14 ELECTRIC FIELDS

H2 Physics 9478

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Learning Outcomes

Candidates should be able to:

(a) recall and use coulomb's law in the form $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ for the electric force between two point charges in free space or air.

(b) recall and use $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ for the electric field strength due to a point charge, in free space or air, to solve problems.

(c) define electric potential at a point as the work done per unit charge by an external force in bringing a small positive test charge from infinity to that point.

(d) use the equation $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ for the electric potential in the field due to a point charge, in free space or air.

(e) show an understanding that the electric potential energy of a system of two-point charges is $U_E = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r}$.

(f) recall that electric field strength at a point is equal to the negative potential gradient at that point and use this to solve problems.

(g) calculate the field strength of the uniform electric field between charged parallel plates in terms of the potential difference and plate separation.

(h) calculate the force on a charge in a uniform electric field. **[in H1 syllabus]**

(i) describe the effect of a uniform electric field on the motion of charged particles. **[in H1 syllabus]**

(j) define capacitance as the ratio of charge stored to the potential difference and use $C = Q/V$ to solve problems.

(k) recall that the electric potential energy stored in a capacitor is given by the area under the graph of potential difference against charge stored, and use this and the equations $U = \frac{1}{2}QV$, $U = \frac{1}{2}\frac{Q^2}{C}$ and $U = CV^2$ to solve problems.

14.1 Coulomb's Law

Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the same torsion balance Cavendish used to meaure the value of the gravitational constant *G* (see Chapter 8). In 1785, he successfully overcame various obstacles after numerous experimentations and established the inverse-square law for electric force between charges which was analogous to Newton's law of gravitation.

https://www.aps.org/publications/apsnews/201606/physicshistory.cfm

The electric force between two point charges in free space or air is given by **Coulomb's law**.

> **Coulomb's Law**
>
> Coulomb's law states that the force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}}$$

where *F* is the magnitude of the electric force between point charges with charges Q_1 and Q_2 that are separated by a distance *r*.

The constant of proportionality is $(4\pi\varepsilon_0)^{-1}$ where $\varepsilon_0 = 8.85\times10^{-12}\ \text{F m}^{-1}$ is the permittivity of free space.

NOTE

- The electric forces that two point charges exert on each other are equal in magnitude and opposite in direction; they constitute an action and reaction pair.
- The electric forces are directed along the line joining the two point charges
- Both Coulombs' law and Newton's law of gravitation are referred to as inverse-square law, due to their $1/r^2$ dependence.
- The major difference between gravitational force and electric (or electrostatic) force is that the former is always an attractive force while the latter can be **attractive or repulsive**. This is because there are two types of charges – positive and negative. Like charges repel, while unlike charges attract as shown in **Fig.14.1**.

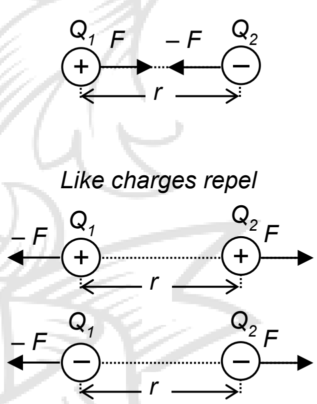


Fig. 14.1 Forces between point charges

❖ Using Coulomb's Law

There are two conditions that form the basis of using Coulomb's law to solve electrostatic problems.

1. Coulomb's law applies only to **point charges**. Two charged objects can be modelled as point charges if they are much smaller than the separation between them.
2. Coulomb's law is a force law and forces are vectors. When more than two charges are present, the resultant force on any of them equals the **vector sum** of the forces exerted by the various individual charges.

Example 1

Two charges Q (+2.0 μC) and q (+1.0 μC) are separated by a distance of 3.0 m.

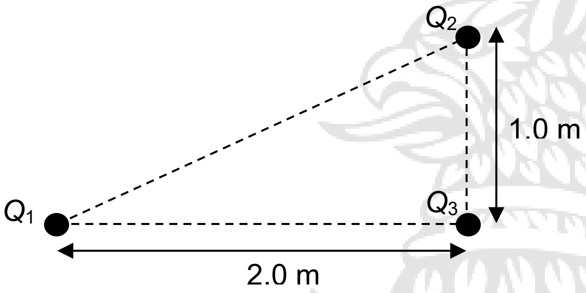
(a) Determine the magnitude and direction of the force acting on q by Q.

(b) If q is negatively charged instead, but with the same magnitude, determine the magnitude and direction of the force on q by Q.

[Solution]

Example 2

Three charges Q_1 = +2.0 μC, Q_2 = +1.0 μC and Q_3 = +3.0 μC are placed at the corners of a right-angle triangle with dimensions as shown.



Determine the magnitude and direction of the resultant force acting on Q_3.

Problem Solving Strategy

1. Identify point charges or objects to be modelled as point charges.
2. Draw free body diagram to show the force vectors on the identified object of interest.
3. Select an appropriate coordinate system e.g., x-y coordinates.
4. Determine the **resultant electric force** on selected object by vector summation.

14.2 Electric Field Strength

Electric forces like gravitational forces are long-range forces which require no contact for one charged particle to exert a force on another. The concept of a field was developed by Michael Faraday (1791 – 1867) to explain how long-range forces operate without physical contact between interacting objects.

A charged particle creates a field of influence around itself that permeates space. Other charged particles present in this field experience a force, which is called an electric force.

Electric Field

An electric field is a region of space in which a charge placed in that region experiences an electric force.

Electric Field Strength

The electric field strength at a point in an electric field is defined as the electric force exerted per unit positive charge placed at that point.

$$\boxed{E = \frac{F}{q}}$$

Electric field strength is a **vector** quantity.

S.I. unit for electric field strength is **N C**$^{-1}$ or **V m**$^{-1}$.

If the electric field strength E at a point in space is known, the electric force F experienced by a charge q placed at that point is given by

$$\boxed{F = qE}$$

Electric force is a **vector** quantity.

S.I. unit for electric force is the **newton (N)**.

NOTE

- The **direction of the electric field strength** is that of the **electric force acting on a small positive test charge** placed at that point.
- The **direction of the electric force** on a charged particle depends on whether its charge is positive or negative (**Fig. 14.2**). A **positive charge** will experience an **electric force** in the same direction as the electric field. A **negative charge** will experience an **electric force** in the opposite direction to the electric field.
- Some A-Level questions refer to electric field strength as just "electric field".

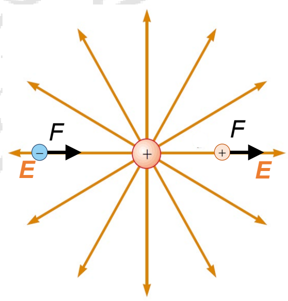


Fig. 14.2

- The formulae above relating the electric force and electric field strength apply to any electric field.
- Two types of electric field will be discussed: non-uniform fields due to point charges and uniform fields produced by a pair of parallel charged plates.

Example 3

A charge of -2.0 μC experiences a force of 8.0 N when placed at a point in an electric field. Determine the electric field strength at that point.

[Solution]

❖ Electric Field due to Point Charges

The electric force acting on point charge q, which is a distance r away from another point charge Q, is given by $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$, according to Coulomb's law.

The electric field strength due to Q at a distance r is given by

$$E = \frac{F}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Hence, the electric field strength (**Fig. 14.3**) due to **a point charge** Q at a distance r away is given by

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

Fig. 14.3

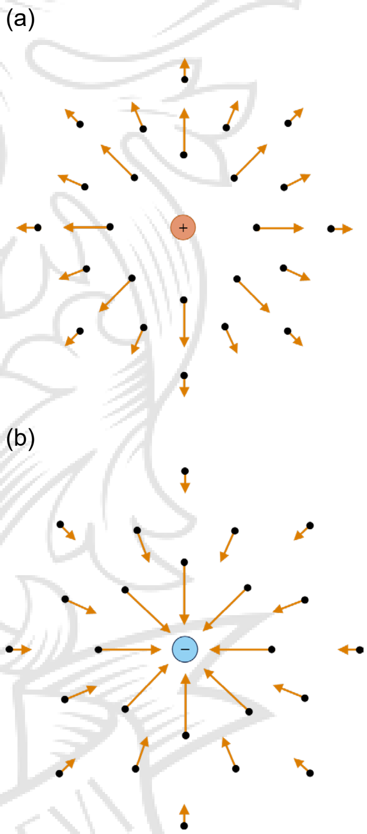


Fig. 14.4 Electric field vectors of an isolated (a) positive and (b) negative point charge. Each arrow represents the electric field direction at that point.

NOTE

- When determining the electric force on charge q, only the field strength due to the other charge (Q) is considered. In other words, the electric field of q does not act on q itself. Similarly, when determining the electric force F on charge Q, only the field strength due to q (i.e. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$) should be considered:

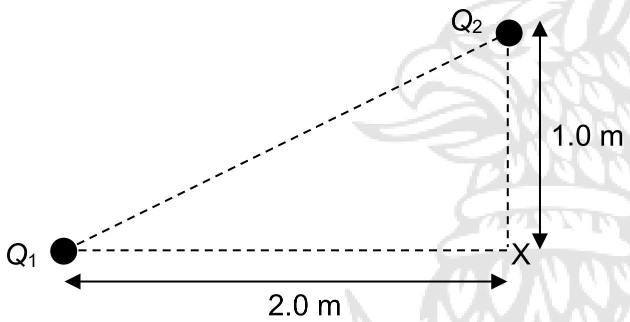
$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = Q\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)$$

- The direction of the electric field strength due to a charged particle depends on the sign of its charge: it points **radially away from a positive point charge**, and **radially towards a negative point charge**. These directions, by definition, are the **directions of the electric forces acting on a small positive test charge** placed at that point. **Fig. 14.4** shows the electric field vectors at different points in the electric field of an isolated positive and negative point charge.

- The **resultant electric field strength** at a point due to more than one charge can be found from the **vector sum** of the individual electric field strength due to each charge at that point.

Example 4

Two charges $Q_1 = +2.0\ \mu C$ and $Q_2 = +1.0\ \mu C$ are placed at the corners of a right-angle triangle with dimensions as shown.



(a) Determine the magnitude and direction of the electric field strength at the third corner X.

(b) If a charge Q_3 of $+3.0\ \mu C$ is placed at corner X, determine the force it experiences.

[Solution]

Problem Solving Strategy

1. Model charged objects as point charges.
2. Identify the point at which you wish to calculate the electric field and draw the electric field vector due to each charge at that point.
3. Select an appropriate coordinate system e.g. x-y coordinates.
4. Determine the resultant electric field by vector summation: $\underset{\sim}{E}_{net} = \sum_{n} \underset{\sim}{E}_i$.

❖ Electric Field Lines

Drawing field lines helps to visualise a field (gravitational, electric or magnetic). Each field is a 3-D pattern, but on paper it is represented by a 2-D slice of the pattern.

The field lines that are drawn around the source of the field (i.e. mass, charge, magnet) are used to denote the directions of the forces experienced by a relevant test object placed within the field.

General guidelines for drawing field lines

1. Direction of a field line indicates the direction of the **resultant force** acting on a small positive test charge.
2. Field lines point from **high to low potential**.
    - Since the direction of the field line is determined by the resultant force acting on a small positive test charge, the field line will always point from a region of high electric potential to a region of low electric potential.
3. Field lines **never cross** one another.
    - The direction of the electric field strength at any point is along the **tangent** to the electric field line at that point and is in the same direction as the field line.
    - If the lines cross one another, it means that there are two tangents at that point. This is not possible as the electric field strength cannot point towards two different directions simultaenously.
4. The **density** of the field lines (number of lines per unit area) represents **field strength**.
    - The spacing between the field lines indicates the relative magnitude of the field strength.

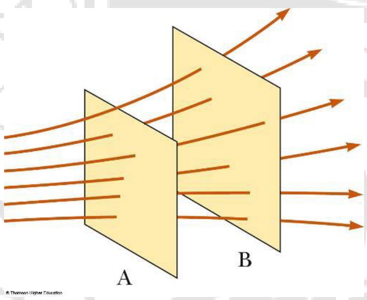


Fig. 14.5

    - Since the spacing of the field lines at region A is closer than at region B, the electric field strength at A has a larger magnitude than that at B.
5. Field lines are **perpendicular to the surface of a conductor**.
    - As the charges on the surface of a conductor will redistribute until there is no potential difference between any two points of the conductor, there will be no electric field within the conductor. Hence, the electric field lines near the surface of a conductor will always be perpendicular to its surface (so that no electric field component that is parallel to the surface exists).

For a single point charge

Fig. 14.6 shows the electric field lines for (a) an isolated positive charge and (b) an isolated negative charge.

Create your own system of charges and observe their field line patterns here.

https://iwant2study.org/lookangejss/05electricitynmagnetism_11efield/ejss_model_twopointcharges01/twopointcharges01_Simulation.xhtml

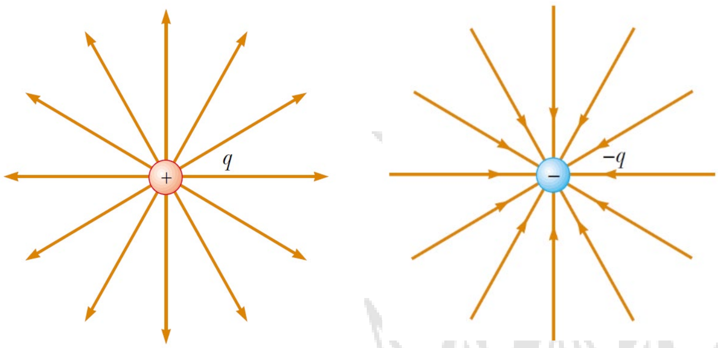
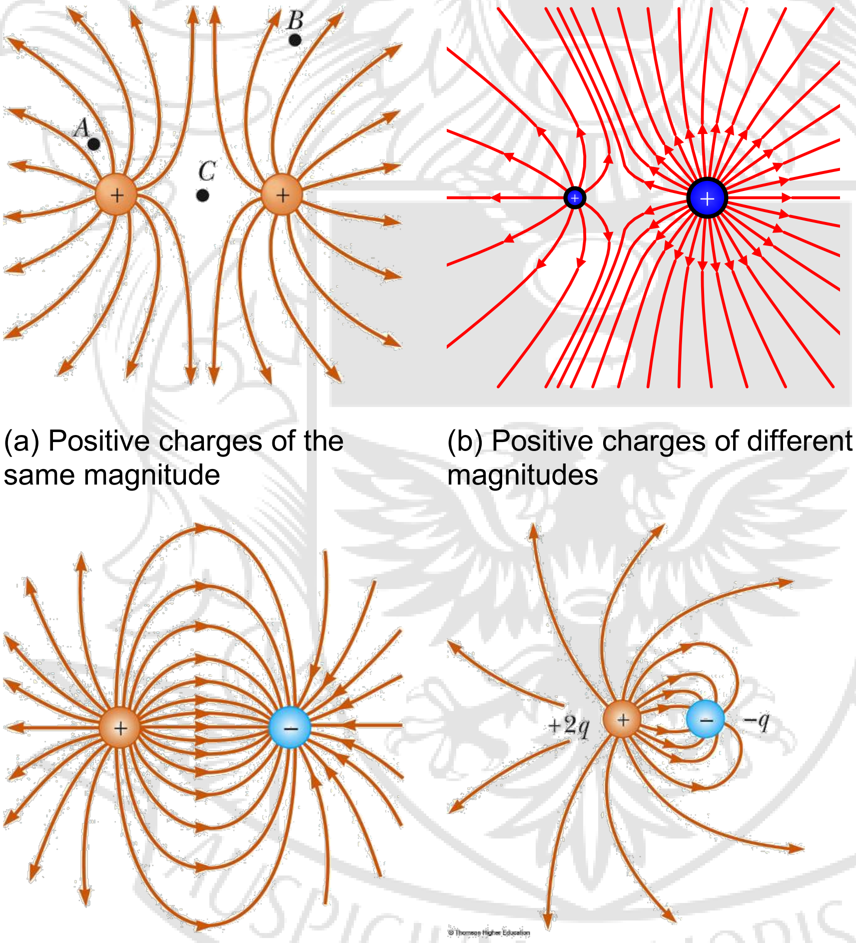


Fig. 14.6

For a system of two-point charges

Fig. 14.7 shows the electric field lines for various configuration of a system of two-point charges.



(a) Positive charges of the same magnitude

(b) Positive charges of different magnitudes

(c) Opposite charges of the same magnitude

(d) Opposite charges of different magnitudes

Fig. 14.7

14.3 Electric Potential Energy and Electric Potential

❖ Electric Potential Energy

The electric force has a special property: the work done against it is independent of the path taken and only depends on the initial and final positions. This is a property of conservative forces.

Electric force, gravitational force and elastic force are conservative forces; drag force and kinetic friction are non-conservative forces.

For this reason, we can define an energy function associated with the positions of the charges within an electric field, called the electric potential energy. This will enable us to simplify the calculation for work done against the electric force (an integration along the path) to the difference in the electric potential energy values between the final and initial positions.

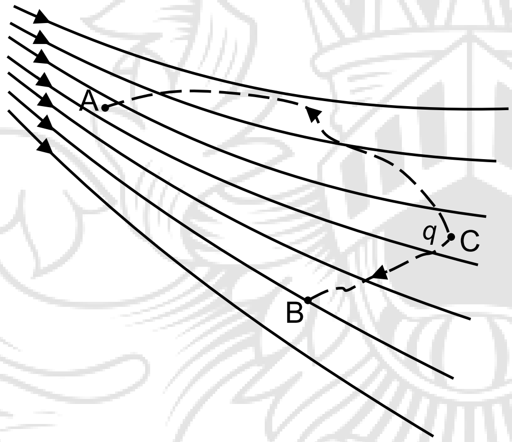


Fig. 14.8

Choosing an arbitrary point C as the point of zero potential (**Fig. 14.8**), the electric potential energy of a charge q at any other point is defined as the work done by an external force in moving q from C to that point.

The point of reference C is conventionally chosen to be at infinity.

> **Electric Potential Energy**
>
> The electric potential energy of a charge at a point in an electric field is defined as the work done by an external force in bringing the charge from infinity to that point.

Electric potential energy is a **scalar** quantity.

S.I. unit for electric potential energy is the **joule (J)**.

Using the definition, the electric potential energy at arbitrary points A and B are

$$U_{\mathrm{A}} = \int_{\infty}^{r_A} F_{\mathrm{ext}}\ \mathrm{d}r = \int_{\infty}^{r_A} \left(-F_{\mathrm{E}}\right)\ \mathrm{d}r$$

$$U_{\mathrm{B}} = \int_{\infty}^{r_B} F_{\mathrm{ext}}\ \mathrm{d}r = \int_{\infty}^{r_B} \left(-F_{\mathrm{E}}\right)\ \mathrm{d}r$$

where the integrations are along arbitrary paths from infinity to points A and B.

NOTE

- In the above definition, the **work done is by an external force** (F_{ext}), where the external force is equal in magnitude but opposite in direction to the electric force (F_{E}) hence net force is zero. The kinetic energy of the charged particle is assumed to be constant.
- One way to look at this definition is that the **work done by the external force only changes the electric potential energy** of the system. Similar considerations go into the definition of the gravitational potential energy.

The **work done by an external force** to move q from A to B can be shown to be **equal to the change in potential energy** between A and B.

To demonstrate this, we can choose the path A → C → B.

Work done by external force to move q from A to B

$$= \int_{r_A}^{r_B} \left(-F_{\mathrm{E}}\right)\ \mathrm{d}r$$

$$= \int_{r_A}^{r_C} \left(-F_{\mathrm{E}}\right)\ \mathrm{d}r + \int_{r_C}^{r_B} \left(-F_{\mathrm{E}}\right)\ \mathrm{d}r$$

$$= -\int_{r_C}^{r_A} \left(-F_{\mathrm{E}}\right)\ \mathrm{d}r + \int_{r_C}^{r_B} \left(-F_{\mathrm{E}}\right)\ \mathrm{d}r$$

$$= -U_{\mathrm{A}} + U_{\mathrm{B}}$$

$$= U_{\mathrm{B}} - U_{\mathrm{A}}$$

$=$ change in electric potential energy,

where we have taken C to be the point of infinity.

❖ Electric Potential

Electric Potential

The electric potential at a point in an electric field is defined as the work done per unit positive charge by an external force in bringing a small positive test charge from infinity to that point.

From the definition of electric potential:

$$V = \frac{1}{q}\int_{\infty}^{r} F_{\text{ext}}\ \mathrm{d}r = \frac{U}{q}$$

$$\boxed{V = \frac{U}{q}}$$

Electric potential is a **scalar** quantity.

S.I. unit for electric potential is the **volt (V)** or **J C^{-1}**.

Given the electric potential V at a point in space, the electric potential energy U of a point charge q placed at that point is

$$\boxed{U = qV}$$

At the point of infinity, both the **electric potential energy and electric potential are zero**.

For moving from one point to another in an electric field, the change in electric potential ΔV , also known as the **potential difference** between the two points, is calculated as

$$\boxed{\Delta V = V_{\text{final}} - V_{\text{initial}}}$$

The corresponding change in potential energy ΔU is

$$\boxed{\Delta U = q\Delta V}$$

The **values of *U*, *V* and *q*** must be substituted into the above equations with their correct signs.

In many situations, the energy gained by a charged particle is small and hence it is convenient to introduce a new unit of energy called the electron-volt (eV).

Using $\Delta U = q\Delta V$,

$$\boxed{1\text{ eV} = 1.60 \times 10^{-19}\text{ C} \times 1\text{ V} = 1.60 \times 10^{-19}\text{ J}}$$

The **electron-volt** is defined as the energy gained by an electron, which carries a charge of magnitude 1.60×10^{-19} C, when it is accelerated through a potential difference of 1 V.

❖ Electric Potential Energy of Point Charges

Consider moving a point charge q from infinity to a point in the electric field produced by another point charge Q. This point is at a distance r away from Q.

The electric force on q at this point is $F_E = \frac{Qq}{4\pi\varepsilon_0 r^2}$ and r is the distance between q and Q.

Using the definition for electric potential energy, we have

$$U = \int_{\infty}^{r} F_{ext}\, dr = \int_{\infty}^{r} (-F_E)\, dr = \int_{\infty}^{r} \left(-\frac{Qq}{4\pi\varepsilon_o r^2} \right) dr = \left[\frac{Qq}{4\pi\varepsilon_o r} \right]_{\infty}^{r} = \frac{Qq}{4\pi\varepsilon_o r}$$

Thus, the potential energy of the system of two point charges is

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r}}$$

NOTE

- The **sign of the potential energy *U*** in the above formula is determined by the **signs of *q* and *Q***. It is positive between two charges of the same sign (due to the repulsion between the charges), and negative between two charges of opposite signs (due to the attraction between them). This is different from the gravitational potential energy between two point masses ($U_{gravitational} = -\frac{Gm_1m_2}{r^2}$), which carries an explicit negative sign because gravitational force is always attractive and the masses are always positive.
- One advantage of choosing infinity as the reference point is the simple form of the above expression. If the reference point is at any finite distance r_C, there would be an additional constant term ($-\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_C}$, which goes to 0 if $r_C \to \infty$) from the integration.

$$U = \int_{r_c}^{r} F_{ext}\, dr = \int_{r_c}^{r} (-F_E)\, dr$$
$$= \int_{r_c}^{r} \left(-\frac{Qq}{4\pi\varepsilon_o r^2} \right) dr$$
$$= \left[\frac{Qq}{4\pi\varepsilon_o r} \right]_{r_c}^{r}$$
$$= \frac{Qq}{4\pi\varepsilon_o r} - \frac{Qq}{4\pi\varepsilon_o r_c}$$

❖ Electric Potential of Point Charges

From the relationship between electric potential V and the expression for electric potential energy U for point charges, the electric potential due to **a point charge *Q*** at a distance r away from Q is

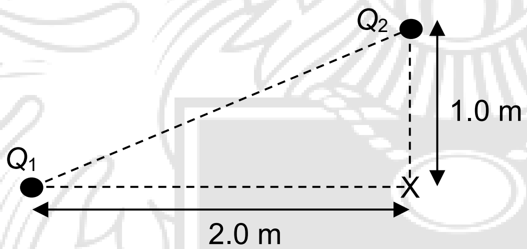
$$\boxed{V = \frac{U}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

- The **sign of the potential *V*** in the above formula is determined by the **sign of *Q***. It is positive when Q is a positive charge and negative when Q is a negative charge.
- If more than one charge is present, the **total electric potential** at a point is the **algebraic sum** of the individual electric potential due to each charge at that point.

The mathematics of electric potentials is much easier than the vector mathematics of electric fields.

Example 5

Two charges Q_1 = +2.0 μC and Q_2 = −2.0 μC are placed at the corners of a triangle with dimensions as shown.



(a) Determine the electric potential at the third corner X.

(b) If a charge Q_3 of +1.0 μC is brought to corner X from infinity, determine the change in electric potential energy of the system.

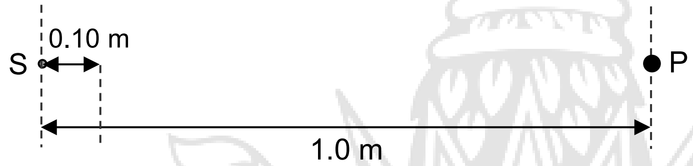
[Solution]

Example 6

A fixed, positive point charge S carries a charge of 1.0×10^{-4} C. A particle P of mass 2.0×10^{-5} kg and charge -1.5×10^{-10} C is released from rest at a distance of 1.0 m from S.

Calculate the speed of particle P when it is at a distance of 0.10 m from S. Neglect any gravitational effect.

[Solution]



Note:

The particle is moving with non-constant acceleration since the electric force varies with $1/r^2$ for point charges.

14.4 Important Relationships

❖ **Relationship between *F* and *U***

The definition of potential energy

$$U = \int_{\infty}^{r} F_{\text{ext}} dr = \int_{\infty}^{r} \left(-F_{\text{E}}\right)\, dr$$

implies that the electric force on a charge at a point is equal to the negative of the derivative of electric potential energy *U* with respect to *r* at that point:

$$\boxed{F_{\text{E}} = -\frac{dU}{dr}}$$

- From a *U*–*r* graph, the **negative of the gradient** of the tangent at a point on the graph gives the **electric force** at that point.
- The negative sign in the formula means that the **electric force points in the direction of decreasing electric potential energy**.

❖ **Relationship between *E* and *V***

Similarly, the definition of the electric potential

$$V = \int_{\infty}^{r} \left(-E\right)\, dr$$

implies that the electric field strength at a point is equal to the negative of the derivative of electric potential *V* with respect to *r* at that point:

$$\boxed{E = -\frac{dV}{dr}}$$

- This equation can also be obtained by dividing both sides of the equation $F_{\text{E}} = -\frac{dU}{dr}$ by charge *q*, since $F_{\text{E}} = qE$ and $U = qV$.
- From a *V*–*r* graph, the **negative of the gradient** of the tangent at a point on the graph gives the **electric field strength** at that point.

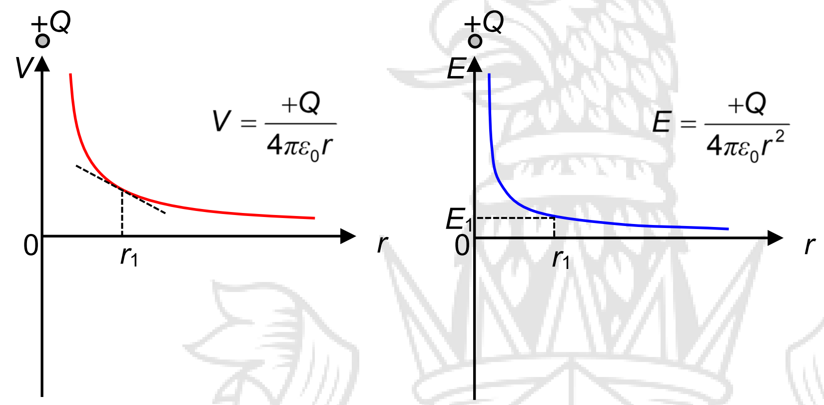
Electric Field Strength At A Point

The electric field strength at a point is equal to the negative potential gradient at that point.

- The negative sign means that the **electric field strength always points in the direction of decreasing potential,** that is **from high potential to low potential**.

❖ *E–r* and *V–r* Graphs of a Point Charge

For a positive point charge Q:

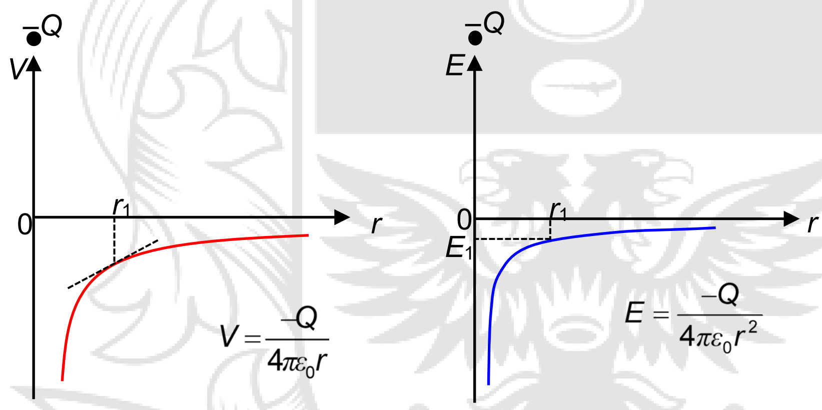


At r_1, since gradient $\frac{dV}{dr}$ of the tangent of the V–r graph is negative, $E_1 = -\frac{dV}{dr}$ is positive.

- V is a scalar
- V is positive at all values of r as charge is positive
- E is a vector
- E is positive at all values of r as it points outwards from $+Q$

Fig. 14.9

For a negative point charge $-Q$:



At r_1, since gradient $\frac{dV}{dr}$ of the tangent of the V–r graph is negative, $E_1 = -\frac{dV}{dr}$ is negative.

- V is a scalar
- V is negative at all values of r as charge is negative
- E is a vector
- E is negative at all values of r as it points inwards towards $-Q$

Fig. 14.10

❖ *E*–*r* and *V*–*r* Graphs of Two Point Charges

Electric potential is a **scalar** quantity. The **total potential** at a point between two point charges is the **algebraic sum** of the potentials due to each point charge at that point.

Electric field strength is a **vector** quantity. The **resultant field strength** at a point between two point charges is the **vector sum** of the field strengths due to each point charge at that point. The **sign** of the resultant field strength indicates its **direction**. Conventionally, positive indicates direction to the right and negative indicates direction to the left.

The relationship $E = -\frac{dV}{dr}$ applies to the graphs for the total potential and resultant field strength between two point-charges. The *V*–*r* (solid line) and *E*–*r* (dotted line) graphs for various two point-charge configurations are shown in **Fig. 14.11** (same sign) and **Fig. 14.12** (opposite signs).

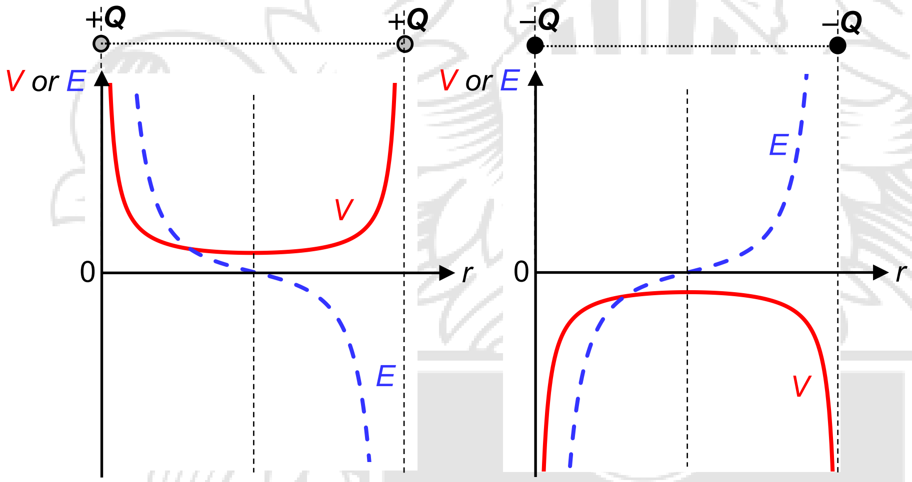


Fig 14.11

For a pair of *like* point charges, there is a point between them where the resultant electric field strength is zero.

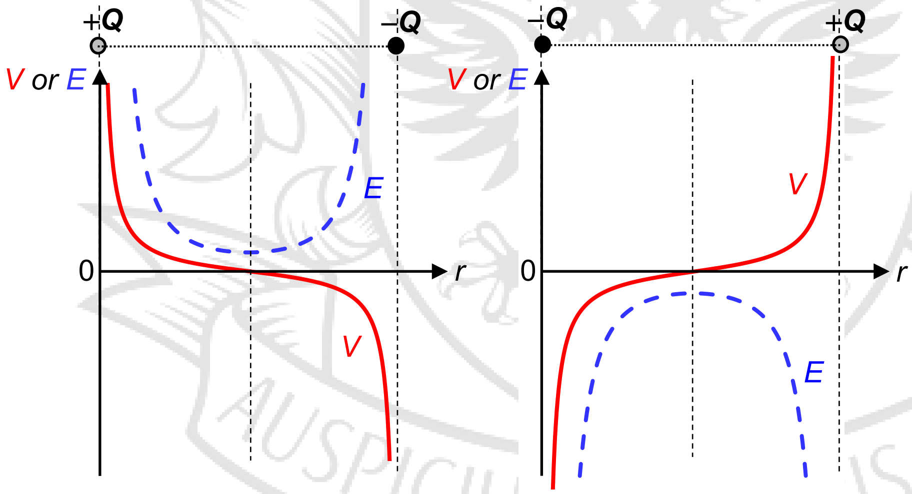


Fig 14.12

For a pair of *unlike* point charges, the resultant electric field strength between them is either always positive or always negative.

❖ Equipotential Lines and Surfaces

The work done W by a force F over a displacement s is given by

$$W = Fs\cos\theta,$$

where θ is the angle between the vectors F and s. If F and s are perpendicular to each other, i.e. $\theta = 90°$, the work done W is zero.

Consider a charged particle q being moved by an external force within an electric field as shown in **Fig 14.13**. The electric force is parallel or antiparallel to the electric field lines. If the path (dotted line) of the charged particle q is perpendicular to the electric field lines at every point, the path will also be perpendicular to the electric force.

The sign of the charge determines the direction of the electric force.

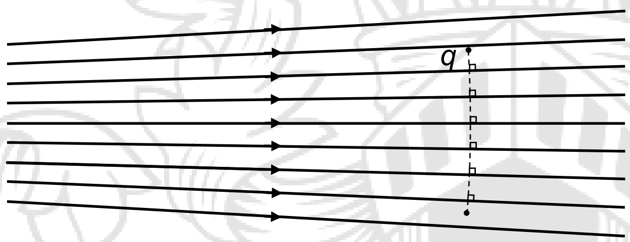


Fig. 14.13

Since the external force is also perpendicular to the path, the work done by the external force is zero. Hence, the change in electric potential or electric potential energy along this path is zero, i.e., the whole path is at the same potential.

Such a path traces out what we call an **equipotential line**, where all points on this line are at the same potential. In 3D space, equipotential lines form equipotential surfaces where all points on the surface have the same potential.

As discussed above, **equipotential lines are perpendicular to electric field lines at every point**. **Movement along an equipotential line or surface requires no work done by the external force**, since such a path is always perpendicular to the electric field lines.

Equipotential lines are like contour lines on a map which trace lines of equal altitude. In this case, the "altitude" is the electric potential. **Dashed lines without arrowheads** are used to represent **equipotential lines**, while solid lines with arrowheads are electric field lines. When a set of equipotential lines are drawn, **the potential difference between consecutive equipotential lines is usually a fixed value**.

❖ Equipotential Lines and Electric Field Lines of Point Charges

In 3D, spherical equipotential surfaces surround a point charge and are centred on the charge. In 2D, the equipotential lines around a point charge are concentric circles centred on the charge (a slice of the 3D spherical equipotential surfaces).

Point charges are sources of non-uniform field.

Fig. 14.14 shows the equipotential lines and electric field lines for an isolated positive point charge. The dashed lines represent potential at equal increments – the equipotential lines **get further apart with increasing distance** from the charge, because the **electric field strength decreases in magnitude**.

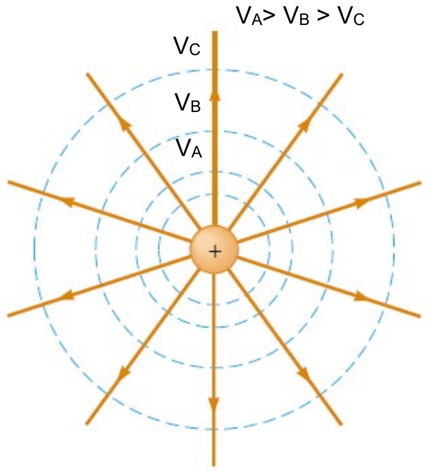


Fig 14.14

In summary,

1. electric field lines are always perpendicular to equipotential lines and surfaces.
2. electric field lines point in the direction of decreasing potential.
3. equipotential surfaces have equal potential differences between them.

14.5 Uniform Electric Fields

Two parallel plates of infinite dimensions, each having a different electric potential, produce a uniform electric field between them.

Just like the uniform gravitational field close to the surface of the Earth, the electric field lines of a uniform field (**Fig. 14.15**) are:

- **parallel** and **evenly spaced,**
- and **perpendicular** to the equipotential lines.

Equipotential lines of equal interval are **equally spaced** since the **field strength is the same everywhere.**

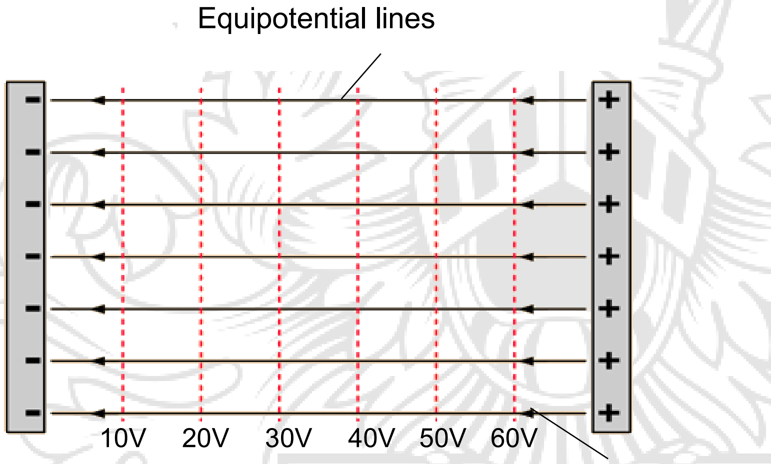
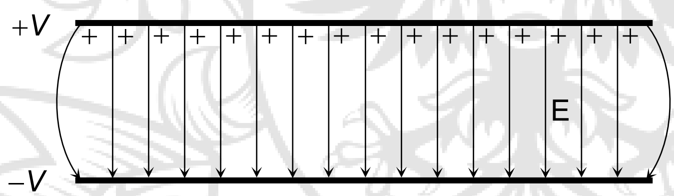


Fig. 14.15

Fig. 14.16 shows the electric field lines between two equally but oppositely charged parallel plates. As real-life plates have finite dimensions, the field at the ends of the plates are not uniform due to the fringing effects. The electric field lines point **from** the plate with **higher potential to** the plate with **lower potential**.



Instead of labelling the plates in terms of charges, we often label them in terms of potential. For e.g. $+V$ and $-V$.

Fig. 14.16 Schematic of parallel plates carrying equal and opposite charge

Since E of a uniform field is constant, the potential gradient $\frac{dV}{dr}$ is also constant. This implies that electric potential V is a linear function of the distance x between the two plates as shown in **Fig. 14.17**.

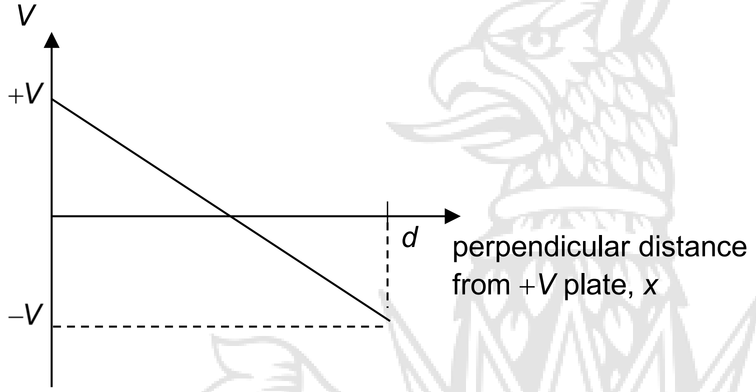


Fig. 14.17

The magnitude of the field strength E can be found by calculating the potential gradient i.e.

$$\boxed{|E| = \left|-\frac{dV}{dx}\right| = \frac{|\Delta V|}{d}}$$

where ΔV is the potential difference between the two plates and d is the perpendicular distance between the plates.

❖ Motion of a Charged Particle in an Uniform Electric Field

When a particle of charge q and mass m is placed in a uniform electric field of field strength E, a constant electric force F acts on the charge where,

$$\boxed{F = qE}$$

If the **constant electric force *F*** is the only force acting on the particle, the **acceleration *a*** of the particle is also **constant**. Thus, the kinematics equations of motion can be applied.

$$\boxed{a = \frac{F}{m} = \frac{qE}{m}}$$

Since the acceleration of the particle is constant, it will move in a parabolic path as shown in **Fig. 14.18**. This is similar to a mass moving in a parabolic path in a uniform gravitational field.

The trajectory of a charged particle in a uniform electric field is a parabola.

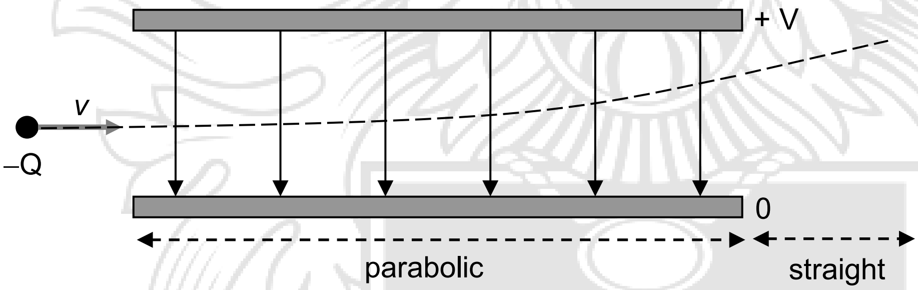


Fig. 14.18

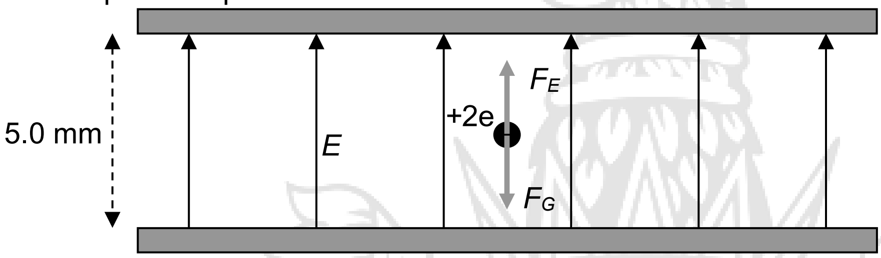
NOTE

1. One common mistake is to apply formulae such as $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ and $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ to uniform fields. These formulae only apply to non-uniform fields of point charges.
2. As the electric force acting on charged particles such as protons and electrons are much larger than the gravitational force acting on them due to their very small masses, i.e. $qE >> mg$, we usually do not consider the weight of these sub-atomic particles when calculating the acceleration or net force acting on them.

Example 7

A particle of mass 2.0×10^{-15} kg and charge $+2e$ remains stationary within two parallel plates maintained at a potential difference V and separated by a vertical distance of 5.0 mm. By considering the forces acting on the particle, determine V.

(elementary charge $e = 1.60 \times 10^{-19}$ C)



[Solution]

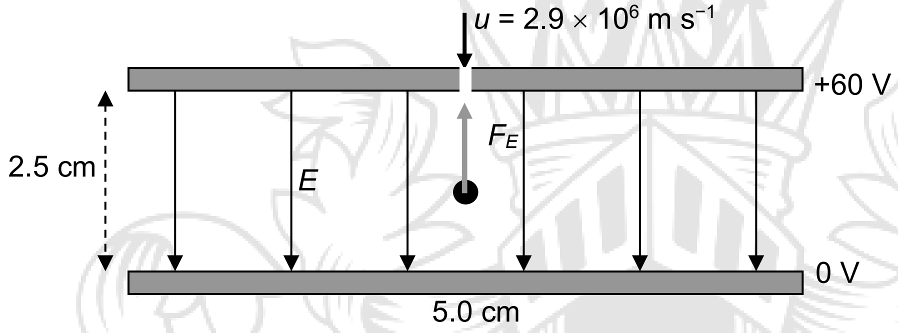
Charge configuration for a parallel plate cannot be approximated as point charges. Coulomb's law **cannot** be used to find the force on the electron.

Example 8

Two horizontal plates of length 5.0 cm are placed 2.5 cm apart in a vacuum. The upper plate is maintained at a potential of +60 V relative to the lower plate.

An electron is injected vertically through a hole in the upper plate with a speed of 2.9×10^6 m s^{-1} towards the lower plate.

Determine the distance s from the upper plate where the electron is momentarily at rest.



(rest mass of electron $m_e = 9.11 \times 10^{-31}$ kg,

elementary charge $e = 1.60 \times 10^{-19}$ C)

[Solution]

For a negative charge, i.e. the electron, the electric force acts in the opposite direction to the electric field.

Example 9

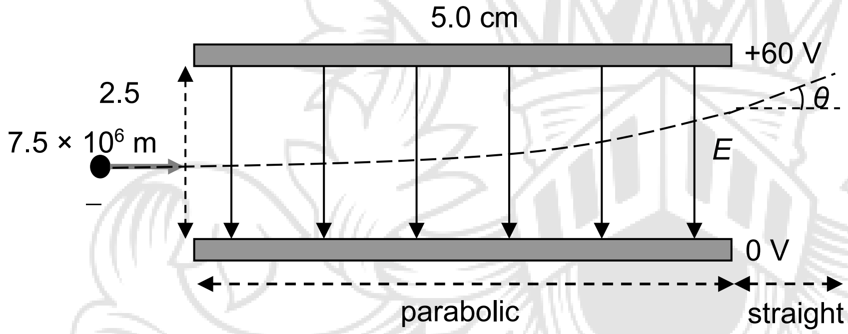
An electron is projected horizontally between the plates in **Example 8** with an initial speed of 7.5 x 10^6 m s^{-1}. Neglecting gravitational and edge effects,

(a) describe and explain the path taken by the electron while moving between the plates and after leaving the plates,

(b) determine the angle the electron makes with the horizontal upon leaving the plates.

(m_e = 9.11 × 10^{-31} kg, e = 1.60 × 10^{-19} C)

[Solution]



14.6 Conductors in Electrostatic Equilibrium

Excess charges added to a conductor will redistribute themselves onto the **surface** of the conductor until **no excess charge is present inside** the conductor. At this point, electrostatic equilibrium is reached, the **electric field inside the conductor is zero** and the **entire conductor is at the same potential**. This behaviour can be understood from the perspectives of the electric field, electric potential, and electric potential energy.

When excess charges are first introduced in a conductor, a non-zero electric field is created. This field exerts an electric force on the charges which drives the charges towards a distribution which weakens the electric field inside the conductor. This redistribution continues until the electric field inside the conductor becomes zero at which point the charges stop moving and electrostatic equilibrium is achieved.

From the electric potential perspective, when the excess charges are initally added, the entire conductor is not at the same potential. The resulting potential gradient (which corresponds to a non-zero electric field, according to $E = -dV/dr$) inside the sphere drives the free electrons to move in a way that reduces their electric potential energy. This means that they will move from regions of lower potential to regions of higher potential, thereby decreasing the potential gradient. This movement continues, reducing the system's energy, until the **entire conductor reaches the same potential**, at which point the electric potential energy is minimized.

Note that the two characteristics of a charged conductor in electrostatic equilibrium, namely "zero field inside" and "entire conductor being at the same potential" are equivalent to each other. This is because $E = -dV\,/\,dr$.

In terms of energy considerations, the potential energy of two point charges q_1 and q_2 a distance r from each other is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$.

This potential energy is positive if the two charges have the same sign and it decreases as the charges move further apart (i.e. as r increases). Thus, by spreading out as far from one another as possible, the charges help to reduce the energy of the system.

❖ **Electric Field Strength and Electric Potential of a Charged Conducting Sphere**

Outside of a **charged solid** or **hollow conducting sphere**, due to its spherical symmetry, it can be shown that the electric field behaves as though **all its charges are concentrated at its centre**. Therefore, we can use the equation for point charges to determine the electric field strength outside the sphere.

Consider a point charge +Q and a sphere with charge +Q. Since the electric field inside the sphere is zero, there are no electric field lines within the sphere. The electric field lines outside the sphere are similar to that of a point charge of equal charge in the centre of the sphere.

Fig. 14.19 shows the +Q point charge in the centre of the +Q sphere. Field lines outside the sphere seem to originate from the centre of the sphere where the point charge is. There are no field lines within the sphere, the dotted lines within the sphere are to show that the field lines meet at the centre of the sphere.

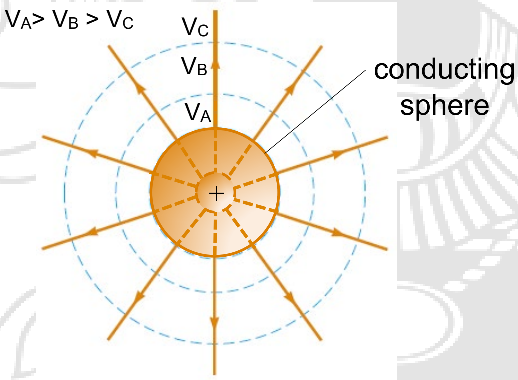


Fig. 14.19

The variation of the electric field strength E, and that of the electric potential V, with distance r for the $+Q$ point charge and the $+Q$ sphere are as shown in **Fig. 14. 20**.

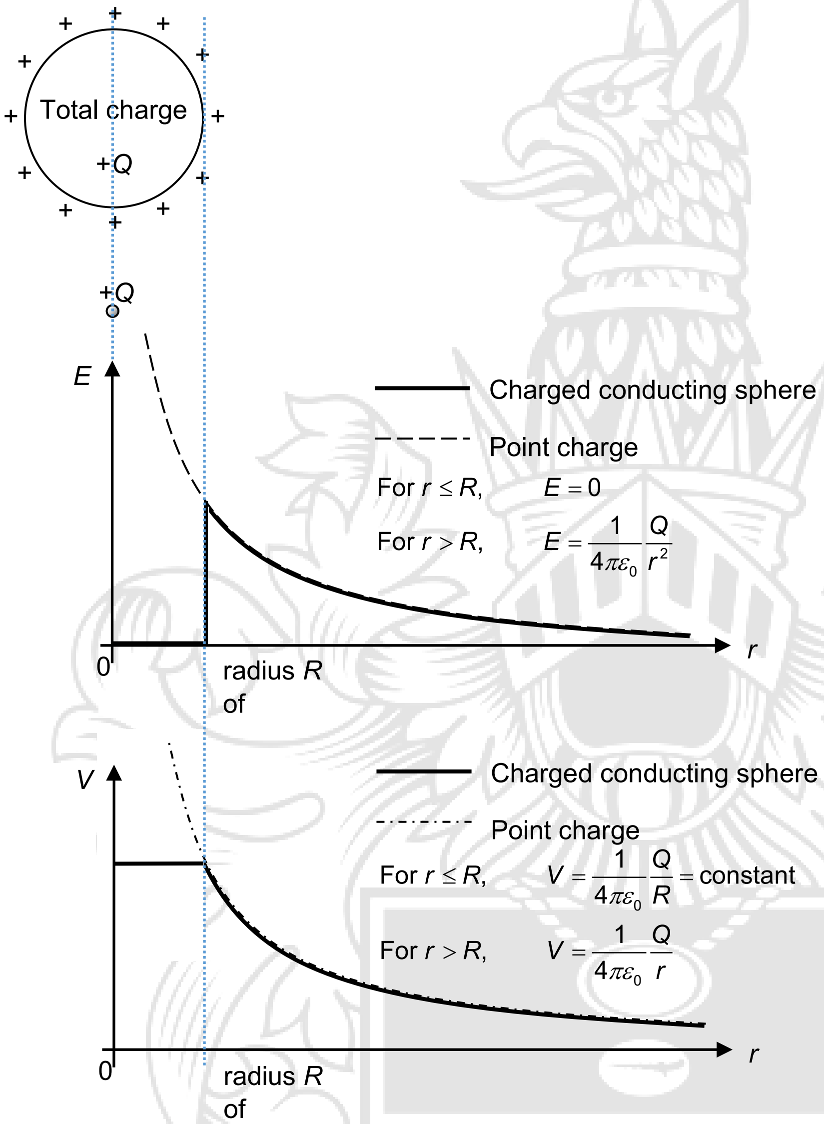


Fig. 14.20

❖ Conductor in an External Electric Field

For an isolated (i.e. not connected in a closed electrical circuit) **solid or hollow conductor** in an external electric field, the free electrons within it will redistribute themselves onto the surface quickly, until the **electric field within the conductor is zero** and the **entire conductor is at the same potential**, at which point the collective movement of electrons will cease (electrostatic equilibrium is achieved). This is due to the same mechanism that resulted in zero field inside a charged conductor, which we described earlier.

The electric field lines near the surface of a neutral conductor placed within a uniform electric field and the electric field lines near the surface of an earthed conductor placed in a non-uniform electric field of a positive charge are shown in **Fig. 14.21** and **Fig. 14.22** respectively.

Therefore,

- there are no electric field lines within a conductor,
- electric field lines must start or end at the surface of the conductor,
- electric field lines are perpendicular to the surface of the conductor since the surface is an equipotential surface – otherwise, the component of the electric field strength that is tangential to the surface will cause the charges to move and redistribute until the net force on them is zero.

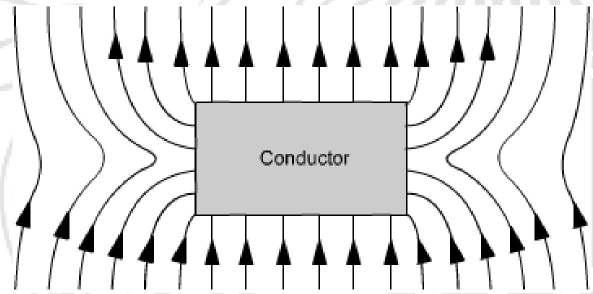


Fig. 14.21

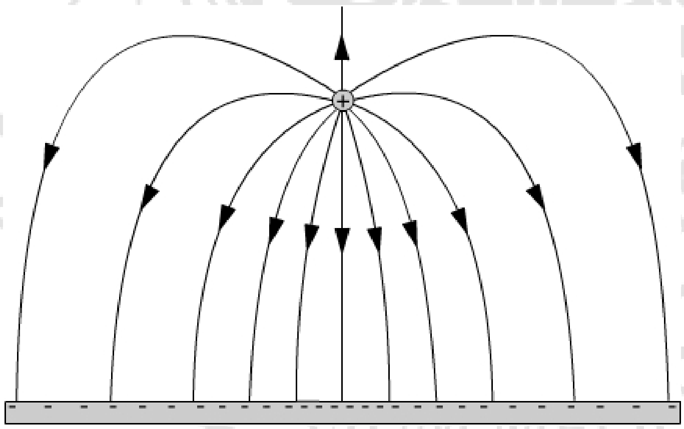


Fig. 14.22

14.7 Capacitance and Capacitors

Capacitors (**Fig. 14.23**) are electrical devices that **store electric charges**. A typical capacitor consists of two conductors separated by an insulator known as a dielectric. Beyond its primary function of storing electric potential energy, capacitors find diverse applications in electronics. Notably, they can be utilized to delay voltage changes when coupled with resistors and to effectively filter out undesirable frequency signals.

Fig. 14.23

In the uncharged state, the charge on both of the conductor plates in the capacitor is zero. During the charging process, a potential difference is set up across the capacitor (e.g. by connecting to a battery) and an electric field is established. This causes positive charges to accumulate on one plate and an equal number of negative charges on the other plate. The potential difference across the capacitor steadily increases as the charge separation continues. The capacitor is fully charged when the potential difference across the capacitor is equal to the potential difference across the battery terminal.

The amount of charge Q that can be stored in a capacitor is directly proportional to the potential difference V across it. The **constant of proportionality *C*** is known as its **capacitance** and is dependent on the geometry of the capacitor as well as the properties of the dielectric. Capacitors with larger capacitance can stored more charges.

Capacitance

Capacitance is defined as the ratio of the magnitude of the charge stored on either conductor to the magnitude of the potential difference between the conductors.

Capacitance is a positive quantity as Q refers to the magnitude of the charge on one of the plates.

$$\boxed{C = \frac{Q}{V}}$$

The S.I. unit of capacitance is the farad (F), or C V^{-1}. Typical values of capacitance range from pico-farads (pF) to milli-farads (mF).

The Farad

One farad is the capacitance of a capacitor such that when the charge on each of its conducting plates has a magnitude one coulomb, the potential difference across them is one volt.

One farad is an enormous capacitance. Typical values for capacitors in devices range from picofarads (1×10^{-12} F) to microfarads (1×10^{-6} F).

The circuit symbol of a capacitor is

Note: A single conductor can be considered a capacitor with its 'other plate' at infinity and zero potential. Its capacitance is then defined as the change in its charge per unit change in its potential.

Example 10

Derive an expression for the capacitance of an isolated spherical conductor of radius R. Hence, by considering the Earth as a spherical conductor, estimate its capacitance.

The radius of the Earth is 6.37×10^6 m.

[Solution]

Note that the capacitance of the earth is 4 orders of magnitude smaller than a Farad.

❖ The Parallel-Plate Capacitor

A parallel-plate capacitor consists of two plates facing each other with a plate separation *d* that is small compared to the size of the plates. This allows us to ignore the fringing of the electric fields at the edge of the plates and the electric field *E* between the plates is considered to be uniform throughout. The potential difference *V* across a parallel-plate capacitor is $V = Ed$.

A schematic of a parallel plate capacitor and the uniform electric field lines of a parallel plate capacitor are shown in **Fig. 14.24(a)** and **(b)** respectively.

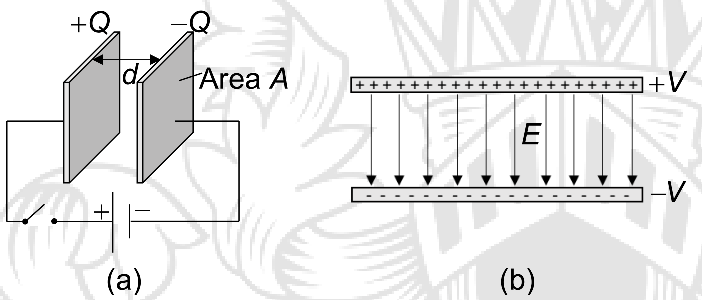


Fig 14.24

The potential difference across an uncharged capacitor is zero. When the switch is closed, an electric field is established in the circuit. The electrons start to move out of the left conducting plate leaving it positively charged and into the right conducting plate leaving it negatively charged. The potential difference across the parallel plates increases until it equals the potential difference between the terminals of the battery. The capacitor is then considered fully charge.

NOTE

The equation for the capacitance of a parallel-plate capacitor is (stated without proof)

$$C = \frac{\varepsilon_0 A}{d},$$

where ε_0 is the permittivity of free space, *A* is the plate area and *d* is the separation of the plates.

Example 11

A parallel-plate capacitor of capacitance 3.0 μF is connected across a 6.0 V battery.

(a) Calculate the charge on either of the plates of the capacitor.

(b) If the plates are separated by a distance of 2.5 mm, calculate the *E* field between the plates.

(c) Explain why the two plates have equal but opposite charge.

[Solution]

❖ Energy Stored in a Capacitor

The amount of electrical energy in a charged capacitor is equal to the work done by an external agent in charging the capacitor. As electrons are being transferred from one plate to another, charges build up and so does the electric field between the plates which opposes the continued transfer. Thus, more work is required to move the same electron across a greater potential difference.

Let dW be the work done in adding a charge dq to a capacitor of charge q. Then

Recall that $U = q\Delta V$.

$$dW = V.dq \ ,$$

where V is the potential difference across the capacitor when its charge is q.

Fig. 14.25 shows the variation of potential difference V across the capacitor with charge Q accumulated during a charging process. The $V-Q$ graph is a straight line with gradient $1/C$.

Since $C = \frac{Q}{V} \Rightarrow V = \left(\frac{1}{C}\right)Q$.

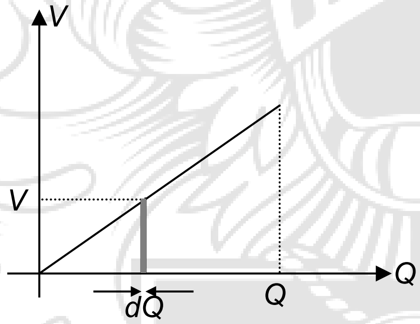


Fig. 14.25

The work $dW = V.dq$ can be given approximately by the area of the shaded rectangle.

Therefore, the total work done in charging the capacitor from zero to a charge Q is given by the **area under the $V-Q$ graph** which is the integral of dW

$$W = \int_0^W dW = \int_0^Q V\,dQ = \int_0^Q \frac{Q}{C}dQ = \frac{1}{2}\frac{Q^2}{C}.$$

Since $Q = CV$, the electrical potential energy U in a capacitor can also be expressed as

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2}$$

Example 12

An uncharged 0.6 μF capacitor is charged to a potential difference of 12 V by a battery.

(a) Calculate the energy stored in the capacitor and the energy supplied by the battery.

(b) Hence, deduce the total heat dissipated in the resistance of the connecting wires and of the battery.

[Solution]

Energy stored in a capacitor is always half of the energy supplied by the battery.

Example 13

A 2.0 μF capacitor and a 3.0 μF capacitor are charged to 0.30 kV and 0.20 kV respectively.

(a) Calculate the energy stored in each of the capacitors.

(b) The two capacitors are then connected with the like charges together,

(i) Calculate the charges on each of the capacitor plates.
(ii) Calculate the new potential difference between the capacitors' plates.
(iii) Calculate the energy stored in the system.
(iv) Account for the disparity between (a) and (b)(iii).

[Solution]

14.8 Electric Field & Gravitational Field: Comparison & Summary

❖ General Relationships

	Electric field	Gravitational field
Strength and Force	$E = \frac{F_E}{q}$	$g = \frac{F_G}{m}$
Force and Potential Energy	$F_E = -\frac{dU_E}{dr}$	$F_G = -\frac{dU_G}{dr}$
Strength and Potential	$E = -\frac{dV}{dr}$	$g = -\frac{d\phi}{dr}$
Potential and Potential Energy	$V = \frac{U_E}{q}$	$\phi = \frac{U_G}{m}$

❖ Point Charges / Point Masses

	Electric field	Gravitational field
Force (vector)	$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$	$F = -G\frac{Mm}{r^2}$
	• can be attracted or repulsive depending on the charges	• M and m are positive values • always attractive (implied by the negative sign)
Field Strength (vector)	$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$	$g = -G\frac{M}{r^2}$
	• points away from positive charge • points towards negative charge	• always points towards the mass
Potential Energy (scalar)	$U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$	$U = -G\frac{Mm}{r}$
	• signs of Q and q must be included in the expression	• M and m are positive values • always negative
Potential (scalar)	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$	$\phi = -G\frac{M}{r}$
	• sign of Q must be included in the expression	• always negative

❖ Uniform Fields

	Electric field between parallel plates	**Gravitational field near Earth surface**
Force	$F = qE$	$F = mg$
Field Strength	$E = \dfrac{F}{q}$	$g = \dfrac{F}{m}$
	• points from higher potential plate to lower potential plate • E is constant in uniform field, i.e. V changes linearly with distance	• always point towards the mass • g is constant in uniform field
Acceleration	$a = \dfrac{F}{m}$	$a_{freefall} = g$ (If we consider the Earth's rotation, $a_{freefall} = g - a_{centripetal}$)
	• a due to electric force is constant • can apply kinematics equations of motion	• a due to gravitational force is constant • can apply kinematics equations of motion
Path of charge / mass	• +ve charge will accelerate towards lower potential plate • –ve charge will accelerate towards higher potential plate • if motion of charge has a perpendicular component to the field, it will move in a **parabolic** path	• mass will accelerate towards Earth's surface • if motion of mass has a perpendicular component to the field, it will move in a **parabolic** path
change in Potential Energy	$U = q\Delta V$	$U = mg\Delta h$
	• sign of q must be included • $\Delta V = V_{\text{final}} - V_{\text{initial}}$ • If U is +ve, there is a gain in electric potential energy • if U is –ve, there is a loss in electric potential energy	• $\Delta h = h_{\text{final}} - h_{\text{initial}}$ • if U is +ve, there is a gain in gravitational potential energy • if U is –ve, there is a loss in gravitational potential energy

14.9 Appendix

❖ Milikan's Oil-Drop Experiment

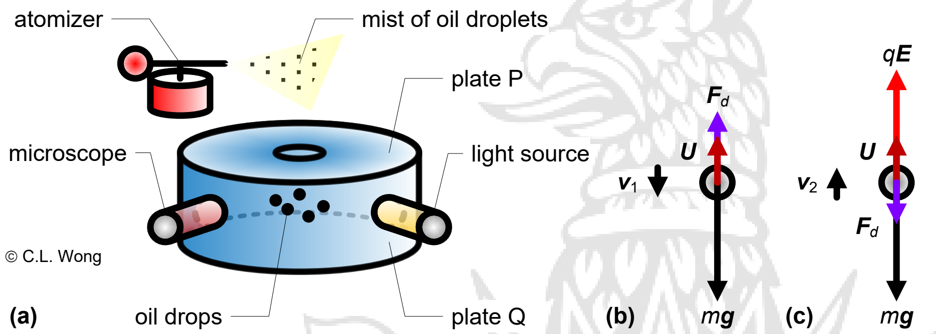


Fig 14.26

Fig. 14.26(a) shows a simplified scheme of the *Millikan's oil-drop experiment*, first used by Robert Millikan and Harvey Fletcher to determine the electric charge of the electron in 1909. By measuring the terminal speeds of the oil droplet between the plates, its electric charge can be determined. Robert Millikan was awarded the Nobel Prize in physics in 1923.

Fig. 14.26(b) shows an oil droplet falling with terminal speed v_1 in the absence of an electric field. An atomizer is used to produce tiny oil droplets which fall through a hole in the upper plate P. Frictional effects in the atomizer caused the oil droplets to be charged; some positively and others negatively. The droplets are illuminated by a light source and observed using a microscope.

First, we can determine the radius r and hence the weight W of an observed oil droplet by allowing it to fall with terminal speed. The drag force F_d acting on the oil droplet is given by Stokes' law:

$$F_d = 6\pi\eta r v \,,$$

where v is the terminal speed and η is the viscosity of air.

When the oil droplet is falling with terminal speed v_1,

$$\tfrac{4}{3}\pi r^3 \rho_{\text{oil}} g = \tfrac{4}{3}\pi r^3 \rho_{\text{air}} g + 6\pi\eta r v_1$$

$$\therefore r = \sqrt{\frac{9\eta v_1}{2g\left(\rho_{\text{oil}} - \rho_{\text{air}}\right)}} \,.$$

Fig. 14.26(c) shows the oil droplet rising with terminal speed v_2 in the presence of an electric field. The uniform electric field is set up between plates P and Q by adjusting the potential difference V between the plates using a potentiometer circuit (not shown). The electric field strength E between the plates is

$$E = \frac{V}{d},$$

where d is the plate separation.

The potential difference V causes the oil droplet to rise with a terminal speed v_2, such that

$$\tfrac{4}{3}\pi r^3 \rho_{\text{oil}} g + 6\pi\eta r v_2 = \tfrac{4}{3}\pi r^3 \rho_{\text{air}} g + qE .$$

The electric charge q on the oil droplet can then be found by measuring v_2.

By repeating the procedure for different oil droplets carrying different charges, Robert Millikan was able to prove that

- the fundamental unit of electric charge e is 1.60×10^{-19} C;
- an electron is negatively charged;
- any charge q is always an integer multiple of e, that is, electric charge is *quantized*.

❖ Linear Accelerator (LINAC)

The most common method of accelerating a charged particle from rest is to use a uniform electric field.

Fig. 14.27

When a particle of charge Q and mass m is accelerated from rest through a potential difference V and distance d, its acceleration is $a = \frac{F_E}{m} = \frac{QE}{m} = \frac{Q}{m}\frac{V}{d}$.

The kinetic energy it gained is $\Delta E_K = QV$,

and its final velocity is $\frac{1}{2}m\left(v_f^{\,2} - v_i^{\,2}\right) = QV \quad \Rightarrow \quad v_f = \sqrt{\frac{2QV}{m}}$, if v_i = 0.

❖ Cathode Ray Oscilloscope (CRO)

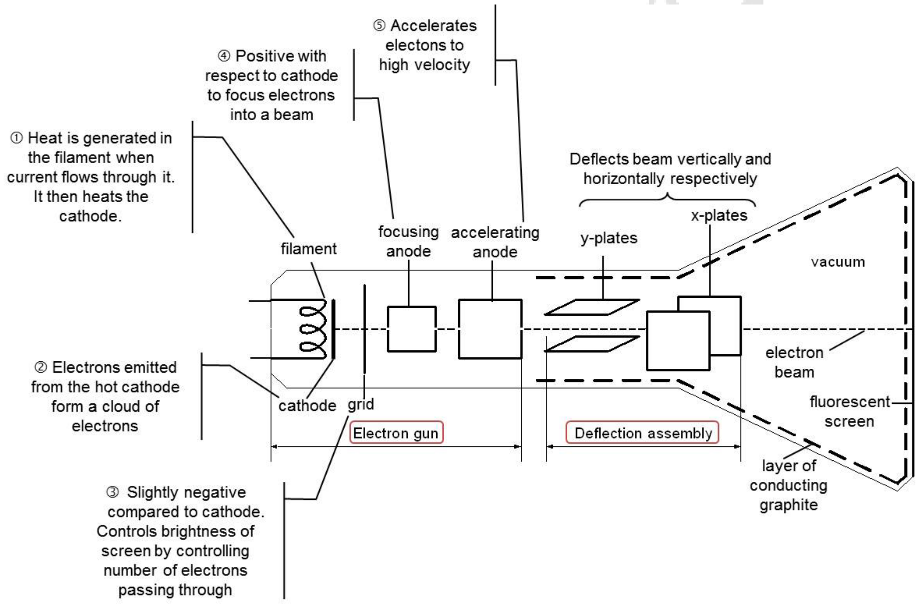


Fig. 14.28 The main features of a cathode ray oscilloscope

Consider a beam of electrons travelling between two horizontal parallel plates as shown in **Fig 14.29**. The electrons experience a constant upward acceleration between the plates, causing them to move in a parabolic path with the component of the velocity perpendicular to the electric field being constant. On exiting the region between the plates, the electrons travel in a straight line.

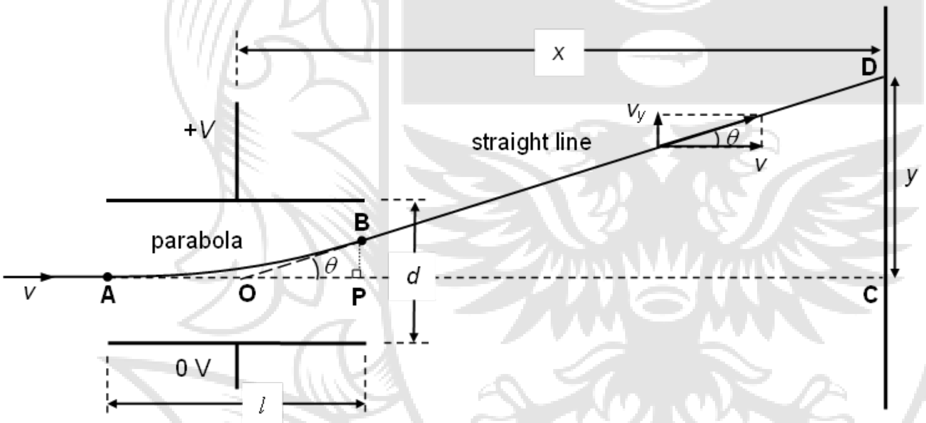


Fig. 14.29 Deflection of an electron beam through parallel plates

Within the parallel plates, an electron will experience an upward electrostatic force. This is the net force acting on the electron (as the weight of the electron is negligible compared to the electrostatic force acting on it).

$$F_E = eE = m_e a \quad \therefore a = \frac{eE}{m_e}.$$

Horizontally, there is no acceleration, hence $u_x = v_x = v$. Using $s_x = u_x t + \frac{1}{2}a_x t^2$ the time taken for the electrons to traverse the length of the plates is

$$l = vt + 0 \quad \Rightarrow \quad t = \frac{l}{v}$$

Considering its vertical motion, its vertical velocity v_y after emerging from the electric field is

$$v_y = u_y + a_y t = 0 + \left(\frac{eE}{m_e}\right)\left(\frac{l}{v}\right) = \frac{eEl}{m_e v}.$$

The electron traverses a parabolic path AB. When it emerges from the electric field, it no longer experiences an electrostatic force and hence travels in a straight line at an angle θ to the horizontal given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{\left(eEl \big/ m_e v\right)}{v} = \frac{eEl}{m_e v^2}.$$

By using $s_y = u_y t + \frac{1}{2}a_y t^2$, the vertical displacement of the electron is

$$\text{BP} = 0 + \frac{1}{2}\left(\frac{eE}{m_e}\right)\left(\frac{l}{v}\right)^2 = \frac{eEl^2}{2m_e v^2}.$$

Since $\tan\theta = \frac{\text{BP}}{\text{OP}} = \frac{eEl}{m_e v^2}$ and $\text{BP} = \frac{eEl^2}{2m_e v^2}$ $\Rightarrow$ $\text{OP} = \frac{eEl^2}{2m_e v^2} \div \frac{eEl}{m_e v^2} = \frac{l}{2}$.

The deflected electron will eventually strike the screen at point D. ΔOBP and ΔODC are similar triangles, hence

$$\frac{y}{x} = \frac{\text{BP}}{\text{OP}} = \tan\theta \quad \Rightarrow \quad y = x\left(\frac{eEl}{m_e v^2}\right) = \frac{xe\left(V\big/ d\right)l}{m_e v^2} = \left(\frac{xel}{m_e v^2 d}\right)V.$$

Hence, deflection y of the electrons is proportional to the p.d. V between the parallel plates.