# Probability

**1** **NYJC Prelim 9758/2023/02/Q6**

In a funfair, a vendor sells lucky draw tickets at $1 per ticket. He accepts payment using $1 or $2 notes only. In the queue for tickets, there are $m$ people each paying with a single $1 note and $n$ people each paying with a single $2 note. Each person in the queue wants to buy a single ticket and each arrangement of people in the queue is equally likely to occur. Initially, the vendor has no money but a large supply of tickets. The vendor will stop selling tickets if he cannot give the required change.

**(a)** In the case of $m \geq 1$ and $n = 1$, find the probability that the vendor can sell one ticket to each person in the queue.

**(b)** By considering the first three people in the queue in the case of $m \geq 2$ and $n = 2$,

**(i)** show that the probability that the vendor is able to sell one ticket to each person in the queue is $\dfrac{m-1}{m+1}$,

**(ii)** find the probability that the vendor is unable to sell any ticket to the third person,

given that the vendor is unable to sell tickets to the first three people in the queue.

[3]

**2** **PJC Prelim 9758/2017/02/Q10**

For events $A$ and $B$, it is given that $\mathrm{P}(A) = \dfrac{11}{20}$ and $\mathrm{P}(B) = \dfrac{1}{2}$.

**(i)** Find the greatest and least possible values of $\mathrm{P}(A \cap B)$. [2]

It is given in addition that $\mathrm{P}(B \mid A') = \dfrac{7}{9}$.

**(ii)** Find $\mathrm{P}(A \cup B)$. [2]

**(iii)** Determine if $A$ and $B$ are independent events. Justify your answer. [2]

**(iv)** Given another event $C$ such that $\mathrm{P}(C) = \dfrac{2}{5}$, $\mathrm{P}(A \cup B \cup C) = \dfrac{19}{20}$,

$\mathrm{P}(A \cap B \cap C) = \dfrac{1}{10}$ and $\mathrm{P}(A \cap C) = 2\mathrm{P}(B \cap C)$, find $\mathrm{P}(A \cap C)$. [3]

**3** **MI Prelim 9758/2020/02/Q8**

A group of $n$ people each takes turn to pick one card, with replacement, from a pack of cards.

**(a)** Assume that there are 12 unique cards in a pack. The probability that all the people in the group picked different cards is denoted by $P$.

**(i)** For the case where $n = 3$, show that $P = \dfrac{55}{72}$. [2]

**(ii)** Find $P$ for the case where $n = 4$ and deduce the smallest value of $n$ such that $P < \dfrac{1}{2}$. [2]

**(iii)** State, with a reason, the least possible value of $n$ such that $P = 0$. [2]

**(b)** Assume now that there are 120 unique cards in the pack. It is given that, for the case where $n = 12$, the probability that all the people in the group picked different cards is 0.56640, correct to 5 decimal places. Find the smallest value of $n$ such that the probability that at least two people picked the same card exceeds $\dfrac{1}{2}$. [4]

**4** **CJC MYE 9758/2021/Q10**

A group of 11 friends, including Alan, Ben and Chris, forms a team to play football friendlies matches. In the team, there has to be 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards. Everyone in the team can play in any position.

(i) Find the number of different teams that the 11 friends can form. [2]

(ii) Find the probability that Alan and Ben play in different positions. [3]

After the match, the team goes to a restaurant for a meal where they all sit at a round table.

(iii) Find the probability that Alan and Ben are seated next to each other. [2]

(iv) Find the probability that Chris sits with either Alan or Ben, given that Alan and Ben are *not* seated next to each other. [3]

**5** **SAJC MYE 9758/2023/01/Q5**

For events $A$ and $B$, it is given that $\mathrm{P}(B) = 0.45$, $\mathrm{P}(B \mid A) = 0.4$ and $\mathrm{P}(A \cup B) = 0.75$.

**(i)** Find $\mathrm{P}(A)$ and $\mathrm{P}(A \cap B)$. [4]

**(ii)** Explain why $A$ and $B$ are not independent. [1]

For a third event $C$, it is given that $\mathrm{P}(C) = 0.4$, and that $B$ and $C$ are mutually exclusive.

**(iii)** Find the greatest and least possible values of $\mathrm{P}(A \cap C)$. [3]

## Vectors

**1** **CJC Promo 9758/2021/Q4**

The points $A(1,0,-2)$, $B(3,-1,-2)$ and $C(-3,7,0)$ lie on plane $p_1$. Another plane $p_2$ has equation $3x-y+2z=3$.

**(i)** Find a vector equation of plane $p_1$ in the form $\mathbf{r}\cdot\mathbf{n}=d$. [3]

**(ii)** Find the acute angle between $p_1$ and $p_2$. [2]

The equation of plane $p_3$ is given to be $-9x+3y-6z=7$.

**(iii)** Find the shortest distance between $p_2$ and $p_3$. [3]

**2** **CJC Promo 9758/2025/Q11**

The plane $\Pi_1$ contains the points $A(1,4,2)$, $B(1,0,5)$ and $C(0,8,-1)$.

**(a)** Find a cartesian equation of $\Pi_1$. [3]

**(b)** Find the shortest distance between the point $P$ (2,1,2) and the plane $\Pi_1$. [2]

A second plane $\Pi_2$ contains the point $D(2,2,3)$ and is perpendicular to the vector $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$. The point $(p,0,q)$ lies in both planes.

**(c)** Find $p$ and $q$. [3]

**(d)** Hence find an equation of the line of intersection of the two planes in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$, where $\lambda$ is a real constant. [2]

**(e)** Find the acute angle between the line $AC$ and the plane $\Pi_2$. [2]

**3** **RI Promo 9758/2024/Q9**

The planes $p$ and $q$ have equations

$$\mathbf{r}=(2+\lambda-2\mu)\mathbf{i}+(-3\lambda+\mu)\mathbf{j}+(3-\lambda+2\mu)\mathbf{k} \quad \text{and} \quad \mathbf{r}.\begin{pmatrix} a \\ -1 \\ b \end{pmatrix}=1 \text{ respectively,}$$

where $a$ and $b$ are constants and $\lambda$ and $\mu$ are parameters.

The line $l$ passes through the point $(5,4,0)$ and is parallel to the vector $-2\mathbf{i}-\mathbf{j}+2\mathbf{k}$. The planes $p$ and $q$ meet in the line $l$.

**(a)** Show that $a=1$ and $b=\frac{1}{2}$. [2]

**(b)** Find the exact acute angle between the planes $p$ and $q$. [3]

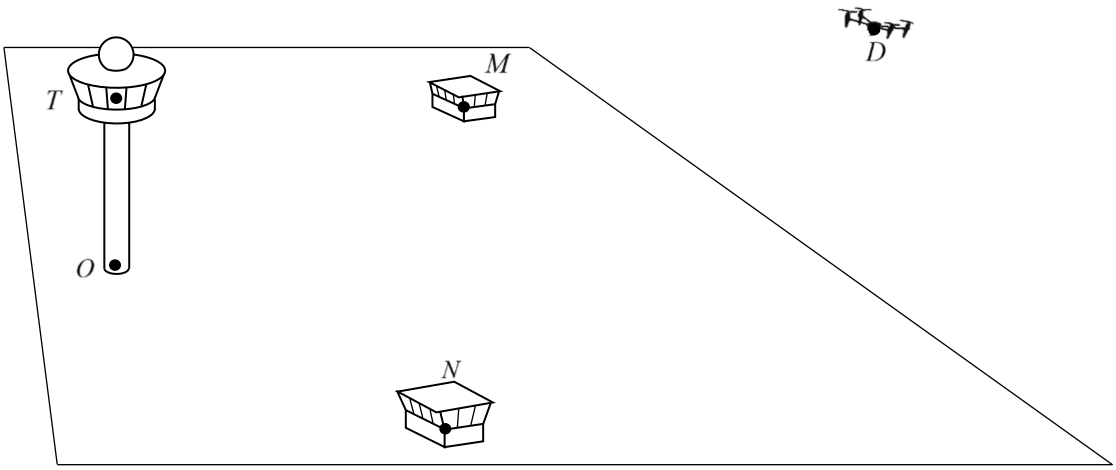
**(c)** Find the distance from the point $A(2,0,3)$ to the plane $q$. Hence deduce the shortest distance from $A$ to $l$. [4]

The plane $q$ is reflected in the plane $p$ to obtain the plane $q'$.

**(d)** Find a Cartesian equation of the plane $q'$. [3]

| | |
|---|---|
| **4** | **HCI JC2 Prelim 9758/2019/01/Q12** |

At an airport, an air traffic control room $T$ is located in a vertical air traffic control tower, 70 m above ground level. Let $O(0,0,0)$ be the foot of the air traffic control tower and all points $(x,y,z)$ are defined relative to $O$ where the units are in kilometres. Two observation posts at the points $M(0.8,0.6,0)$ and $N(0.4,-0.9,0)$ are located within the perimeters of the airport as shown.




An air traffic controller on duty at $T$ spots an errant drone in the vicinity of the airport. The two observation posts at $M$ and $N$ are alerted immediately. A laser rangefinder at $M$ directs a laser beam in the direction $\begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix}$ at the errant drone to determine $D$, the position of the errant drone. The position $D$ is confirmed using another laser beam from $N$, which passes through the point $(0.8, 0.75, 0.3)$, directed at the errant drone.

**(i)** Show that $D$ has coordinates $(0.56,-0.24,0.12)$. [4]

A Drone Catcher, an anti-drone drone which uses a net to trap and capture errant drones, is deployed instantly from $O$ and flies in a straight line directly to $D$ to intercept the errant drone.

**(ii)** Find the acute angle between the flight path of the Drone Catcher and the horizontal ground. [2]

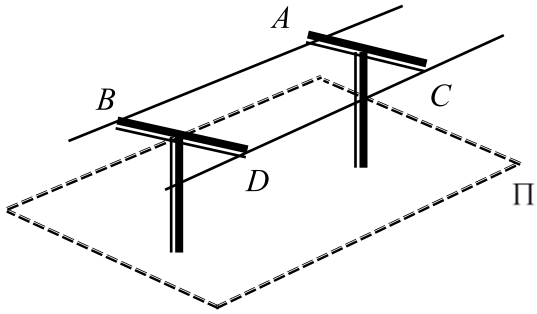
At the same time, a Jammer Gun, which emits a signal to jam the control signals of the errant drone, is fired at the errant drone. The Jammer Gun is located at a point $G$ on the plane $p$ containing the points $T$, $M$ and $N$.

**(iii)** Show that the equation of $p$ is $\mathbf{r}\cdot\begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = -6.72$. [3]

It is also known that the Jammer Gun is at the foot of the perpendicular from the errant drone to plane $p$.

**(iv)** Find the coordinates of $G$. [3]
**(v)** Hence, or otherwise, find the distance $GD$ in metres. [2]

| | |
|---|---|
| **5** | **ACJC JC2 Prelim 9758/2024/01/Q11** |




At a ski resort, engineers are installing cables for a new cable car system to transport skiers to ski slopes. The system involves installing cables running between support towers. Cables are laid in straight lines and the widths of cables can be neglected. The cable *AB* is used to transport skiers up the slope and another parallel cable *CD* is used to transport skiers down the slope. Straight lines are used to represent the cables and a plane $\Pi$ is used to model the ski slope.

The cable *AB* has vector equation $\mathbf{r} = \begin{pmatrix} 5 \\ -5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$ where $\lambda \in \mathbb{R}$ and $-5 < \lambda < 15$.

The parallel cable *CD* passes through the point $(3, -18, 10)$. The cartesian equation of the ski slope $\Pi$ is $x - 2y + 3z = 5$.

**(i)** Find the distance between the cables *AB* and *CD*. [2]

**(ii)** The length of the cable from point *A* to point *B* is 100 units. Find the length of the projection of *AB* on the ski slope $\Pi$. [3]

**(iii)** Find the coordinates of a point *P* on the cable *AB* which is at a perpendicular distance of $2\sqrt{14}$ units from the ski slope $\Pi$. [3]

**(iv)** There is a viewing gallery on the mountain that overlooks the ski slope. The engineers wish to install a huge glass window plane at the viewing gallery. The window plane is perpendicular to the ski slope and is parallel to the cable *AB*. Given that $(10, 10, 20)$ is a point on the window plane, find the cartesian equation of the window plane. [3]

**(v)** Due to a power outage while testing the system, a cable car got stuck on the cable *AB* at the point $Q$ with coordinates $(-7, 7, 25)$. The maintenance team wishes to reach the point $Q$ as quickly as possible. Find the coordinates of the point on the ski slope closest to the point $Q$ from where the team should launch their operation. [3]

## Complex Numbers

| | |
|---|---|
| **1** | **VJC JC2 MYE 9758/2021/01/Q2**<br>**Do not use a calculator in answering this question.**<br>The complex numbers $z$ and $w$ satisfy the simultaneous equations<br>$$z^*+2w=5-4\mathrm{i} \quad\text{and}\quad |z|^2-\mathrm{i}w=2-2\mathrm{i}$$<br>Find $z$ and $w$ in cartesian form $x+\mathrm{i}y$, showing your working. [4] |
| **2** | **NJC JC2 MYE 9758/2021/01/Q6**<br>One of the roots of the equation $4z^4+az^3+23z^2+bz+15=0$, where $a$ and $b$ are real, is $2+\mathrm{i}$.<br><br>**(i)** Find the values of $a$ and $b$ and the other roots, giving your answers in the form $x+\mathrm{i}y$ where $x$ and $y$ are exact values. [5]<br>**(ii)** The roots of the equation are represented by $z_1$, $z_2$, $z_3$, $z_4$ such that $-\pi<\arg(z_1)<\arg(z_2)<\arg(z_3)<\arg(z_4)\le\pi$. The points $A$, $B$, $C$ and $D$ represent the complex numbers $z_1$, $z_2$, $z_3$, $z_4$ respectively. Sketch the points $A$, $B$, $C$ and $D$ on an Argand diagram and state the shape of the quadrilateral $ABCD$. [3] |
| **3** | **RI JC2 Prelim 9758/2024/02/Q2+ TJC Prelim 9758/2025/P1/Q8(b)**<br>**(a)** The function f is defined by $\mathrm{f}(z)=z^4+Az^3+Bz^2+Cz+45$, where $A$, $B$ and $C$ are real numbers. Given that $2+\mathrm{i}$ is a root of $\mathrm{f}(z)=0$ and $(z-k)^2$ is a factor of $\mathrm{f}(z)$, where $k$ is a positive real number, find the values of $A$, $B$, $C$ and $k$. [5]<br>**(b)** It is given that $z_1=2-\mathrm{i}$ and $z_3=-3+2\mathrm{i}$. On an Argand diagram, mark the points $A$ and $C$ representing $z_1$ and $z_3$ respectively. [1]<br>The points $B$ and $D$ on the Argand diagram represent complex numbers $z_2$ and $z_4$ respectively. Given that $ABCD$ is a square, labelled in an anti-clockwise direction, find $z_2$ and $z_4$. [4] |
| **4** | **DHS JC2 Prelim 9758/2019/02/Q4(modified)**<br>The complex number $w=1+\mathrm{i}$ and $z=-\dfrac{\sqrt{6}}{2}+\dfrac{\sqrt{2}}{2}\mathrm{i}$.<br>**(i)** Find the argument and modulus of both $w$ and $z$. State the find the exact real and imaginary parts of $w+z$. [3]<br><br>**(ii)** On the same Argand diagram, sketch the points $P$, $Q$, $R$ representing the complex numbers $z$, $w$ and $z+w$ respectively. State the geometrical shape of the quadrilateral $OPRQ$. [3]<br><br>**(iii)** Referring to the Argand diagram in part **(ii)**, find $\arg(w+z)$ and show that $\tan\left(\frac{11}{24}\pi\right)=\dfrac{a+\sqrt{2}}{\sqrt{6}+b}$ where $a$ and $b$ are constants to be determined. [2] |

**Answers**

**Probability**

**1**. (a) $\dfrac{m}{m+1}$ (b)(ii) $\dfrac{1}{m+2}$

**2**. (i) Least value of $\mathrm{P}(A \cap B)=\dfrac{1}{20}$, Greatest value of $\mathrm{P}(A \cap B)=\dfrac{1}{2}$

(ii) $\dfrac{9}{10}$, (iii) $A$ and $B$ are not independent events . (iv) $\mathrm{P}(A \cap C)=0.3$

**3.** (a)(ii) 5 (a)(iii) 13 (b) 14

**4.** (i) 34650 (ii) 0.764 (iii) $\dfrac{1}{5}$ (iv) $\dfrac{5}{12}$

**5.** (i) $\mathrm{P}(A)=0.5, \mathrm{P}(A \cap B)=0.2$ (iii) Min = 0.15, Max = 0.3

**Vectors**

**1.** (i) $\underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}=11$ (ii) 1.12rad or 64.0° (iii) 1.43 units

**2.** (a) $3y + 4z = 20$ (b) $\dfrac{9}{5}$ (c) $p=2, q=5$ (d) $\mathbf{r}=\begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}+t\begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}, t \in \mathbb{R}$,

(v) $3.7^{\circ}$

**3**. (b) 45° (c) $\dfrac{5}{3}, \dfrac{5}{3}\sqrt{2}$ (d) $x+2y+2z=13$

**4**. (ii) 11.1° (iv) $(0.547,-0.237,0.00325)$ (v) 117 m

**5**. (i) 13.1 units (iii) 98.1 units (iii) $P(7,-7,4)$ (iv) $12x+9y+2z=250$

(v) $\left(-\dfrac{21}{2}, 14, \dfrac{29}{2}\right)$

**Complex Numbers**

**1.** (a) $z=1-2\mathrm{i}, w=2-3\mathrm{i}$ or $z=1+1.5\mathrm{i}, w=2-1.25\mathrm{i}$

**2.** (i) $a=-16$ and $b=-12$, $2-\mathrm{i}$, $\dfrac{\sqrt{3}}{2}\mathrm{i}$ and $-\dfrac{\sqrt{3}}{2}\mathrm{i}$ (ii) *ABCD* is a trapezium

**3.** (a) $A=-10, B=38, C=-66, k=3$ (b) $z_2=1+3\mathrm{i}$, $z_4=-2-2\mathrm{i}$

**4.** (i) $|w|=\sqrt{2}, \arg(w)=\dfrac{\pi}{4}$, $|z|=\sqrt{2}, \arg(z)=\dfrac{5\pi}{6}$, $\operatorname{Re}(w+z)=1-\dfrac{\sqrt{6}}{2}, \operatorname{Im}(w+z)=1+\dfrac{\sqrt{2}}{2}$

(ii) *OPRQ* is a rhombus (iii) $\therefore a=2, b=-2$