18 ELECTROMAGNETIC INDUCTION

H2 Physics 9478

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Learning Outcomes

Candidates should be able to:

(a) define magnetic flux as the product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density.

(b) show an understanding of and use the concept of magnetic flux linkage.

(c) recall and use $\phi = BA$ and $N\phi = NBA$ to solve problems, where N is the number of turns.

(d) infer from appropriate experiments on electromagnetic induction:
  (i) that a changing magnetic flux can induce an e.m.f. ;
  (ii) that the direction of the induced e.m.f. opposes the change producing it ;
  (iii) the factors affecting the magnitude of the induced e.m.f.

(e) recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law.

(f) explain simple applications of electromagnetic induction.

(g) show an understanding of the principle of operation of a simple iron-core transformer and recall and solve problems using $\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_P}{I_s}$ for an ideal transformer.

Introduction

We began our study on the relationship between electricity and magnetism in the previous chapter where we learned that an electric current (or moving charges) produces a magnetic field. As a result, when a current-carrying conductor or a moving charge is within a magnetic field, it will experience a force.

How electromagnetic induction was discovered

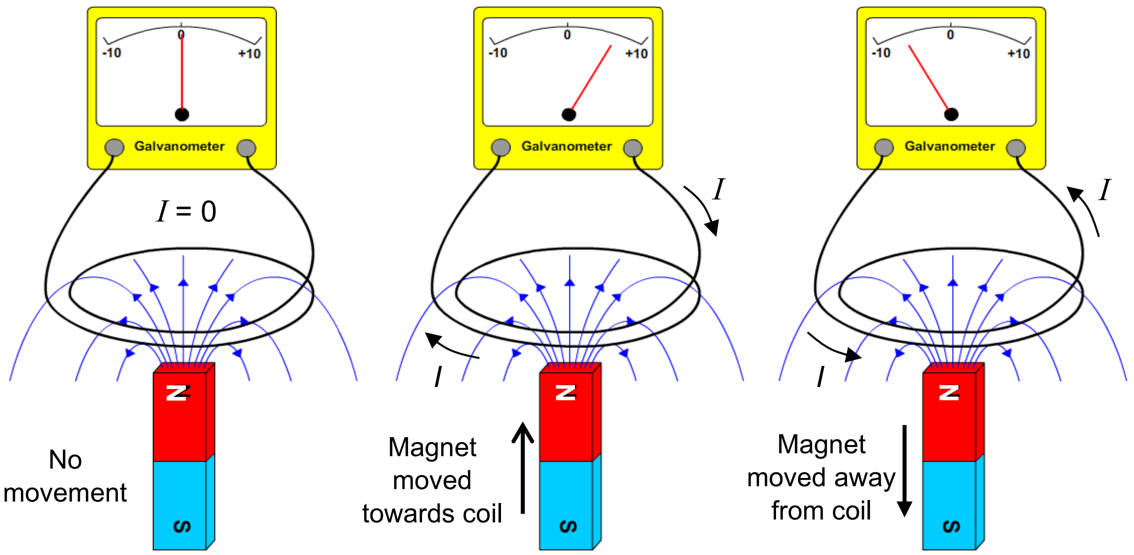
In this chapter, we will see that the converse happens. Experiments conducted by Michael Faraday in England in 1831 and independently by Joseph Henry in the United States the same year demonstrated that an e.m.f. could be induced by a changing magnetic field. This effect is known as **electromagnetic induction.**

This discovery greatly revolutionized the production of electricity and has a significant impact on our daily lives.

18.1 Faraday's Law of Electromagnetic Induction

❖ Observing Electromagnetic Induction

Electromagnetic induction can be demonstrated easily using the laboratory apparatus shown in Fig. 18.1.



(a) (b) (c)

Fig. 18.1

A bar magnet is moved towards or away from a coil of wire which is connected to a sensitive galvanometer. It is found that:

- **no current** is generated when the magnet is kept stationary relative to the coil (Fig. 18.1(a)),
- **a current is induced** in the coil when the magnet is moved towards (Fig. 18.1(b)) or moved away (Fig. 18.1(c)) from the coil, and the **directions** of the currents in the two cases are opposite.
- the **magnitude** of the induced current increases as the magnet moves faster.

It is also found that

- a current is induced when the coil is moved towards or away from the magnet.
- a current is induced when the coil is pulled at 2 ends such that its cross sectional area decreases.

The observations in the simple demonstration in Fig. 18.1 show that a current is induced as long as there is some change in the magnetic field through the coil. Such an induced current must be produced by an induced e.m.f. This forms the basis of Faraday's law of electromagnetic induction.

Faraday's law of electromagnetic induction

Faraday's law of electromagnetic induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.

The law can be expressed mathematically as:

$$\text{induced e.m.f.} = -\frac{d(N\phi)}{dt}$$

Formula

where $N\phi$ is the **magnetic flux linkage** of a coil or circuit.

To understand the law, we need to know the meaning of magnetic flux and magnetic flux linkage.

18.2 Magnetic Flux and Magnetic Flux Linkage

❖ Magnetic Flux, ϕ

Consider an area A where a uniform magnetic field of magnetic flux density B passes through at an angle θ to the normal of the area.

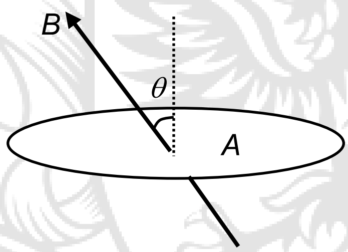


Fig. 18.2

Magnetic Flux

Magnetic flux is defined as the product of the magnetic flux density (through an area) and the cross-sectional area perpendicular to the direction of the magnetic flux density.

Magnetic flux can be thought of as the number of magnetic field lines passing through an area. This clearly depends on

(i) B, how strong the magnetic flux density is (stronger fields are represented by closer lines),
(ii) A, how big the area is, and
(iii) θ, the angle between the area and the magnetic field.

The expression for magnetic flux ϕ is

$$\phi = BA\cos\theta$$

Formula

where θ is the angle that the magnetic flux density vector B makes with the normal to the area A. See Fig. 18.3 and Fig. 18.4.

Mathematically, this relation can also be expressed in vector notation as $\phi = \vec{B} \cdot \vec{A}$, where $\vec{A}$ is the area vector that has a magnitude that is given by the area A and a direction that is normal to the area.

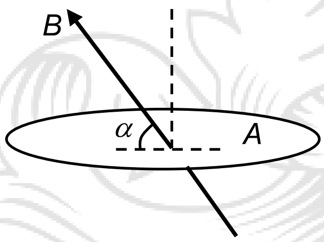


Fig. 18.3

If the angle that B makes with the plane of A is given instead,
$\phi = BA \sin \alpha$

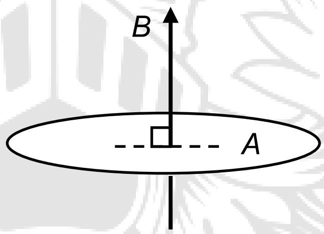


Fig. 18.4

If B passes perpendicularly through A, i.e. $\alpha = 90^{\circ}$ or $\theta = 0^{\circ}$,
$\phi = BA$

❖ Magnetic Flux Linkage, $N\phi$

Suppose we have a coil of N turns and uniform cross-sectional area.

> **Magnetic Flux Linkage**
>
> The magnetic flux linkage of a coil is defined as the product of the magnetic flux through the coil and the number of turns of the coil.

The expression of magnetic flux linkage is

$$\text{magnetic flux linkage} = N\phi$$

Formula

The S.I. unit for both ϕ and $N\phi$ is the weber (Wb).

Since $N\phi = NBA\cos\theta$, changing any of the quantities N, B, A and θ will change the magnetic flux linkage of a coil. Making reference to Faraday's law, e.m.f. can be induced in a coil or circuit when one or more of the following changes are made:

NOTE!!

- magnitude and direction of B varies with time.
- area A of the coil or circuit varies with time.
- angle θ between B and the normal to the plane varies with time.

Example 1

A coil A experiences a change in magnetic flux linkage from 0 to 0.20 Wb at a constant rate in 2.0 s. Another coil B experiences a change in magnetic flux linkage from 0 Wb to 0.50 Wb at a constant rate in 10.0 s. Calculate the magnitude of induced e.m.f. in each coil.

Solution

Induced emf in coil A =

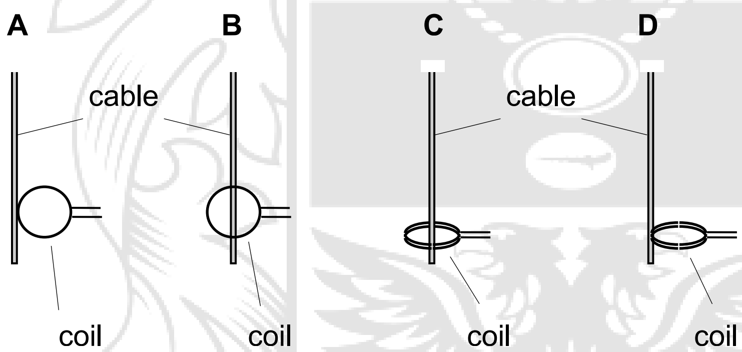
Induced emf in coil B =

It can be seen that although coil B experiences a ___________ change in magnetic flux linkage, it has a ____________ induced e.m.f. The size of the induced e.m.f. is determined by the ________ of change of magnetic flux, not solely on how big the change in magnetic flux is.

Example 2

Large alternating currents in a cable can be measured by monitoring the e.m.f. induced in a small coil situated near the cable. This e.m.f. is induced by the varying magnetic field set up around the cable.

In which arrangement of coil and cable will the e.m.f. induced be a maximum?



Solution

The coil in ___ has the largest amount of magnetic flux through it. Hence, it will experience the largest change in magnetic flux linkage when the direction of current in the cable changes.

The net magnetic flux through ___ is zero because the magnetic field in one half is pointing ______ of the page, while that in the other half is pointing ______ the page.

The magnetic flux through C and D is also ______ as the magnetic field is ___________ to the plane of the coil.

Hence the answer is ___ .

18.3 Direction of Induced e.m.f. (or Current)

Faraday's law allows us to calculate the magnitude of the induced e.m.f. The direction of this induced e.m.f. can be determined in 3 ways:

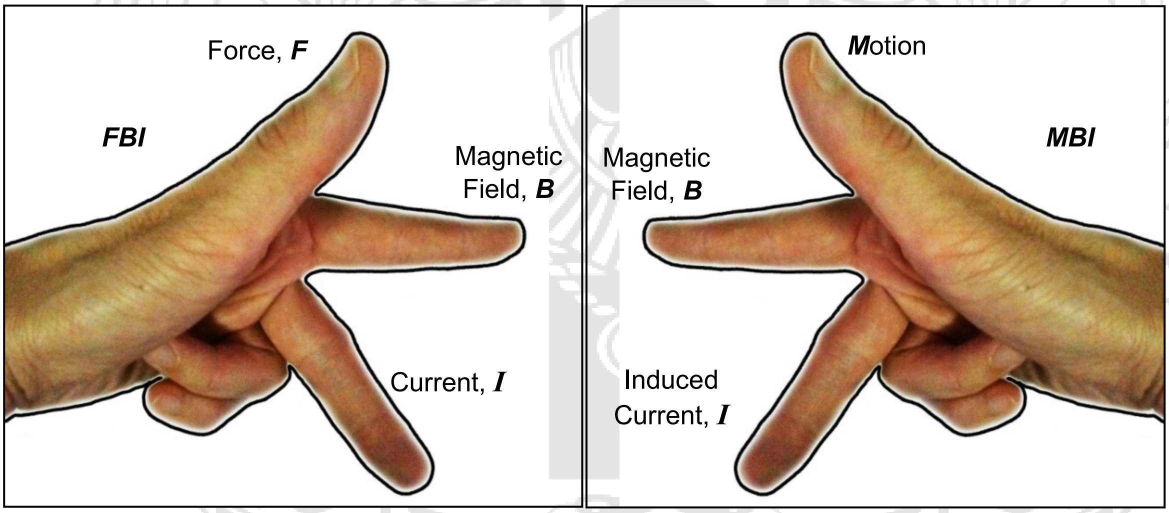
1. Fleming's right-hand rule *(this is typically used as a quick check and should not be used to explain the direction of induced e.m.f.)*.
2. Lenz's Law.
3. From first principles *(to be covered in section 18.4)*.

Faraday's law states that there will be an induced e.m.f. when there is a change in magnetic flux linkage. This does not always mean that there will be an induced current. An induced current exists only when there is a closed circuit.

NOTE!!

❖ 1. Fleming's Right-Hand Rule

This is similar to Fleming's Left-Hand Rule, except that you use your RIGHT HAND. The thumb denotes the direction of motion (of conductor), while the first two fingers denote the directions of the magnetic field and the induced current, respectively.



(a) Fleming's Left-hand rule
(to find direction of force)

(b) Fleming's Right-hand rule
(to find direction of induced current)

Fig. 18.5

❖ 2. Lenz's Law

In the demonstration in Fig. 18.1, the direction of the induced current was observed to depend on the direction of movement of the magnet.

- In Fig. 18.1(b), when the north pole of the magnet is pushed towards the coil, the induced current in the coil is in a direction to produce a north pole facing the magnet so as to oppose its motion.
- In Fig. 18.1(c), when the north pole of the magnet is pulled away from the coil, the induced current in the coil is in a direction to produce a south pole facing the magnet once again to oppose its motion.

It appeared that **the induced current is always in a direction so as to oppose the change that produces it.**

Lenz's Law

Lenz's law states that the direction of induced e.m.f. is such as to cause effects to oppose the change producing it.

Lenz's Law is a consequence of the **principle of conservation of energy**. (See Example 3 below.)

Example 3

With reference to Fig. 18.1(b) and Fig. 18.1(c),

(a) use Lenz's Law to determine the direction of the induced current in the coil,

(b) explain how the direction of current is consistent with the principle of conservation of energy.

Solution

(a) Fig. 18.1(b): as the North pole of the magnet **approaches** the coil, a ________-pole is induced at the end of the coil that is facing the incoming magnet (repulsion).

This is so as to oppose the ____________ in the magnetic flux linkage through the coil.

Using right-hand grip rule with the thumb pointing downwards, the direction of induced current is _________________ as shown.

Fig. 18.1(c): as North pole of the magnet **moves away** from the coil, a _________-pole is induced at the end of the coil that is facing the receding magnet (attraction). This is to oppose the _______________ in the magnetic flux through the coil. Using right-hand grip rule with the thumb pointing upwards, the direction of induced current is ___________________ as shown.

(b) Due to Lenz's law, the induced current in the coil is in a direction such that a _____________ force is produced to _____________ the movement of the magnet (either to repel or attract).

A force has to be applied to overcome this magnetic force. The **work done by this applied force** gives rise to the electrical energy generated.

In other words, _____________ energy is converted to _______________ energy. This is a consequence of the principle of conservation of energy.

Example 4

In Fig. 18.6, an induced current is momentarily produced in the secondary coil when current in the primary coil is switched on or off.

(a) Using Faraday's law, explain why, in the secondary coil,
  (i) an induced current is produced,
  (ii) this induced current exists only momentarily.

(b) Using Lenz's law, determine the directions of the induced current through the galvonometer in the secondary coil when
  (i) the switch connected to the primary coil is closed,
  (ii) the switch connected to the primary coil is opened.

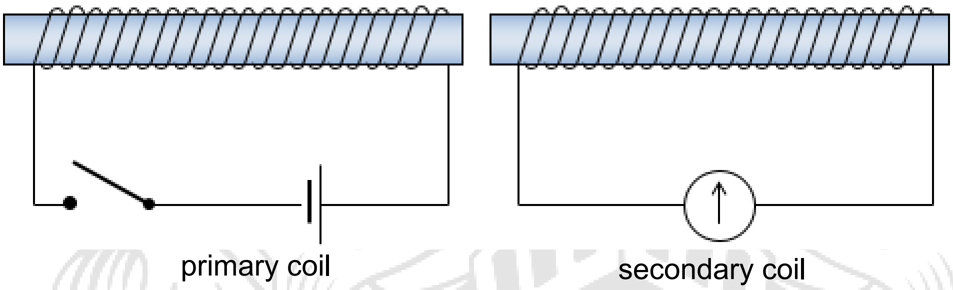


Fig. 18.6

Solution

(a)(i) When the switch is closed or opened, an ______________ or ______________ current flows through the ______________ coil, producing a change in the magnetic flux of the primary coil. The ______________ coil experiences a change in magnetic flux linkage. By Faraday's Law, an e.m.f. is induced in the secondary coil which drives an induced current through the closed circuit.

(ii) When the current (and hence magnetic field around the primary coil) in the primary coil reaches a ____________________, the secondary coil no longer experiences a changing magnetic flux linkage. Hence, there is no induced e.m.f. or induced current in the secondary coil

(b)(i) The secondary coil experiences an ____________ in magnetic flux linkage. The induced current is in a direction to produce a __________ pole on its left to _________ the primary coil so as to oppose this increase in magnetic flux linkage. Hence, the direction of the induced current is ___________ through the galvanometer.

(ii) The secondary coil experiences a _____________ in magnetic flux linkage. The induced current in the secondary coil is in a direction to produce a south pole on its left to ___________ the primary coil so as to oppose this decrease in magnetic flux linkage. Hence, the direction of the induced current is ___________ through the galvanometer.

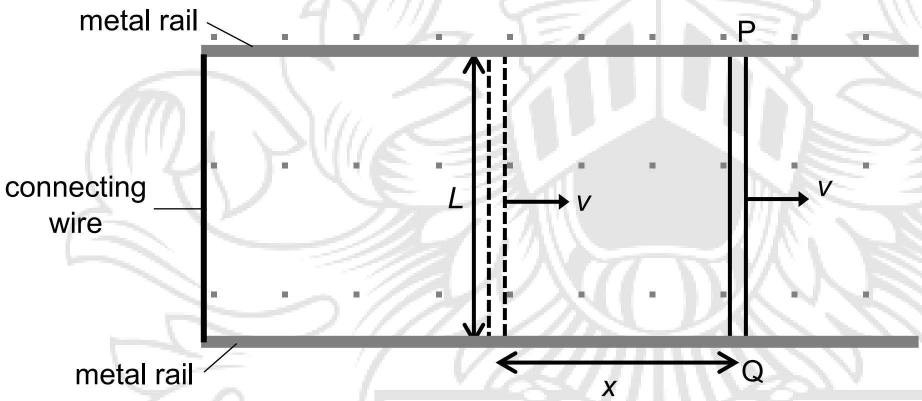
❖ 3. Determining Direction of Induced e.m.f. from First Principles

The third method requires an understanding of the cause of the motion of charge carriers in a conductor when there is relative motion between the conductor and a magnetic field. This is described in detail in section 18.4.

18.4 Metal Rod Moving across a Uniform Magnetic Field

❖ Induced e.m.f.

Fig. 18.7 shows a metal rod PQ of length L sliding along two frictionless metal rails with constant velocity v to the right across a uniform magnetic field of flux density B acting perpendicularly out of the plane of the paper. The metal rails are connected by a connecting wire such that the rod, the rails and wire form a closed circuit.



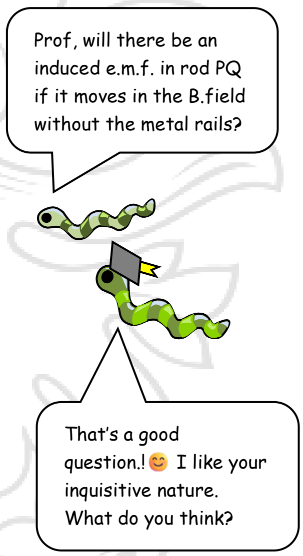


Fig. 18.7

The distance travelled by the rod in time t is x. As the rod cuts across the magnetic field, the change in magnetic flux linkage through the circuit in time t is given by

$$\Delta(N\phi) = NBA - 0 = BLx$$

By Faraday's Law, the magnitude of the induced e.m.f. across the rod is given by

$$|E| = \left|-\frac{\Delta(N\phi)}{\Delta t}\right| = BL\frac{x}{t} = BLv$$

The above expression tells us that the magnitude of the induced e.m.f., and hence current, depends on the speed at which the rod is moving across the magnetic field, the magnetic flux density and the length of the rod in the magnetic field (and between the rails).

❖ Direction of Induced Current

We can use the methods stated in section 18.3 to determine the direction of the induced current.

1. Fleming's Right Hand Rule

- Thumb – points to the right to indicate direction of motion of the rod
- First finger – points out of the paper to represent direction of B
- Second finger – points in the direction PQ along the rod to represent direction of the induced current

As the rod is a source of e.m.f., end Q is at higher potential with respect to end P.

In exams, Fleming's right-hand rule cannot be used to explain the direction of induced e.m.f. or induced current.

NOTE!!

2. Lenz's Law

As the rod moves to the right, there is an increase in the magnetic flux through the circuit (as area is increasing). By Lenz's law, the induced current would give rise to a magnetic force (to the left) on the rod that opposes its rightward movement. Using **Fleming's Left Hand rule**, the induced current is in the direction P to Q along the rod.

The leftward magnetic force will cause the rod to slow down. In order to keep the rod in constant motion, an external force that is of the same magnitude as the induced magnetic force has to be applied.

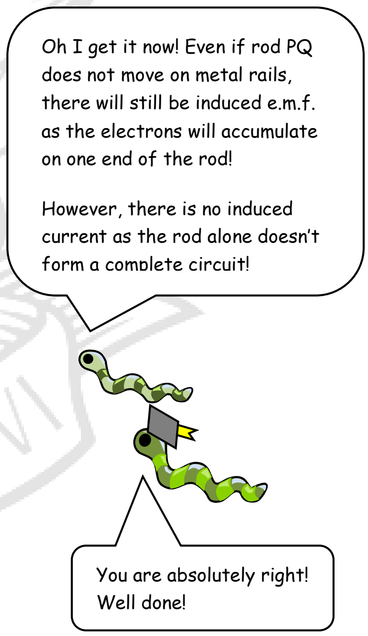
NOTE!!

Alternatively, this can be explained using energy approach. Due to induced current in the circuit, electrical energy is dissipated (I^2R). By the principle of conservation of energy, the kinetic energy of the rod decreases. In order to maintain the motion of the rod, work has to be done by an external force on the rod.

3. From First Principles

As the rod moves to the right, the free electrons inside the rod would also be moving towards the right as shown in Fig. 18.8.

Since the electrons are moving towards the right, the direction of conventional current due to the motion of these electrons is towards the left as shown in Fig. 18.9.



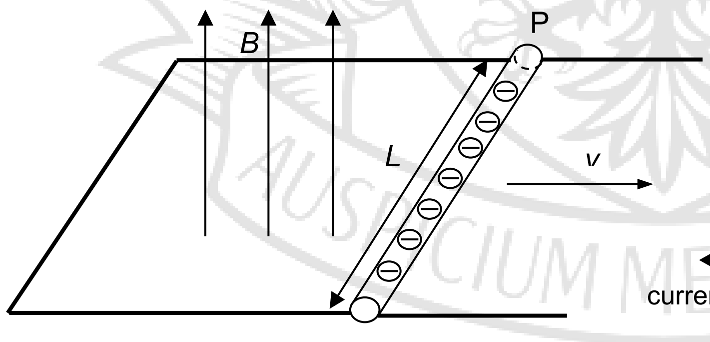


Fig. 18.8

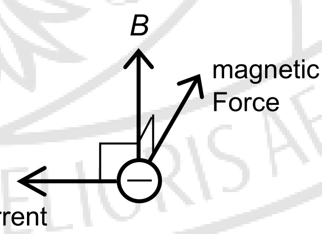


Fig. 18.9

Using Fleming's Left-Hand Rule, we can deduce that the electrons in the metal rod will experience a magnetic force in the direction from Q to P as shown in Fig. 18.9.

As a result, the electrons in the rod move towards end P. This is equivalent to a conventional current in the direction from P to Q. Hence, a current is induced in a clockwise direction around the circuit.

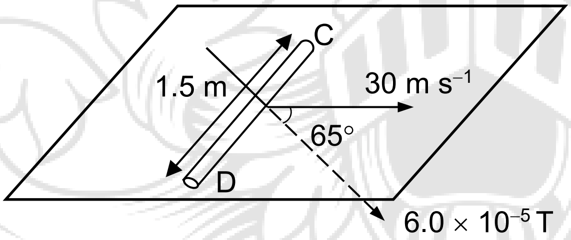
This is consistent with Fleming's Right-Hand Rule and Lenz's Law.

Example 5

A 1.5 m rod CD is moving at 30 m s^{-1} horizontally to the right through a region where the Earth's magnetic field of flux density 6.0×10^{-5} T acts downwards at 65° to the horizontal.

(a) Calculate the induced e.m.f. across the rod.

(b) Which end of CD is at a higher potential?



Solution

(a)

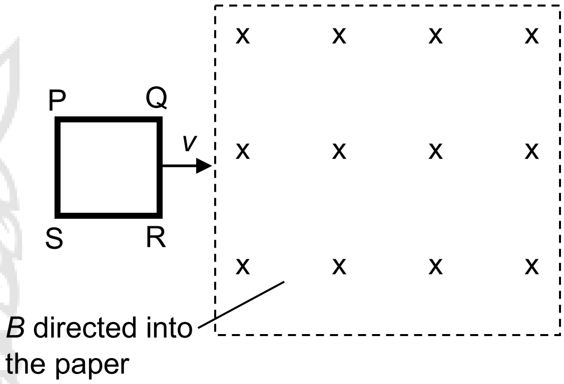
(b) Induced current would tend to be in the direction from ____ to ___. Hence, ____ would be at a higher potential. (There is no induced current in the moving rod as it does not form a complete circuit)

OR

Using Fleming's Left-Hand Rule, the electrons in the rod would be pushed towards D. Hence ___ would be at a higher potential.

Example 6

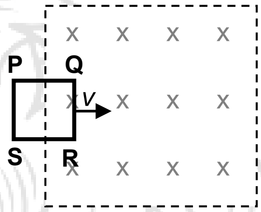
A square wire frame PQRS with sides L of 0.10 m, moves with a constant velocity v of 0.20 m s^{-1} into a region of uniform magnetic flux density B of 1.0×10^{-2} T acting into the plane of the paper as shown. The resistance of the frame is 5.0 Ω.



Deduce the magnitude and direction of the induced current in the frame when
(a) QR just enters the field,
(b) the whole frame moves within the field, and
(c) QR just exits the field.

Solution

(a) When QR just enters the field, the e.m.f. E induced along QR is given by ____________. Hence,



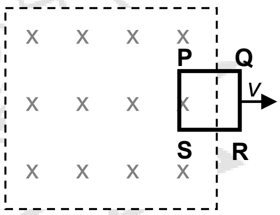
$I =$

By Lenz's law, the induced current will produce a leftward force on QR to oppose its motion. Using Fleming's Left-Hand Rule, the direction of induced current is from __ to __, i.e., _________________ around the frame.

OR The frame experiences an increasing magnetic flux as it enters the field. By Lenz's law, the induced current will produce a magnetic field pointing out of the page to oppose the increase in magnetic flux. Using the right-hand grip rule (thumb pointing out of page), the direction of the induced current is anticlockwise around the frame.

(b) When the whole frame is within the field, the magnetic flux through PQRS is _____________. Hence, there is no induced e.m.f. and therefore no induced current.

(c) When QR just exits the field, an e.m.f. is induced along PS. The magnitude of the induced current is the ___________ as (a), i.e., 4.0×10^{-5} A.

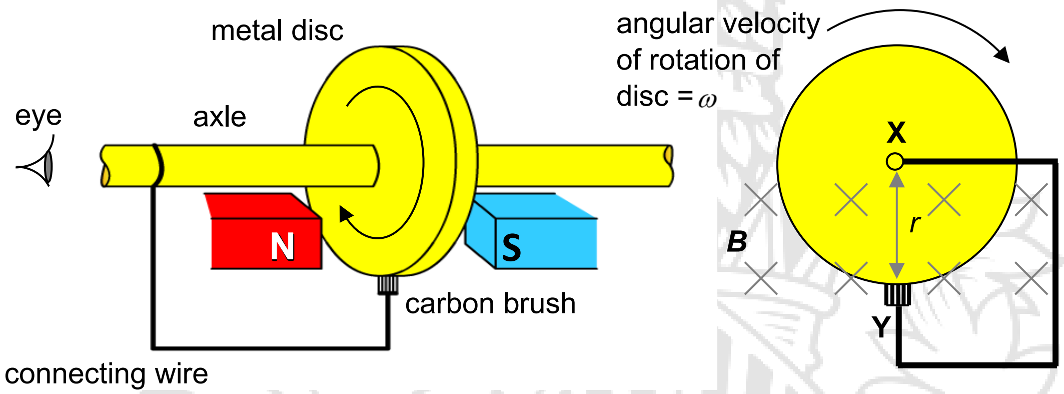


By Lenz's law, the induced current will produce a leftward force on PS to oppose its motion. Using Fleming's Left-Hand Rule, the direction of induced current is from ___ to ____, i.e., ________________ around the frame.

OR The frame experiences a decreasing magnetic flux as it exits the field. By Lenz's law, the induced current produces a magnetic field pointing into the page to oppose the decrease in magnetic flux. Using the right-hand grip rule (thumb pointing into page), the direction of the induced current is clockwise around the frame.

18.5 Rotating Disc in a Uniform Magnetic Field

A metal disc of radius *r* is mounted on an axle and rotates with angular velocity ω in a clockwise direction as shown in Fig. 18.10. With magnets installed beneath the axle, the bottom part of the disc between the axle and carbon brush is like a straight conductor of length *r* that continuously cuts the magnetic field of flux density *B* between the poles of the magnets.



(a) **(b) from the eye's view**

Fig. 18.10

Assume that the disc is made up of straight radial conductors each of length *r*.

Consider one such conductor connected between X and Y as shown in Fig. 18.10(b). At this instant, this conductor is moving to the left (disc is rotating clockwise).

At point Y of the rod, it is moving with a speed of $v = r\omega$ and at point X of the rod, its speed is 0. Hence the average speed of the rod is

$$\frac{v+0}{2} = \frac{r\omega}{2}$$

By Faraday's law, the magnitude of the induced e.m.f. between the axle (X) and rim (Y) of the disc is

$$E = BLv_{\text{average}} = Br\left(\frac{r\omega}{2}\right) = \frac{1}{2}Br^2\omega$$

Alternatively, since $\omega = 2\pi f$, we have

$$E = \frac{1}{2}Br^2(2\pi f) = B\pi r^2 f = BAf$$

where *A* is the area of the disc and *f* is the frequency of rotation.

Similar to the moving rod example in Section 18.4, the disc will start to slow down when there is induced current in the disc.

Example 7

Referring to Fig. 18.10(b), determine the direction of the induced current through the rotating disc.

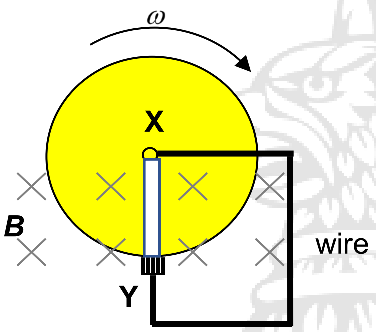


Fig. 18.10(b)

Solution

1. Fleming's Right Hand Rule

Using Fleming's Right Hand Rule, the induced current is in the direction ____ to __ through the conductor, in an __________________ direction through the connecting wire.

2. Lenz's Law

The induced current is in a direction so as to produce a _________ to _______________ the clockwise rotation of the disc. Using Fleming's Left Hand Rule, in order to produce a magnetic force to act towards the right, the induced current must be in the direction X to Y through the conductor.

3. First Principles

The free electrons in the conductor XY are moving to the left as the disc rotates clockwise. This means that the conventional current is to the right. Using Fleming's Left Hand Rule, the magnetic ____________ on the electrons acts ______________. An electric field is set up across XY, such that ____ is at a higher potential with respect to ____. Hence, induced current is in the direction X to Y through the conductor.

18.6 Rotating Coil in a Uniform Magnetic Field (AC Generator)

A coil of N turns, each of area A, is being rotated at a constant angular speed ω in a uniform magnetic field of flux density B as shown in Fig. 18.11.

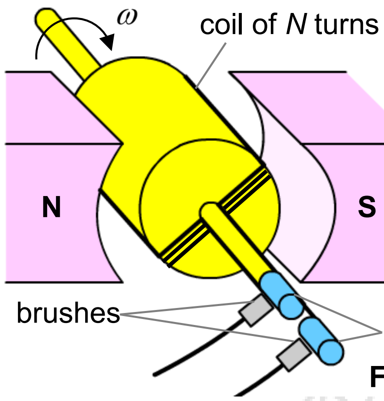


Fig. 18.11

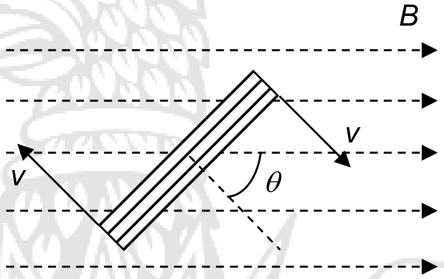


Fig. 18.12

In Fig. 18.12, the normal to the plane of the coil makes an angle θ with the magnetic field at time t ($\theta = 0°$ when t = 0 s). The magnetic flux linkage through the coil is

$$N\phi = NBA\cos\theta = NBA\cos\omega t$$

By Faraday's law,

$$E = -\frac{d(N\phi)}{dt} = -\frac{d(NBA\cos\omega t)}{dt} = NBA\omega\sin\omega t$$

This expression shows that the induced e.m.f. in the coil varies sinusoidally with time.

The maximum induced e.m.f. is

$$E_{max} = NBA\omega$$

Alternating current is generated by electromagnetic induction using this method.

Fig. 18.13 shows the variation with time t of the magnetic flux linkage $N\phi$ and induced e.m.f. E in the coil as the it rotates 1 complete cycle.

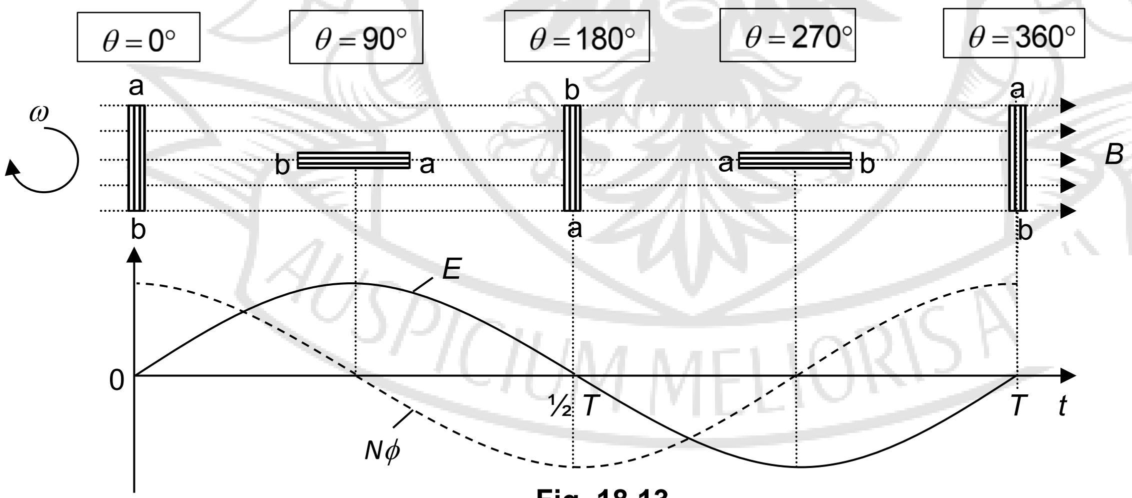
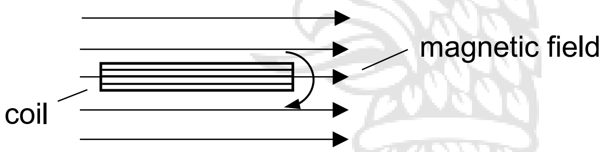


Fig. 18.13

Example 8

An AC generator consists of a coil of eight turns of wire each of area 0.090 m^2 and a total resistance of 12.0 Ω. The coil rotates in a uniform magnetic field of flux density 0.500 T at a constant frequency of 60.0 Hz. Figure shows the position of the loop with respect to the field at t = 0 s.



(d) Determine
  (i) the angular frequency of rotation ω,
  (ii) the maximum induced e.m.f. E_{max},
  (iii) the maximum induced current I_{max}.

(e) Using the values in **(a)**, state expressions to represent the variation with time t of the induced e.m.f. E and current I through the loop.

Solution

(a) (i)

(ii)

(iii)

(b) At t = 0 s, there is ________ magnetic flux linkage. Hence, the variation of magnetic flux linkage with time is given by $N\phi = NBA\sin(377t)$.

Since $E = -\dfrac{d(N\phi)}{dt}$,

$E = -136\cos(377t)$

$I = -11.3\cos(377t)$

❖ **Back e.m.f.**

In the previous chapter, we saw how electrical energy is converted into mechanical energy in an electric motor. The torque acting on a current-carrying coil in a uniform magnetic field causes it to rotate.

However, the magnetic flux linkage of the coil changes as it rotates. According to Faraday's law, an e.m.f. would be induced in the coil. By Lenz's law, this induced e.m.f. always acts to reduce the current supplied to the coil or the motor. This is known as the **back e.m.f.**, and its magnitude increases as the rotational speed of the coil increases.

As such, the effective current I_{net} in the coil would decrease as the rotational speed increases.

$$I_{net} = I_{\text{supplied to motor}} - I_{\text{due to back e.m.f.}}$$

Example 9

A potential difference of 120 V is applied to a motor having coils that have a total resistance of 10 Ω. When the motor is running at its maximum speed, the back e.m.f. is 70 V. Find the current in the coils when

(a) the motor is first turned on,
(b) the motor has reached maximum speed.

Solution

(a) When the motor is first turned on, there is no back e.m.f.

(b) When the motor has reached maximum speed,

18.7 Practical Applications of Electromagnetic Induction

❖ AC Generators

As discussed earlier, alternating current (AC) is generated by rotating a coil in a uniform magnetic field. Today, almost all power stations generate electricity using AC generators. The mechanical energy required to rotate the coil can be derived from a variety of sources (e.g. falling water, burning coal, etc).

Alternating current is preferred to direct current (DC) in the distribution of electricity because using AC is much more efficient due to the ease of stepping up and stepping down the transmission voltage.

❖ Wireless Charging

Many of our portable devices (e.g. smartwatches, smartphones, electric toothbrush, iron, portable vacuum cleaner, etc) make use of wireless charging. This is made possible by the same principle behind transformers.

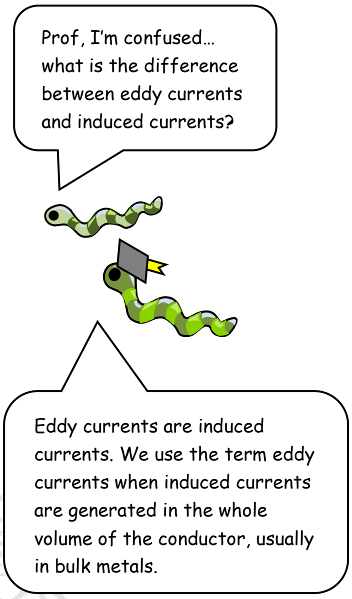
The wireless charging station consists of a primary coil. When an alternating current passes through the primary coil, a changing magnetic flux induces an e.m.f. in a secondary coil which is situated within the device to be charged.

❖ Eddy Currents and their Applications

Eddy currents are loops of induced current that circulate in a conductor of extended size when there is a change in the magnetic flux through the conductor.

By Lenz's law, eddy currents is in a direction that oppose the change that gives rise to them.

For example, in Fig, 18.14(a), if a rectangular aluminum metal plate is set swinging between the poles of a strong magnet, it soon comes to rest because the eddy currents circulating inside the metal plate oppose its motion.



If many slots are cut into the plate, as shown in Fig. 18.14(b), the plate oscillates for a longer time before coming to rest. Eddy currents are considerably reduced in the slotted plate as they cannot flow across the many air gaps formed by the slots.

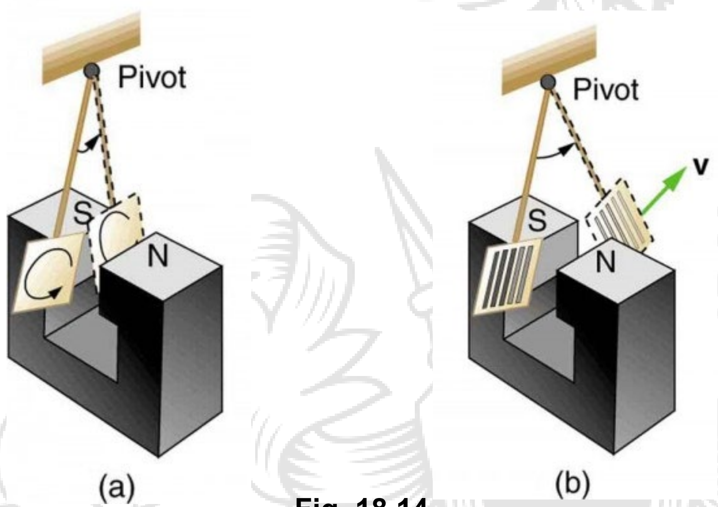


Fig. 18.14

❖ Induction Furnace

Eddy currents provide a useful means of heating a metallic substance (which is inaccessible to other ways of heating) to a very high temperature in an induction furnace. The sample is placed in the centre of a coil that carries a high frequency current. This is known as induction heating.

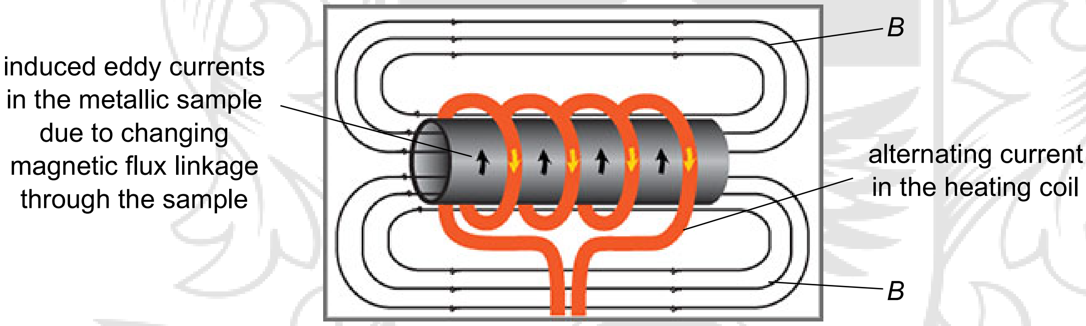


Fig. 18.5

❖ Induction Cookers

An induction cooker consists of a coil of copper wire underneath the ceramic surface. When there is an alternating current in the coil, it produces an alternating magnetic field all around it.

When a frying pan made of ferromagnetic material is placed on the cooking zone, the changing magnetic flux through the base of the pan induces eddy currents in the pan. Heat from the pan then transfers to the liquid or food in it.

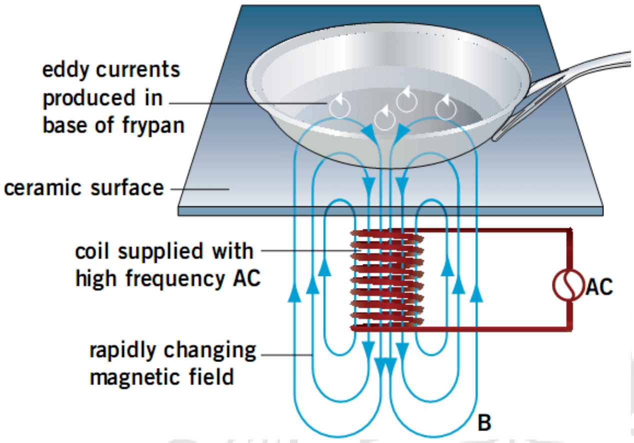


Fig. 18.16

Some advantages of induction cookers over gas stoves:
- It is deemed safer as there is no naked flame or flammable gas.
- It is more energy efficient, hence more cost-savings.
- It enables even cooking as the eddy currents are induced throughout the base of the pot, eliminating the occurrence of hot spots which may burn or scorch the food.

❖ Eddy Current / Electromagnetic Braking

Eddy current braking in vehicles is an application of the rotating disc in a magnetic field discussed in Section 18.5.

The setup is similar to that shown in Fig. 18.10, except that the magnetic field is produced by an electromagnet. When the vehicle is moving, the braking disc rotates. When the brakes are applied, the electromagnet is activated. As the rotating disc now cuts the magnetic field, eddy currents are induced in the disc. These eddy currents generate a magnetic force that acts as the braking force. This is an example of an application of Lenz's law.

One form of electromagnetic braking is regenerative braking as the induced current can be channelled back to charge up the car battery.

How regenerative braking system works.

This frictionless braking system has the following advantages over conventional brakes:
- lesser heat is produced.
- easier maintenance as there is lesser wear and tear of parts.
- braking force is not affected by wet conditions.
- the setup in Fig. 18.10 can be modified into a regenerative braking system where energy can be recovered during braking.

Regenerative brakes are very useful for slowing down high speed vehicles such as trains. However, as the disc slows down, the induced eddy currents would become smaller, making the braking less efficient. Hence, it is not used in the parking brake of a car.

- **Metal Detectors**

A metal detector consists of a coil of wire known as the transmitter coil. When an alternating current passes through the transmitter coil, a changing magnetic field is set up. When the coil is placed over a metal object, eddy currents are induced in the object. As a result, the object produces its own magnetic field.

Inside the metal detector lies another coil known as the receiver coil. The magnetic field of the object induces a current in the receiver coil, which causes a loudspeaker to produce a sound.

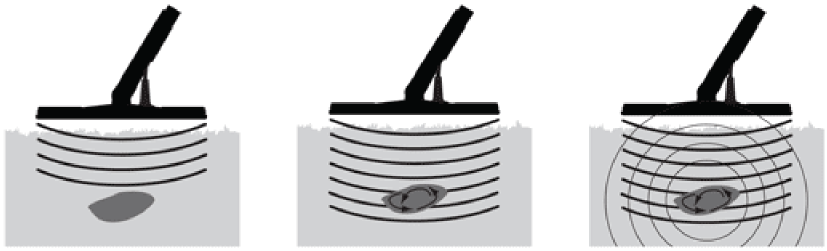


Fig. 18.17

18.8 Transformers

A transformer is another application of electromagnetic induction that is widely used in power transmission and electrical appliances.

Function: It uses mutual electromagnetic induction to vary (either step up or step down) an alternating voltage.

Structure: Fig. 18.18 shows a schematic of a simple step up transformer.

It consists of two coils of wire wound around the same soft iron core.

The coil that is connected to the a.c. voltage source is known as the primary coil and it has N_P turns.

The coil that is connected to the load is known as the secondary coil and it has N_S turns.

The soft iron core concentrates the magnetic flux density through both the primary and secondary coils, and it is typically made from 'E' and 'I' shaped laminated sheets so that induced current cannot flow across the sheets.

Principle: The a.c. voltage source (primary voltage) V_P causes an alternating current I_P in the primary coil.

This sets up a varying magnetic field in the iron core which links the primary coil with the secondary coil.

In accordance to Faraday's law of electromagnetic induction, the changing magnetic field induces an alternating e.m.f. across each turn of wire in both the primary and secondary coils.

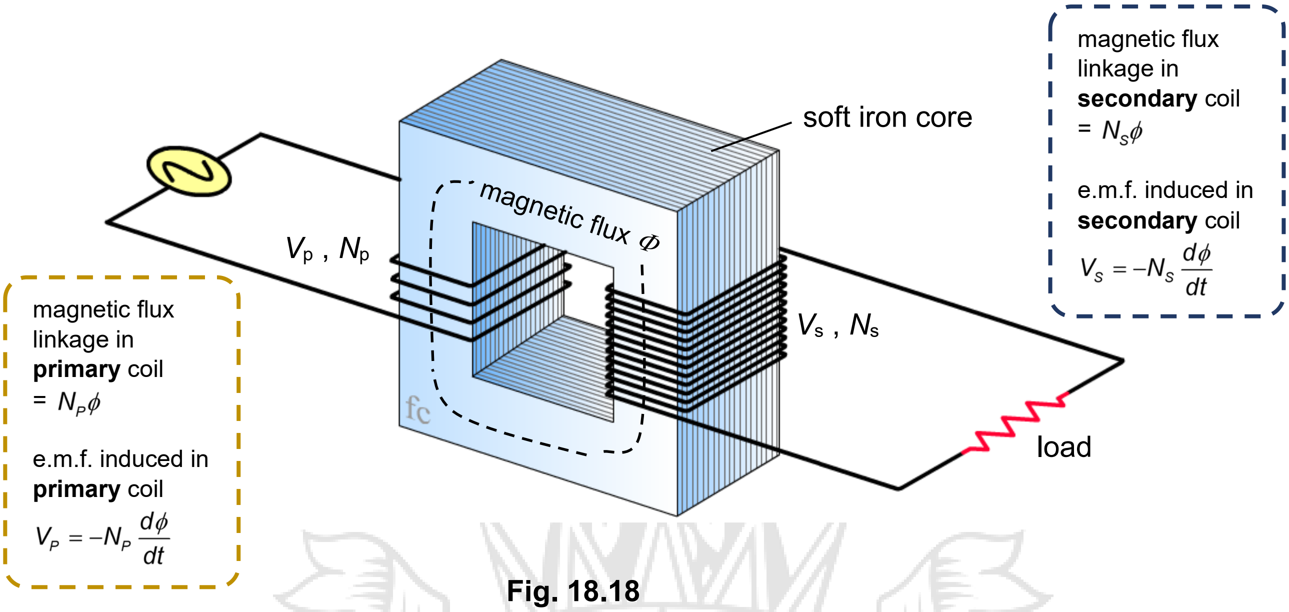


Fig. 18.18

In an ideal transformer (i.e.100% efficient), during operation,

- there is no energy loss in the coils and in the core, and
- there is no flux leakage.

Hence, **the magnetic flux ϕ through each turn is the same in both the primary and secondary coils**.

Since magnetic flux ϕ through each turn is the same for both the primary and secondary coils, the rate of change of magnetic flux is the same.

$$\frac{V_P}{N_P} = \frac{V_S}{N_S} \Rightarrow \frac{N_S}{N_P} = \frac{V_S}{V_P}$$

By selecting an appropriate turns-ratio $\frac{N_S}{N_P}$, the desired output or secondary voltage can be obtained from a given input or primary voltage.

Step-up transformer: $N_S > N_P$, $V_S > V_P$
Step-down transformer: $N_S < N_P$, $V_S < V_P$

In an ideal transformer, since there is no energy loss in both the core and the coils,

$$\text{input power} = \text{output power}$$

$$I_P V_P = I_S V_S$$

$$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$

Therefore,

$$\boxed{\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}}$$

Formula

❖ **Power Loss in Transformers (see Appendix C for details)**

Cause of power loss in a transformer	Design features to reduce power loss
Joule heating of copper wires of the coils.	Thick copper wires of low resistance are used, particularly for the coil carrying high current at low voltage.
Heating effect due to **eddy currents** induced in the iron core.	The iron core is made of laminated sheets, cutting across the path of eddy currents, to increase resistance to current flow.
Hysteresis loss Energy is used in magnetising the iron core and reversing this magnetisation every time the primary current reverses. This heats up the iron core.	The iron core is made of soft iron ("annealed iron" which has high values of permeability and lower reluctance), which can be easily magnetised and demagnetised by the magnetic field of the primary coil.
Magnetic flux leakage Some of the magnetic field lines produced by the primary coil do not link with the secondary coil, reducing the e.m.f. induced in the secondary coil.	The presence of iron core maximises the magnetic flux linkage between the primary and secondary coils. Magnetic flux leakage can also be minimized by having a good design. In the 'E-I' shaped iron core, the secondary coil is wound on top of both the primary coil and the iron core to form a closed loop.

❖ **Transmission of electrical power**

When it comes to heating and lighting applications, a.c. has no practical advantage over d.c. because the heating effect of a current is independent of its direction. In fact, many practical systems like planes, light rail system, computers and consumer electronics are powered by direct current.

The main advantage of using a.c. is that it can be stepped up or down easily using a transformer compared to d.c. for high voltage transmission. High voltage transmission of electrical power is more efficient due to low power losses in the cables.

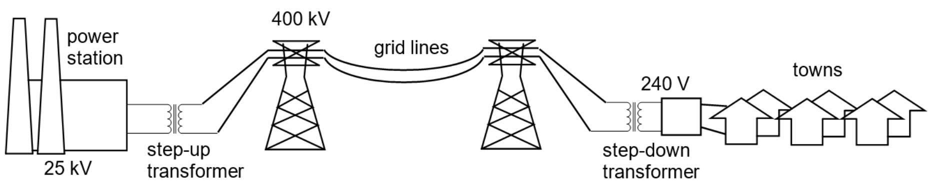


Fig. 18.19

Consider a transmission line supplying electrical power P from a generating (power) station to consumers (e.g. homes, commercial buildings, factories). Fig. 18.20 shows a simplified schematic of the power transmission system.

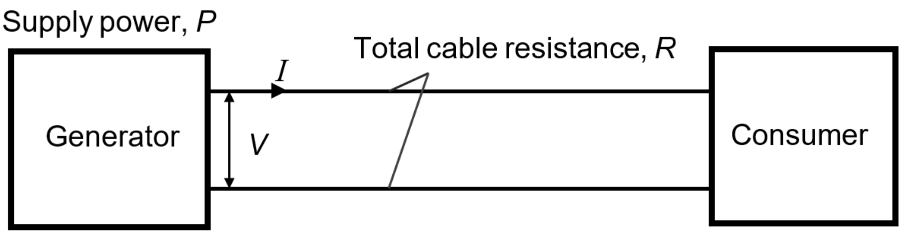


Fig. 18.20

Since the transmission cables run for kilometers, some power loss due to joule heating is expected.

For a constant power supplied by the generator P and transmission voltage V, the current in the cables is

$$I = \frac{P}{V}$$

If total resistance of the cables is R, the power loss across the cables is

$$P_{loss} = I^2R = \left(\frac{P}{V}\right)^2 R$$

Hence electrical power is transmitted (over long distances) at **high voltage** so as to minimise power lost in the cables.

The potential drop across the cable is calculated by $V_{cable} = IR = \left(\frac{P}{V}\right)R$.

$$V_{cable} \neq V \text{, and } P_{loss} = I^2R = \frac{V_{cable}^2}{R}$$

Example 10

An ideal transformer has 550 turns and 30 turns in the primary and secondary coils, respectively.

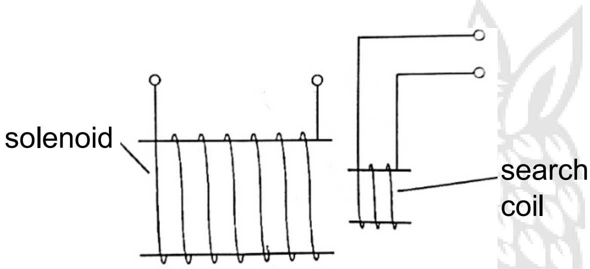
(a) What is the maximum output potential difference if V_p is 3.3 kV?

(b) What is the maximum primary current required if a maximum current of 11 A is drawn by a resistive load?
(note: the load is connected to the secondary coil)

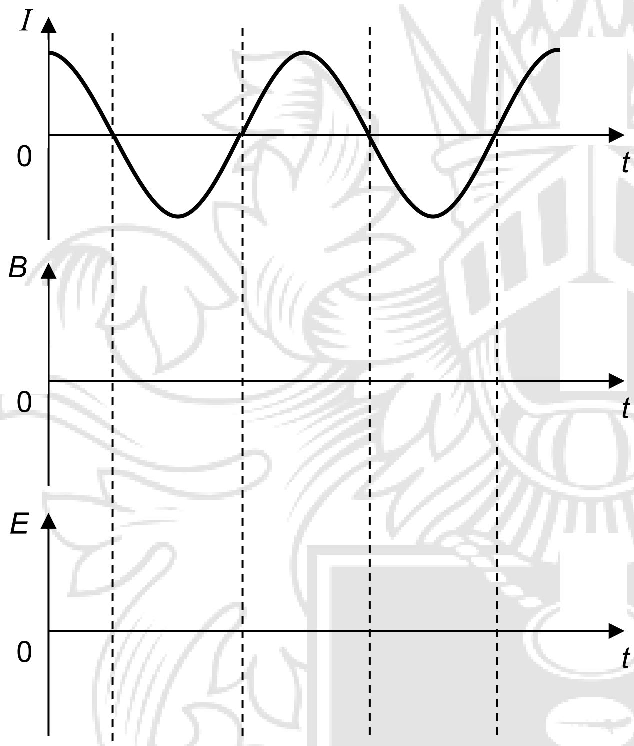
Solution

Example 11 [J93/II/4(part)]

A current-carrying solenoid is placed near to a search coil as shown.



The variation with time *t* of the current I in the solenoid is shown below.



$I = I_{max} \cos \omega t$

For the solenood, $B = \mu_o n I$,
where *n* is the number of turns per unit length
$B \propto I$
$B = B_{max} \cos \omega t$

For the search coil, $E \propto -\dfrac{dB}{dt}$

$E = E_{max} \sin \omega t$

(a) Sketch, on the given axes above, the variation with time *t* of
(i) the magnetic flux density *B* in the solenoid,
(ii) the e.m.f. *E* induced in the search coil.

(b) Describe and explain the effect on the amplitude and frequency of *E* if, separately,
(i) a ferrous core is slowly introduced into the solenoid,
(ii) the frequency of the current in the solenoid is increased, whilst maintaining the same amplitude.

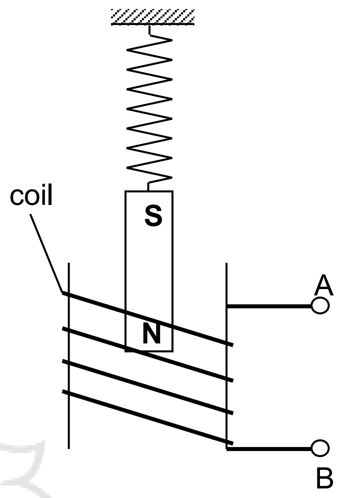
Solution

(b)(i) The ferrous core would cause the magnetic flux density *B* in the solenoid to ________________________, but the frequency is unchanged. Hence, *E* ________________________, but frequency remains the ____________.

(ii) An increased frequency of current in the solenoid leads to an increased rate of change of ______________________ in the search coil. Hence, *E* will have a ______________amplitude and frequency.

Example 12 [J94 P3/Q3]

The north pole of a magnet is placed inside a coil of wire, as shown. The terminals of the coil are connected to the Y-plates of a cathode ray oscilloscope (c.r.o.) which may be assumed to have infinite input resistance.



(a) Sketch a graph to show how the induced e.m.f. in the coil will vary with time *t* when the magnet oscillates in the coil. Mark relevant times (for example, t_1, t_2, t_3, t_4) on the *t*-axis of your graph.

(b) Use Faraday's law to explain the shape of your graph.

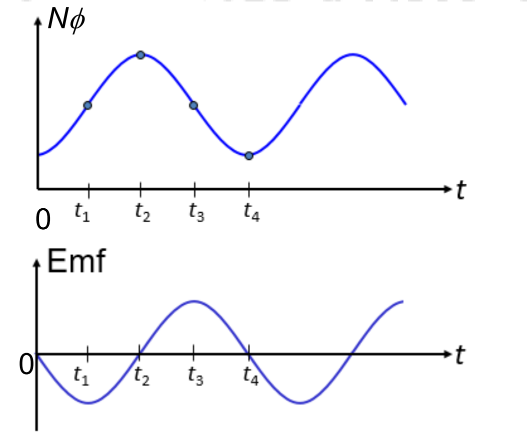
(c) A high resistance resistor is now connected in parallel with the c.r.o. between the points A and B. Draw a second graph to show how the e.m.f. will vary with time *t*.

(d) Explain, in terms of the principle of conservation of energy, why this graph is different from your first graph.

(e) Describe, with the aid of a sketch graph, the changes which would occur to the shape of the graph drawn in **(c)** if the resistance of the resistor is reduced to a very low value.

Solution

(a) Assume that the magnet is moved vertically up a certain displacement and released from rest.



0 s: The magnet is momentarily at rest at the highest point of its oscillations. There is minimum overlapping of the magnet and the coil. The flux linkage in the coil is ________________.

$\therefore \dfrac{d(N\phi)}{dt} = 0 \quad \Rightarrow \quad$ induced e.m.f. $= 0$

t_1 : A point somewhere in between the points of maximum overlapping and minimum overlapping of the magnet and the coil. The rate of change of magnetic flux linkage in the coil is ________________.

$\therefore \dfrac{d(N\phi)}{dt} = \text{maximum} \Rightarrow$ maximum induced e.m.f.

t_2 : The magnet is momentarily at rest at the lowest point of its oscillation. There is maximum overlapping of the magnet and the coil. The flux linkage in the coil is ________________.

$\therefore \dfrac{d(N\phi)}{dt} = 0 \quad \Rightarrow \quad$ induced e.m.f. $= 0$

t_3 : Similar to t_1 , but the magnet is moving out of the coil. E.m.f. is also ________________, but of opposite sign to that at t_1.

$\therefore \frac{d(N\phi)}{dt} = \text{maximum} \Rightarrow$ maximum induced e.m.f.

t_4 : The magnet is back at the highest point of its oscillation.

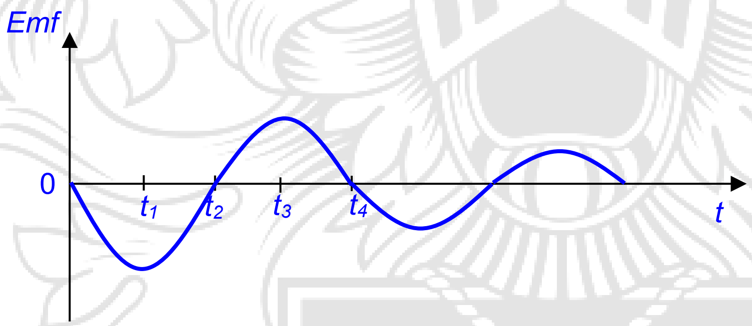
(b) <u>From $t = 0$ to t_2</u>

The magnetic flux linkage of the coil is ________________ as it moves into the coil. Hence, there is an induced e.m.f. according to Faraday's Law. This e.m.f. peaks at t_1 , where $\frac{d(N\phi)}{dt}$ is maximum.

<u>From t_2 to t_4</u>

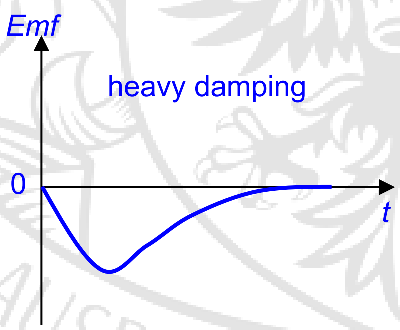
The magnetic flux linkage of the coil is ________________ as the magnet moves out of the coil. Again, e.m.f. is induced, but in the ________________ direction. The e.m.f. peaks at t_3, where $\frac{d(N\phi)}{dt}$ is maximum.

(c)



(d) A ____________ current flows through the resistor and hence ____________ is produced in the resistor at the expense of ____________ mechanical energy. The oscillation dies off slowly (due to light damping) and the magnitude of the induced e.m.f. becomes ____________ with time.

(e)



If the resistance is very low, the induced current is very ____________ and this leads to a ____________________ of heat dissipation. Hence, there is a ____________________ of damping and there may not be any oscillations due to ____________ damping.

Summary

1. Magnetic flux is defined as the product of the magnetic flux density (through an area) and the cross-sectional area perpendicular to the direction of the magnetic flux density.
2. The magnetic flux linkage of a coil is defined as the product of the magnetic flux through the coil and the number of turns of the coil.
3. Magnetic flux and magnetic flux linkage are measured in weber (Wb).
4. Faraday's law of electromagnetic induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.
5. Lenz's law states that the direction of induced e.m.f. is such as to cause effects to oppose the change producing it.
6. Mathematically, induced e.m.f. E is given by:

$$E = -\frac{d(N\phi)}{dt}$$

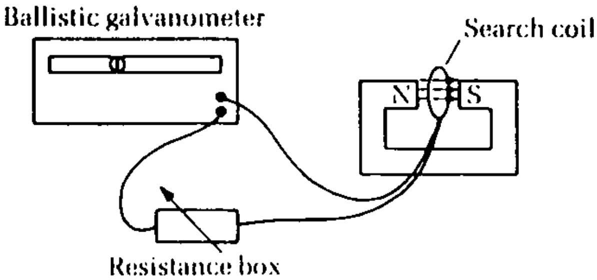
When this is applied to different situations, different expressions are obtained:

	Situation	Magnitude of induced e.m.f.
1	Moving rod	$E = BLv$
2	AC generator (rotating coil at constant angular speed)	$E = NBA\omega \sin\omega t$
3	Rotating disc	$E = BAf = ½ Br^2\omega$
4	Average e.m.f.	$\langle E \rangle = \left\| -\frac{\Delta N\phi}{\Delta t} \right\| = \frac{\left\| (N\phi)_{final} - (N\phi)_{initial} \right\|}{\Delta t}$

To find the direction of the induced e.m.f. (or current), use either Fleming's Right-Hand Rule, Lenz's Law or consider the direction of the magnetic force experienced by the charge carriers in the conductor.

❖ Measurement of the Magnetic Flux Density Using a Search Coil

A search coil can be used to measure the magnetic flux density of a magnetic field by placing it in the magnetic field to be measured so that the magnetic flux density is at right angles to the plane of the search coil.



If N = number of turns in the search coil and A = area of the coil, then the initial magnetic flux linkage is $\Phi_0 = NBA$

The search coil is then quickly removed from the magnetic field in time t and the magnetic flux linkage is now $\Phi_t = 0$

This sudden change in the magnetic flux linkage causes a current to be induced in the search coil. When charges flow through the ballistic galvanometer, the first 'throw' θ produced by the change in flux is noted.

The charge Q driven through the coil is proportional to θ. That is, $Q = k\theta$, where k is found by calibrating the ballistic galvanometer.

From $I=\frac{dQ}{dt}$, $Q = \int_0^t I dt$.

From $E = IR$, $I = \frac{E}{R}$, where E is the induced e.m.f. and R the resistance of the search coil.

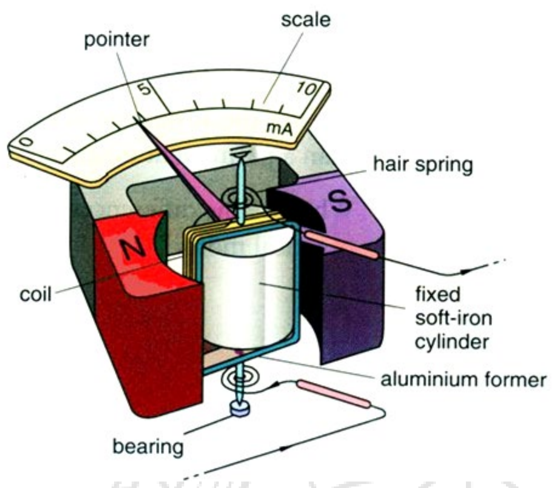
Therefore,

$$Q = \int_0^t I dt = \int_0^t \frac{E}{R} dt = -\frac{1}{R}\int_0^t \frac{d\Phi}{dt} dt = -\frac{1}{R}\int_0^t d\Phi = \frac{1}{R}\left[\Phi_0 - \Phi_t\right] = \frac{NBA}{R}$$

Hence, $\frac{NBA}{R} = k\theta$ and $B = \frac{k\theta R}{AN}$.

Appendix B

❖ Application of EMI in Protection of Moving Coil Meters



Suspension-type meters are very sensitive and easily damaged by careless handling. To protect the meter when not in use, the meter terminals should be connected together. When the meter is moved carelessly, the motion of its coil in the magnetic field creates induced current in the coil. The induced current causes magnetic forces to act on the coil to oppose its motion relative to the field. So movement of the sensitive coil relative to the magnet is greatly reduced by connecting the terminals together.

If the coil is wound on a metal frame, current is induced in the frame too, so that also helps to prevent the coil from swinging too freely. Also, when in use, induced current in the frame serves to slow the frame down, which stops the meter pointer from over-shooting.

❖ Power Loss in Transformers

The power loss in transformers are solely electrical in nature as transformers are static devices with no moving parts. Thus, there is no loss of mechanical power (e.g. due to friction of moving parts).

The causes of power loss in transformers are as listed below in detail.

1. Joule heating losses

Cause

Joule heating, also known as ohmic heating and resistive heating, is the process by which the passage of an electric current through a conductor produces heat.

For transformers, the copper wires used for the primary and secondary coils heat up when currents pass through them, resulting in power losses.

Design Feature

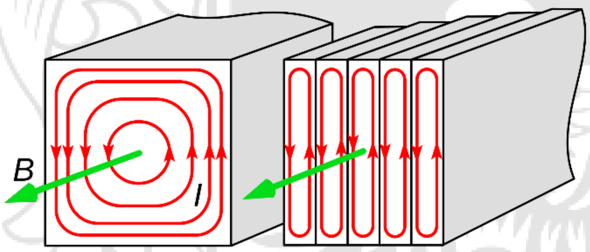
Thick copper wires are used particularly for coils carrying large current at low voltage.

2. Eddy current losses

Cause

The alternating current supplied to the primary coil sets up an alternating magnetic flux in the transformer core and the secondary coil, and induces e.m.f. in these parts.

The induced e.m.f. in the core produces small circulating currents called eddy currents, which cause power loss due to joule heating.



Design Feature

Joule heating is minimised by using a laminated core, i.e., one built up of insulated iron sheets or laminae.

The large electrical resistance of each lamina, due to an insulating varnish, effectively confines the eddy currents to individual lamina. The eddy-current paths are now narrower (the effective resistance of eddy-current paths is considerably increased), and the eddy currents and their heating effects are greatly reduced. This is illustrated in the figure above.

3. Hysteresis losses

Cause

Hysteresis loss results in energy lost as heat due to reversal of magnetization in the transformer core every time the primary current reverses.

This loss depends upon the volume and grade of the iron, frequency of magnetic reversals and the magnitude of the magnetic flux density.

Design Feature

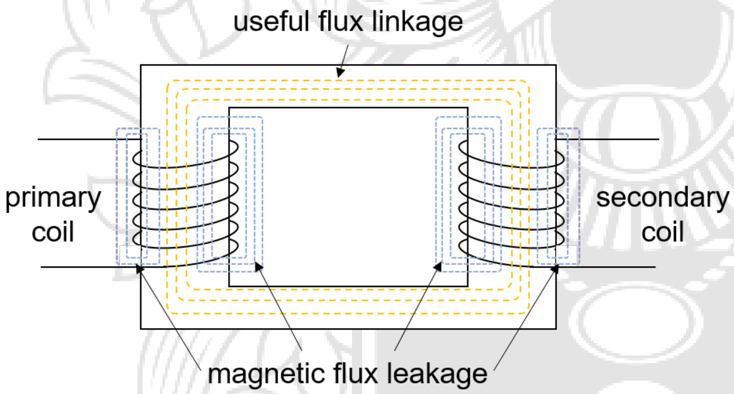
Core made of soft iron has low hysteresis. That is, it can be easily magnetised and demagnetised by the magnetic field caused by alternating current in the primary winding.

4. Magnetic flux leakage

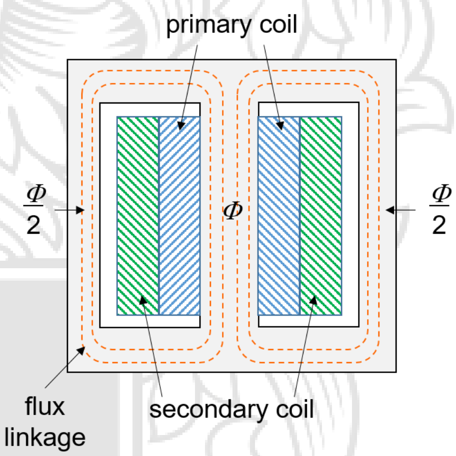
Cause

Magnetic flux leakage happens when not all of the magnetic flux is used to do useful work.

In an ideal transformer, all magnetic flux will link with both the primary and secondary windings, but in practice, this is impossible. Figure (a) illustrates the magnetic flux around a transformer.



(a)



(b)

Design Feature

The presence of an iron core increases the magnetic flux linkage through both coils and reduces the flux leakage between them.

Flux leakage can be minimised if both coils could be made to occupy the same space by placing secondary and primary in a concentric manner in the "E-I" shaped iron core as shown in Figure (b).

NOTETAKING

"Be less curious about people and more curious about ideas."