16 CIRCUITS

H2 Physics 9478

Content **Page**

Learning Outcomes

Candidates should be able to:

(a) recall and use appropriate circuit symbols.

(b) draw and interpret circuit diagrams containing sources, switches, resistors (fixed and variable), ammeters, voltmeters, lamps, thermistors, light-dependent resistors, diodes, capacitors and any other type of component referred to in the syllabus.

(c) define the resistance of a circuit component as the ratio of the potential difference across the component to the current in it, and solve problems using the equation $V = IR$.

(d) recall and solve problems using the equation relating resistance to resistivity, length and cross-sectional area, $R = \frac{\rho l}{A}$.

(e) sketch and interpret the *I-V* characteristics of various electrical components in a d.c. circuit, such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor.

(f) explain the temperature dependence of the resistivity of typical metals (e.g. in a filament lamp) and semiconductors (e.g. in an NTC thermistor) in terms of the drift velocity and number density of charge carriers respectively.

(g) show an understanding of the effects of the internal resistance of a constant source of e.m.f. on the terminal potential difference and output power.

(h) solve problems using the formula for the combined resistance of two or more resistors in series.

(i) solve problems using the formula for the combined resistance of two or more resistors in parallel.

(j) solve problems involving series and parallel arrangements of resistors for a constant source of e.m.f., including potential divider circuits which may involve NTC thermistors and light-dependent resistors.

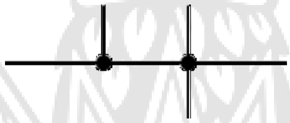
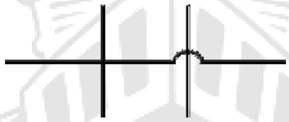
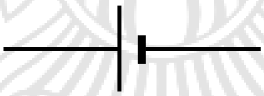
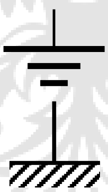
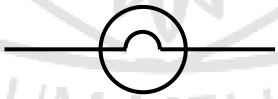
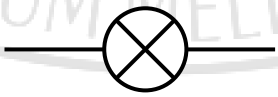
(k) solve problems using the formulae for the combined capacitance of two or more capacitors in series and in parallel.

(l) describe and represent the variation with time, of quantities like current, charge and potential difference, for a capacitor that is charging or discharging through a resistor, using equations of the form $x = x_0 \exp(-t/\tau)$ or $x = x_0[1 - \exp(-t/\tau)]$, where $\tau = RC$ is the time constant.

16.1 Circuit symbols and diagrams

❖ **Circuit Symbols**

The following is a list of circuit symbols used in the syllabus.

Category	Component	Circuit Symbol	Function of Component
Wires and Connections	Wire		To pass current from one part of a circuit to another.
	Wires that are connected		This symbol is used in circuit diagrams where wires cross to show that they are connected (joined).
	Wires that are not connected		In complex circuit diagrams it is often necessary to draw wires crossing even though they are not connected. You may prefer to use the 'hump' symbol shown on the right, because the simple crossing on the left looks like connection.
Power Supplies	Cell		Supplies electrical energy. Single cells are often wrongly called batteries; strictly, a battery comprises two or more cells joined together. The side with the longer line is the positive terminal.
	Battery		Supplies electrical energy. A battery is more than one cell joined together. The side with the longer line is the positive terminal.
	D.C. supply		Supplies electrical energy.
	A.C. supply		Supplies electrical energy.
	Earth (ground)		A connection to earth. For many electronic circuits, this is the 0 V (zero volt) of the power supply, but for mains electricity and some radio circuits, it really means the earth. It is also known as ground.
Lamps	Lamp		A transducer which converts electrical energy to light. This symbol is used for a lamp providing illumination, e.g. a car headlamp or torch bulb.
	Indicator or light source		

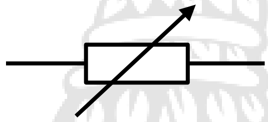
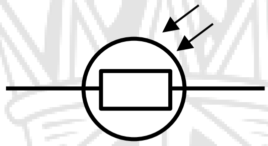
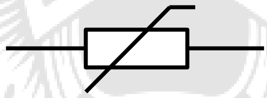
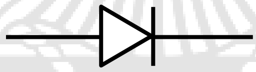
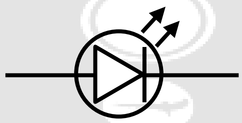
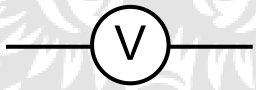
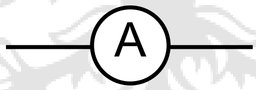
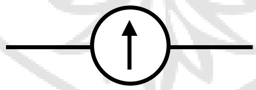
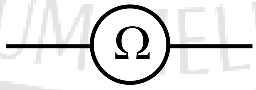
Category	Component	Circuit Symbol	Function of Component
Switches	On-Off switch		An on-off switch allows current to flow only when it is in the closed (on) position.
Resistors	Fixed resistor		A resistor restricts the flow of current, for example to limit the current passing through a device.
	Variable resistor		This type of variable resistor (e.g. a rheostat) is usually used to vary current through a circuit.
	Light Dependent Resistor (LDR)		An LDR is a semiconductor device. Its resistance decreases from several 10^6 Ω in the dark to several 10^2 Ω as the brightness (illumination) of light falling on it increases.
	Thermistor		A thermistor is a semiconductor device. The resistance of most thermistors decreases from 10 kΩ to 2 kΩ as its temperature increases from – 55 °C to 70 °C. Such thermistors are called negative temperature coefficient or N.T.C. thermistors.
Diodes	Diode		A device which only allows current to flow in one direction.
	Light Emitting Diode (LED)		A transducer which converts electrical energy to light.
Capacitors	Capacitor		A capacitor is used for storing electric charge.
Meters	Voltmeter		A voltmeter is used to measure potential difference.
	Ammeter		An ammeter is used to measure current.
	Galvanometer		A galvanometer is a very sensitive meter which is used to detect tiny currents, and changes in direction of current.
	Ohmmeter		An ohmmeter is used to measure resistance. Most digital multimeters (DMMs) have an ohmmeter setting.

Fig. 16.1

❖ Circuit Diagrams

Circuit diagrams are drawn with wires and electrical components. Only essential features of a circuit are drawn, and all electrical devices are represented by standard symbols.

The figure below shows an actual circuit arrangement with its circuit diagram. Note that inter-connected wires must be indicated with a dot (•).

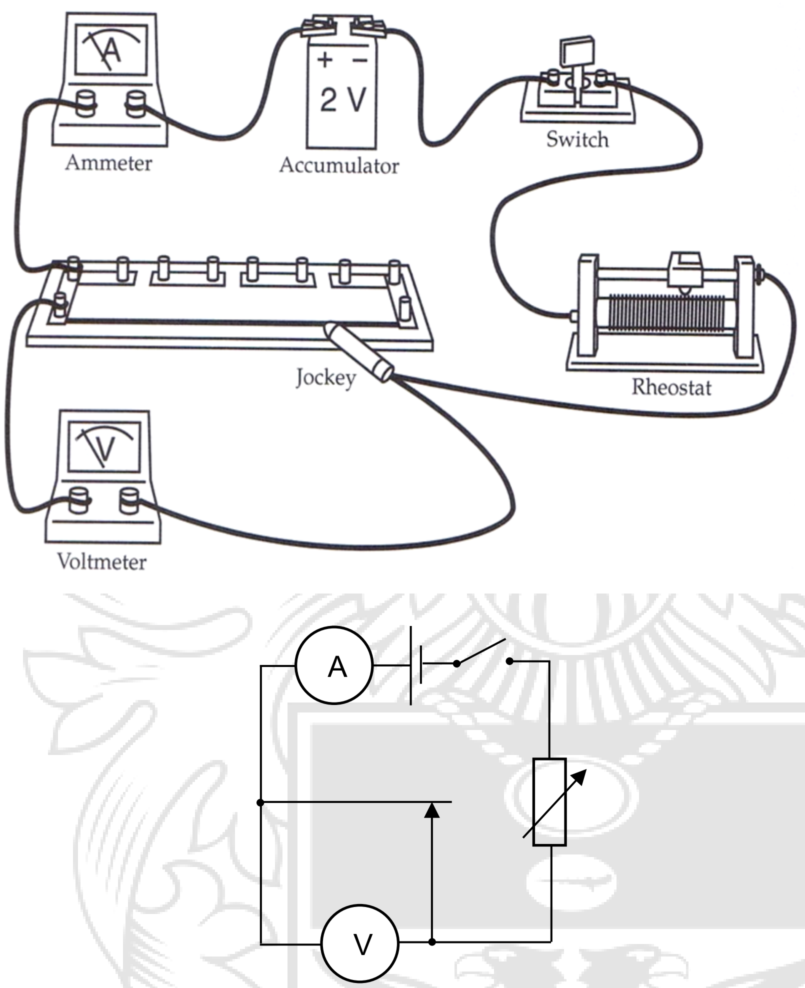


Fig. 16.2

16.2 Resistance, resistivity, and internal resistance

❖ Resistance *R* and its unit.

> **Definition**
>
> The **resistance** of a circuit component is defined as the ratio of the potential difference across it to the current in it.

In equation form, electrical resistance is given by

$$R = \frac{V}{I}$$

The S.I. unit for resistance is the **ohm,** Ω.

The **ohm** is the resistance of a circuit component when a potential difference of one volt across it causes a current of one ampere to flow through it ($1\,\Omega = 1\,\text{V A}^{-1}$).

Determining *R*

A simple way to determine the resistance of a circuit component is to connect

1. an **ammeter**, assumed to have no resistance, **in series** with the component and

2. a **voltmeter**, assumed to have an infinite resistance, **in parallel** with the component as shown in Fig. 16.3.

The ammeter measures the current flowing through the component and the voltmeter measures the p.d. across it. The resistance can thus be determined using $R = \frac{V}{I}$.

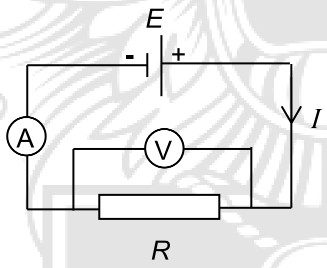


Fig. 16.3

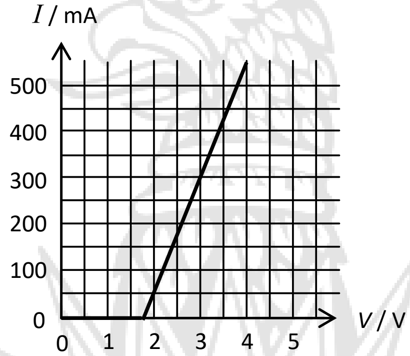
Example 1

A car stereo system draws a current of 400 mA when connected to a 12.0 V battery.
(a) Calculate the resistance of the stereo system.
(b) The stereo is left playing from the battery for several hours while the engine is turned off. The radio can continue to operate until the current drops to 320 mA. Calculate the p.d. across the stereo when it stops playing.

Solution

Example 2

The graph below shows the relationship between the direct current I in a certain conductor and the potential difference V across it. When V < 1.8 V, the current is negligible. Determine the resistance of the conductor when the p.d. is 3.0 V.



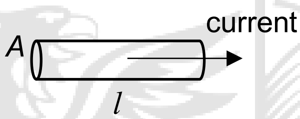
Solution

❖ **Resistivity ρ of Conductors and its unit**

The resistance of a uniform[1] conductor (at a given temperature) is

(i) directionally proportional to its length l, and
(ii) inversely proportional to its cross-sectional area A,

$$R \propto \frac{l}{A}$$



or $$R = \rho \frac{l}{A}$$

where the constant of proportionality ρ is called the resistivity of the material.

The S.I. unit for resistivity is the **ohm metre,** Ω m.

Every material has a characteristic resistivity. Good electrical conductors have low resistivities and insulators have high resistivities.

[1] 'Uniform conductor' here refers to a conductor that is made of a single type of material.

❖ Semi-conductors

The resistance of pure semiconductors is between that of a conductor and that of an insulator. By introducing controlled amounts of impurities, their resistivities can be significantly reduced. These materials are then used to make electronic components which are necessary in the fabrication of computer chips.

❖ Resistivities of common materials

The following table shows a list of resistivities of some common materials at 20°C.

Type	Material	Resistivity, ρ / Ω m
Conductors	Silver	1.6×10^{-8}
	Copper	1.7×10^{-8}
	Gold	2.4×10^{-8}
	Aluminium	2.8×10^{-8}
	Tungsten	5.6×10^{-8}
	Iron	10×10^{-8}
Pure Semiconductors	Germanium	0.60
	Silicon	2300
Insulators	Glass	10^{7} to 10^{10}
	Rubber	10^{13} to 10^{16}
	Sulphur	10^{15}

Fig. 16.4

* Note: The resistivity of semiconductors greatly depends on the amount and type of impurities added.

Example 3

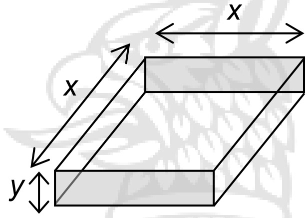
Calculate the resistance of a tungsten filament of length 4.0 cm and diameter 0.020 mm at 20°C.

Solution

Example 4

A sample of resistive material is prepared in the form of a thin square slab of side x. For a given thickness y, the resistance between opposite edge faces of the sample (shown shaded in the figure below) is:

A proportional to x^2
B proportional to x
C independent of x
D inversely proportional to x
E inversely proportional to x^2



Solution

❖ I-V Characteristics of Circuit Components

By looking at how the current I through a material varies with the potential difference V (p.d.) across it, it is possible to differentiate between ohmic and non-ohmic materials/devices.

Ohmic Conductors (Metallic conductor at constant temperature).

Experimentally, it has been found that if the **temperature** of a metallic conductor is **kept constant**, **its resistance remains constant**. That is, the metallic conductor is known as an ohmic conductor.

Circuit symbol for an ohmic resistor.

The I-V characteristic for an ohmic conductor, e.g., a resistor at a constant temperature, is a **straight-line graph passing through the origin** as shown in Fig. 16.5:

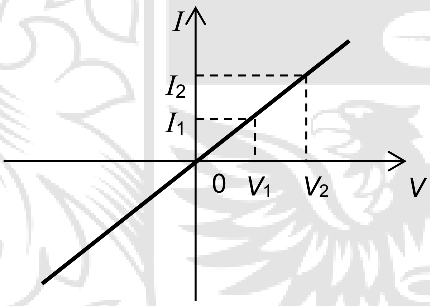


Fig. 16.5

- any increase in p.d. produces a proportionate increase in current (i.e. $I \propto V$)
- ratio $\frac{V}{I}$ is constant
- resistance,

$$R = \frac{V_1}{I_1} = \frac{V_2}{I_2} = \text{ constant}$$

The temperature of the conductor can be kept constant by immersing the conductor in a constant temperature bath.

Non-Ohmic conductors and effect of temperature.

When the temperature of a conductor is increased its resistivity will change due to the following two effects:

1. the number density of charge carriers may increase, and
2. the frequency of collision between charge carriers and lattice ions will increase.

Effect 1:
Some electrons will gain enough energy and break free from the atoms to become mobile or free charge carriers, and this increases the number density of charge carriers, thus resulting in a decrease in resistivity.

Effect 2:
The lattice ions gain thermal energy and vibrate faster and with a larger amplitude. The charge carriers thus collide more frequently with these lattice ions, resulting in an increase in resistivity.

Two examples of non-ohmic conductors are:

(a) Filament lamp

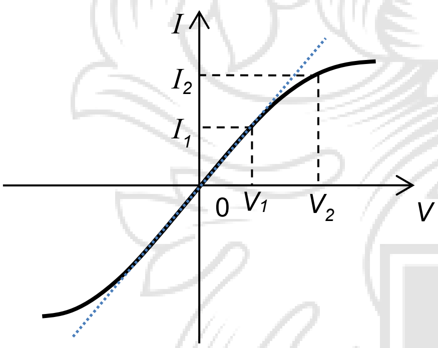


Fig. 16.9

- as current increases, temperature increases
- resistivity and hence resistance increases i.e. $R = \frac{V_1}{I_1} < \frac{V_2}{I_2}$
- ratio $\frac{V}{I}$ increases
- I–V curve gets more gentle

Effect 1 (number density of electrons)	**Effect 2** (collision frequency of electrons with lattice ions)	**Overall result**
• In a metal, the electrons are already free and mobile at room temperature. • As temperature increases, there is no appreciable increase in the number density of free electrons.	• Ions vibrate faster and with greater amplitude, hence frequency of collision between lattice ions and electrons increases. • Movement of charge carriers slows down.	Effect 2 is more significant. Resistivity increases.

(b) Thermistor (NTC)

A thermistor is made of a semiconductor, which is a poor electrical conductor at relatively low temperatures due to the small number density of free electrons compared to that in metals.

Effect 1 (number density of charge carriers)	**Effect 2** (collision frequency of charge carriers with lattice ions)	**Overall result**
• As temperature increases, more charge carriers acquire energy to break free from their bonds. • Number density of charge carriers increases significantly.	• Frequency of collision between lattice ions and charge carriers increases. • Movement of charge carriers slows down.	Effect 1 is more significant. Resistivity decreases.

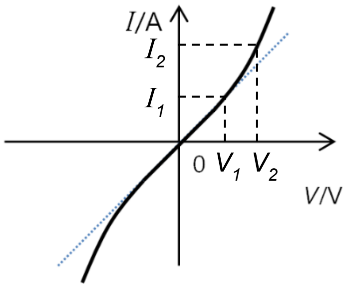


Fig. 16.10

- as current increases, temperature increases
- resistivity, and hence resistance, decreases. i.e. $R = \frac{V_1}{I_1} > \frac{V_2}{I_2}$
- ratio $\frac{V}{I}$ decreases
- I–V curve gets steeper.

Resistance vs Temperature curves.

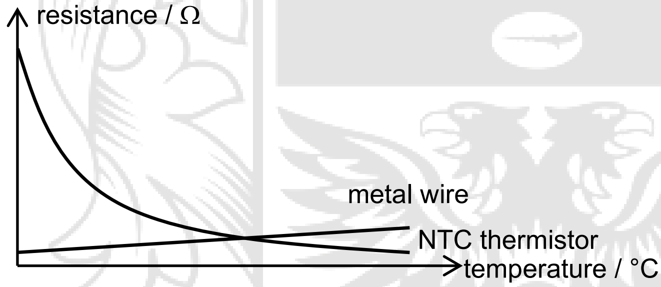


Fig. 16.11

(c) Diodes

A diode is a semiconductor device which allows current to flow in one direction only.

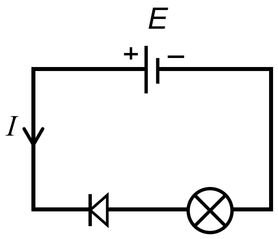


Fig. 16.6

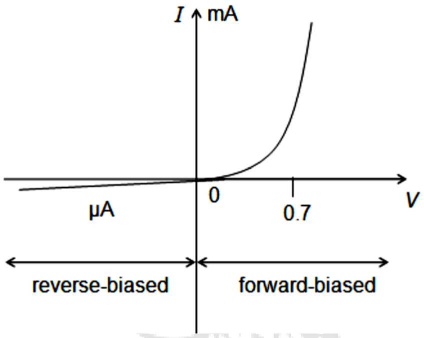


Fig. 16.7

Reverse-bias

- Current through the diode is very small (a few μA and appears to be zero on the graph).
- Resistance of diode is very high.

Forward-bias

- Current through a diode increases very rapidly when $V > 0.7$ V (turn-on voltage),
- Resistance of diode is very low.

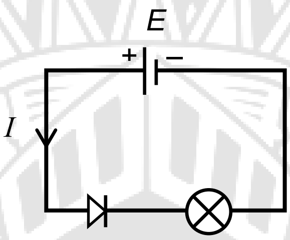


Fig. 16.8

❖ Internal Resistance *r* of a Source of emf and Its Effects

Consider a simple circuit consisting of a cell and a resistor. [The terms cell and battery are used interchangeably here.]

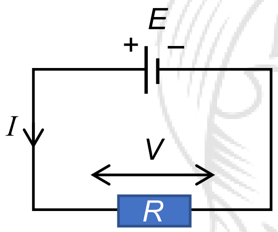


Fig. 16.12

In an ideal circuit,
- the e.m.f. source would have no internal resistance,
- the connecting wires would have no resistance, and
- the terminal p.d. V of the cell = e.m.f. E of the cell.

However, **real** cells have internal resistance *r*. Hence, not all electrical energy generated is available to the external load *R*. Some of this energy is lost as thermal energy within the cell due to its internal resistance. As a result, the terminal p.d. is not equal to the e.m.f. of the source.

Model of a battery

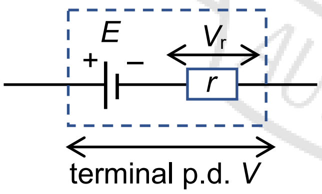


Fig. 16.13

A **real** battery can be modelled as follows
- a resistance *r*, which is
- connected in series to an ideal e.m.f. source as shown in the dashed box.

For a circuit with a **battery** of e.m.f. E, an internal resistance r and a terminal p.d. V:

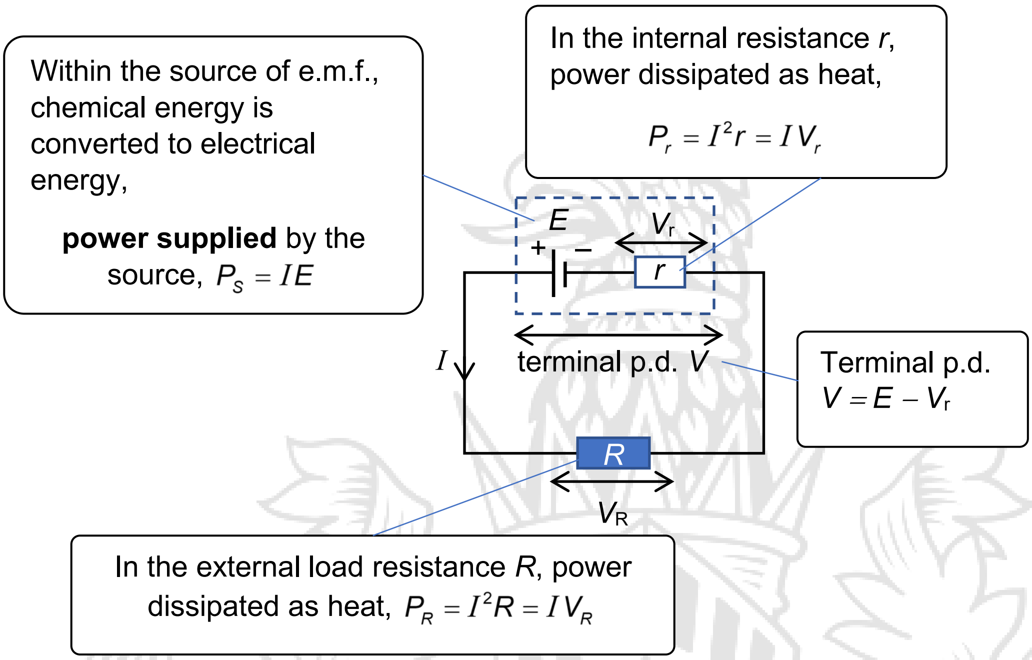


Fig. 16.14

By the principle of conservation of energy,
Power **supplied** by source = Power **dissipated** in r and R

$$P_S = P_r + P_R$$

$$IE = I^2 r + I^2 R$$

$\boxed{E = I(r+R)}$ OR $\boxed{E = Ir + V_R}$ OR $\boxed{V_R = E - Ir}$

Terminal p.d. $V = V_R = E - Ir$

Graph of terminal p.d. V against current I:

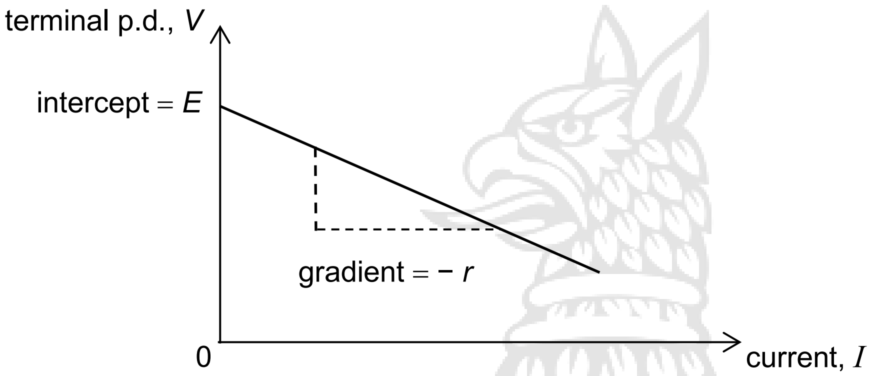


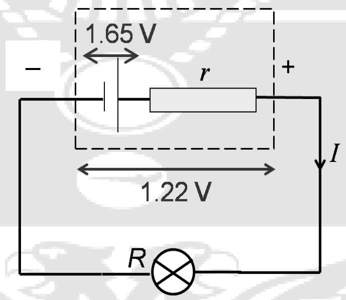
Fig. 16.15

When **no current** flows (i.e. open circuit),	When a current I flows,
▪ p.d. across the internal resistance is 0 ▪ hence terminal p.d. V = e.m.f. E of the source	▪ p.d. across the internal resistance r is Ir ▪ hence terminal p.d. decreases by Ir ▪ terminal p.d. $V = V_R = E - Ir$

Example 5

A high-resistance voltmeter reads 1.65 V when connected across a dry-cell in an open circuit and 1.22 V when the same cell is supplying a current of 0.30 A through a lamp. Calculate

(a) the e.m.f. of the cell,
(b) the internal resistance of the cell, and
(c) the resistance of the lamp.



Solution

❖ Power output or (power delivered to load)

A battery with e.m.f. E and internal resistance r is connected to a load of resistance R as shown in Fig. 16.16.

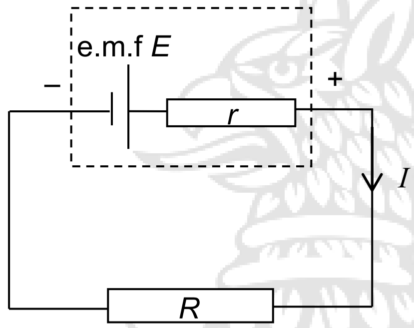


Fig. 16.16

$$E = I(r+R) \quad \Rightarrow I = \frac{E}{(r+R)}$$

Hence power delivered to the external load R is

$$P_R = I^2 R = \left[\frac{E}{(r+R)}\right]^2 R \quad \Rightarrow P_R = \frac{E^2 R}{(r+R)^2}$$

Maximum Power Theorem (Maximum Power Delivered to Load)

To find maximum power delivered to the external load R, differentiating the above equation w.r.t R,

$$\frac{dP_R}{dR} = 0$$

$$\frac{E^2(r+R) - 2E^2 R}{(r+R)^3} = 0$$

$$E^2(r+R) - 2E^2 R = 0$$

$$(r+R) - 2R = 0$$

$$R = r$$

It can be concluded that a source of e.m.f. delivers the maximum amount of power to an external load when the **resistance of the load is equal to the internal resistance** of the source. This is the <u>maximum power theorem</u>.

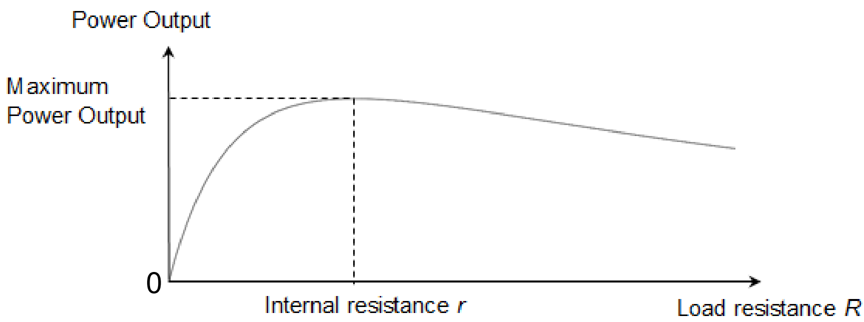


Fig. 16.17

Example 6

A cell drives a current of 1.50 A through a 3.08 Ω resistor and 2.00 A through a 2.00 Ω resistor, respectively. Calculate

(a) the e.m.f. of the cell,

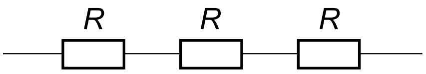
(b) the internal resistance of the cell, and

(c) the maximum heating effect it can develop in an external load.

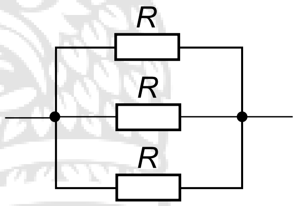
Solution

❖ Combination of Resistors

The following are four possible combinations of three resistors:



resistors in **series**



resistors in **parallel**

resistors in a **combination of series and parallel** arrangement

Fig. 16.18

Equivalent or effective resistance

It is possible to replace a combination of resistors in any given circuit with a single resistor without altering the p.d. and current across the terminals of the combination. The resistance of this single resistor is called the *equivalent or effective resistance* of the combination.

❖ Resistors in Series

Consider 3 resistors of resistances R_1, R_2 and R_3 connected in series.
Current I through each resistor is the same.

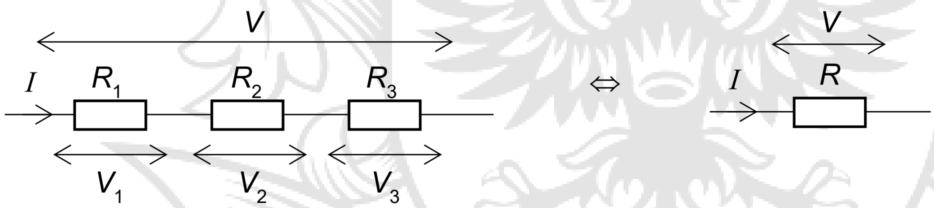


Fig. 16.19

$$V_1 = IR_1 \qquad V_2 = IR_2 \qquad V_3 = IR_3$$

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 \\ \frac{V}{I} &= R_1 + R_2 + R_3 \quad \cdots\cdots(1) \end{aligned}$$

$$V = IR$$

$$R = \frac{V}{I} \quad \cdots\cdots(2)$$

Comparing equations (1) & (2), the equivalent resistance R is

$$R = R_1 + R_2 + R_3$$

Hence for a case of n resistors in series, the equivalent resistance R is

$$R = R_1 + R_2 + \ldots + R_n$$

❖ **Resistors in Parallel**

Consider 3 resistors of resistances R_1, R_2 and R_3 connected in parallel.

Potential difference V across each resistor is the same.

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$I_3 = \frac{V}{R_3}$$

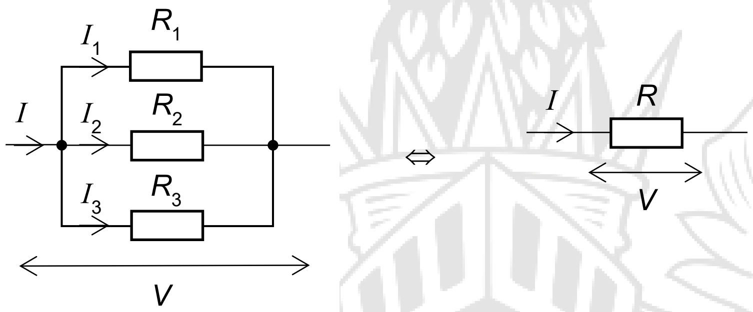


Fig. 16.20

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \cdots\cdots(3)$$

$$V = IR$$

$$\frac{1}{R} = \frac{I}{V} \quad \cdots\cdots(4)$$

Comparing equations (3) & (4), the equivalent resistance R is

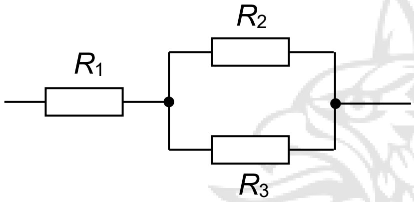
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Hence for a case of n resistors in parallel, the equivalent resistance R is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Example 7

Three resistors with R_1 = 2.0 Ω, R_2 = 4.0 Ω and R_3 = 6.0 Ω are connected as shown below.



Calculate

(a) the equivalent resistance of the combination,

(b) the current that passes through the combination if a potential difference of 8.0 V is applied to the terminals,

(c) the potential difference across each resistor and the current flowing through each resistor.

Solution

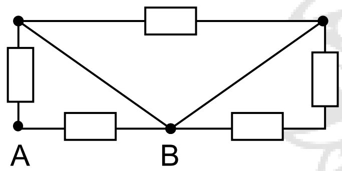
❖ **Simplification of D.C. Circuits**

Strategy for simplifying D.C. circuits:

1. Identify and label junctions or points in the circuit that are of the same potential.
2. Remove redundant circuit components, i.e., where
   - $I_{component} = 0$ or
   - $\Delta V_{component} = 0$
3. Redraw equivalent circuit.

Example 8

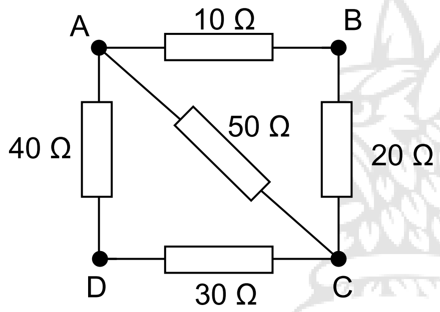
Five resistors, each of resistance *R*, are connected as shown. Find the equivalent resistance between points A and B.



Solution

Example 9

Five resistors are connected as shown.



Determine the resistance between

(i) points A and C, and

(ii) points A and D.

Solution – for (i)

For (ii)

❖ Combination of E.M.Fs. of Cells

In many devices that use batteries, such as portable radios and flashlights, it is insufficient to use just one cell at a time. Cells are usually grouped together in a serial arrangement to increase the voltage or in a parallel arrangement to increase current.

1) Cells in series

For two cells in series, there are two possible combinations:

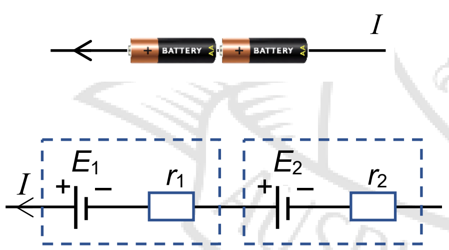


Fig. 16.21

Total e.m.f. $E = E_1 + E_2$

Total internal resistance $r = r_1 + r_2$

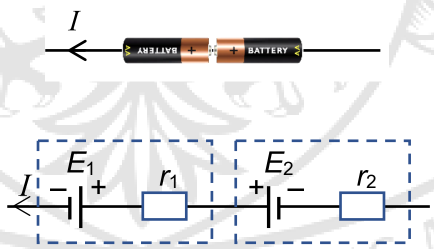


Fig. 16.22

Total e.m.f. $E = |E_2 - E_1|$

Total internal resistance $r = r_1 + r_2$

2) Cells in parallel

For two identical cells with e.m.f. E_1 and internal resistance r_1 connected in parallel:

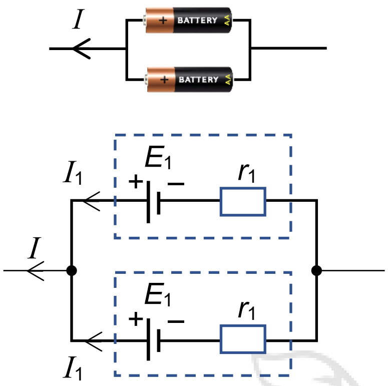


Fig. 16.23

Total e.m.f. $E = E_1$

Total internal resistance r

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_1} = \frac{2}{r_1}$$

$$r = \left(\frac{2}{r_1}\right)^{-1} = \frac{1}{2} r_1$$

❖ Potential Dividers and Potentiometer

A potential divider is an arrangement of two or more resistors connected in *series* across an applied potential difference (p.d.), with the output p.d. V_{out} being taken across one of the resistors.

The potential divider is used to produce an output p.d. V_{out} that is a ***fraction*** of the voltage supply V_S. The reduced output p.d. V_{out} may then be connected to a load, such as a lamp.

The circuit below shows a simple potential divider circuit, which consists of two known resistors of resistances R_1 and R_2 connected in series to a voltage supply V_S.

The output p.d. V_{out} is taken across resistor R_1.

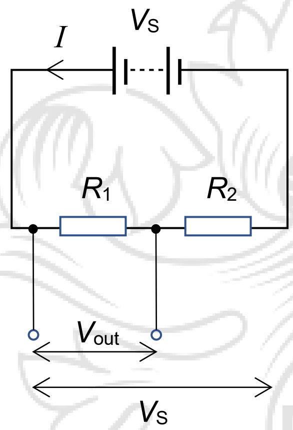


Fig. 16.24

Current $I = \dfrac{V_S}{R_1 + R_2}$

P.d. across R_1,

$$V_{out} = IR_1 = \left(\frac{V_S}{R_1 + R_2}\right)R_1$$

Rearranging the RHS of the equation, $V_{out} = \left(\dfrac{R_1}{R_1 + R_2}\right)V_S$

This shows that V_{out} is a fraction of V_S.

To supply a **continuously variable potential difference** from zero to the full potential difference of the supply V_S, a resistor with a sliding contact may be used as shown in Fig. 16.25.

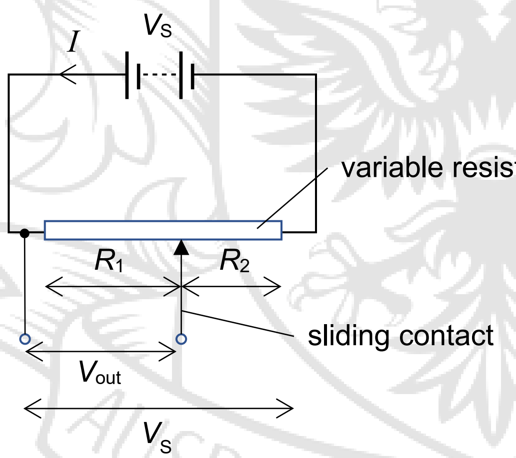


Fig. 16.25

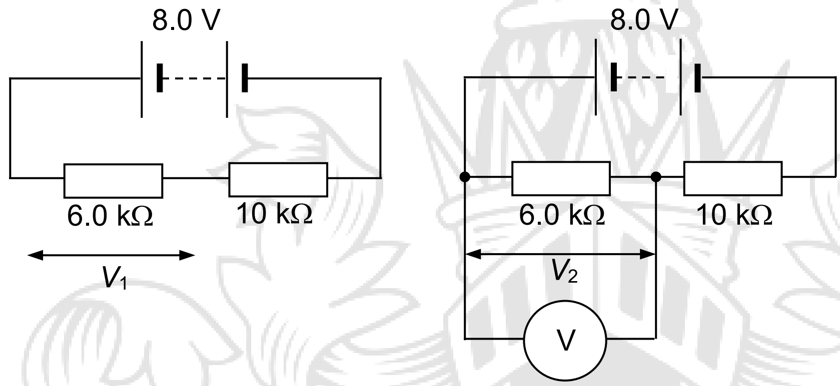
Example 10

A 10 kΩ and a 6.0 kΩ resistor are connected in series to a 8.0 V battery. The potential difference across the 6.0 kΩ resistor is measured by connecting a voltmeter of resistance 75 kΩ across it. Calculate

(a) the p.d. V_1 across the 6.0 kΩ resistor without the voltmeter,

(b) the p.d. V_2 across the 6.0 kΩ resistor in the presence of the voltmeter,

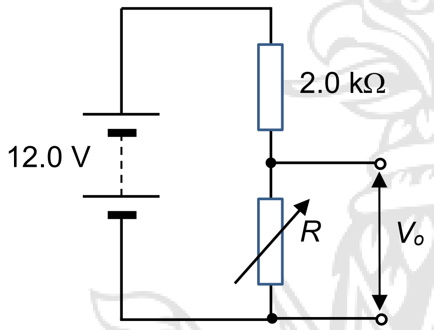
(c) the resulting percentage error in the measurement taken by the voltmeter.



Solution

Example 11

A potential divider circuit which can be used to provide a variable output potential difference V_o is shown in the figure below. The resistance R of the variable resistor can be varied from 1.0 kΩ to 10 kΩ. Calculate the range of output potential difference V_o that can be produced.



Solution

❖ **Potential at a Point**

If the potential at a point in an electric circuit is known (similar to specifying a point of reference for GPE = *mgh* with *h* = 0), the potential at any other point in the circuit can be found. This is usually done by connecting a point to earth, which assigns 0 V to that point. In the circuit below, point A is earthed.

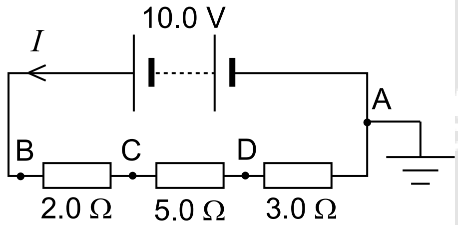


Fig. 16.26

To find the potential at different points in the circuit:

1) Find the potential difference across each resistor.

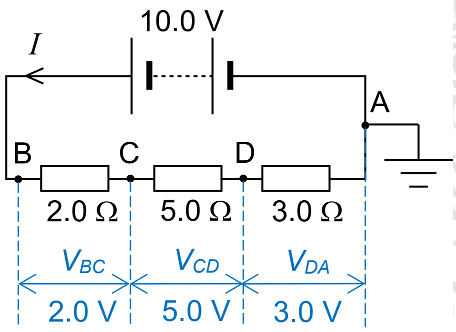


Fig. 16.27

Using the potential divider principle,

$$V_{BC} = \left(\frac{2.0}{2.0+5.0+3.0}\right)(10.0) = 2.0 \text{ V}$$

$$V_{CD} = \left(\frac{5.0}{2.0+5.0+3.0}\right)(10.0) = 5.0 \text{ V}$$

$$V_{DA} = \left(\frac{3.0}{2.0+5.0+3.0}\right)(10.0) = 3.0 \text{ V}$$

2) Identify the known potentials

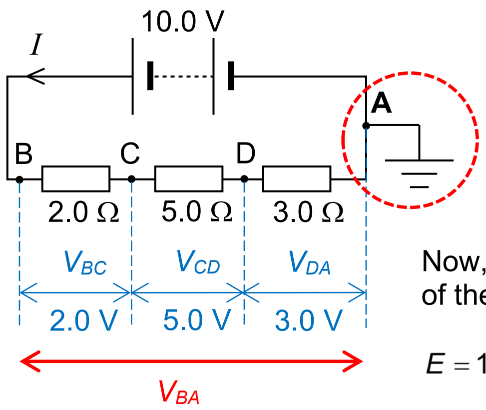


Fig. 16.28

Since A is **earthed**, it is assigned a potential of **0 V**

i.e. $V_A = 0$ V

Now, look along the upper path of the circuit,

$E = 10 \text{ V} = V_{BA} = V_B - V_A$

Then, potential at B (with reference to A) is $V_B = 10$ V

3) **Determine the potentials at the other points** based on the p.d. calculated in step 1 and the identified potentials of B and A as determined in step 2.

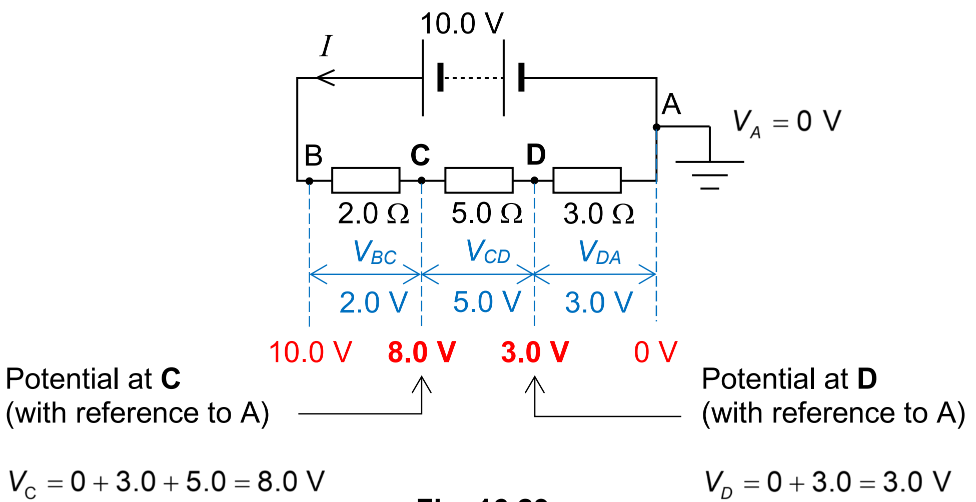


Fig. 16.29

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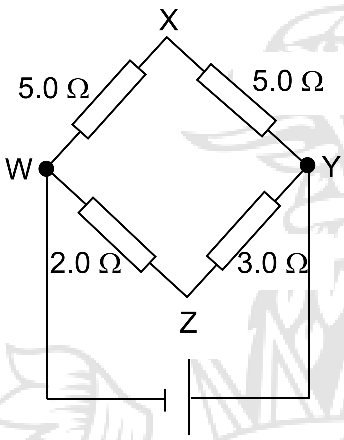
The potential difference between points W and Y is 5.0 V.
What is the potential of X with respect to Z?



A – 2.50 V
B – 0.50 V
C + 0.50 V
D + 2.50 V

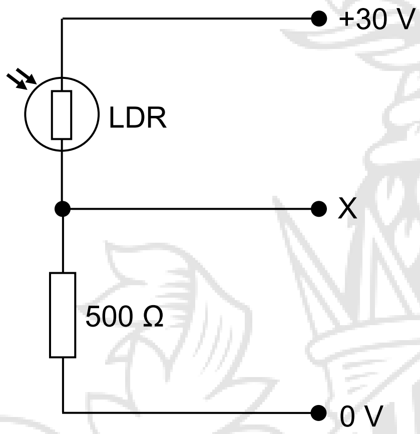
Solution

- **Use of a Light Dependent Resistor (LDR) in a potential divider circuit**

A light-dependent resistor (LDR) can be used in a potential divider to provide a potential difference which is dependent on the **illumination** that it is exposed to.

Example 13

A LDR and a 500 Ω resistor form a potential divider between potential lines held at +30 V and 0 V as shown. The resistance of the LDR is 1.0 kΩ in the dark, but drops to 100 Ω in bright light. Determine the corresponding change in the potential at X.



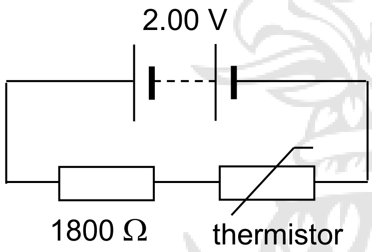
Solution

❖ **Use of a Thermistor in a potential divider circuit**

In a similar manner, a thermistor can be used in a potential divider to provide a potential difference which is dependent on **temperature**.

Example 14

The temperature change due to an expansion of a gas can be detected using a small thermistor. The thermistor is connected in series with a fixed resistor of resistance 1800 Ω and a 2.00 V battery of negligible internal resistance as shown.



Initially, the thermistor has resistance 1800 Ω and after the gas expands, its resistance is 1910 Ω. Calculate the potential difference across the thermistor before and after the gas has expanded.

Solution

❖ **Capacitors in Series**

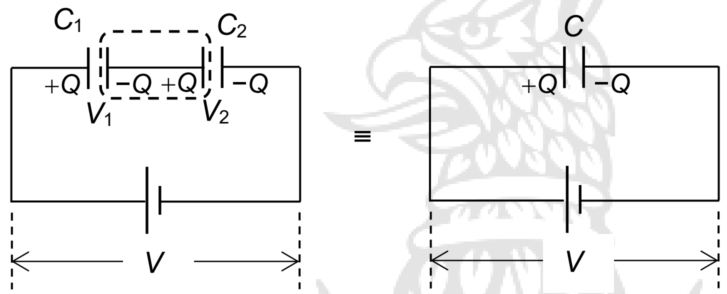


Fig. 16.30

When two capacitors are connected in series, the ***same charge*** Q is stored in both capacitors*. The potential difference across the combination is the sum of the potential differences across each capacitor:

$$V = V_1 + V_2$$

The capacitors connected in series may be regarded as a single or equivalent capacitor whose effective capacitance C can be determined using the equation $Q = CV$, where $V = V_1 + V_2$ and Q is the charge stored in this equivalent capacitor.

From $V = V_1 + V_2$, it follows that

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} \qquad \Rightarrow \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

So, in general, for any number of capacitors connected in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

*since the right plate of capacitor C_1 and the left plate of capacitor C_2 are connected, they form a single conductor. Before the two capacitors are charged, this single conductor is neutral (ie., uncharged). When the left plate of C_2 gains positive charge $+Q$ from C_1, the right plate of C_1 should have a charge of $-Q$ as charge is conserved. Hence both capacitors C_1 and C_2 have the same charge.

❖ **Capacitors in Parallel**

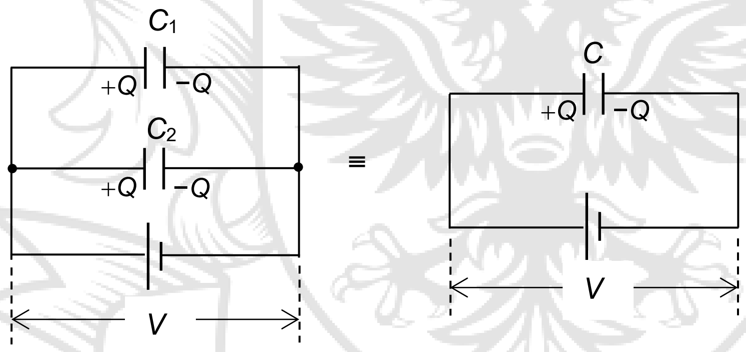


Fig. 16.31

When two capacitors are connected in parallel, the ***same potential difference*** V is applied across both capacitors. The charge stored in the combination is the sum of the charges stored in the capacitors:

$$Q = Q_1 + Q_1$$

Since $Q = CV$,

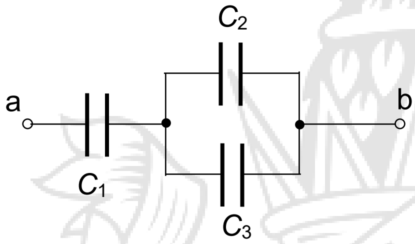
$$CV = C_1V + C_2V \qquad \Rightarrow \qquad C = C_1 + C_2$$

So, in general, for any number of capacitors connected in parallel,

$$C = C_1 + C_2 + C_3 + \ldots$$

Example 15

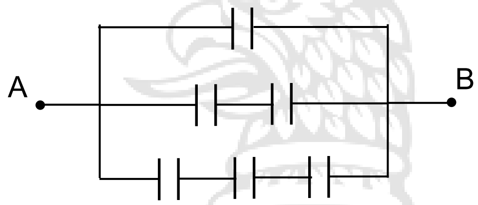
In the capacitor arrangement shown, the capacitances of the capacitors are C_1 = 3.0 μF, C_2 = 2.0 μF, C_3 = 4.0 μF. Determine the effective capacitance across points a and b.



Solution:

Example 16

Determine the effective capacitance between the points A and B in the arrangement of capacitors shown. The capacitance of each capacitor is 2.0 μF.



Solution:

❖ **Charging and discharging of a capacitor**

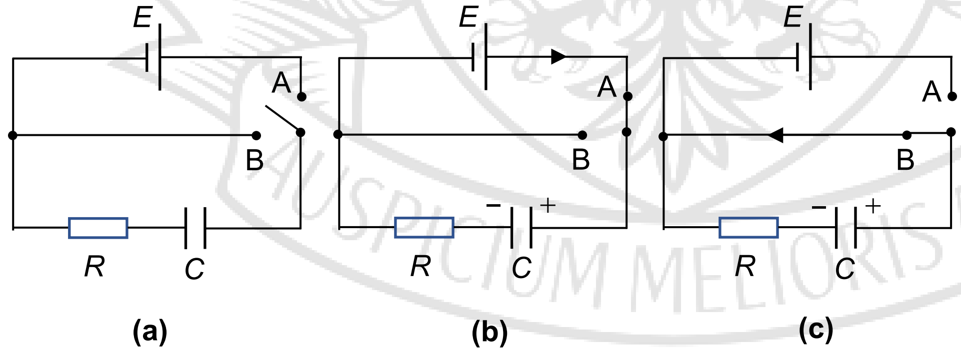


Fig. 16.32

Charging of a capacitor

Fig. 16.32 shows a simple ***RC circuit***. The capacitor is initially uncharged. At time $t = 0$, the switch is thrown to A in Fig. 16.32 **(b)** and current in the circuit charges the capacitor. During charging, charge is transferred between the plates through the connecting wires until the capacitor is fully charged. Applying Kirchhoff's voltage law (or 2nd law or loop rule) to the charging circuit,

$$E - V_C - V_R = 0 \Rightarrow E - \frac{q}{C} - IR = 0\,, \qquad \text{--- (**)}$$

where q/C is the p.d. across the capacitor, IR is the p.d. across the resistor and I is the instantaneous current in the circuit. Initially, at time $t = 0$, $q = 0$ and the current is a maximum, with a value of

$$I_0 = \frac{E}{R}.$$

Finally, when the capacitor is fully charged to its maximum value of Q_0, $I = 0$ and $V_R = IR = 0$.

$$E - \frac{Q_0}{C} - 0 = 0 \Rightarrow E = \frac{Q_0}{C}$$

Since $I = \dfrac{dq}{dt}$, then from equation (**) above,

$$\frac{dq}{dt} = \frac{E}{R} - \frac{q}{RC}.$$

The solution to this first-order differential equation is

$$q(t) = Q_0(1 - e^{-t/RC}).$$

The instantaneous p.d. across the capacitor is

$$V_C(t) = \frac{q(t)}{C} = \frac{Q_0}{C}\left(1 - e^{-t/RC}\right) = E\left(1 - e^{-t/RC}\right)$$

and the instantaneous current is

$$I(t) = \frac{dq}{dt} = \frac{Q_0}{RC}e^{-t/RC} = \frac{E}{R}e^{-t/RC} = I_0 e^{-t/RC}.$$

The ***time constant*** τ of the RC circuit is $\tau = RC$. It represents the time interval required for the current to change by 63.2% (or $1 - e^{-1}$) of its initial value.

The graphs below (Fig. 16.33) show the variations of the p.d. V_C across the capacitor and the current I through the circuit with time t during charging.

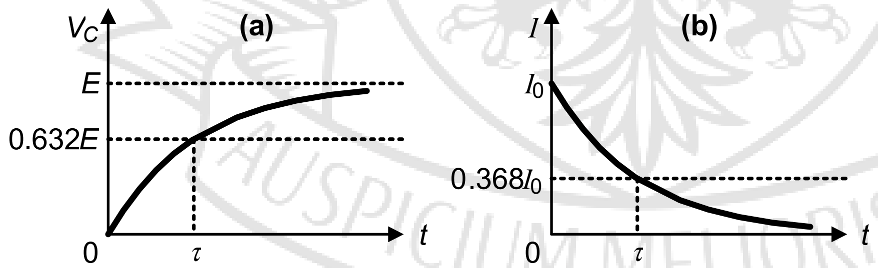


Fig. 16.33

Discharging of a Capacitor

The switch is now connected to B at time $t = 0$ as shown in Fig. 16.32 **(c)** and the capacitor begins to discharge through the resistor. At time *t*, the current in the circuit is I and the charge on the capacitor is *q*. Since *E* = 0, eqn (**) becomes

> Note that d*q*/d*t* is negative as charge is being lost from the capacitor as time goes on. Thus
> $$I = -\frac{dq}{dt}$$

$$\frac{q}{C} = IR = -\frac{dq}{dt} \cdot R$$

since $I = -\frac{dq}{dt}$.

$$\therefore \frac{dq}{dt} = -\frac{q}{RC}$$

The solution to this first-order differential equation is

$$q(t) = Q_0 e^{-t/RC}$$

The instantaneous p.d. across the capacitor is

$$V_C(t) = \frac{q(t)}{C} = \frac{Q_0}{C} e^{-t/RC} = E\, e^{-t/RC}$$

and the instantaneous current is

$$I(t) = \frac{dq}{dt} = \frac{Q_0}{RC} e^{-t/RC} = \frac{E}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

The graphs below (Fig. 16.34) show the variations of the p.d. V_C across the capacitor and the current I through the circuit with time *t* during discharging.

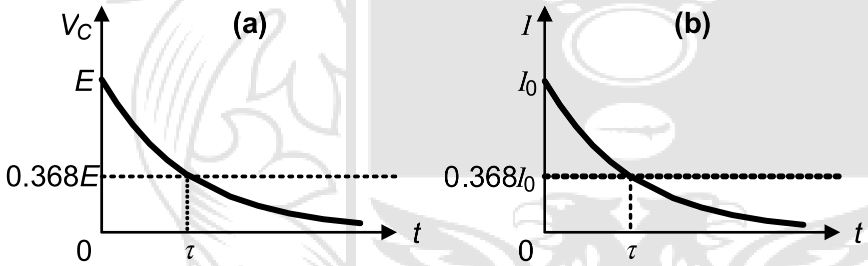
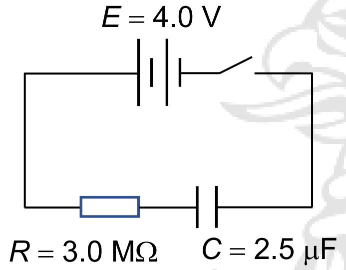


Fig. 16.34

Example 17

A 3.0 MΩ resistor and a 2.5 μF capacitor are connected in series with an emf source of 4.0 V. At 2.0 s after the switch is closed, determine the rates at which



(a) the charge of the capacitor is increasing, and
(b) energy is being stored in the capacitor.

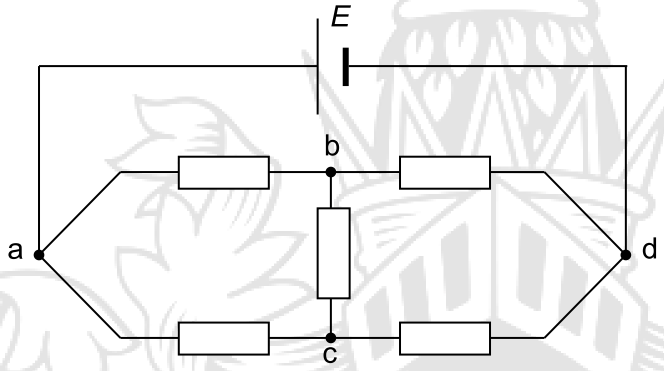
Solution:

16.5 Appendix

- **Kirchhoff's Laws**

Kirchhoff's Laws are useful in solving circuit problems. Kirchhoff's First Law, also known as the Point Rule, is applied to junctions in a circuit. A branch point (junction) is a point in a network at which three or more conductors are joined. Kirchhoff's Second Law, also known as the Loop Rule, is applied to loops in a circuit. A loop is any closed conducting path.

In the circuit below, the junctions are: a, b, c and d.
The loops are: abca , bdcb , abdca , a*E*dba, a*E*dca.



Kirchhoff's First Law (Point Rule) states that:

The algebraic sum of the currents toward any branch point of a network is zero.

At a junction, $\boxed{\Sigma I = 0}$

This law is based on the observation that no charge accumulates at a junction and hence total charge per unit time entering a junction equals total charge per unit time leaving the same junction.

Kirchhoff's Second Law (Loop Rule) states that:

The algebraic sum of e.m.f.s in any loop of a network equals the algebraic sum of the *iR* products in the same loop.

$$\boxed{\Sigma E = \Sigma iR}$$

This law is based on the principle of conservation of energy. Recall that e.m.f. and p.d. are terms used for energy per unit charge. A source of e.m.f. is a source of electrical energy whereas p.d. represents a sink of electrical energy where electrical energy is converted to other forms of energy. Imagine the journey of a coulomb of charge round a circuit loop. The total energy it delivers to the components in the loop (i.e. the sum of p.d.s round the loop) is equal to the total electrical energy which is supplied to it (i.e. the net e.m.f.).

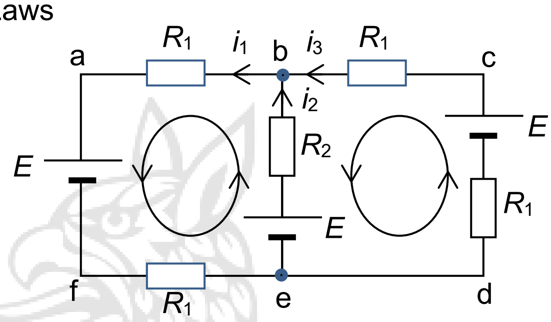
Application of Kirchhoff's Laws:

If there are n junctions in a circuit, Kirchhoff's First Law can be applied to (n – 1) junctions to obtain equations that are distinct from each other. Similarly, if there are n loops, Kirchhoff's Second Law can be applied to (n – 1) loops to form distinct equations. Then, if there are N unknowns to be solved, the number of equations needed is N and are formulated using Kirchhoff's First and Second Laws.

Example on application of Kirchhoff's Laws

Determine the three currents in the circuit given that:

$R_1 = 1.0\ \Omega$
$R_2 = 2.0\ \Omega$
$E_1 = 2.0$ V
$E_2 = E_3 = 4.0$ V



The circuit has only two junctions: b and e.

Applying Kirchhoff's Point Rule to junction b:

total current entering the junction = total current leaving the junction

$i_2 + i_3 = i_1$ ··· (1)

[Note: Application of Kirchhoff's Point Rule to junction e will also yield equation (1).]

Since there are three unknowns to be solved, another two equations are needed and these can be obtained by applying Kirchhoff's Loop Rule to two out of the following three possible loops: afeba, bedcb and afedcba.

Consider loop afeba:

E_2 (acting alone) will supply current in the direction of the loop while E_1 (acting alone) will supply a current in the opposite direction to the loop. Hence, net e.m.f. in the direction of the loop is given by:

$\Sigma E = E_2 - E_1 = 4.0 - 2.0 = 2.0$ V

The currents i_1 and i_2 are both in the direction of the loop and the iR products due to each of the currents flowing through the resistors are assigned positive values. Hence,

$\Sigma iR = i_1R_1 + i_1R_1 + i_2R_2 = 2.0\ i_1 + 2.0\ i_2$

Equating $\Sigma E = \Sigma iR$ for this loop:

$2.0\ i_1 + 2.0\ i_2 = 2.0$

$i_1 + i_2 = 1.0$ ··· (2)

Consider loop bedcb:

E_3 (acting alone) will supply current in the direction of the loop while E_2 (acting alone) will supply a current opposite in direction to the loop. Hence, net e.m.f. in the direction of the loop is given by:

$\Sigma E = E_3 - E_2 = 4.0 - 4.0 = 0$

The current i_3 is in the direction of the loop and the iR products due to the current i_3 passing through the resistors are assigned positive values. However, the current i_2 is opposite in direction to that of the loop, and so the iR products due to i_2 are assigned negative values.

$\Sigma iR = i_3R_1 + i_3R_1 - i_2R_2 = 2.0\ i_3 - 2.0\ i_2$

Hence, $\Sigma E = \Sigma iR$ for this loop leads to:

$i_3 - i_2 = 0$ ··· (3)

Solving equations (1), (2) and (3):

$i_1 = 0.67$ A , $i_2 = 0.33$ A and $i_3 = 0.33$ A

❖ The Potentiometer

A potentiometer is a set-up (an example is shown in Fig. 16.38) used to measure a potential difference (p.d.) without drawing any current (unlike a voltmeter), and it does this by balancing its p.d. (from a variable potential divider) against the p.d. to be measured. Balance is achieved when no current flows in the galvanometer. A potentiometer can be used to determine the e.m.f. and internal resistance of a given cell and can also be used to compare the e.m.f.s of different cells.

In practice, the potentiometer set-up in Fig. 16.38 is in the form shown in Fig. 16.40. The resistances R_1 and R_2 of the two resistors correspond to the resistances of the lengths AC and CB of the uniform wire AB respectively. Wire AB is called the potentiometer wire. In Fig. 16.40, the potential difference across any section of the wire will be directly proportional to the length of the wire, provided the wire is of a uniform cross-sectional area, and a uniform current flows through the wire.

To understand the meaning of "balancing two p.d.s", consider the following example: when two cells are connected as shown in Fig. 16.35, the total e.m.f. in the circuit is $E_1 + E_2$. Thus, the current in the circuit is $I = \dfrac{E_1 + E_2}{R}$.

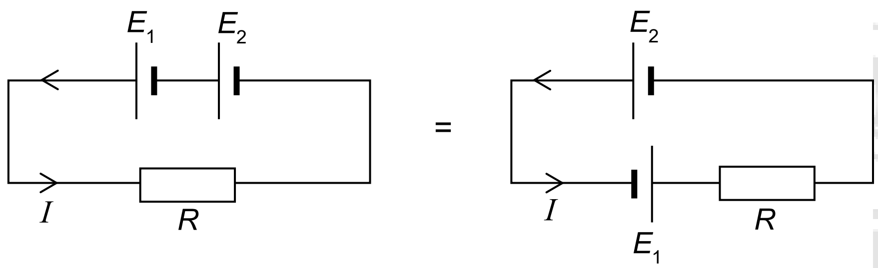


Fig. 16.35

But when the two cells are connected as shown in Fig. 16.36, the total e.m.f. is $E_1 - E_2$. If $E_1 = E_2$, then the total e.m.f. is zero. Thus, we say that the two e.m.f.s balance each other. When this happens, there will be no current in the circuit (i.e. $I = 0$).

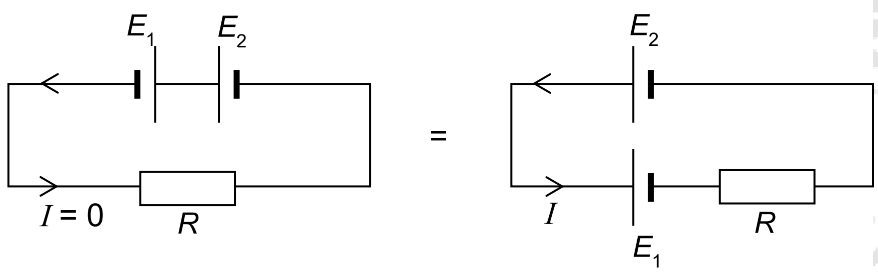


Fig. 16.36

As shown in Fig. 16.37 and Fig. 16.38, E_2 can be replaced with a potential divider circuit with an e.m.f. source E and resistors R_1 and R_2. When the galvanometer detects no current, we can conclude that V_1 is equal to E_2.

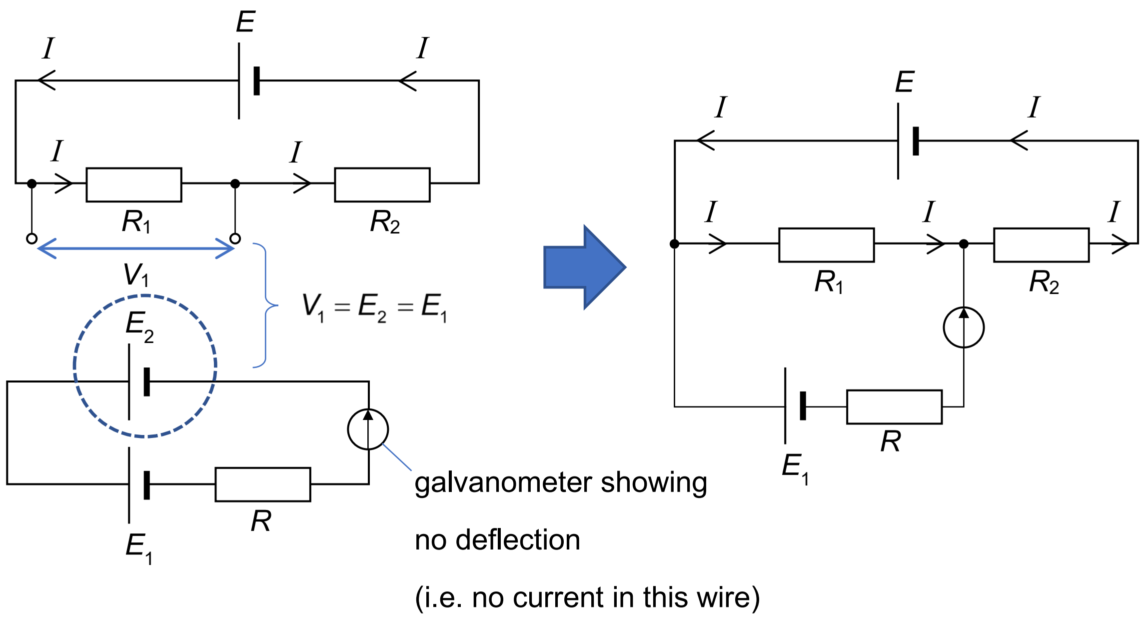


Fig. 16.37 **Fig. 16.38**

If the value of E_1 was not known earlier, we now know that it is just the p.d. V_1 across R_1. In other words, V_1 (or E_1) can now be determined accurately using the potential divider principle.

A more practical form of this instrument is to use a resistance wire AB in place of the two resistors, R_1 and R_2, with a sliding contact C connected through a galvanometer to the e.m.f. source that is to be measured, as shown in Fig. 16.40.

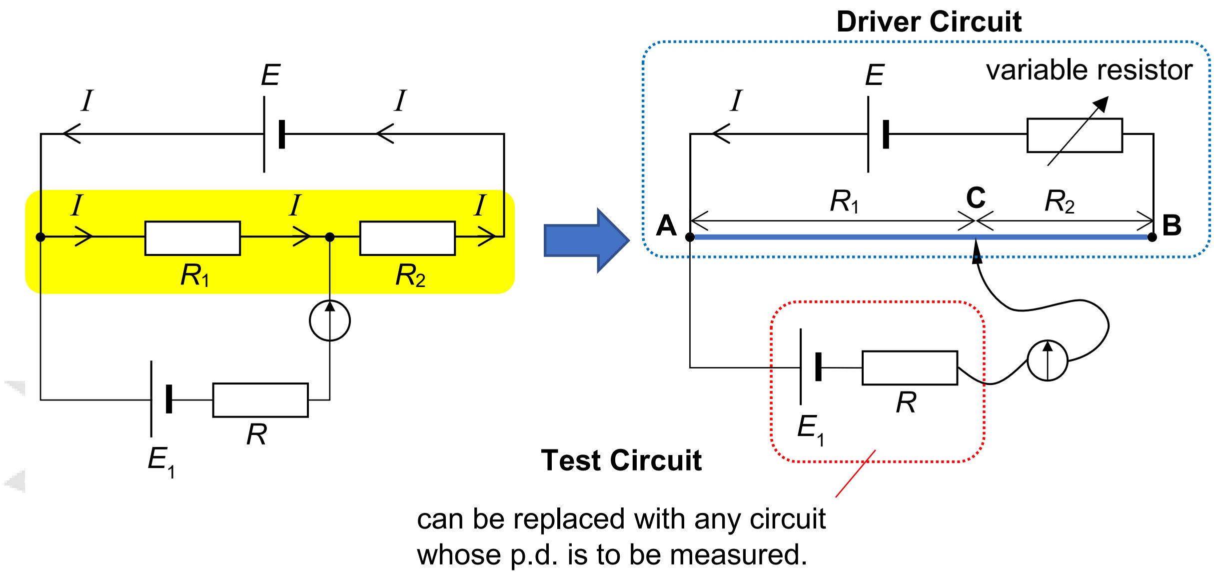


Fig. 16.39 **Fig. 16.40**

At this juncture, let us replace the symbols R_1, R_2, V_1 and V_2 with R_{AC}, R_{BC}, V_{AC} and V_{BC} respectively instead.

When the contact C is first placed (or made) randomly at a point along wire AB, the pd V_{AC} across AC may not be equal to E_1, and hence cannot balance each other. A current will thus flow through the galvanometer and it will show a non-zero deflection.

The contact is then shifted along the wire AB until the galvanometer shows no deflection. This means that V_{AC} balances E_1, and they are equal to each other. The point on the wire when this happens is called the "**balance point**".

Balance Length

When C is at the **balance point**, the length L_{AC} is known as the ***balance length***.

Applying the Potential Divider Principle

Assumptions

- Wire has uniform cross-sectional area and resistivity.
- Potential difference across wire AB remains constant with time.

For a uniform wire of length L, $R = \dfrac{\rho L}{A} \quad \Rightarrow R \propto L$

Since the same current flows through the entire wire when the potentiometer is at balance,

$$V_{AC} = \frac{R_{AC}}{R_{AB}} \times V_{AB} = \frac{L_{AC}}{L_{AB}} \times V_{AB}$$

$$\therefore \frac{V_{AC}}{V_{AB}} = \frac{R_{AC}}{R_{AB}} = \frac{L_{AC}}{L_{AB}}$$

Hence, the p.d. being measured $= V_{AC} = kL_{AC}$

where $k = \dfrac{V_{AB}}{L_{AB}} =$ p.d. per unit length of the uniform wire.

*[**TIP:** In solving problems involving the potentiometer, it is always useful to first determine the potential difference V_{AB} across the entire length of the resistance wire AB.]*

Comparing e.m.f.s

The circuit shown in Fig. 16.41 is used to compare the e.m.f.s of two cells.

[Note: Since two balance lengths are to be determined corresponding to the e.m.f.s of the two cells, we will call these balance lengths L_1 and L_2 corresponding to E_1 and E_2 respectively.]

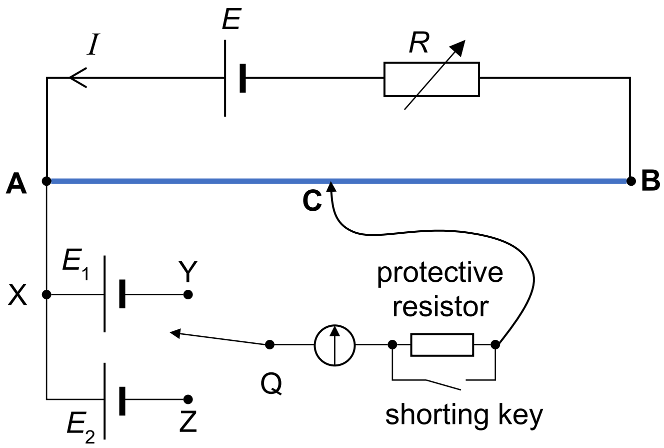


Fig. 16.41

Note:
To ensure that the balance point can be found with high *precision*, a very sensitive galvanometer is used.
Such a galvanometer should be connected in series with a **protective resistor** (with a high resistance). This prevents relatively high currents from flowing through the galvanometer when in off-balance situations.
The shorting key is left open until an approximate balance point has been found.

The key is then closed to short out the protective resistance so that the galvanometer is made very sensitive. The sliding contact C is then moved by small amounts until an accurate balance point is found.

Determining the balance length L_1 for E_1

With E_1 in the circuit (i.e. key Q connected to point Y), the balance point is located by *tapping* the jockey on the resistance wire until the galvanometer registers zero current. The balance length L_1 is then recorded.

Since at balance point, no current is drawn from either cell, the terminal p.d.s across their terminals are their e.m.f.s.

Thus, $E_1 = \frac{L_1}{L_{AB}} \times V_{AB}$

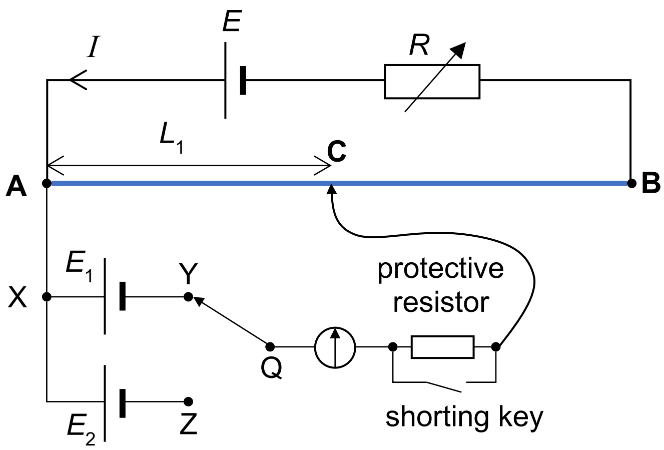


Fig. 16.42

Determining the balance length L_2 for E_2

With E_2 in the circuit (i.e. key Q is connected to Z), the new balance length L_2 is similarly determined: $E_2 = \frac{L_2}{L_{AB}} \times V_{AB}$

Thus, $\frac{E_1}{E_2} = \frac{L_1}{L_2}$

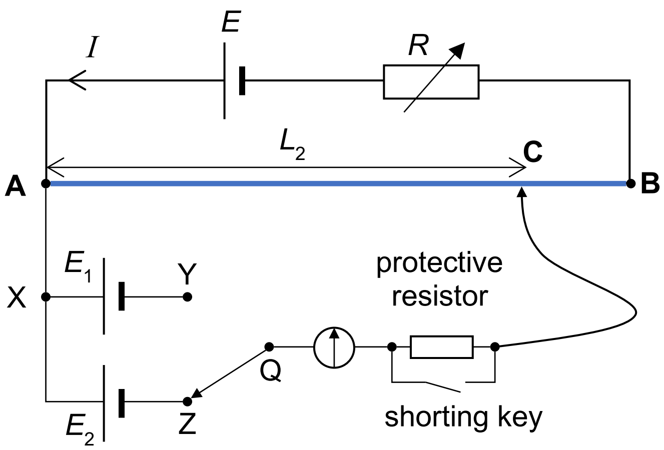


Fig. 16.43

If the e.m.f. of one of the cells is to be determined, the other cell should be a standard cell which has an accurately known e.m.f. The Weston standard cell, which has an e.m.f. of 1.018638 V, when new, is often used.

Potentiometer vs Voltmeter

When a potentiometer is used to compare the e.m.f.s of cells, no error is introduced by the internal resistances of the cells because no current flows through the cells at balance point and their terminal p.d.s are therefore equal to their respective e.m.f.s.

On the other hand, a real voltmeter has a finite resistance and draws a small current from the cell. The terminal p.d. measured is thus less than the emf of the cell (see Example 5). Thus, the measured values are slightly lower than the true e.m.f.s.

Since no current flows from the cell at the balance point, the potentiometer may thus be considered to be a voltmeter with an infinite resistance (that is, the potentiometer is an ideal voltmeter).

Practical Details

1. The <u>positive terminals of the cells</u> being compared, E_1 and E_2, must be <u>connected in the same direction as the driver cell</u> E. Otherwise, no balance point can be found.

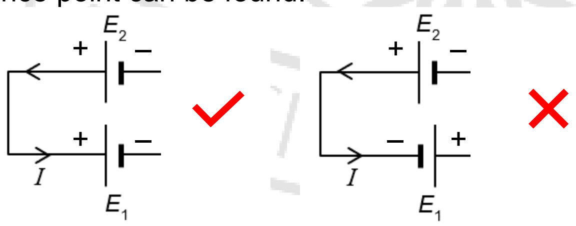


Fig. 16.44

2. The accuracies of the measurements of the balance lengths are greatest when L_1 and L_2 are large. A preliminary measurement should be made to determine which of L_1 and L_2 is the larger. The rheostat in the driver circuit should then be adjusted so that the larger of the two balance lengths L_1 and L_2 is close to the full length L_{AB}. The rheostat setting must be kept the same for the measurement for both balance lengths.
3. The current through the potentiometer wire must be the same throughout the experiment.
4. To ensure that the balance point can be found with high *precision*, a very sensitive galvanometer is used.

   To protect the galvanometer from high currents before an accurate balance point is found, it is connected in series with a high resistance resistor (known as a protective resistor). With the shorting key left open, an approximate balance point is first located. Next, the shorting key is then closed to short out the protective resistor. The galvanometer is now very sensitive to small potential differences across it and very small currents flow in it. By shifting the sliding contact C until the galvanometer shows no deflection, an accurate balance point can then be found.

Comparing resistances of two resistors

Determining balance length for V_1

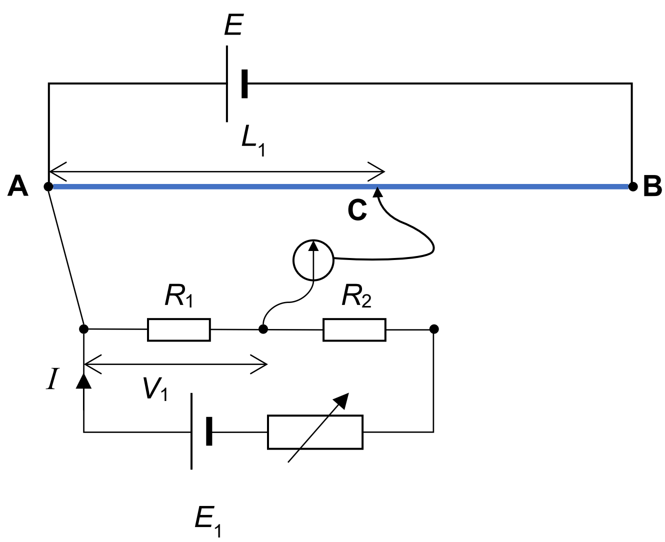


Fig. 16.45

To compare the resistances R_1 and R_2, the length L_1 of wire AB whose p.d. is equal to that across R_1 is first determined.

The **balance point** is at point C and the **balance length** is L_1.

$$\text{p.d. across } R_1 = V_1 = V_{AC} = \frac{L_1}{L_{AB}} \times V_{AB} = IR_1$$

Determining balance length for V_2

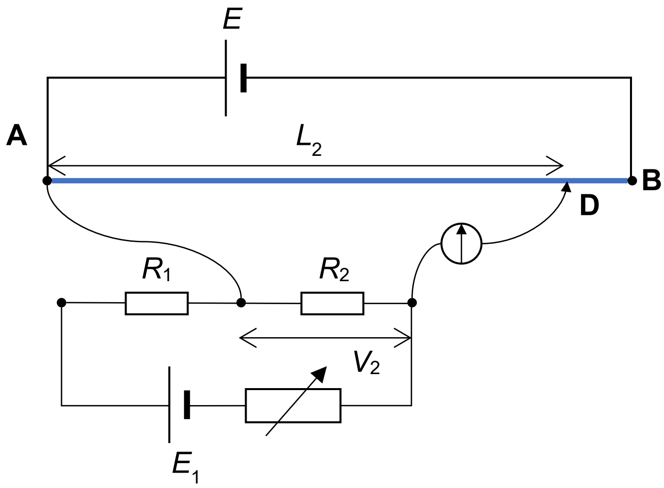


Fig. 16.46

Next, the length L_2 of wire AB whose p.d. is equal to that across R_2 is determined.

The **balance point** is at point D and the **balance length** is L_2.

p.d. across $R_2 = V_2 = V_{AD}$

$$= \frac{L_2}{L_{AB}} \times V_{AB}$$
$$= IR_2$$

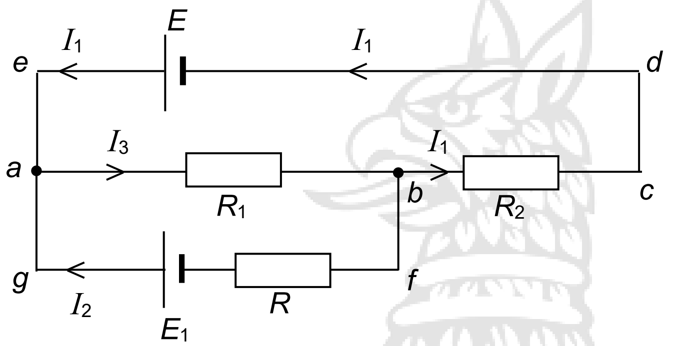
Since V_{AB}, L_{AB} and I are constant,

$$\frac{R_1}{L_1} = \frac{R_2}{L_2} \quad \Rightarrow \frac{R_1}{R_2} = \frac{L_1}{L_2}$$

If the resistance of either R_1 or R_2 is known, the resistance of the other unknown resistor can be determined.

❖ Applying Kirchhoff's Law in Potentiometer Circuits

Consider the following potentiometer circuit.



Kirchhoff's first law applied to junction *a* yields the equation

$$I_1 + I_2 = I_3 \quad (1)$$

Kirchhoff's second law applied to the driver circuit *acdea* yields the equation

$$E - I_3R_1 - I_1R_2 = 0 \quad (2)$$

and to the circuit *abfga* yields

$$E_1 - I_3R_1 - I_2R = 0 \quad (3)$$

In principle, one can solve the above three simultaneous equations and obtain expressions for the three currents in terms of the emf's and resistances. But for our current purpose, it suffices to consider the following three scenarios:

Scenario 1: $I_3R_1 < E_1$

This happens when R_1 is too small (or if the jockey is in contact with a point to the left of the balance point if a resistance wire is used instead of two resistors). From Eq. (3):

$I_2R = E_1 - I_3R_1 > 0$

Hence $I_2 > 0$, which means the current is flowing from *f* to *g*, just as illustrated in the circuit diagram above.

Scenario 2: $I_3R_1 > E_1$

This happens when R_1 is too large (or if the jockey is in contact with a point to the right of the balance point if a resistance wire is used instead of two resistors). Again from Eq. (3):

$I_2R = E_1 - I_3R_1 < 0$

Hence $I_2 < 0$, which means the current is flowing from *g* to *f*, opposite to the direction illustrated in the circuit diagram.

Scenario 3: $I_3R_1 = E_1$

This happens when R_1 is of just the right resistance (or if the jockey is in contact with the balance point). From Eq. (3), one obtains

$I_2R = E_1 - I_3R_1 = 0$

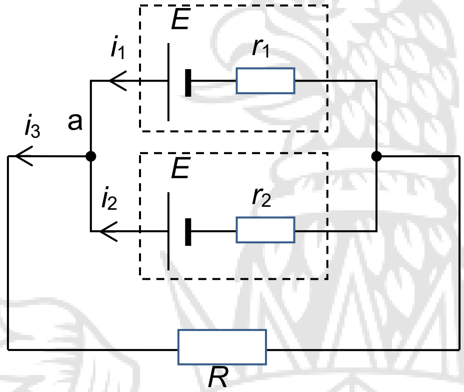
Hence $I_2 = 0$: there is no current in the branch containing E_1 and R in this case! Eq. (1) then says $I_1 = I_3$. Substitute E_1 for I_3R_1 in Eq. (2) yields

$$I_1 = I_3 = \frac{E - E_1}{R_2} = \frac{E_1}{R_1} = \frac{E}{R_1 + R_2},$$

where the second last equality comes from $I_3R_1 = E_1$, and the last comes from substituting I_3 for I_1 in Eq. (2).

- **Total e.m.f. and total internal resistance for unlike cells connected in parallel.**

Consider two cells, one with e.m.f. E_1 and internal resistance r_1 and the other with e.m.f. E_2 and internal resistance r_2, connected in parallel. The combination is connected to an external resistance R, as shown below.



Applying Kirchhoff's Point Rule to junction a,

$i_3 = i_1 + i_2$ ··· (1)

Applying Kirchhoff's Loop Rule for the loop containing E_1, r_1 and R,

$E_1 = i_1r_1 + i_3R$ ··· (2)

Applying Kirchhoff's Loop Rule for the loop containing E_2, r_2 and R,

$E_2 = i_2r_2 + i_3R$ ··· (3)

Multiplying equation (2) by r_2 and equation (3) by r_1 and then adding them:

$E_1r_2 + E_2r_1 = (i_1 + i_2)\, r_1r_2 + i_3R(r_1 + r_2)$

Since $i_3 = i_1 + i_2$, the above equation becomes:

$E_1r_2 + E_2r_1 = i_3\,[r_1r_2 + R(r_1 + r_2)]$

$$i_3 = \frac{E_1r_2 + E_2r_1}{r_1r_2 + R(r_1 + r_2)}$$

p.d. across R , $V_R = i_3R = \dfrac{(E_1r_2 + E_2r_1)R}{r_1r_2 + R(r_1 + r_2)}$

If $r_1 = r_2 = r$, then $V_R = \dfrac{(E_1 + E_2)rR}{r^2 + 2rR} = \dfrac{(E_1 + E_2)R}{r + 2R}$

If $r << R$, then $V_R = \dfrac{(E_1 + E_2)}{2}$

Hence, if the internal resistances of the cells are negligible, the total e.m.f. of the cells when connected in parallel is the average of the e.m.f.s of the cells.

NOTETAKING

"I am no poet, but if you think for yourselves, as I proceed, the facts will form a poem in your minds."