6 COLLISIONS

H2 Physics 9478

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Learning Outcomes

Candidates should be able to:

(a) recall that impulse is given by the area under the force-time graph for a body and use this to solve problems.

(b) state the principle of conservation of momentum.

(c) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required).

(d) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(e) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Introduction

In the topic *Fields and Energy*, we have seen that total energy is always conserved within an isolated system, and bodies within the system transfer energies between each other when they interact. During their interactions (such as in collisions), these bodies exert forces on each other, hence transferring momenta too. We will learn about the conservation law associated with momentum, and how it can be applied to interacting bodies.

6.1 Impulse and Momentum Change

❖ Momentum and Newton's Second Law of Motion

Recall that the momentum p of a body is defined as the product of its mass m, and its velocity v.

$$p = mv$$

Newton's Second Law of Motion states that the rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the same direction as the resultant force. In S.I. units, the proportionality constant is 1. Hence,

$$F_{\text{net}} = \frac{dp}{dt}$$

In this topic, we will examine the effect of applying a resultant force on a body over a duration of time, and see how this concept, together with Newton's Third Law of Motion naturally lead to the principle of conservation of momentum.

❖ Impulse

Consider a resultant force F_{net} acting on a body. The graph in Fig. 6.1 shows how F_{net} varies with time t.

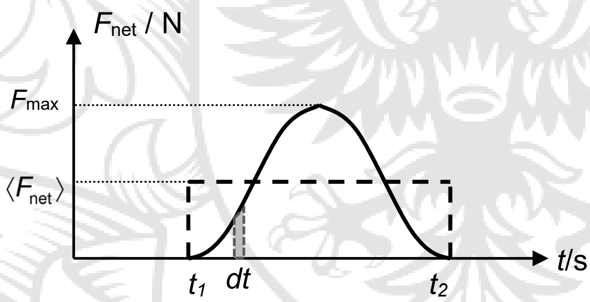


Fig. 6.1

From Newton's Second Law, $F_{\text{net}} = \frac{dp}{dt}$. During a small time interval dt, the momentum of the body changes by $dp = F_{\text{net}}dt$.

To determine the change in momentum Δp over a time interval t_1 to t_2,

$$\int_{p_i}^{p_f} dp = \int_{t_1}^{t_2} F_{\text{net}} dt$$

$$p_f - p_i = \Delta p = \int_{t_1}^{t_2} F_{\text{net}} dt$$

where $\int_{t_1}^{t_2} F_{\text{net}} dt$ is the area under the $F_{net} - t$ graph from t_1 to t_2.

Since the resultant force may be varying, it is sometimes necessary to define an average resultant force $\langle F_{\text{net}} \rangle$. The average resultant force $\langle F_{\text{net}} \rangle$ over the same time interval, $\Delta t = t_2 - t_1$, produces the same change in momentum as the varying resultant force.

$$\Delta p = \left\langle F_{\text{net}} \right\rangle \Delta t$$

Impulse

Impulse is the product of the average resultant force $\langle F_{net} \rangle$ acting on the body and the time interval Δt for which the force acts.

$$\text{impulse} = \left\langle F_{net} \right\rangle \Delta t$$

Hence impulse of the force acting on a body over a time interval Δt is equal to its change in momentum over the same time interval Δt.

$$\text{impulse} = \Delta p = \left\langle F_{net} \right\rangle \Delta t = \int_{t_1}^{t_2} F_{net} dt$$

Impulse is a vector. It has the same S.I. unit as momentum.

S.I. unit for impulse: kg m s^{-1} or N s

❖ Concept of Impulse

Have you wondered why we have to buckle our seat belts in cars? Or why cars crumple during a car crush?

The crumpling of a car during a car crash is not due to the poor quality of the car frame! It is a safety feature designed to reduce the impact of the crash on the passengers in it.

To reduce the injury on passengers, the average resultant force acting on them should be reduced. For a car travelling at a particular speed (which is also the passenger's speed), the impulse on the passenger or the change in momentum of the passenger due to the car crash is constant as it reduces his / her initial speed to zero.

From $\text{impulse} = \left\langle F_{net} \right\rangle \Delta t$, the average resultant force acting on the passenger can be reduced if the time taken for the crash, Δt is increased. That is why cars crumple during the crash; it increases the duration of the crash!

Similarly, the air bag also increases the time taken for the passenger's momentum to reduce to zero and therefore reduce the force acting on the passenger based on the Newton's second law of motion.

Impulse and Change in Momentum

www.youtube.com/watch?v=FOIrZF4WHXg

Example 1 (constant force)

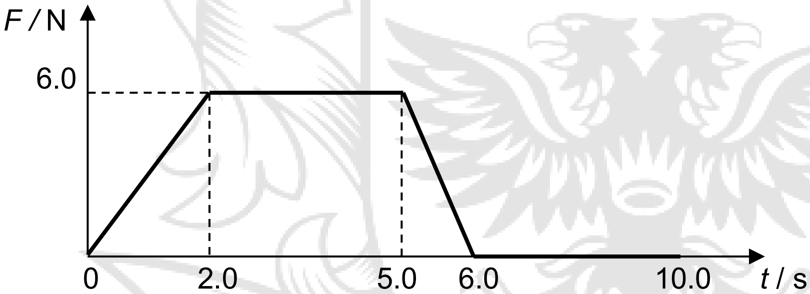
A block of mass 2.0 kg is moving with an initial velocity of 5.0 m s^{-1} towards the right on a smooth horizontal surface. A constant force of 10 N towards the right is then applied on it for 2.0 s.

(a) Calculate the change in momentum experienced by the block.
(b) Determine the final velocity of the block.
(c) Determine the final velocity of the block if its initial velocity is 5.0 m s^{-1} towards the *left*.

Recall that velocity is a vector quantity.

Example 2 (time varying force)

A body of mass 3.0 kg is moving with an initial speed of 1.0 m s^{-1} when it is acted upon by a force *F* which varies with time *t* as shown below:



If the force acts in the same direction as the initial velocity of the body, what is the body's velocity at *t* = 10.0 s?

Example 3 (average of a time varying force)

A man throws a ball of mass 0.30 kg with a horizontal speed of 20 m s^{-1}.

(a) His hand is in contact with the ball for a time interval of 0.20 s while throwing the ball. Calculate the average force he exerts on the ball.

(b) The ball thrown by the man hits a vertical wall at right angles with a speed of 20 m s^{-1} and bounces back horizontally with a speed of 15 m s^{-1}. The ball is in contact with the wall for 0.050 s. Determine the magnitude of the average force exerted by the wall on the ball.

Recall that momentum is a vector quantity.

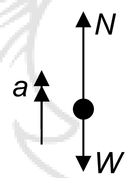
Example 4

A parachutist of weight *W* strikes the ground with her knees bent and comes to rest with a deceleration of 3*g*.

(a) Determine, in terms of *W*, the force exerted on her by the ground during landing.

(b) Suggest and explain what may happen if the parachutist did not bend her knees upon landing?

Recall the safety features of a car.



6.2 Conservation of Momentum and Energy

❖ Principle of Conservation of Momentum

In closed system of bodies (one where no resultant external force is acting), when the bodies interact with one another, their individual momentum may change but the total momentum of the bodies remains constant. This is the principle of conservation of momentum.

The Principle of Conservation of Momentum states that

when bodies in a system interact, the total momentum of the system remains constant, provided no net external force acts on it.

An external force is a force acting on the system from the outside.

This principle is a consequence of Newton's Second Law and Third Law of Motion. The principle will always hold if there is no net external force acting in the direction in which momentum is considered.

Whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

❖ Types of interactions between bodies

We can broadly classify interactions between bodies under two categories using energy considerations:

- Collisions: when total kinetic energy remain the same or decrease after the interaction
- Super-elastic interactions: when total kinetic energy increase after the interaction

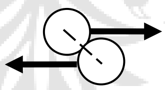
6.3 Collisions

Collisions can occur in any planes and in any directions. In particular, we consider collisions made *head-on* (Fig. 6.2(a)) and at a *glancing angle* (Fig. 6.2(b)).

A *head-on* collision is one where the velocities of the bodies before and after collision are along the line joining their centres of mass.

head-on collision

Fig. 6.2(a)



collision at glancing angle

Fig. 6.2(b)

In a collision, two bodies come close to each other and interact by exerting a force F (action-reaction pair) on each other. The time interval Δt over which this interaction occur is relatively short. Since $F = \frac{\Delta p}{\Delta t}$, for a certain change in momentum during the collision, the force of interaction F is relatively large.

This large force causes the motion of at least one of the colliding bodies to change so abruptly that we can make a clean distinction between "before the event" and "after the event". The laws of conservation of momentum and energy permit us to learn much about such processes by studying the "before" and "after" situations.

❖ Conservation of Momentum during Collisions

Consider two bodies, A and B, moving along the same straight line. Body A, of mass m_A and initial velocity u_A, collides with body B, of mass m_B and initial velocity u_B. After the collision, body A and body B move off with final velocities v_A and v_B respectively. This is shown in Fig. 6.3.

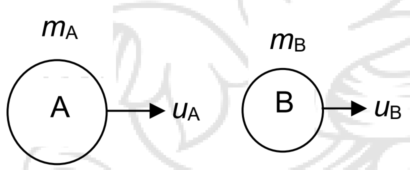


Fig. 6.3(a)

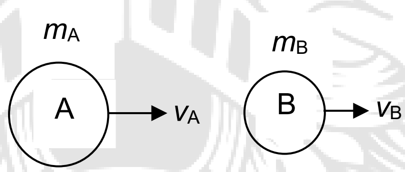


Fig. 6.3(b)

During the brief collision, bodies A and B exert large forces on one another. By Newton's Third Law of motion, the force F_{BA} exerted by B on A is equal and opposite to the force F_{AB} exerted by A on B as shown in Fig. 6.4.

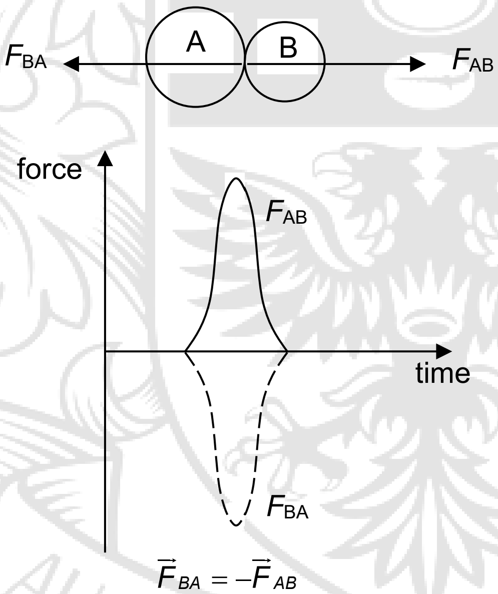


Fig. 6.4

Since the time interval during which the force acted on B is equal to the time during which the force of reaction acted on A, the impulse on A is equal and opposite to that on B.

$$\int_{t_i}^{t_f} F_{BA}\mathrm{d}t = -\int_{t_i}^{t_f} F_{AB}\mathrm{d}t$$

Since impulse = change in momentum

$$\Delta p_A = -\Delta p_B$$
$$m_A v_A - m_A u_A = -\left(m_B v_B - m_B u_B\right)$$

Re-arranging the equation,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

total initial momentum of bodies = total final momentum of bodies

The total momentum of the system of colliding bodies is always conserved in all types of collisions, if there is no net external force acting on the system. However, the momentum of each individual body may change.

❖ Conservation of Energy during Collisions

The total energy in a collision is always conserved. The total energy could include energy stores like kinetic energy, elastic potential energy, sound energy and internal energy.

However, whether kinetic energy remains the same before and after collision depends on the type of collision.

❖ Different Types of Collision

Collisions can be any one of the following 3 types:

1. Perfectly elastic collision
2. Inelastic collision
3. Completely inelastic collision

Perfectly Elastic Collision

In a perfectly elastic collision or elastic collision, both the total momentum and the total kinetic energy of the system *after* the collision are the same as their respective values before the collision.

In the process of the collision, the kinetic energy lost by one body is entirely transferred to the other body so that the total kinetic energy of the colliding bodies after the collision remains the same as the kinetic energy before the collision. Note that *during* the collision, the total kinetic energy does not remain constant as part of the energy is in the form of potential energy. However, there is no energy loss to the surroundings during the conversion of kinetic energy before the collision to potential energy during the collision and then to kinetic energy after the collision. This results in no change in the total kinetic energy before and after the collision.

Inelastic Collision

In an inelastic collision, the total momentum of the system is conserved but the total kinetic energy of the system after the collision is less than the kinetic energy before the collision.

During the collision, some of the kinetic energy is converted into sound energy or internal energy when the objects are deformed. Therefore, in an inelastic collision, the total kinetic energy decreases.

Completely Inelastic Collision

A completely inelastic collision is like an inelastic collision in which total momentum of the system is conserved but the total kinetic energy is not.

In addition, the bodies stick together (coalesced) after collision such that their final velocities are the same.

❖ Analysis of Elastic Collision in One Dimension (Head-on)

Consider a head-on elastic collision between two bodies of masses m_1 and m_2 initially moving with velocities u_1 and u_2 as shown in Fig. 6.5(a). After the collision they move off with velocities v_1 and v_2 as shown in Fig. 6.5(b).

Before collision

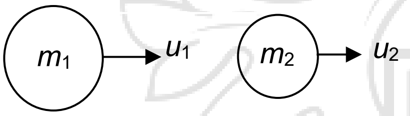


Fig. 6.5(a)

After collision

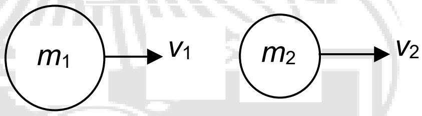


Fig. 6.5(b)

Take direction to the right as positive.

Applying the principle of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

Since the collision is an elastic collision, the total kinetic energy of the bodies remains constant.

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \qquad (2)$$

Eliminating m_1 and m_2 from equations (1) and (2) [refer to Appendix],

$$u_1 - u_2 = v_2 - v_1$$

relative speed of approach = relative speed of separation

u_1, u_2, v_2 and v_1 are all velocities in the equation $u_1 - u_2 = v_2 - v_1$. If we choose the direction to the right to be the positive direction, then a negative sign must be included in the numerical substitution of the equation if the body is moving to the left.

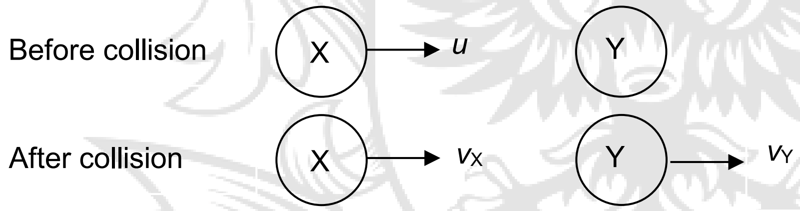
❖ Solving Collision Problems

When solving collision problems, first identify the type of collision. Then apply the relevant laws.

Type of collision	Laws to apply
Elastic	1. Conservation of momentum $\sum p_{initial} = \sum p_{final}$ 2. Relative speed of approach = Relative speed of separation $u_1 - u_2 = v_2 - v_1$ **OR** Conservation of kinetic energy $\sum KE_{initial} = \sum KE_{final}$
Inelastic	1. Conservation of momentum $\sum p_{initial} = \sum p_{final}$
Completely Inelastic	1. Conservation of momentum $\sum p_{initial} = \sum p_{final}$ Final velocities of both bodies are the same.

Example 5

An ice-puck X, traveling with velocity u, strikes head-on with an identical stationary ice-puck Y, as shown below. The collision is elastic. Determine the velocities of X and Y immediately after the collision.

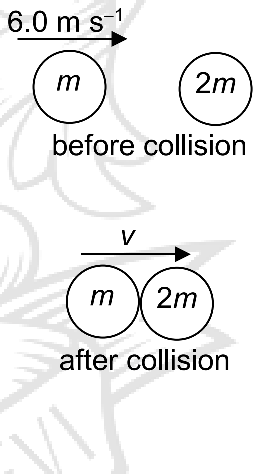


Example 6

A 2.0 kg body moving with velocity 6.0 m s^{-1} collides head-on with a 1.0 kg body moving with velocity 10 m s^{-1} in the opposite direction. The collision is elastic. Calculate the velocity of each body after the collision.

Example 7

A body of mass *m* moving with a velocity of 6.0 m s^{-1} makes a head-on collision with a stationary body of mass 2*m*. The particles coalesce and move off with a common velocity. Determine the velocity of the bodies after the collision.



6.4 Collisions with increase in total kinetic energy

In certain interactions, the total kinetic energy of the system increases due to the conversion of potential energy store into kinetic energy store.

Some examples:

1. Separation of two bodies with a compressed spring in between
2. Recoil
3. Explosion

By the principle of conservation of energy, although the kinetic energy of the system has increased, the total energy of the system before the interaction must be equal to the total energy of the system after the interaction.

❖ Separation of Two Bodies with Compressed Spring

Two bodies are held together at rest with a horizontal compressed spring in between them. Consider the two bodies and the spring as a system.

When the bodies are released, the horizontal force by the compressed spring will cause the two bodies to move in opposite directions. The elastic potential energy of the spring is converted to kinetic energy of the bodies, causing the kinetic energy of the system to increase.

Since the bodies are at rest before they are released, the total initial momentum of the system is zero.

Immediately after the two bodies are released, since the total momentum of the system is conserved, the total momentum of the bodies must remain as zero. This means the bodies must move in directly opposite directions with the same increase in momentum. This ensures that the resultant momentum of the system is zero after release.

Example 8

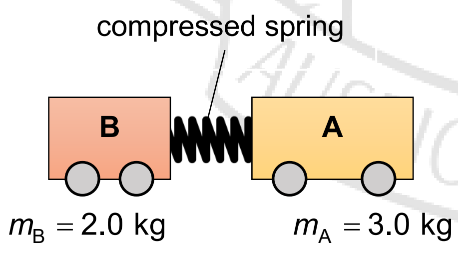
The figure below shows two trolleys A and B are held at rest, with a light compressed spring between them. The trolleys are now released and the 3.0 kg trolley moves with a velocity of 1.0 m s^{-1} to the right after separation. Calculate

(a) the velocity of the 2.0 kg trolley after separation,

(b) the total kinetic energy of the trolleys after separation.

(c)

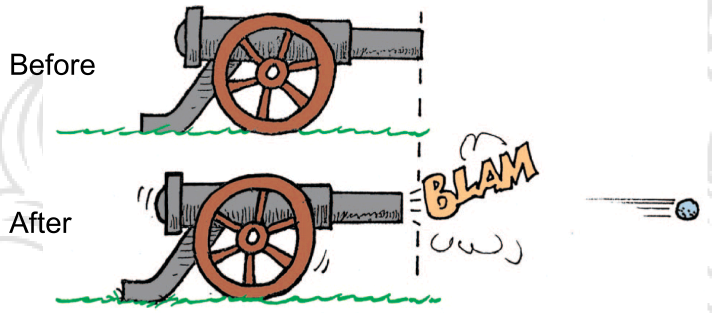
Neglect the mass of the spring and any friction forces.



❖ Recoil

A gun carrying a cannon ball is initially at rest. Consider the gun and the cannon ball as a system.

When the cannon ball is fired horizontally, the potential energy in the gun is converted to kinetic energy of the cannon ball and the gun, causing the kinetic energy of the system to increase.



Since gun and the cannon ball are at rest before the cannon ball is fired, the total initial momentum of the system is zero.

Immediately after the cannon ball is fired horizontally, since the total momentum of the system is conserved, the total momentum of the gun and cannon ball must always remain as zero.

Since the momentum of the cannon ball increases in the rightward direction, the momentum of the gun must increase by the same magnitude in the leftward direction (recoil). This ensures that the resultant momentum of the system is zero after cannon ball is fired.

Recoil is the backward motion of the gun when it fires the cannon ball in the forward direction.

Example 9

A shell of mass 1.6 kg is fired with a velocity of 250 m s^{-1} from an artillery of mass 1000 kg. If the gun is free to move, calculate its recoil velocity.

❖ Explosion

When a stationary body explodes, large forces act on the fragments causing them to be ejected at high speeds in different directions. Consider the exploding body and fragments as a system.

The energy released by the explosion is carried away mainly as kinetic energies of the fragments. Since the body is stationary before the explosion, it is evident that the total kinetic energy of the system increases after the explosion. Other stores of energies produced from the explosion would include sound and thermal.

Immediately after the explosion, each fragment possesses momentum but since the forces that they exert on each other are internal forces, the total momentum of the system is conserved. Hence the total momentum of the fragments is equal to the momentum of the body before explosion, which is zero.

Although in most explosions, gravitational force is present as a net external force on the exploding body and fragments, the effect the gravitational force has on the momentum of the system is negligible. This is because the explosive force is often very large and the duration of the explosion is very small. The effect on the momenta of the fragments by the explosive force is much more significant and the principle of conservation of momentum may be applied to the system immediately before and after the explosion.

Example 10

A 10 kg rock initially at rest is broken into 2 pieces after a small explosion. If a 6.8 kg piece of rock flies to the right at 12 m s^{-1}, calculate the velocity of the other piece of rock.

6.5 Appendix

❖ Elastic Collisions in One Dimension

Consider an elastic collision of two particles of masses m_1 and m_2 initially moving with velocities u_1 and u_2 as shown in Fig. 6.6(a). After the collision they move off with velocities as shown in Fig. 6.6(b).

Before collision

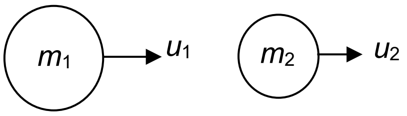


Fig. 6.6(a)

After collision

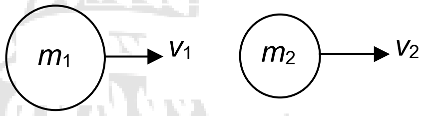


Fig. 6.6(b)

Taking motion to the right as positive.

Applying the principle of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \ldots(1)$$

Since the collision is an elastic collision, the total kinetic energy of the bodies remains constant.

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \quad \ldots(2)$$

$$\Rightarrow \quad m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2 \quad \ldots(3)$$

Re-arranging equations (1) and (3):

$$m_1\left(u_1 - v_1\right) = m_2\left(v_2 - u_2\right) \quad \ldots(4)$$
$$m_1\left(u_1^2 - v_1^2\right) = m_2\left(v_2^2 - u_2^2\right) \quad \ldots(5)$$

Dividing equation (5) by equation (4):

$$\frac{u_1^2 - v_1^2}{u_1 - v_1} = \frac{v_2^2 - u_2^2}{v_2 - u_2}$$
$$\frac{\left(u_1 + v_1\right)\left(u_1 - v_1\right)}{u_1 - v_1} = \frac{\left(v_2 + u_2\right)\left(v_2 - u_2\right)}{v_2 - u_2}$$
$$u_1 + v_1 = v_2 + u_2$$
$$u_1 - u_2 = v_2 - v_1 \quad \cdots(6)$$

relative speed of approach = relative speed of separation

To obtain expressions for v_1 and v_2 in terms of m_1 , m_2 , u_1 and *u_2* :
From eqn (6), $v_2 = u_1 - u_2 + v_1$. Substituting expression into eqn (1):

$$m_1 v_1 + m_2(u_1 - u_2 + v_1) = m_1 u_1 + m_2 u_2$$
$$(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2 u_2$$
$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \qquad \cdots(7)$$

Similarly substituting $v_1 = v_2 - u_1 + u_2$ into eqn (1) and then expressing v_2 in terms of the masses and initial velocities give

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \qquad \cdots(8)$$

Cases of Interest: [deduced from equations (7) and (8)]

Cases	Final velocities	Remark
<u>Case 1:</u> $m_1 = m_2$	$v_1 = u_2$ $v_2 = u_1$	In a one-dimensional elastic collision of two particles of equal mass, the particles simply exchange velocities during collision.
If $u_2 = 0$	$v_1 = 0$ $v_2 = u_1$	When a particle collides with a second identical particle initially at rest, the first particle is stopped cold and the second particle took off with the original velocity of the first particle.
<u>Case 2:</u> $m_1 << m_2$ and $u_2 = 0$ (i.e. m_1 can be approximated to zero)	$v_1 \approx -u_1$ $v_2 \approx 0$	When a light particle collides with a massive particle at rest, the light particle rebounds with little change in speed whilst the massive particle remains approximately at rest.
<u>Case 3:</u> $m_1 >> m_2$ and $u_2 = 0$ (i.e. m_2 can be approximated to zero)	$v_1 \approx u_1$ $v_2 \approx 2u_1$	When a massive particle collides with a light particle at rest, the velocity of the massive particle is virtually unchanged whilst the light particle moves off with approximately twice the velocity of the incident particle.

❖ **Reference Links**

Impulse and change in momentum

www.youtube.com/watch?v=FOIrZF4WHXg