15 CURRENTS

H2 Physics 9478

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Learning Outcomes

Candidates should be able to:

(a) show an understanding that electric current is the rate of flow of charge and solve problems using $I = \frac{Q}{t}$.

(b) derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity.

(c) recall and solve problems using the equation for potential difference in terms of electrical work done per unit charge, $V = \frac{W}{Q}$.

(d) recall and solve problems using the equations for electrical power $P = VI$, $P = I^2R$ and $P = \frac{V^2}{R}$.

(e) distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations.

(f) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage. **[not in H1 syllabus]**

(g) represent a sinusoidal alternating current or voltage by an equation of the form $x = x_0 \sin \omega t$. **[not in H1 syllabus]**

(h) deduce that the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current. **[not in H1 syllabus]**

(i) distinguish between r.m.s. and peak values, and recall and use $I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}$ and $V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}$ for the sinusoidal case. **[not in H1 syllabus]**

(j) explain the use of a single diode for the half-wave rectification of an alternating current. **[not in H1 syllabus]**

Introduction

Direct current (d.c.) is the unidirectional flow of electric charge. This flow of electric charge in a constant direction distinguishes it from the alternating current (a.c.). Such direct currents are produced by sources such as batteries and solar cells.

Direct current may also be obtained from an alternating current supply by the use of a rectifier, which allows current in one direction only. Similarly, direct currents can also be converted into alternating currents with the use of an inverter.

The first commercial electric power transmission that was developed by Thomas Edison in the late 19th century used direct current. Today, direct current is used in rail transport, electric vehicles, general electrical appliances, as well as to charge batteries.

15.1 Current and drift velocity

❖ Charge and electric current

The SI unit of charge is the **coulomb** (C), named after the French physicist Charles-Augustine de Coulomb (1736–1806).

Charge is related to the electric current I by the equation:

$$I = \frac{Q}{t}$$

where Q is the amount of charge that passes through the cross section at any point of a conductor in time t.

Direction of current is the direction of flow of positive charges. Hence, electrons moving to the left gives rise to current to the right.

Shock and Awe: The Story of Electricity

Electric current
Electric current is the rate of flow of charge.

Electric current is measured in **ampere** (A), named after the French physicist André Ampère (1775–1836).

1 C = 1 A s, i.e., coulomb is an ampere second.

If the rate at which charge flows varies with time, the **instantaneous current I** is defined as

$$I = \frac{dQ}{dt}.$$

When a constant current I flows through a cross-section of a conductor for a duration t, the amount of electrical charge Q passing through it is

$$Q = It.$$

A continuous conducting path, i.e., a complete circuit, is required for there to be an electric current in a circuit.

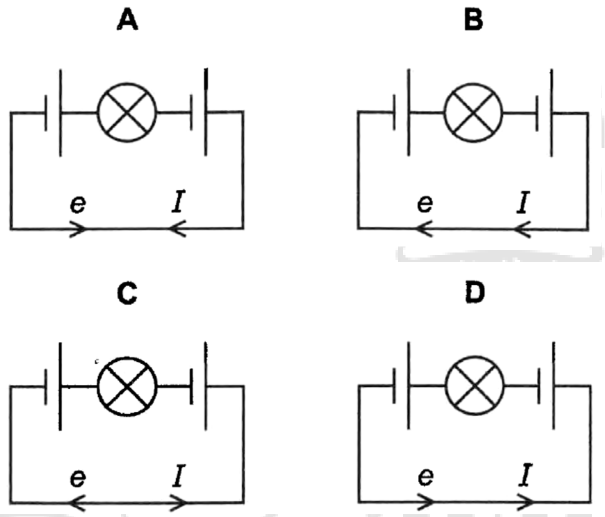
If the current flow is not constant,

$$Q = \int I \, dt.$$

By convention, the <u>direction of current is defined as the direction of flow of positive charge</u>, regardless of the actual sign of the charged carriers. Positive charges flow from the positive terminal to the negative terminal outside of a cell (i.e. from negative to positive terminal within a cell).

Example 1

Which diagram shows the direction of the current I and the direction of electron flow *e* in a circuit?



[Solution]
Direction of current is the direction in which positive charges move, i.e., in the ______________ direction.

Electrons move in the _____________ direction, i.e., in the ________________ direction, as they move from the __________terminal to the _________ terminal outside of the cells.

Answer:

❖ Drift velocity

In the absence of an electric field, the conduction or free electrons in a metal conductor move randomly and undergo frequent collisions with the positive ions that form the crystal lattice. The electrons do not, on average, move in any particular direction and there is no current. The magnitude of the velocity of the random motion is of the order of 10^6 m s^{-1}.

Number density is the number of charge carriers per unit volume.

When an electric field is applied to the conductor, the electrons continue to move randomly but with a net motion or drift in the direction opposite to that of the electric field. This motion is described in terms of the **drift velocity** of the electrons. The magnitude of the drift velocity is of the order of 10^{-4} m s^{-1} or less.

Drift Velocity Derivation - A Level Physics

Consider a current-carrying conductor with charge carriers each of charge q, number density of charge carriers n and cross-sectional area A. Assume that all charge carriers move with the same drift velocity v. In time t, each charge carrier in the conductor travels a distance ℓ, where $\ell = vt$. Hence, charge carriers within a cylinder of length ℓ and volume V will pass through a cross-section A of the conductor in time t.

A light bulb turns on almost immediately after a switch is closed (despite such a slow drift velocity) because closing the switch establishes an electric field almost immediately throughout the circuit, causing the free electrons in the entire circuit to move almost instantly. On the other hand, it may take hours for a charge to move all the way round a circuit.

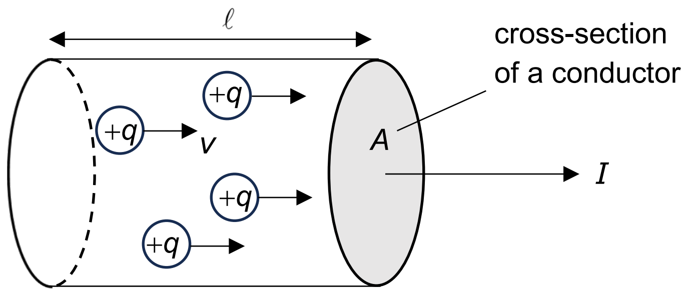


Fig. 15.1

From the definition of electric current,

$$
\begin{aligned}
I &= \frac{Q}{t} \\
&= \frac{Nq}{t} \quad (\because Q = Nq) \\
&= \frac{(nV)q}{t} \quad \left(\because n = \frac{N}{V}\right) \\
&= \frac{n(A\ell)q}{t} \quad (\because V = A\ell) \\
&= nAvq \quad \left(\because v = \frac{\ell}{t}\right)
\end{aligned}
$$

where N is the total number of charge carriers that pass through the cross section of the conductor in time t.

15.2 Potential difference, electromotive force and power

❖ Potential difference (p.d.)

Potential difference

The **potential difference *V*** between two points in a circuit is defined as the amount of electrical energy per unit charge that is converted to other forms of energy when charge passes from one point to the other.

Electric potential difference is colloquially known as voltage as it is measured in volts.

1 V = 1 J C^{-1}, i.e., volt is joule per coulomb.

The SI unit for electric potential and electric potential difference is the **volt** (V), named after Italian physicist Alessandro Volt (1745–1827).

Electrical work *W* is done when charges *Q* are moved across two points with potential difference *V*, transferring energy from electrical from to other forms. Mathematically,

$$V = \frac{W}{Q}$$

❖ Electromotive force (e.m.f.)

The sources of energy that cause charges to move in electric circuits are called sources of electromotive force. A source of e.m.f. is a device that maintains a potential difference across a conductor. Examples of sources of e.m.f. are batteries, electric generators and solar cells.

Electromotive force is not a force.

A cell is a source of constant e.m.f. and not a constant source of current or terminal voltage.

Electromotive force (e.m.f.)

The **electromotive force (e.m.f.) of a source** is defined as the amount of electrical energy per unit charge that is converted *from* other forms of energy when charge passes through the source.

A cell does not create or destroy any charge, i.e., a cell does not supply or absorb electrons, nor does a resistor absorb or destroy charge.

E.m.f. shares the same SI unit as p.d., i.e., the volt (V).

❖ Comparing p.d. and e.m.f

p.d. *V* across an external load	e.m.f. *E* of a source
$V = \frac{W}{Q}$	$E = \frac{W}{Q}$
W is the energy converted **from electrical energy** to other forms	*W* is the energy converted **from other forms** to electrical energy

❖ Electrical power

Electrical power is the electrical work done per unit time.

$$
\begin{aligned}
P &= \frac{dW}{dt} \\
&= VI \quad \left(\because W = QV \text{ and } \frac{dQ}{dt} = I\right) \\
&= I^2R \quad (\because V = IR) \\
&= \frac{V^2}{R}
\end{aligned}
$$

The units of electrical and mechanical powers are the same, i.e., the watt.

The SI unit of electrical power is the watt (W), named after the English inventor James Watt.

$1\ \text{W} = 1\ \text{J s}^{-1}$

Example 2

A 12 V car battery is connected to a lamp labelled 12 V, 24 W. The lamp shines at normal brightness for 25 hours.

Determine the amount of charge that passes through the lamp in this time.

Remember to convert the unit of time from hour to second as 1 C = 1 A s and not 1 A h.

[Solution]

Example 3

During a lightning strike, there is a current of 300 kA between a charged cloud and the Earth for 5.0 ms. The lightning releases 450 MJ of energy.

Calculate the potential difference between the cloud and the Earth.

[Solution]

15.3 Power supplies: d.c. and a.c.

A direct current has charges that flow in one direction only. Examples:

The value of direct current does not have to be constant.

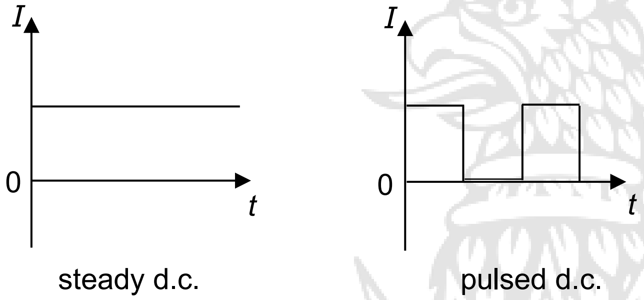


Fig. 15.2

An alternating current is one in which the flow of charges reverses in direction. Examples:

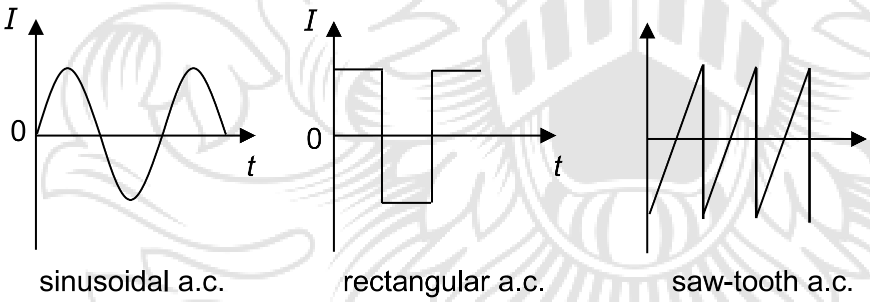


Fig. 15.3

❖ **Terminology**

Quantity	**Description**
period T	time taken to complete one cycle
frequency f	number of cycles per unit time
peak value I_0 (or V_0)	amplitude / maximum value of current (or voltage)

Example 4

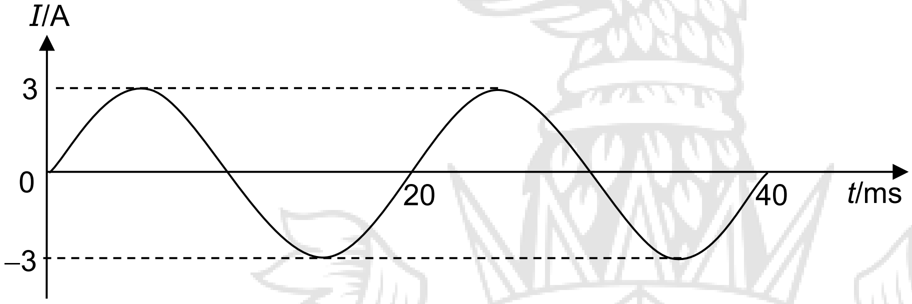
For the a.c. shown below, determine the

(a) peak current

(b) period

(c) frequency

(d) angular frequency



[Solution]

❖ Sinusoidal alternating current

The most commonly encountered a.c. takes a sinusoidal form as shown in Fig. 15.4. The waveform can be expressed by the equation:

$$I = I_0 \sin \omega t$$

where ω is the angular frequency ($\omega = 2\pi f$).

As potential difference is related to current by the equation $V = IR$,

$$\begin{aligned} V &= (I_0 \sin \omega t) R \\ &= (I_0 R) \sin \omega t \\ &= V_0 \sin \omega t \quad (\because V_0 = I_0 R) \end{aligned}$$

Hence, the alternating current and alternating p.d. are in phase.

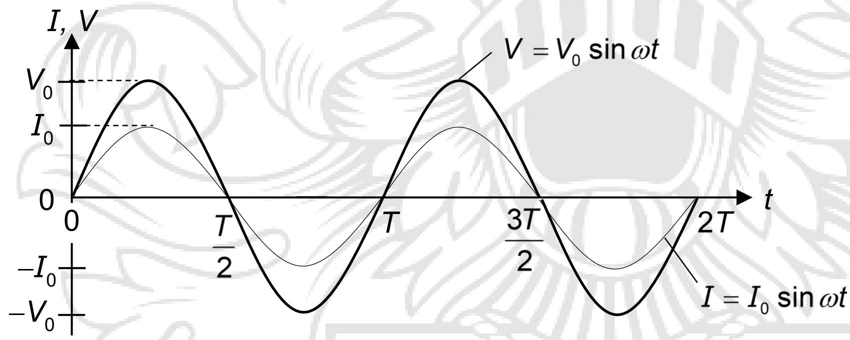


Fig. 15.4

❖ **Instantaneous power**

The instantaneous power P is given by the following equations,

$$\begin{aligned} P &= VI \\ &= (V_0 \sin \omega t)(I_0 \sin \omega t) \\ &= V_0 I_0 \sin^2 \omega t \\ &= P_0 \sin^2 \omega t \quad (\because P_0 = V_0 I_0) \end{aligned}$$

or

$$\begin{aligned} P &= I^2 R \\ &= (I_0 \sin \omega t)^2 R \\ &= (I_0^2 R) \sin^2 \omega t \\ &= P_0 \sin^2 \omega t \quad (\because P_0 = I_0^2 R) \end{aligned}$$

or

$$\begin{aligned} P &= \frac{V^2}{R} \\ &= \frac{(V_0 \sin \omega t)^2}{R} \\ &= \left(\frac{V_0^2}{R}\right) \sin^2 \omega t \\ &= P_0 \sin^2 \omega t \quad \left(\because P_0 = \frac{V_0^2}{R}\right) \end{aligned}$$

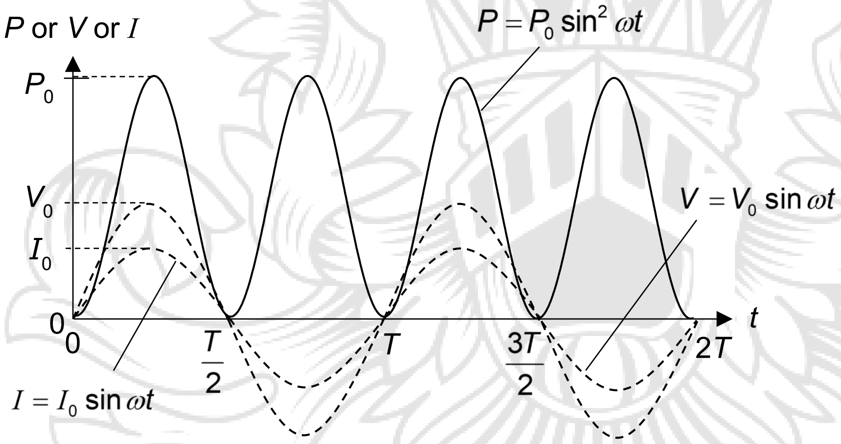


Fig. 15.5

Powers are always greater than or equal to zero.

The period of power is **half** that of the sinusoidal a.c. and hence its frequency is **twice** the frequency of the a.c.

❖ **Mean power**

The mean power $\langle P \rangle$ or $\bar{P}$ can be found by considering the $P - t$ graph in Fig. 15.6.

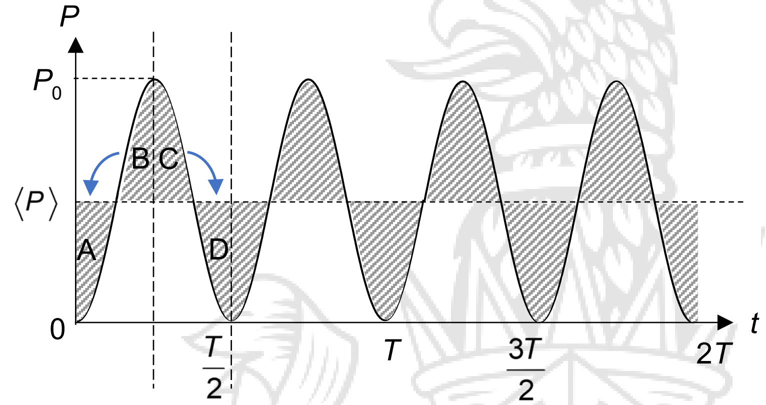


Fig. 15.6

Mean power $\langle P \rangle$ of an a.c. is the constant power that gives the same area under the $P - t$ graph as the a.c. in one cycle.

Graphically, $\langle P \rangle = \dfrac{\text{area under the } P - t \text{ graph in time } t}{t}$.

For the shaded regions A, B, C and D to be of equal area (so that B can fit into A and C into D), the horizontal line cutting across the graph is half the maximum power, i.e., $\dfrac{P_0}{2}$.

Hence, the area under $P - t$ graph from $t = 0$ to $t = T/2$ is equal to the area of the rectangle with base $T/2$ and height $\dfrac{P_0}{2}$.

Thus, the mean power in a resistive load is **half** the maximum power for a **sinusoidal** alternating current, i.e.,

$$\langle P \rangle = \frac{P_0}{2}.$$

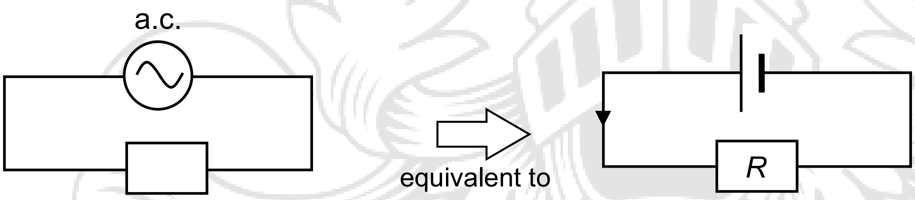
❖ Root-mean-square value

For a sinusoidal a.c., the mean value of current $\langle I \rangle$ is **zero**. However, heat is dissipated when there is an a.c. in a resistor, implying that the mean value of an a.c. cannot be used to calculate the mean power of the a.c.. Instead, one should use the root-mean-square value:

$$\langle P_{ac} \rangle = \langle I_{ac}^2 R \rangle = \langle I_{ac}^2 \rangle R = \left(\sqrt{\langle I_{ac}^2 \rangle} \right)^2 R = \left(I_{\text{r.m.s.}} \right)^2 R$$

where $I_{\text{r.m.s.}} = \sqrt{\langle I_{ac}^2 \rangle}$ is known as the root-mean-square current of the a.c..

In Fig. 15.7, the diagram on the left shows a resistor R connected to an a.c. source, while the diagram on the right shows the same resistor connected to a d.c. source. P_R is the power dissipated in R.



$$P_R = \langle P_{ac} \rangle = \left(I_{\text{r.m.s.}} \right)^2 R \qquad P_R = \langle P_{dc} \rangle = I_{dc}{}^2 R$$

Fig. 15.7

Suppose the resistor in both circuits dissipates heat at the same rate, we can then equate the average power dissipated as:

$$\langle P_{ac} \rangle = P_{dc}$$

$$\left(I_{\text{r.m.s.}} \right)^2 R = I_{dc}^2 R$$

$$\therefore I_{\text{r.m.s.}} = I_{dc}$$

Root-mean-square value $I_{\text{r.m.s.}}$ (or $V_{\text{r.m.s.}}$)

The root-mean square value is the constant direct current (or voltage) that produces the same power in a resistor as the alternating current (or alternating voltage)

Meters used for a.c. and alternating voltage are nearly always calibrated to read r.m.s. values.

Currents and voltages in power distribution systems are always described in terms of r.m.s. values.

Similarly, current and voltage ratings in electrical appliances / supplies are also expressed in r.m.s. values.

The alternating electrical mains supply to households in Singapore is 230 V, 50 Hz.

To determine the r.m.s. value:
1. Consider one period of the alternating quantity.
2. Square the quantity.
3. Find the mean of the squared quantity.
4. Take the square root of the mean-square quantity.

For a **sinusoidal** a.c., the r.m.s. value of the current $I_{\text{r.m.s.}}$ or $\sqrt{\langle I^2 \rangle}$ can be found by applying these steps to Fig, 15,8.

The average value of $\sin^2 \omega t$ over a full cycle is ½.

The equations $I_{\text{r.m.s.}} = \dfrac{I_0}{\sqrt{2}}$ and $V_{\text{r.m.s.}} = \dfrac{V_0}{\sqrt{2}}$ are for the **sinusoidal** case.

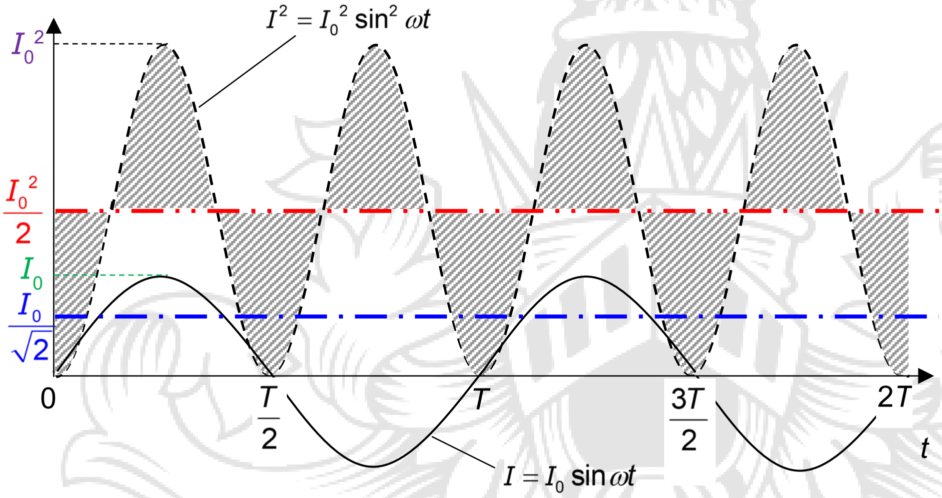


Fig. 15.8

Step 1:
Consider one period of the a.c. I–t graph.

Step 2:
Square the I–t to obtain the I^2–t graph (- - - - - graph).
The peak of the I^2–t graph is I_0^2.

Step 3:
Find the mean of the I^2–t graph (– · · – · · – graph).
Similar to the computation of mean power, the mean of the I^2–t graph is half the peak value, i.e., $\dfrac{I_0^{\,2}}{2}$.

Step 4:
Take the square root of the mean-square value to determine the $I_{\text{r.m.s.}}$ value (– · – · – · graph).

Hence, $I_{\text{r.m.s.}}$ of a sinusoidal a.c. is related to the peak current I_0 by:

$$I_{\text{r.m.s.}} = \sqrt{\frac{I_0^{\,2}}{2}} = \frac{I_0}{\sqrt{2}}$$

Similarly,

$$V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}$$

From the equation of mean power, $\langle P \rangle = \dfrac{P_0}{2}$,

$$\langle P \rangle = \frac{V_0 I_0}{2} \quad (\because P_0 = V_0 I_0)$$
$$= V_{\text{r.m.s.}} I_{\text{r.m.s}}$$

or

$$\langle P \rangle = \frac{I_0^{\,2} R}{2} \quad (\because P_0 = I_0^{\,2} R)$$
$$= I_{\text{r.m.s.}}^{\;2} R$$

or

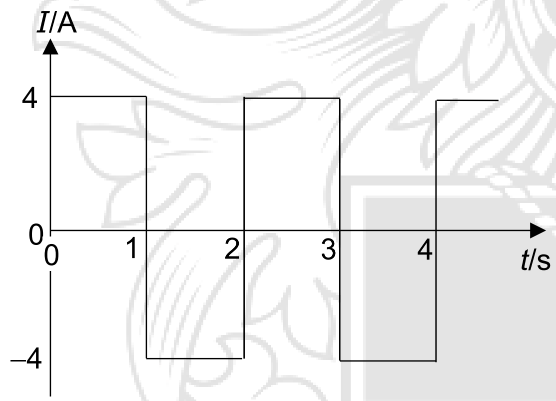
$$\langle P \rangle = \frac{V_0^{\,2}}{2R} \quad \left(\because P_0 = \frac{V_0^{\,2}}{R}\right)$$
$$= \frac{V_{\text{r.m.s.}}^{\;2}}{R}$$

The three equations for mean power can be used for all types of a.c. in circuits with resistors only.

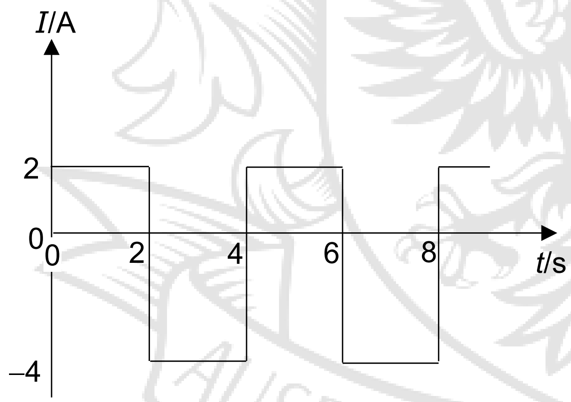
Example 5

Determine the mean value and r.m.s. value for each of the alternating current shown.

(a)



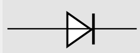
(b)



[Solution]

❖ Rectification

Although electrical power is transmitted as a.c., many appliances require the use of d.c. The process of converting a.c. to d.c. is known as rectification and this is done using diodes, which conduct current in one direction but not in the opposite direction.



circuit symbol of diode

The arrow indicates the direction in which the diode conducts.

A diode is forward-biased when it is connected to a power supply in such a way that there is current in it. It is reverse-biased if there is little or no current in it. An ideal diode has zero resistance in forward bias and infinite resistance in reverse bias.

forward-biased (there is current) reverse-biased (no current)

Fig. 15.9

Half-wave Rectification

The a.c. supply to be rectified is connected in series with the diode and the equipment that requires d.c. (represented by resistor R in Fig. 15.9).

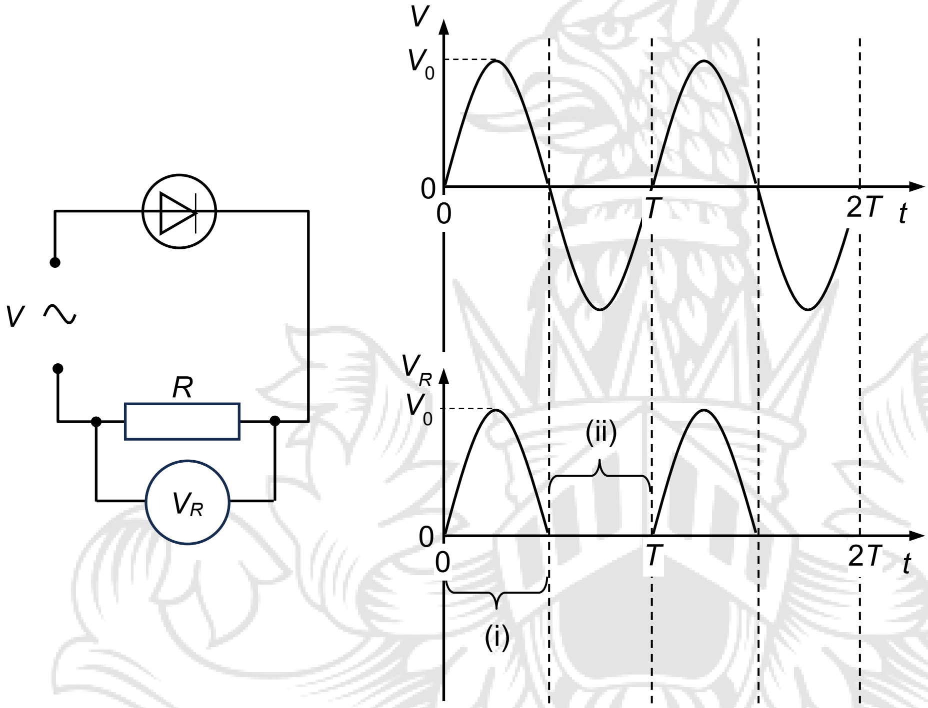


Fig. 15.10

The current through load R is in one direction only.

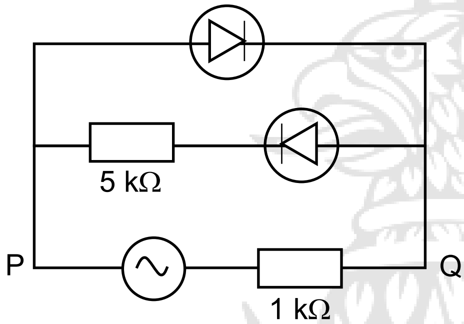
For the first half-cycle (i), when the diode is forward-biased, there is current in the circuit and a potential difference V_R across R.

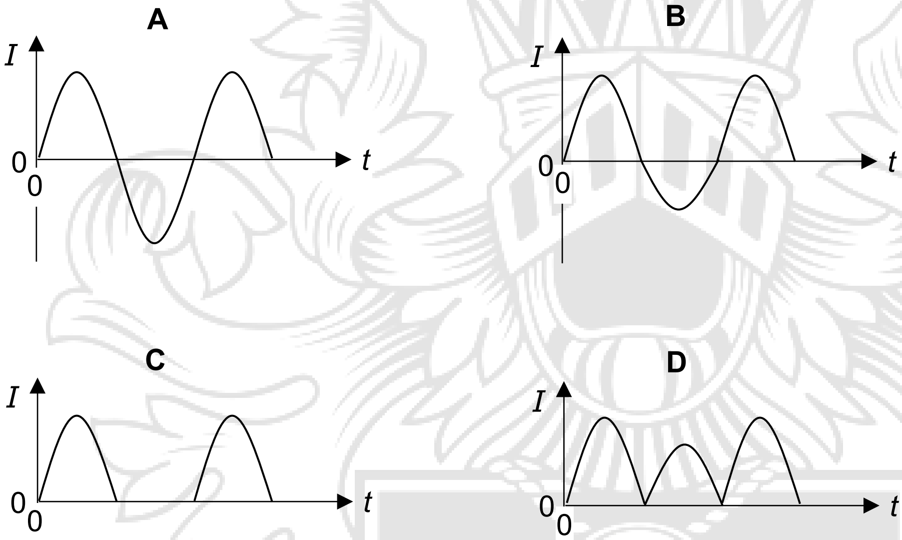
For the second half cycle (ii), when the diode is reverse-biased, there is no current in the circuit and the potential difference across R is zero.

This is repeated for each cycle of the a.c.

Example 6

Which of the following graphs below represents the variation of current I with time t through PQ?





[Solution]

$I = I_0 \sin \omega t$

For the first half of the cycle (assume P is at a higher potential than Q), there is no current in the middle branch. Hence, __________.

For the second half of the cycle (Q is then at a higher potential than P), there is no current in the upper branch. Hence, ___________.

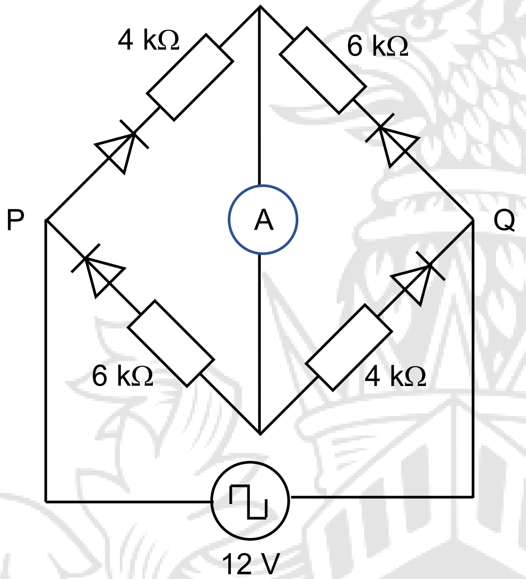
Since R_{eff} for the second half of the cycle is greater, peak value of current I_0 is smaller.

Answer:

Example 7

The diagram shows a diode-resistor network. The network is connected to a 12 V a.c. source with a **square** waveform.

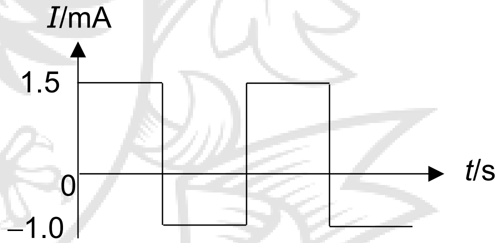
Determine the r.m.s reading on the ammeter.



[Solution]

For the first half of the cycle, when P has a higher potential than Q and current flows through the 4 kΩ resistors,

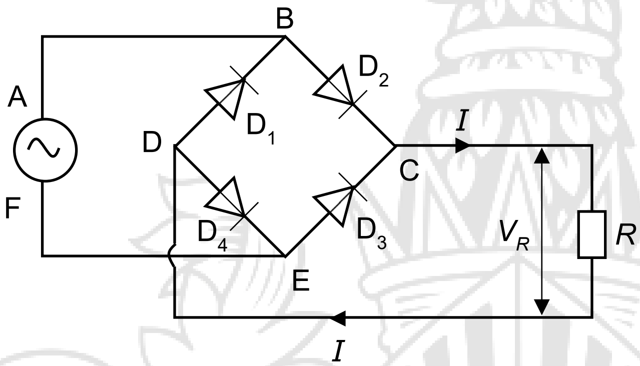
For the second half of the cycle, when Q has a higher potential than P and current flows through the 6 kΩ resistors,

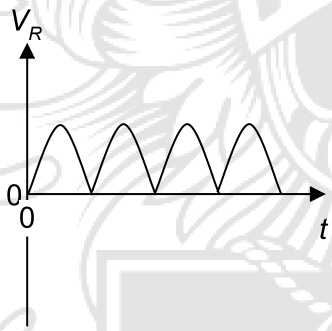


15.4 Appendix

❖ Full-wave rectification

To improve the transmission of power, full-wave rectification could be introduced by using a full-wave (or bridge) rectifier. The circuit and output voltage for this layout is shown below.





Only diodes D_2 and D_4 conduct during one half of the cycle (when A is positive with respect to F) and only diodes D_3 and D_1 conduct in the other half (when F is positive with respect to A).

The current through load R is always from C to D.

❖ Mathematical derivation of r.m.s. value for a sinusoidal a.c.

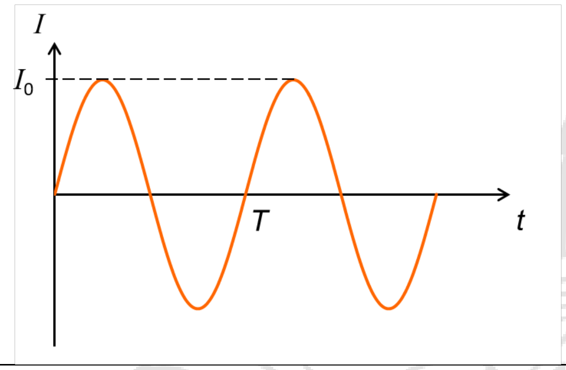
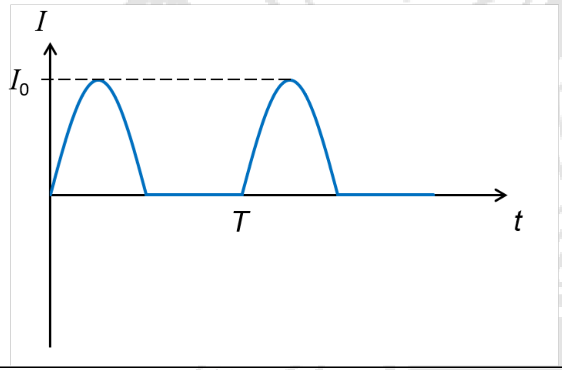
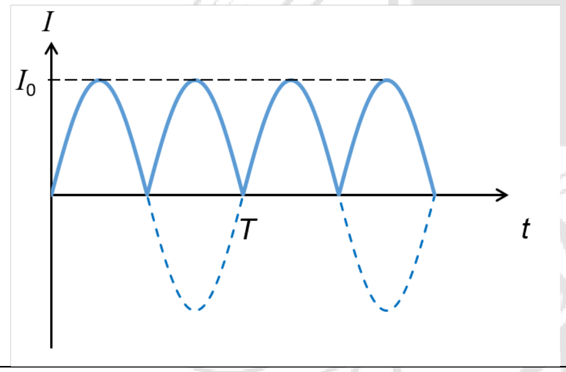
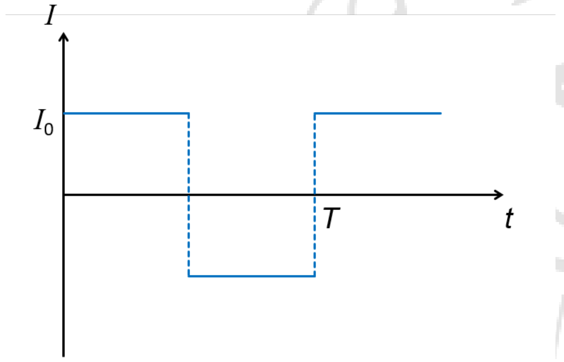
$$
\begin{aligned}
x_{\text{r.m.s.}} &= \sqrt{\frac{1}{T}\int_0^T (x_0 \sin \omega t)^2 \, dt} \\
&= \sqrt{\frac{x_0^{\,2}}{T}\int_0^T \sin^2 \omega t dt} \\
&= \sqrt{\frac{x_0^{\,2}}{2T}\int_0^T 1-\cos(2\omega t)\, dt} \qquad \left(\because \sin^2 x = \frac{1-\cos(2x)}{2}\right) \\
&= \sqrt{\frac{x_0^{\,2}}{2T}\left[t - \frac{\sin(2\omega t)}{2\omega}\right]_0^T} \\
&= \sqrt{\frac{x_0^{\,2}}{2\cancel{T}}(\cancel{T})} \\
&= \frac{x_0}{\sqrt{2}}
\end{aligned}
$$

- **Reference Links**

Physics for Scientists and Engineers with Modern Physics 10th edition (Serway and Jewett)

Fundamentals of Physics Extended 10th edition (Halliday and Resnick)

- **Summary**

Variation of current I with time t	Waveform	$I_{\text{r.m.s.}}$	$\langle I \rangle$
	sinusoidal	$\dfrac{I_0}{\sqrt{2}}$	0
	half-wave rectified	$\dfrac{I_0}{2}$	$\dfrac{I_0}{\pi}$
	full-wave rectified	$\dfrac{I_0}{\sqrt{2}}$	$\dfrac{2I_0}{\pi}$
	rectangle/square	I_0	0

NOTETAKING

"The measure of greatness in a scientific idea is the extent to which it stimulates thought and opens up new lines of research."