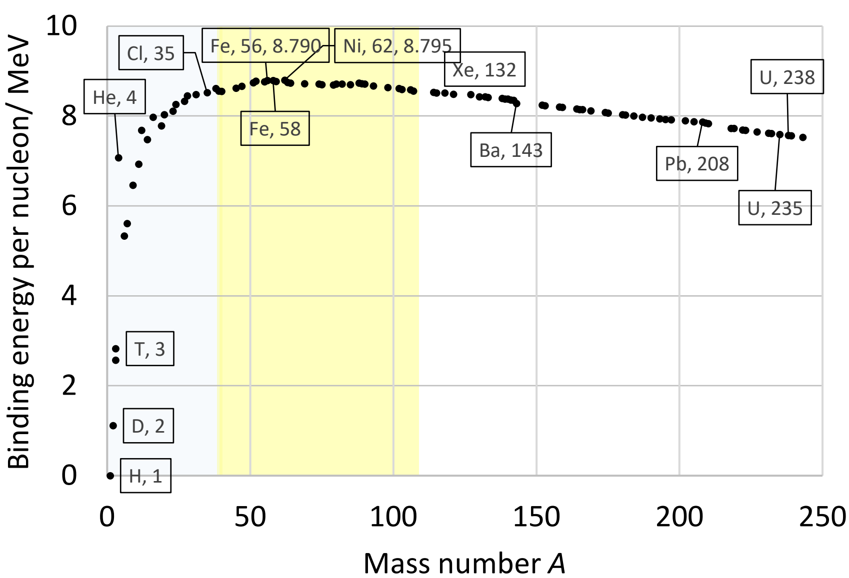


Fig. 20.12 Binding energy per nucleon with nucleon number

The following observations can be made from **Fig. 20.12**:

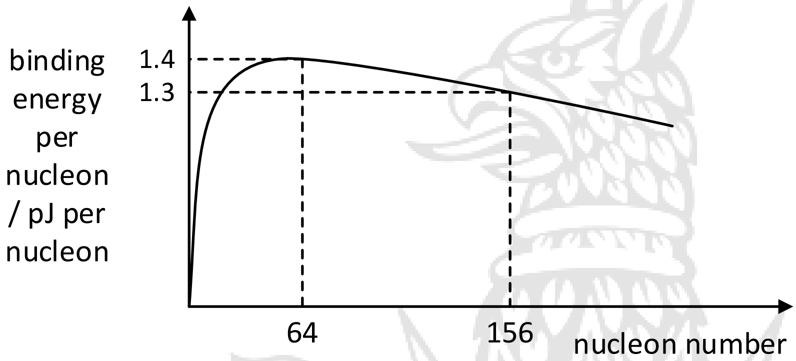
- Light elements from deuterium ($^{2}_{1}$H) to iron ($^{56}_{26}$Fe) have **increasing** binding energy per nucleon. In this region, the increase in attractive nuclear forces per nucleon (as more nucleons are added) outweighs the increase in Coulomb repulsion between protons.
- $^{4}_{2}$He (7.084 MeV) is unusually stable amongst light nuclei.
- Elements with intermediate mass number ($^{38}_{18}$Ar to $^{103}_{45}$Rh) are relatively stable (high binding energy per nucleon).
- $^{56}_{26}$Fe (8.790 MeV), $^{58}_{26}$Fe (8.792 MeV) and $^{62}_{28}$Ni (8.795 MeV) are the three most stable nuclei.
- Heavier nuclei exhibit **decreasing** binding energy per nucleon. In this region, the Coulomb repulsion between the protons becomes more prominent.

Binding energy of these nuclei are typically greater than 8.6 MeV.

$^{56}_{26}$Fe is more abundant than $^{62}_{28}$Ni and $^{58}_{26}$Fe in the universe.

Example 11 [J98/I/29]

The graph shows how the binding energy per nucleon varies with the nucleon number for naturally occurring nuclides.



Calculate the binding energy of the nuclide $^{156}_{64}\text{Gd}$.

[Solution]

❖ Nuclear Fission

Nuclear Fission

Nuclear fission is the splitting of a heavy nucleus into two lighter nuclei.

The fission of a heavy nucleus is typically initiated by a slow (or thermal) neutron and the fission products include two or three slow neutrons and gamma ray photons as shown in Fig. 20.13. An enormous amount of energy is released during nuclear fission of a very heavy nucleus.

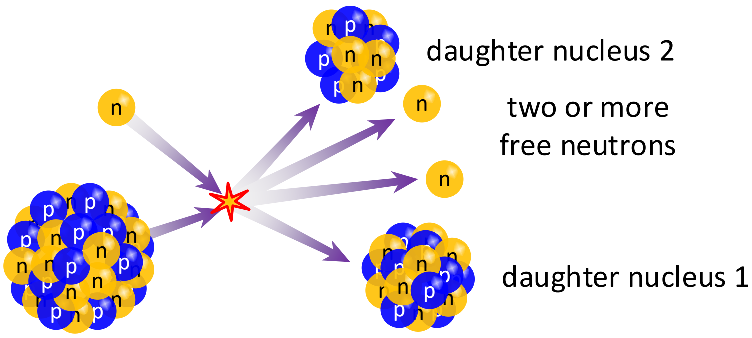


Fig. 20.13 Fission of a heavy nucleus into two lighter nuclei

Since the slow neutrons produced in one fission reaction can each trigger another fission reaction, a chain reaction is very quickly set up. This can easily get out of control (this is the case in an exploding atomic bomb where a huge amount of energy is released) unless steps are taken to control the rate of the reactions (this is done in a nuclear reactor).

The release of energy via nuclear fission of heavy nuclei can be explained by the concept of **binding energy per nucleon**.

We can visualise fission as a two-step process: the disintegration of the parent nucleus and the formation of the daughter nuclei.

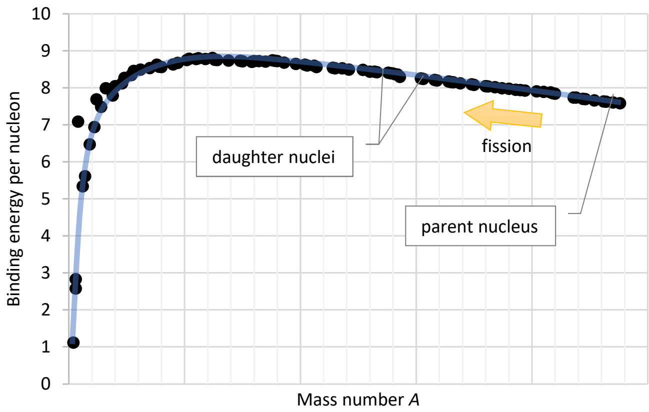


Fig. 20.14

Fig. 20.14 shows that the lighter "daughter" nuclei have **higher** binding energy per nucleon than the "parent" nucleus. This means that the formation of the daughter nuclei releases more energy than that required to completely disintegrate the parent nucleus. The energy E released in a fission reaction can be shown to be equal to the difference in binding energies of the reactants and the products:

$$E = \text{B.E.}_{\text{products}} - \text{B.E.}_{\text{reactants}}$$

*See **Appendix** for derivation of this equation.

❖ Nuclear Chain Reaction

In 1933, Leó Szilárd hypothesized that neutron-driven fission of heavy nuclei could create a **nuclear chain reaction** (Fig. 20.15). Assuming that the number of secondary neutrons produced by fission is greater than one, then each reaction can initiate additional reactions and an exponentially increasing number of reactions can be achieved in a chain reaction. His hypothesis led to the possibility of yielding large amounts of energy for civilian (nuclear power plant) and military use (atomic bomb).

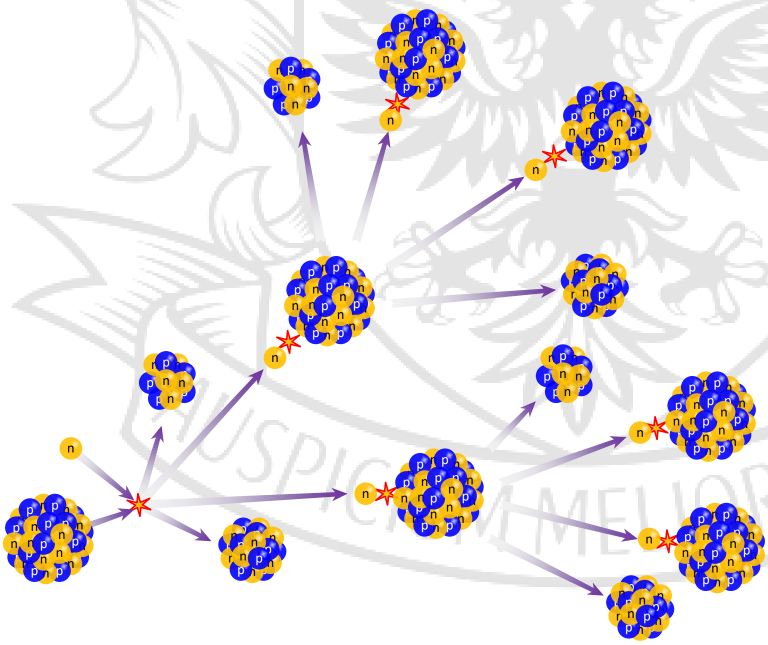


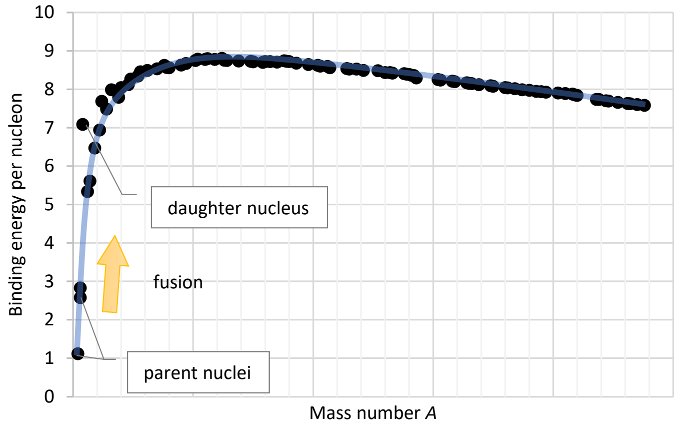
Fig. 20.15 Nuclear chain reaction

❖ Nuclear Fusion

Nuclear Fusion

Nuclear fusion occurs when two light nuclei combine to form a nucleus of greater mass.

Energy is released during nuclear fusion of very light nuclei.



Hydrogen nucleus has zero binding energy (not shown).

Fig. 20.16 Fusion of two lighter nuclei to form a heavier nucleus

Fig. 20.16 shows that energy is released as lighter parent nuclei combine to form a heavier daughter nucleus.

The highest possible energy released per nucleon is by fusing the nuclei of the lightest known element, hydrogen.

Protons must have very high energy to overcome mutual Coulomb repulsion. This would require a temperature in excess of millions of Kelvin.

The Sun derives its power from the fusion of hydrogen nuclei (actually protons) to form a helium nucleus through a **proton-proton cycle**. The cycle consists of three distinct nuclear reactions and begins with the fusion of protons to form deuterium with the release of 0.42 MeV of energy:

$$ {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + {}^{0}_{1}\text{e} + \nu \qquad (1)$$

The deuterium nucleus then fuses with a proton to form a helium-3 nucleus with the release of 5.49 MeV of energy:

$$ {}^{2}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + \gamma \qquad (2)$$

γ is a gamma ray photon.

The helium-3 nuclei from two such reactions then combine to give a helium-4 nucleus and a pair of protons, releasing 12.86 MeV of energy in this reaction:

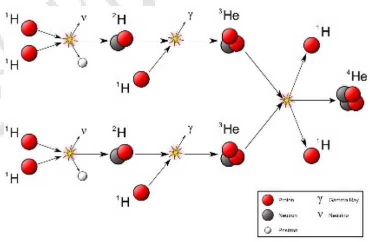
$$ {}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + 2\,{}^{1}_{1}\text{H} \qquad (3)$$

Lastly, the positron from (1) will annihilate a single electron to produce two gamma ray photons of 1.022 MeV in total:

This is the opposite of pair production.

$$ {}^{0}_{1}\text{e} + {}^{0}_{-1}\text{e} \rightarrow 2\gamma \qquad (4)$$

In one proton-proton cycle, reactions (1), (2) and (4) occur twice while reaction (3) once (see diagram on the right). The net effect of the above reactions (1) to (4) is to convert four protons and two electrons into a single helium-4 nucleus:



$$ 4\,{}^{1}_{1}\text{H} + 2\,{}^{0}_{-1}\text{e} \rightarrow {}^{4}_{2}\text{He}$$

with the release of 26.7 MeV of energy.

To calculate the energy released in a nuclear reaction, we can use either one of the two equations:

$$\text{energy released} = \left(m_{\text{reactants}} - m_{\text{products}}\right)c^2$$

or

$$\text{energy released} = \text{B.E.}_{\text{products}} - \text{B.E.}_{\text{reactants}}$$

Which equation to use depends on the information given in the question.

Examples 20.12 to 20.16 illustrate the use of the equations above to various situations to calculate the energetics of the nuclear reactions involved.

Example 12: Energy Released in Alpha Decay

A stationary nucleus of ${}^{238}_{92}\text{U}$ decays into a Thorium nucleus by the emission of an α-particle. Calculate

(a) the energy released, and

(b) the kinetic energy of the alpha particle.

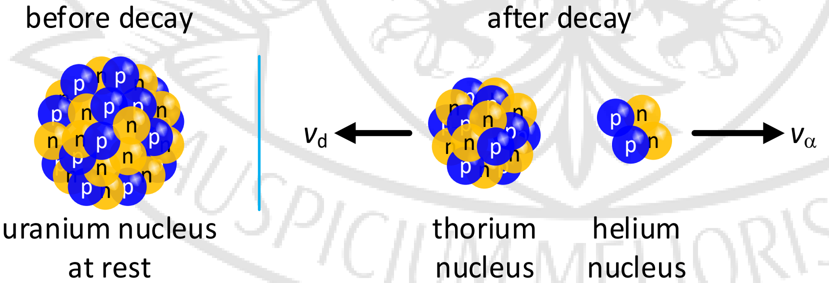
Masses of nuclei:

$m_{\text{U-238}} = 238.000281u$; $m_{\text{Th-234}} = 233.994192\ u$; $m_{\alpha} = 4.001505\ u$

[Solution]

(a)

(b) Since the total momentum of the system is conserved, and since there are only two particles involved in the decay (daughter nucleus and α-particle), these two particles will move in opposite directions after the decay. The two particles are said to recoil from each other.



By the principle of conservation of momentum, the thorium nucleus and the α-particle have equal but opposite momentum.

$p = \sqrt{2mE}$

The daughter nucleus should carry away the rest of the energy release of 0.07 MeV.

Example 13: Energy Released in Beta Decay

Calculate the energy released in the β decay of helium-6.

Masses of nuclei:

$m_{\text{He-6}} = 6.017788\ u$; $m_{\text{Li-6}} = 6.013475\ u$; $m_e = 0.000549\ u$

[Solution]

$${}^{6}_{2}\text{He} \rightarrow {}^{6}_{3}\text{Li} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$$

Example 14: Energy Released in a Nuclear Reaction

The discovery of the proton took place after the first induced nuclear reaction was carried in a laboratory by Rutherford. The equation for this reaction is

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

Calculate the energy released in the above reaction given that the rest masses of the atoms are:

$m_{\text{N-atom}} = 14.003074\ u$; $m_{\text{He-atom}} = 4.002603\ u$;

$m_{\text{O-atom}} = 16.999131\ u$; $m_{\text{H-atom}} = 1.007825\ u$

[Solution]

Decrease in mass

$\Delta m =$

$=$

$=$

Energy released

$E =$

This energy release is also known as the Q-value, the reaction energy. Q is positive for exothermic reaction and negative for endothermic reactions.

From the energy point of view, this reaction is not likely to happen due to the energy shortfall. So how did this reaction occur for Rutherford to discover the proton? The answer lies with the α-particle. The energy shortfall in the reaction was provided by the kinetic energy of the α-particle.

Atomic masses may be used in the calculation for energy release as there is an equal number of electrons on both sides of the equation. The electron binding energies of the atoms may be neglected.

The **negative** sign means that the total mass of the products is greater than that of the reactants. In other words, there is an increase in mass.

Example 15: Energy Released in a Nuclear Fission

Write down the nuclear equation for the neutron-induced fission of plutonium-239 ($^{239}_{94}\text{Pu}$) that results in barium-143 ($^{143}_{56}\text{Ba}$) and strontium-95 ($^{95}_{38}\text{Sr}$), and identify other particles that may be produced in this reaction. Calculate the energy released given that the rest masses of the atoms are:

$m_{\text{Pu-atom}} = 239.052165\ u$, $m_{\text{Ba-atom}} = 142.920625\ u$,
$m_{\text{Sr-atom}} = 94.919351\ u$

[Solution]

This is a neutron-induced fission reaction:

$$^{239}_{94}\text{Pu} + {}^{1}_{0}\text{n} \rightarrow {}^{143}_{56}\text{Ba} + {}^{95}_{38}\text{Sr} + 2\,{}^{1}_{0}\text{n}$$

Decrease in mass

$\Delta m =$

$=$

Energy released

$E =$

Example 16: Energy Released in a Nuclear Fusion

Although ordinary hydrogen is abundant in the universe, the fusion between two hydrogen nuclei does not occur readily. Fusion research has therefore focused on reactions involving the heavier hydrogen isotopes. The deuterium-deuterium (D-D) reaction is one such possible reaction:

$$^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{1}_{0}\text{n}$$

Calculate the energy released in this reaction given the binding energy per nucleon of deuterium and helium-3 are 1.114 MeV and 2.576 MeV respectively.

[Solution]

To calculate the energy released, we need to compute the binding energies of $^{2}_{1}\text{H}$ and of $^{3}_{2}\text{He}$.

Hence the energy released is

Free neutrons have zero binding energy.

20.5 Appendix

❖ Radiation Detectors

A radiation detector is a device used to detect, track and/or identify ionizing particles such as those produced by radioactive decay or cosmic radiation. In this section, we will briefly discuss the operation of a few commonly used radiation detectors, namely, the Wilson cloud chamber, the Geiger-Müller tube and the scintillation counter.

The Wilson Cloud Chamber

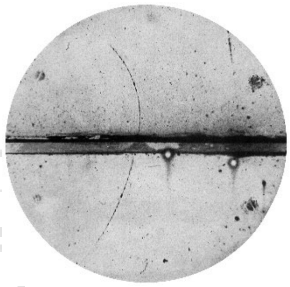


Fig. 20.17 Cloud chamber photo that proved the existence of positron.

Modern cloud chamber

Invented by Charles Wilson in 1911, a diffusion cloud chamber is a particle detector that can show the path of a passing ionising radiation (see Fig. 20.5 on pg. 13).

A cloud chamber consists of a sealed chamber with a large temperature gradient between the top plate (at room temperature) and cold bottom plate. The bottom plate is cooled to about –78°C by a layer of dry ice beneath it. A felt disc, soaked with isopropyl alcohol, is placed near the top plate. The alcohol then vaporises and diffuses downwards. Near the bottom plate, the alcohol vapour condenses but just above the bottom plate, a layer of supersaturated alcohol vapour is formed.

A radioactive source, placed inside this chamber, emits ionising radiation. These ionizing radiations produce ions as they pass through the supersaturated vapour. Since alcohol molecules are polar, the supersaturated vapour is attracted towards the ions and condensation occurs. These condensations produced a trail of visible alcohol droplets which represents the path of the radiations.

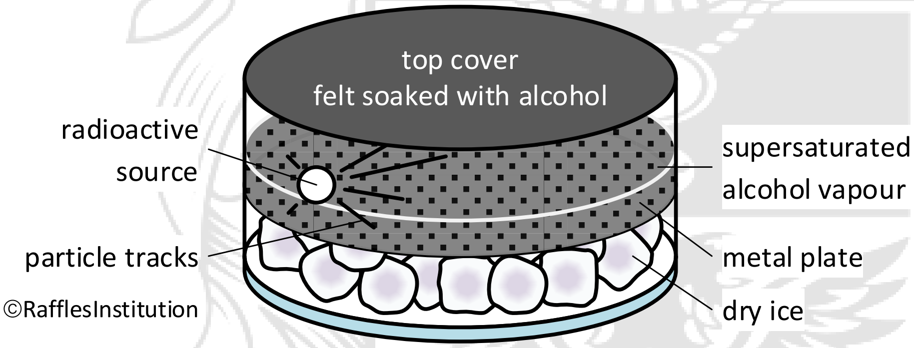


Fig. 20.18 A schematic diagram of a cloud chamber

Invented by Donald Glaser in 1952, the bubble chamber works on a similar principle as a cloud chamber except that it uses a super-heated liquid. The bubble chamber was the detector of choice for many decades and superseded the cloud chamber as it is capable of tracking more energetic ionising radiations.

The Geiger-Müller Tube

A Geiger-Müller (GM) tube is a low cost and widely used radiation detector. Invented by Hans Geiger and Walther Müller in 1928, it consists of a metal chamber filled with a low-pressure inert gas such as argon. A potential difference of a few hundred volts is applied between the wall of the tube (cathode) and a thin wire in the centre (anode).

A Geiger-Müller (GM) counter

When ionising radiation enters the tube through the mica window, it ionizes the argon atoms to create pairs of positively charged ions and electrons. The strong electric field accelerates the positive ions towards the cathode and electrons towards the anode. Close to the anode, the electrons gain sufficient energies to cause additional ionization of the argon atoms. This creates an electron avalanche and a large output pulse is detected from a single ionising radiation. The disadvantage of a GM counter is that it is unable to measure the energy of the ionising radiation or distinguish the type of ionising radiation.

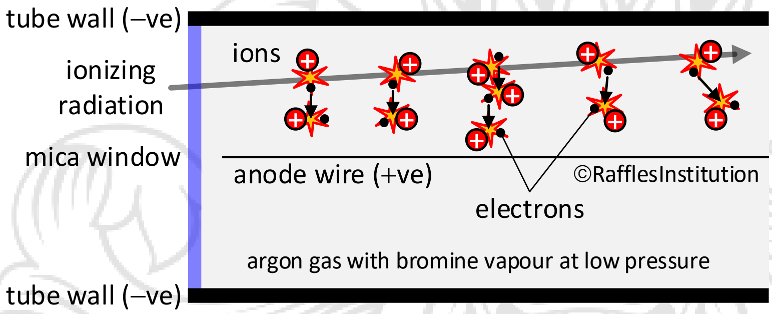


Fig. 20.19 A schematic diagram of a Geiger-Müller tube

Meanwhile, the positively charged ions acquire electron to become argon atoms upon reaching the cathode. These atoms may subsequently emit photons which can cause further ionisation and produce undesirable secondary pulses. Hence, a quenching agent, such as bromine, may be added to minimise secondary pulses. The addition of a quenching agent, however, renders the tube inactive for a short period of time. During this “dead time”, the tube is unable to detect the arrival of new ionising radiation, hence limiting the accuracy and the maximum count rate.

The GM tube is connected to a counter (to measure cumulative count) or a rate-meter (to measure count rate).

The Scintillation Counter

A scintillation counter consists of a scintillator and a photomultiplier. Invented by Sir Samuel Curran in 1944, a scintillator emits photons when ionizing radiation strikes the scintillating crystal. These photons cause the emission of electrons (photoelectric effect) at the photocathode. They are then electrostatically accelerated towards the first dynode of the photomultiplier tube. On impact with the dynode, a single electron releases a number of secondary electrons which are in turn accelerated to strike the second dynode. At each dynode, more electrons are released so that the current is amplified. The resultant output is an electrical pulse which is passed to the processing electronics.

A scintillation counter

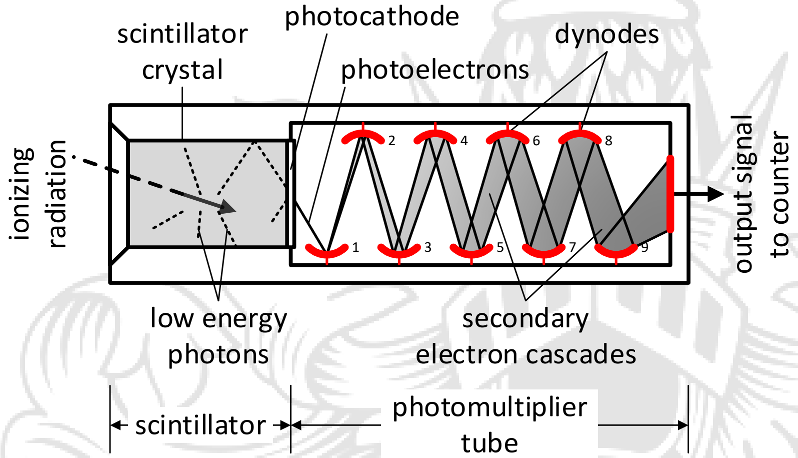


Fig. 20.20 A schematic diagram of a scintillation counter

Unlike a GM tube, a scintillator can measure both the intensity and the energy of the incident radiation. It can also distinguish between different ionising radiation through the use of different scintillating crystal – zinc sulphide for α-particles, plastic scintillators for β-particles and sodium iodide for γ-ray photons.

❖ Experimental Determination of Decay Constant

The decay constant or half-life of a radioactive nuclide is usually determined experimentally. The technique described here is suitable for the determination of half-lives which range from a few minutes to a few days.

Alternative technique is required for determination of very short half-lives (< milli-second) or very long half-lives (> months or years).

Before measuring the count rate of a radioactive source, we first measure the count rate C_B due to **background radiation** with a detector, such as Geiger-Müller (GM) tube connected to a rate-meter. This background count rate C_B must be subtracted from the total count rate C_T when a radioactive source is placed at a fixed position in front of the GM tube.

The count rate C is a **fraction** of the activity A of a radioactive sample as radiations of the decay are emitted in all directions and not all radiations are counted by the detector.

$$C_S = C_T - C_B$$

Once the count rate C_S due to the source is computed, we can then proceed with one of two ways to determine the decay constant.

Method 1

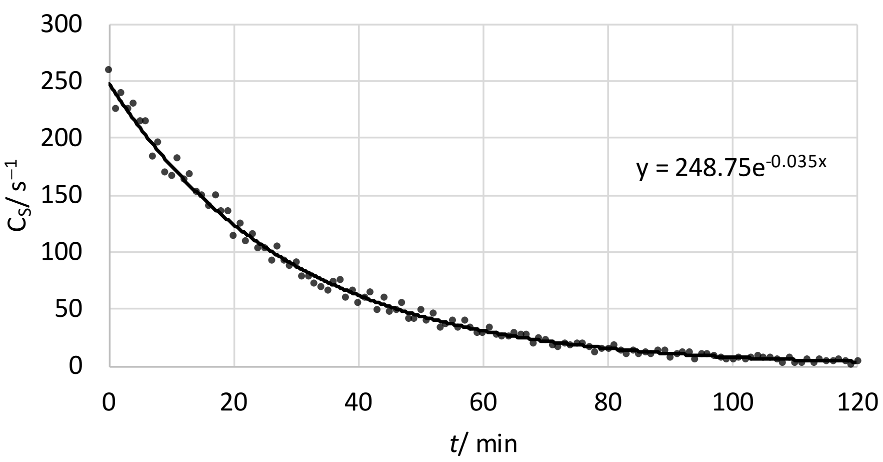


Fig. 20.21 The variation with time t of the count rate C_S

In **Fig. 20.21**, an exponential line of best fit is drawn since we know that the count rate C_s varies exponentially with time,

$$C_s = C_{s0} e^{-\lambda t} \quad (1)$$

We read off C_s at two specific times, say, at $t = 0$ and $t = 60$ s ,

$$C_{S0} = 250 \text{ s}^{-1}; \; C_S = 30 \text{ s}^{-1}$$

Using eqn. (1), we then determine that the decay constant is -0.035 min^{-1} and the half-life of radioactive source is 20 min.

Method 2

Taking natural logarithm on both sides of eqn. (1), we obtain

$$\ln C_s = \ln C_{s0} - \lambda t$$

So, a straight line with a gradient of $-\lambda$ would be obtained if $\ln C_s$ is plotted against time t as shown in Fig. 20.22.

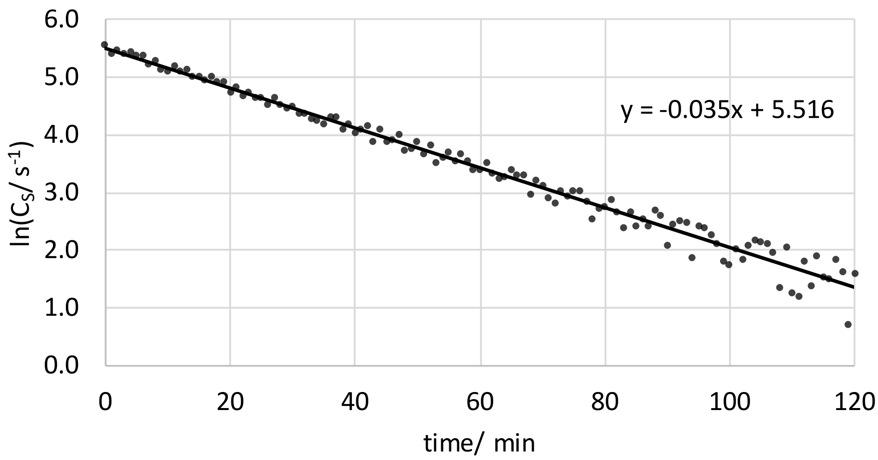


Fig. 20.22 The variation with time t of $\ln C_S$

❖ THE NUCLEAR REACTOR

The extensive use of fossil fuels (coal, oil or gas) to generate electrical power has resulted in elevated global temperature and caused alarming rise in sea level and unpredictable weather patterns. Furthermore, the rapid depletion of the fossil fuels has made it imperative for mankind to explore alternative sources of power.

To satisfy the enormous energy needs of mankind, we have been harnessing nuclear power for decades. With major advances in technology and design, nuclear power plants have been made safer but the Chernobyl accident in 1986 and the Fukushima disaster in 2011 are painful reminders of what can happen when things go wrong.

Advantages of nuclear power

- Significantly lower CO_2 emission than fossil-fuel power plant
- Relatively low fuel consumption (30 tons per year of uranium vs. 9000 tons per year of fossil fuel per power plant)
- Abundant amount of uranium in earth crust
- Spent uranium can be enriched and re-used
- Low operating cost

Disadvantages of nuclear power

- Risk of meltdown and the catastrophic consequences
- Storage of increasing amount of nuclear waste
- Nuclear power plants are expensive to build and decommission
- Proliferation of nuclear weapon from enriched fuel

In this section, we will further explore the inner workings of pressurized-water reactor (PWR) power plant.

The other types being boiling water reactor and supercritical water reactor.

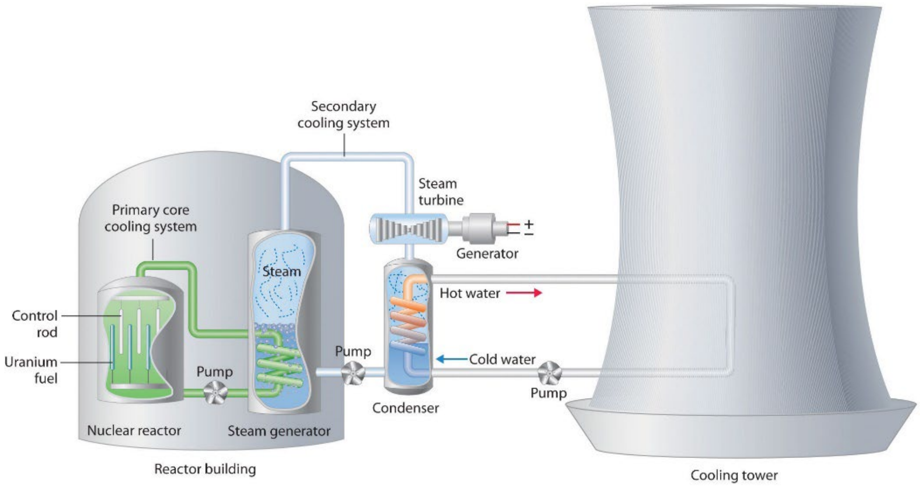


Fig. 20.22 Schematics of a pressurized-water reactor

In **Section 20.4**, we learnt that heavy nuclei such as uranium-235 ($^{235}_{92}U$) or plutonium-239 ($^{239}_{94}Pu$) can be made to fission by bombarding with slow thermal neutron. In the process, the neutrons released can cause further fission of the heavy nuclei. This is known as nuclear chain reaction.

Energy of a fast neutron is between 1-2 MeV while that of slow thermal neutron is 0.025 eV.

A typical nuclear reactor may contain as many as 150 to 250 assemblies of 200 to 300 fuel rods. Each of these rods consists of ceramic pallets of enriched uranium dioxide. So, each nuclear reactor may contain as much as 80 to 100 tons of uranium.

Most modern nuclear reactors use light water (H_2O) as moderator, but carbon graphite or heavy water (D_2O) may be used instead.

During fission of the uranium nuclei, a tremendous amount of heat is released and more energetic neutrons are produced. However, these neutrons are too fast to effectively cause further fission so they must be slowed down sufficiently with a moderator (water surrounding the fuel rods served as a moderator and a coolant to prevent a meltdown). As fast neutrons pass through the moderator, they experience multiple collisions with the hydrogen atoms in the water. As a result, these neutrons lose energy and slow down.

When $K > 1$, the reactor is supercritical and a nuclear meltdown may occur. When $K < 1$, the reactor is subcritical and the reaction will eventually die out.

An important parameter describing the operation of a nuclear reactor is the reproduction constant *K*, defined as the average number of neutrons from each fission event that cause another fission event. In a typical uncontrolled fission of uranium-235, an average of 2.5 neutrons are released from each fission event, i.e., $K = 2.5$. For sustained and controlled chain reaction, it is important to ensure *K* is close to 1, i.e., the excess neutrons must be removed from the reactor using control rods. The depth at which the control rods are inserted into the nuclear reactor will control the rate of nuclear fission; deeper to reduce the rate of reaction, shallower to increase the rate of reaction.

Control rods are usually made of boron, silver, indium and cadmium which are capable of absorbing neutron without undergoing nuclear fission.

Now the heat released must be converted to electrical energy. In PWR, water in the reactor is heated to 588 K by the fuel rod. This heated water remains liquid due to the extremely high pressure of 15.5 Mpa in the primary cooling system. It is then pump through a heat exchanger (steam generator) where heat is conducted through the copper piping to turn water in the secondary cooling loop into steam. The steam then drives the steam turbine which in turn drives an electrical generator. It then passes through the condenser where it condenses back to water and before being circulated back to the heat exchanger.

In the heat exchanger, water in the secondary cooling system is never in contact with the primary cooling system and hence is not contaminated by any radioactive material.

One safety feature of PWR is its use of water as a moderator. Neutrons are more effectively "moderated" as the neutrons will experience more frequent collisions with water molecules. However, any increase in temperature of the reactor will cause water to expand, thereby reducing its ability to moderate the neutrons and decreasing the reactivity of the reactor. Therefore, the reactor will stablise itself to a temperature set by the position of the control rods. This makes PWR inherently safer compared to other reactor design.

❖ **Derivation of Equation** $E = \text{B.E.}_{\text{products}} - \text{B.E.}_{\text{reactants}}$

We will use the **fission reaction** in **Example 15**:

$$^{239}_{94}\text{Pu} + ^{1}_{0}\text{n} \rightarrow ^{143}_{56}\text{Ba} + ^{95}_{38}\text{Sr} + 2^{1}_{0}\text{n}$$

Decrease in mass

$$\Delta m = \left(m_{\text{Pu}} + m_n\right) - \left(m_{\text{Ba}} + m_{\text{Sr}} + 2m_n\right) = m_{\text{Pu}} - m_{\text{Ba}} - m_{\text{Sr}} - m_n$$

However,

$$m_{\text{Pu}} = \left(239 - 94\right) m_n + 94 m_p - \frac{\text{B.E.}_{\text{Pu}}}{c^2} = 145 m_n + 94 m_p - \frac{\text{B.E.}_{\text{Pu}}}{c^2}$$

$$m_{\text{Ba}} = \left(143 - 56\right) m_n + 56 m_p - \frac{\text{B.E.}_{\text{Ba}}}{c^2} = 87 m_n + 56 m_p - \frac{\text{B.E.}_{\text{Ba}}}{c^2}$$

$$m_{\text{Sr}} = \left(95 - 38\right) m_n + 38 m_p - \frac{\text{B.E.}_{\text{Sr}}}{c^2} = 57 m_n + 38 m_p - \frac{\text{B.E.}_{\text{Sr}}}{c^2}$$

Combining these equations:

$$\Delta m = -\frac{BE_{Pu}}{c^2} + \frac{BE_{Ba}}{c^2} + \frac{BE_{Sr}}{c^2}$$

Therefore, the energy released is

$$E = \Delta m\, c^2 = \text{B.E.}_{\text{products}} - \text{B.E.}_{\text{reactants}}$$

❖ Danger of Neutron Radiation

Neutrons are high-speed subatomic particles that have an exceptional ability to penetrate other materials, including human bodies. Amongst the different types of ionising radiations (such as α, β and γ-radiations), neutrons are the only ones capable of turning a nucleus radioactive. This process, called neutron activation, is used to produce many of the radioactive sources that are used in medical, academic, and industrial applications (including oil exploration).

Besides its ability to transform our tissues into a radioactive mess, neutron is also highly capable of knocking out protons from the hydrogen atom in water molecules and thereby breaking its chemical bonds. This proton then ionises any atoms along its path, causing irreparable damage to cells. Damaged cells may die or become abnormal which can result in cancer. Because our body is composed mostly of water, neutrons is up to 10 times more damaging compared to beta radiations or gamma-rays.

Because of their exceptional ability to penetrate other materials, neutrons can travel great distances in air and require very thick hydrogen-containing materials (such as concrete or water) to block them. Fortunately, however, neutron radiation primarily occurs inside a nuclear reactor, where many feet of water provide effective shielding.

Source: https://www.nrc.gov/about-nrc/radiation/rad-health-effects.html

Refer to the following links for more in-depth discussion:

http://en.wikipedia.org/wiki/Nuclear_reactor

http://en.wikipedia.org/wiki/Nuclear_weapon

https://en.wikipedia.org/wiki/Neutron_radiation

ATLAS – The Particles Strike Back

https://www.youtube.com/watch?v=iYRQpcJVQx8

NOTETAKING

"If an apple is magnified to the size of the earth, then the atoms in the apple are approximately the size of the original apple."