

Sec 4 Exp AM Prelim Exam Paper 1 Marking Scheme

1(a)	$(x + 3)(x - 1)$ $f(1) = 0$ $1 + 4 + 2 + a - b = 0$ $a - b = -7 \text{ -----}(1)$ $f(-3) = 0$ $(-3)^4 + 4(-3)^3 + 2(-3)^2 + a(-3) - b = 0$ $81 - 108 + 18 - 3a - b = 0$ $-3a - b = 9 \text{ -----(2)}$ $(1) - (2): \quad 4a = -16$ $\qquad\qquad a = -4$ $\qquad\qquad b = 3$	[M1] [A1] [A1]
1(b)	Long division $\begin{array}{r} x^2 + 2x + 1 \\ x^2 + 2x - 3 \overline{\Bigg } x^4 + 4x^3 + 2x^2 - 4x - 3 \\ -(x^4 + 2x^3 - 3x^2) \\ \hline 2x^3 + 5x^2 - 4x - 3 \\ -(2x^3 + 4x^2 - 6x) \\ \hline x^2 + 2x - 3 \\ -(x^2 + 2x - 3) \\ \hline 0 \end{array}$ $f(x) = (x^2 + 2x - 3)(x^2 + 2x + 1)$ Other factor is $x^2 + 2x + 1$.	[M1] [A1]
Qn 1	(a) $a = -4,$ $b = 3$ (b) the other factor: $x^2 + 2x + 1$	

2(a)	Let the perpendicular distance be x. $\frac{1}{2}(4\sqrt{2} + 8)x = 18$ $(4\sqrt{2} + 8)x = 36$ $x = \frac{36}{8 + 4\sqrt{2}}$ $= \frac{36}{8 + 4\sqrt{2}} \times \frac{8 - 4\sqrt{2}}{8 - 4\sqrt{2}}$ $= \frac{288 - 144\sqrt{2}}{64 - 32}$ $= 9 - \frac{9}{2}(\sqrt{2}) \text{ cm}$	[M1] [M1] [A1]
2(b)	$\sin 45^\circ = \frac{x}{PS}$ $\frac{1}{2} = \frac{9 - \frac{9}{2}\sqrt{2}}{PS}$ $PS = \sqrt{2} \left(9 - \frac{9}{2}\sqrt{2} \right)$ $= 9\sqrt{2} - \frac{9}{2}(2)$ $= -9 + 9\sqrt{2}$	[M1] [A1]
Qn 2	(a) $9 - \frac{9}{2}(\sqrt{2}) \text{ cm}$ (b) $PS = -9 + 9\sqrt{2} \text{ cm}$	

3(a)	$\frac{dy}{dx} = \frac{1}{2}e^{2x}(2) - \frac{1}{3}e^{-3x}(-3)$ $= e^{2x} + \frac{1}{e^{3x}}$ $= \frac{e^{2x}(e^{3x}) + 1}{e^{3x}}$ $= \frac{e^{5x} + 1}{e^{3x}} \quad (\text{shown})$	[M1] [M1]
3(b)	at P, $\frac{dy}{dx} = \frac{e^{5\ln 2} + 1}{e^{3\ln 2}} = \frac{33}{8}$ $\therefore$ gradient of normal $= -\frac{8}{33}$ equation of normal is $y - \frac{55}{24} = -\frac{8}{33}(x - \ln 2)$ $y = -\frac{8}{33}x + \frac{8}{33}\ln 2 + \frac{55}{24}$	[M1] [M1] [A1]
3(c)	$Q\left(0, \frac{8}{33}\ln 2 + \frac{55}{24}\right)$	[A1]
Qn 3	(b) $y = -\frac{8}{33}x + \frac{8}{33}\ln 2 + \frac{55}{24}$ (c) $Q\left(0, \frac{8}{33}\ln 2 + \frac{55}{24}\right)$	

4(a)	$\frac{dy}{dx} = 6x^2 - 18x$ $\text{let } \frac{dy}{dx} = 0,$ $6x^2 - 18x = 0$ $6x(x - 3) = 0$ $x = 0 \quad \text{or} \quad x = 3$ $y = 13 \quad \quad \quad y = -14$ $\therefore (0, 13) \text{ and } (3, -14)$	[M1] [M1] [A2]
4(b)	$\frac{d^2y}{dx^2} = 12x - 18$ $\text{at } x = 0, \quad \frac{d^2y}{dx^2} = -18 < 0$ $\therefore (0, 13) \text{ is maximum point.}$ $\text{at } x = 3, \quad \frac{d^2y}{dx^2} = 18 > 0$ $\therefore (3, -14) \text{ is minimum point.}$	[M1] [A1] [A1]
Qn 4	(a) (0, 13) and (3, -14) (b) (0, 13) max point; (3, -14) min point.	

5(a)	$\frac{d}{dx}(2x \cos 2x) = 2x(-\sin 2x)(2) + 2 \cos 2x$ $= 2 \cos 2x - 4x \sin 2x$	[M1] [A1]
5(b)	$\int (2 \cos 2x - 4x \sin 2x) dx = 2x \cos 2x + c$ $\int 2 \cos 2x dx - \int 4x \sin 2x dx = 2x \cos 2x + c$ $\int 4x \sin 2x dx = \int 2 \cos 2x dx - 2x \cos 2x + c$ $= 2 \left(\frac{\sin 2x}{2} \right) - 2x \cos 2x + c$ $= \sin 2x - 2x \cos 2x + c$ so, $\int_0^{\frac{\pi}{2}} (1 - 4x \sin 2x) dx = \left[x - \sin 2x + 2x \cos 2x \right]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{2} - \pi$ $= -\frac{\pi}{2}$	[M1] [M1] [M1] [A1]

6(a)	$\frac{dy}{dx} = (x-2) \times 3(2x-5)^2(2) + (2x-5)^3$ $= (2x-5)^2 [6x-12+2x-5]$ $= (2x-5)^2 (8x-17)$	[M1] [A1]
6(b)	$\frac{dy}{dx} < 0, \text{ so } (2x-5)^2 (8x-17) < 0$ since $(2x-5)^2 > 0$ for all values of x, then $(8x-17) < 0$. $8x-17 < 0$ $x < \frac{17}{8}$	[M1] [A1]
6(c)	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $0.35 = (2x-5)^2 (8x-17) \times \frac{dx}{dt}$ when $x = 3$, $0.35 = 7 \times \frac{dx}{dt}$ $\frac{dx}{dt} = 0.35 \div 7 = 0.05 \text{ units/s}^2$	[M1] [A1]
6(d)	$z = y^2$ $\frac{dz}{dy} = 2y$ $\frac{dz}{dt} = \frac{dz}{dy} \times \frac{dy}{dt}$ $= 2y \times \frac{dy}{dt}$ $= 2(x-2)(2x-5)^3 \times \frac{dy}{dt}$ when $x = 3$, $\frac{dz}{dt} = 2(1)(1)^3 \frac{dy}{dt}$ $= 2 \frac{dy}{dt}$	[M1] [A1]
Qn 6	(a) $(2x-5)^2 (8x-23)$ (b) $x < \frac{23}{8}$ (c) 0.05 units/s^2	

7(a)	$\sqrt{125^x} = \frac{5^{1-x}}{25}$ $\sqrt{5^{3x}} = \frac{5^{1-x}}{5^2}$ $5^{\frac{3}{2}x} = 5^{1-x-2}$ $\Rightarrow \frac{3}{2}x = -x - 1$ $x = -\frac{2}{5}$ $\therefore \sqrt{125^x} = 0.3807 = 0.381 \text{ (3s.f.)}$	 [M1] [M1] [A1] [A1]
7(b)	$\log_2 x - \log_{16} x$ $= \log_2 x - \frac{\log_2 x}{\log_{16} x}$ $= \log_2 x - \frac{\log_2 x}{4}$ $= \log_2 x - \log_2 x^{\frac{1}{4}}$ $= \log_2 x^{\frac{3}{4}}$ $= \frac{3}{4} \log_2 x$ $= \frac{3}{4} \left(\frac{\lg x}{\lg 2} \right) = \frac{3 \lg x}{4 \lg 2}$	 [M1] [M1] [A1] [M1]
Qn 7	(a) 0.381	

8(a)	$T_{r+1} = {}^{12}C_r (2x)^{12-r} \left(\frac{1}{2x^3} \right)^r$	[B1]
8(b)	$T_{r+1} = {}^{12}C_r (2x)^{12-r} \left(\frac{1}{2x^3} \right)^r$ $= {}^{12}C_r (2)^{12-r-r} (x)^{12-4r}$ Term with x^4: $12 - 4r = 4$ $r = 2$ Coef. of $x^4 = {}^{12}C_2 (2)^8 = 16896$ Term with x^0: $12 - 4r = 0$ $r = 3$ Coef. of $x^0 = {}^{12}C_3 (2)^6 = 14080$ Term independent of $x = 1(16896) + (-1)(14080)$ $= 2816$	[M2] [M1] [M1] [A1]
8(c)	$n = 12 - 4r$ $4r$ is even $\Rightarrow n = 12 - 4r$ is even. Therefore, there are no terms containing x^n whereby n is odd.	[M1] [A1]
Qn 8	(a) $T_{r+1} = {}^{12}C_r (2x)^{12-r} \left(\frac{1}{2x^3} \right)^r$ (b) 2816 (c) There are no terms containing x^n whereby n is odd.	

9(a)	$LHS = \frac{\sec^2 x}{1 + \sec^2 x}$ $= \frac{1}{\frac{1}{\sec^2 x} + 1}$ $= \frac{1}{\cos^2 x + 1}$ $= \frac{1}{\frac{\cos 2x + 1}{2} + 1}$ $= \frac{2}{\cos 2x + 3} = RHS$	[M1] [M1] [A1]
9(b)	$\frac{\sec^2 2x}{1 + \sec^2 2x} = \frac{4}{7}$ $\frac{2}{\cos 4x + 3} = \frac{4}{7}$ $14 = 4 \cos 4x + 12$ $\cos 4x = \frac{1}{2}$ $ref \angle = \frac{\pi}{3}$ $4x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$	[M1] [M1] [A1] [A1]
9(c)	$-2\pi \leq x \leq 2\pi \Rightarrow -8\pi \leq 4x \leq 8\pi$ $\therefore \text{No. of solutions} = 16$	[A1]
Qn 9	(b) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$ (c) 16 solutions	

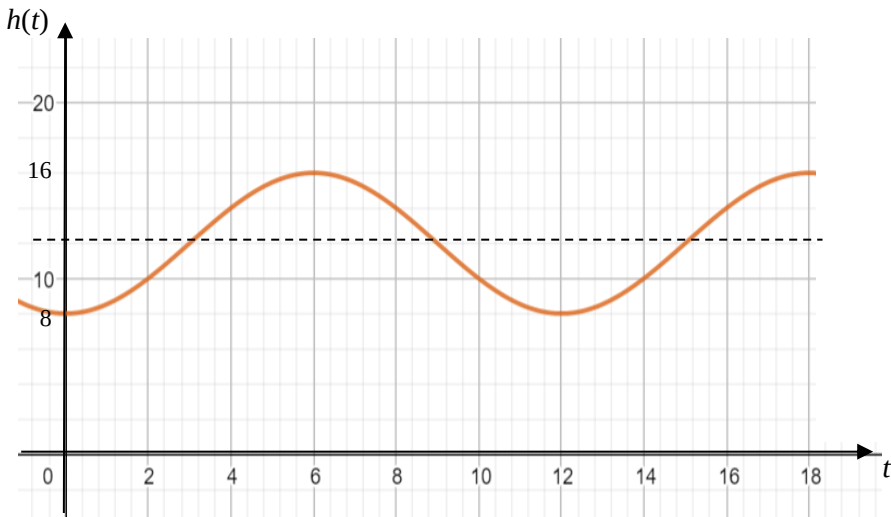
10(a)	$\cos \theta = \frac{OB}{4} \Rightarrow OB = 4 \cos \theta$ $\angle ADO = \theta$ $\sin \theta = \frac{OA}{7} \Rightarrow OA = 7 \sin \theta$ $\therefore AB = OA + OB = 7 \sin \theta + 4 \cos \theta$	[M1] [A1]
10(b)	$AB = R \sin(\theta + \alpha)$ $R = \sqrt{7^2 + 4^2} = \sqrt{65}$ $\alpha = \tan^{-1}\left(\frac{4}{7}\right) = 29.744$ $\therefore AB = \sqrt{65} \sin(\theta + 29.7^\circ)$	[M1] [M1] [A1]
10(c)	Max length of AB = $\sqrt{65}$ Max occurs when $\sin(\theta + 29.744^\circ) = 1$ $\sin(\theta + 29.744^\circ) = 1$ <i>ref</i> $\angle = 90^\circ$ $\theta + 29.744^\circ = 90^\circ$ $\theta = 90^\circ - 29.744$ $= 60.256^\circ = 60.3^\circ \text{ (1dp)}$	[A1] [M1] [A1]
10(d)	$\sqrt{65} \sin(\theta + 29.744^\circ) = \sqrt{55}$ $\sin(\theta + 29.744^\circ) = \frac{\sqrt{55}}{\sqrt{65}}$ <i>ref</i> $\angle = 66.906^\circ$ $\theta + 29.744^\circ = 66.906^\circ$ $\theta = 66.906^\circ - 29.744$ $= 37.162 = 37.2^\circ \text{ (1dp)}$	[M1] [M1] [A1]
Qn 10	(b) $AB = \sqrt{65} \sin(\theta + 29.7^\circ)$ (c) Max AB = $\sqrt{65}$; $\theta = 60.3^\circ$ (d) $\theta = 37.2^\circ$	

12(a)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>ln y</td><td>0.64</td><td>0.83</td><td>1.04</td><td>1.24</td><td>1.44</td></tr></table> Table of values [1] Plot points [1] Smooth Curve [1]	x	1	2	3	4	5	ln y	0.64	0.83	1.04	1.24	1.44	
x	1	2	3	4	5									
ln y	0.64	0.83	1.04	1.24	1.44									
10(b) 12(b)	$y = ae^{bx-1}$ $\ln y = \ln a + \ln e^{bx-1}$ $\ln y = (bx - 1) + \ln a$ $\ln y = bx + \ln a - 1$ gradient = b vertical intercept = $\ln a - 1$ From the graph, $\ln a - 1 = 0.43$ $\ln a = 1.43$ $a = e^{1.43}$ $= 4.1786 = 4.18 \text{ (3s. f)}$ $Grad = \frac{1.34 - 0.54}{4.5 - 0.5} = 0.2$ $\therefore b = 0.2$	[M1] [A1] [A1]												
11(c) 12(c)	$2.46 = ae^{bx-1}$ $y = 2.46 \Rightarrow \ln y = 0.9$ From graph, when $\ln y = 0.9$, $x = 2.3$	[M1] [A1]												
Qn 11 12	(b) $a = 4.18$; $b = 0.2$ (c) $x = 2.3$													

Sec 4 Exp AM Prelim Exam Paper 2 Marking Scheme

1(a)	$y = 2x^2 - 4x + 7$ $= 2(x^2 - 2x + 7/2)$ $= 2[x^2 - 2x + (-1)^2 + 7/2 - (-1)^2]$ $= 2[(x - 1)^2 + 5/2]$ $= 2(x - 1)^2 + 5$	[M1] [M1] [A1]
1(b)	$2(x - 1)^2 \geq 0$ $2(x - 1)^2 + 5 > 0$ $\Rightarrow y > 0$ Therefore, curve lies completely above the x-axis.	[M1] [A1]
Qn 1	(a) $y = 2(x - 1)^2 + 5$	
2(a)	$I = \frac{a}{b + 2^{20-t}}$ When $t = 16$, $I = 3600$ $3600 = \frac{a}{b + 2^4}$ $3600b + 57600 = a \quad (1)$ When $t = 20$, $I = 9000$ $9000 = \frac{a}{b + 1}$ $9000b + 9000 = a \quad (2)$ sub (1) into (2) $9000b + 9000 = 3600b + 57600$ $5400b = 48600$ $b = 9$ $a = 90000$	[M1] [M2] [A1]

2(b)	$\frac{90000}{9 + 2^{20-t}} = 100$ $90000 = 900 + 100(2^{20-t})$ $89100 = 100(2^{20-t})$ $891 = 2^{20-t}$ $\lg 891 = (20-t) \lg 2$ $t = 20 - \frac{\lg 891}{\lg 2}$ $t = 10.2007$ $\Rightarrow n = 11$	[M1] [M1] [A1]
2(c)	$t \rightarrow \infty, 2^{20-t} \rightarrow 0$ $\Rightarrow I \rightarrow \frac{90000}{9} = 10000$	[A1]
Qn 2	(b) $n = 11$ (c) $I \rightarrow 10000$	
3(a)	$\sin 75^\circ = \sin (30^\circ + 45^\circ)$ $= \sin 30 \cos 45 + \cos 30 \sin 45$ $= \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right) + \frac{\sqrt{3}}{2} \left(\frac{1}{\sqrt{2}} \right)$ $= \frac{1 + \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{\sqrt{2} + \sqrt{6}}{4}$	[M1] [M1] [A1]
3(b)	$h(t) = -4\cos bt + 12$ $\text{Period} = \frac{2\pi}{b}$ $\text{Period} = 12$ $\frac{2\pi}{b} = 12$ $\Rightarrow b = \frac{\pi}{6}$	[M1] [A1]

3(c)		Curve[2] axes/ label [1]
4(a)	$\frac{d}{dx} \left(\frac{\ln x}{x^4} \right) = \frac{x^4 \left(\frac{1}{x} \right) - (\ln x)(4x^3)}{x^8}$ $= \frac{x^3 - 4x^3 \ln x}{x^8}$ $= \frac{1 - 4 \ln x}{x^5}$ $= \frac{1}{x^5} - \frac{4 \ln x}{x^5}$	M1 M1 A1
4(b)	$\int \left(\frac{1}{x^5} - \frac{4 \ln x}{x^5} \right) dx = \frac{\ln x}{x^4} + c$ $\int \frac{1}{x^5} dx - \int \frac{4 \ln x}{x^5} dx = \frac{\ln x}{x^4} + c$ $\int \frac{4 \ln x}{x^5} dx = \int \frac{1}{x^5} dx - \frac{\ln x}{x^4} + c$ $\int \frac{\ln x}{x^5} dx = -\frac{1}{16x^4} - \frac{\ln x}{4x^4} + c$	M1 M1 A1

4(c)	$\int f'(x) dx = \int \frac{\ln x}{x^5} dx$ $f(x) = -\frac{1}{16x^4} - \frac{\ln x}{4x^4} + c$ sub $\left(1, \frac{3}{4}\right)$ into $f(x)$, $\frac{3}{4} = -\frac{1}{16} + c$ $c = \frac{13}{16}$ $\therefore f(x) = \frac{1}{16x^4} - \frac{\ln x}{4x^4} + \frac{13}{16}$	M1 M1 A1
Qns 4	(b) $\frac{1}{16x^4} - \frac{\ln x}{4x^4} + c$ (c) $f(x) = \frac{1}{16x^4} - \frac{\ln x}{4x^4} + \frac{11}{16}$	
5(a)	$y = -x^2 + 5x - 4$ $\frac{dy}{dx} = -2x + 5$ let $\frac{dy}{dx} = 2$, then, $x = \frac{3}{2}$ and $y = \frac{5}{4}$ so, $P\left(\frac{3}{2}, \frac{5}{4}\right)$ let $\frac{dy}{dx} = -2$ then, $x = \frac{7}{2}$ and $y = \frac{5}{4}$ so, $Q\left(\frac{7}{2}, \frac{5}{4}\right)$	M1 A1 A1

5(b)	equation of l_1 is $y - \frac{5}{4} = 2\left(x - \frac{3}{2}\right)$ $y = 2x - \frac{7}{4}$ equation of l_2 is $y - \frac{5}{4} = -2\left(x - \frac{7}{2}\right)$ $y = -2x + \frac{33}{4}$ $2x - \frac{7}{4} = -2x + \frac{33}{4}$ $4x = 10$ $x = \frac{5}{2} \text{ and } y = \frac{13}{4}$ $\therefore R\left(\frac{5}{2}, \frac{13}{4}\right)$	M1 A1
5(c)	Shaded area $= 2 \times \left[\frac{1}{2} \left(\frac{5}{4} + \frac{13}{4} \right) (1) - \int_{\frac{3}{2}}^{\frac{5}{2}} (-x^2 + 5x - 4) dx \right]$ $= 2 \times \left[\frac{9}{4} - \left[\frac{-x^3}{3} + \frac{5x^2}{2} - 4x \right]_{\frac{3}{2}}^{\frac{5}{2}} \right]$ $= 2 \times \left[\frac{9}{4} - \left[\frac{5}{12} - \left(-\frac{3}{2} \right) \right] \right]$ $= \frac{2}{3} \text{ units}^2$	M2 M1 A1
Qns 5	(b) $R\left(\frac{5}{2}, \frac{13}{4}\right)$ (c) $\frac{2}{3} \text{ units}^2$	
6(a)	$5x^2 + 12x > 3x + 2$ $5x^2 + 9x - 2 > 0$ $(5x - 1)(x + 2) > 0$ $x < -2 \text{ or } x > \frac{1}{5}$	[M1] [M1] [A1]

6(bi)	$y = mx + c \quad (1)$ $x^2 + y^2 = 8 \quad (2)$ Sub (1) into (2), $x^2 + (mx + c)^2 = 8$ $x^2 + m^2x^2 + 2mcx + c^2 - 8 = 0$ $(1 + m^2)x^2 + 2mcx + c^2 - 8 = 0$ Does not intersect $\Rightarrow D=0$ $(2mc)^2 - 4(1 + m^2)(c^2 - 8) < 0$ $4m^2c^2 - 4(c^2 - 8 + m^2c^2 - 8m^2) < 0$ $4m^2c^2 - 4c^2 + 32 - 4m^2c^2 + 32m^2 < 0$ $32m^2 - 4c^2 + 32 < 0$ $8m^2 + 8 < c^2$	 [M1] [M1] [M1] [A1]
6(bii)	$y = 2x + 5$ $m = 2, c = 5$ $8m^2 + 8 = 8(2)^2 + 8 = 40$ $c^2 = 25$ Since $8m^2 + 8$ is not $< c^2$, $y = 2x + 5$ intersect the curve.	 } [M1] [A1]
Qn 6	(a) $x < -2$ or $x > \frac{1}{5}$ (b) The line intersect the curve.	
7(a)	$x^2 + 2x + 1 + y^2 - 8y + (-4)^2 = 8 + 1 + 16$ $(x + 1)^2 + (y - 4)^2 = 25$ Centre $C(-1, 4)$ Radius = 5 units	 [M1] [A1] [A1]

7(b)	Grad of CP = -4 Grad of tangent at P = $\frac{1}{4}$ $y = \frac{1}{4}x + c$ $(-2, 8)$ $8 = \frac{1}{4}(-2) + c$ $c = \frac{17}{2}$ $y = \frac{1}{4}x + \frac{17}{2}$	[M1] [M1] [A1]
7(c)	Centre = $(-1, -2)$ Radius = 5 Equation: $(x + 1)^2 + (y + 2)^2 = 25$	[M1] [A1]
7(d)	Diameter is the diagonal of the biggest square that can be drawn inside the circle. Let x – side of the square. $x^2 + x^2 = 10^2$ $2x^2 = 100 \Rightarrow x = \sqrt{50}$ Area = $\sqrt{50} \times \sqrt{50} = 50 \text{ units}^2$	[M1] [A1]
Qn 7	(a) Centre C $(-1, 4)$ (b) $y = \frac{1}{4}x + \frac{17}{2}$ Radius = 5 units (c) $(x + 1)^2 + (y + 2)^2 = 25$ (d) 50 units^2	
8(a)	$\frac{8x^3 + 3x + 1}{x(2x + 1)^2} = \frac{8x^3 + 3x + 1}{4x^3 + 4x^2 + x} = 2 + \frac{1 + x - 8x^2}{4x^3 + 4x^2 + x}$	B1

8(b)	so, $\frac{1+x-8x^2}{x(2x+1)^2} = \frac{A}{x} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$ $1+x-8x^2 = A(2x+1)^2 + Bx(2x+1) + Cx$ let $x=0$, $1=A$ let $x=-\frac{1}{2}$, $-\frac{3}{2} = -\frac{C}{2}$ $C=3$ let $x=1$, $-6=9A+3B+C$ $-6=9+3B+3$ $B=-6$ $\therefore \frac{8x^3+3x+1}{x(2x+1)^2} = 2 + \frac{1}{x} - \frac{6}{2x+1} + \frac{3}{(2x+1)^2}$	M1 M3 A1
8(c)	$\int_1^3 \frac{8x^3+3x+1}{x(2x+1)^2} dx = \int_1^3 \left(2 + \frac{1}{x} - \frac{6}{2x+1} + \frac{3}{(2x+1)^2} \right) dx$ $= \left[2x + \ln x - 3 \ln(2x+1) - \frac{3}{2(2x+1)} \right]_1^3$ $= 1.046596 + 1.795836$ $= 2.8424$ $= 2.84 \text{ (3 s.f.)}$	M1 M1 M1 A1
Qns 8	(a) $2 + \frac{1+x-8x^2}{4x^3+4x^2+x}$ (b) $2 + \frac{1}{x} - \frac{6}{2x+1} + \frac{3}{(2x+1)^2}$ (c) 2.84	
9(a)	$10^2 = (2y)^2 + (3x)^2$ $100 = 4y^2 + 9x^2$ $y = \sqrt{\frac{100-9x^2}{4}}$	M1 A1
9(b)	$S = \frac{1}{2}(5x)(2y) + 4xy$ $= 9xy$ $= 9x \left(\sqrt{\frac{100-9x^2}{4}} \right)$ $= \frac{9}{2}x\sqrt{100-9x^2} \text{ (shown)}$	M1 M1 A1

10(a)	$9\sin\left(\frac{t}{2}\right) = 0$ $\sin\left(\frac{t}{2}\right) = 0$ basic angle = 0 rad so, $\frac{t}{2} = 0, \pi$ $t = 0, 2\pi$ $\therefore t = 2\pi$ sec	M1 M1 A1
10(b)	Maximum velocity when $\sin\left(\frac{t}{2}\right) = 1$. $\frac{t}{2} = \frac{\pi}{2}$ $t = \pi$ sec	M1 A1
10(c)	$s = \int 9\sin\frac{t}{2} dx$ $s = 9\left(\frac{-\cos\frac{t}{2}}{\frac{1}{2}}\right) + c$ $s = -18\cos\frac{t}{2} + c$ when $t = 0, s = 0, \quad c = 18$ hence, $s = -18\cos\frac{t}{2} + 18$ when $t = 2\pi, \quad s = -18\cos\pi + 18$ $s = 36$ m when $t = 10, \quad s = -18\cos 5 + 18$ $s = 12.894$ m Dist. travelled in first 10 sec $= 36 + (36 - 12.894)$ $= 59.1059$ $= 59.1$ m (3 s.f.)	M1 M1 A1

10(d)	$s = -18\cos\frac{t}{2} + 18$ let $s = 0$, $-18\cos\frac{t}{2} + 18 = 0$ $\cos\frac{t}{2} = 1$ basic angle = 0 $\frac{t}{2} = 0 \text{ (rej)}, \quad 2\pi$ $\therefore t = 4\pi$	M1 M1 A1
Qns 10	(a) $t = 2\pi$ sec (b) $t = \pi$ sec (c) 59.1 m (d) $t = 4\pi$	

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CLASS

INDEX NO.

Solution		
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ST. PATRICK'S SCHOOL PRELIMINARY EXAMINATION 2022

**SUBJECT : Additional Mathematics
(4049/01)**

DATE : 11 AUGUST 2022

LEVEL : Secondary 4 Express

DURATION : 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Score	/90

This question paper consists of 19 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}bc \sin A$$

1. Find the range of values of m for which the graph of the straight line $y = mx - 1$ is always below that of $y = 2x^2 + 7$ for all the real values of x . [4]

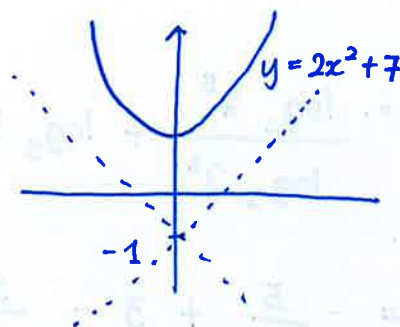
~~4~~

$$mx - 1 < 2x^2 + 7$$

$$2x^2 + 7 > mx - 1 \quad (M1)$$

$$2x^2 + 7 - mx + 1 > 0$$

$$2x^2 - mx + 8 > 0$$



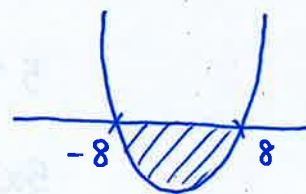
Since there's no intersection, $D < 0$.

$$a = 2, b = -m, c = 8$$

$$(-m)^2 - 4(2)(8) < 0 \quad (M1)$$

$$m^2 - 64 < 0$$

$$(m - 8)(m + 8) < 0 \quad (M1)$$



$$\therefore -8 < m < 8 \quad (A1)$$

2. (i) Simplify $\log_4 32 + \log_5 125 - \log_{\sqrt{3}} 27$.

[3]

$$\begin{aligned}
 & \frac{\log_2 32}{\log_2 4} + \log_5 125 - \frac{\log_3 27}{\log_3 \sqrt{3}} \quad (\text{mi}) \\
 &= \frac{\log_2 2^5}{\log_2 2^2} + \log_5 5^3 - \frac{\log_3 3^3}{\log_3 3^{\frac{1}{2}}} \\
 &= \frac{5}{2} + 3 - \frac{3}{\frac{1}{2}} \quad (\text{mi}) \\
 &= \frac{5}{2} + 3 - 6 \\
 &= -\frac{1}{2} \quad (\text{AI})
 \end{aligned}$$

(ii) Solve $5 + 2e^y = 3e^{-y}$.

[4]

Let x be e^y .

$$5 + 2x = 3(x)^{-1} \quad (\text{mi})$$

$$5 + 2x = \frac{3}{x}$$

$$5x + 2x^2 = 3$$

$$2x^2 + 5x - 3 = 0 \quad (\text{mi})$$

$$(2x - 1)(x + 3) = 0$$

$$x = \frac{1}{2}$$

or $x = -3$

(AI)

$$e^y = \frac{1}{2}$$

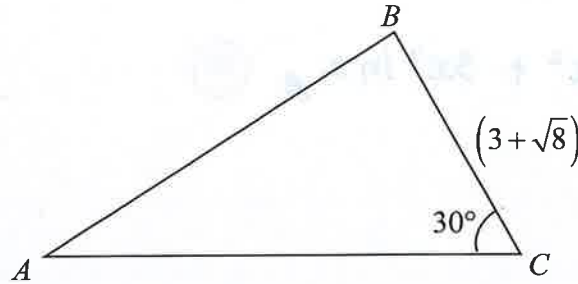
$$e^y = -3$$

$$y = \ln \frac{1}{2}$$

(AI)

$$y = \ln(-3) \\ = (\text{rej})$$

3. In the diagram, area of triangle ABC is $(14+9\sqrt{2}) \text{ cm}^2$, $BC = (3+\sqrt{8}) \text{ cm}$ and angle $ACB = 30^\circ$. Find AC in the form of $p(q-r\sqrt{2})$. [5]



$$A = \frac{1}{2} (AC)(3+\sqrt{8}) \sin 30^\circ$$

$$14+9\sqrt{2} = \frac{1}{2} (AC)(3+\sqrt{8}) \left(\frac{1}{2}\right) \quad (m1)$$

$$4(14+9\sqrt{2}) = (AC)(3+\sqrt{8})$$

$$56+36\sqrt{2} = (AC)(3+\sqrt{8})$$

$$AC = \frac{56+36\sqrt{2}}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} \quad (m1)$$

$$= \frac{168 - 56\sqrt{8} + 108\sqrt{2} - 36(4)}{9-8} \quad (m1)$$

$$= 24 - 112\sqrt{2} + 108\sqrt{2}$$

$$= 24 - 4\sqrt{2}$$

$$= 4(6-\sqrt{2}) \quad (A1)$$

4. (i) Find $\frac{d}{dx}(x^3 \ln x)$.

[2]

$$\begin{aligned}\frac{d}{dx}(x^3 \ln x) &= x^3 \left(\frac{1}{x}\right) + \ln x (3x^2) \quad (M1) \\ &= x^2 + 3x^2 \ln x \quad \# \quad (A1)\end{aligned}$$

(ii) Hence, find $\int x^2 \ln x dx$.

[3]

$$\int x^2 + 3x^2 \ln x dx = [x^3 \ln x] + C_1 \quad (M1)$$

$$\int 3x^2 \ln x dx = x^3 \ln x - \int x^2 dx + C_2$$

$$\int 3x^2 \ln x dx = x^3 \ln x - \left[\frac{x^3}{3}\right] + C_2 \quad (M1)$$

$$3 \int x^2 \ln x dx = x^3 \ln x - \frac{x^3}{3} + C_2$$

$$\int x^2 \ln x dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C \quad \# \quad (A1)$$

5. Find the equation of the tangent to the curve $y = e^{1+\sin x}$ at the point where $x = \pi$, leaving your answer in terms of π and e . [5]

$$\frac{dy}{dx} = \cancel{1+\sin x} \cos x (e^{1+\sin x}) \quad (M1)$$

when $x = \pi$,

$$\begin{aligned} \frac{dy}{dx} &= \cos \pi (e^{1+\sin \pi}) \\ &= (-1)(e^1) \\ &= -e \quad (M1) \end{aligned}$$

when $x = \pi$,

$$\begin{aligned} y &= e^{1+\sin \pi} \\ &= e \end{aligned}$$

$$\therefore \text{equation of tangent} \Rightarrow y - e = -e(x - \pi) \quad (M2)$$

$$y - e = -ex + \pi e$$

$$y = -ex + \pi e + e \neq \quad (A1)$$

6. It is given that $\tan \theta = -2\sqrt{2}$ and $\sin \theta > 0$. $\rightarrow$ lies in 2nd/1st.

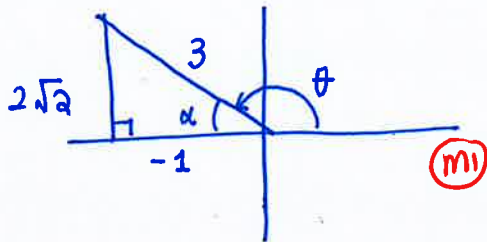
Find the exact value of

(i) $\cos \theta$,

$\rightarrow$ lies in 2nd/4th

$\therefore \theta$ is in 2nd.

[2]



$$\therefore \cos \theta = -\frac{1}{3} \quad (A1)$$

(ii) $\tan 2\theta$,

[2]

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2(-2\sqrt{2})}{1 - (-2\sqrt{2})^2} \quad (M1)$$

$$= \frac{-4\sqrt{2}}{1 - [8]} = \frac{4\sqrt{2}}{7} \quad (A1)$$

(iii) $\sin \frac{1}{2}\theta$.

[3]

$$\sin 2\left(\frac{1}{2}\theta\right) = 1 - 2 \cos^2\left(\frac{1}{2}\theta\right) - 1$$

$$\sin \theta = 2 \cos^2\left(\frac{1}{2}\theta\right) - 1$$

$$\frac{1}{2\sqrt{2}} = 2 \left[\cos^2 \frac{1}{2}\theta \right] - 1$$

$$\frac{1}{2\sqrt{2}} + 1 = 2 \left[\cos^2 \frac{1}{2}\theta \right]$$

$$\frac{-1 + 2\sqrt{2}}{2\sqrt{2}} = 2 \left[\cos^2 \frac{1}{2}\theta \right]$$

$$\cos 2\left(\frac{\theta}{2}\right) = 1 - 2 \sin^2\left(\frac{\theta}{2}\right) \quad (M1)$$

$$-\frac{1}{3} = 1 - 2 \sin^2\left(\frac{\theta}{2}\right)$$

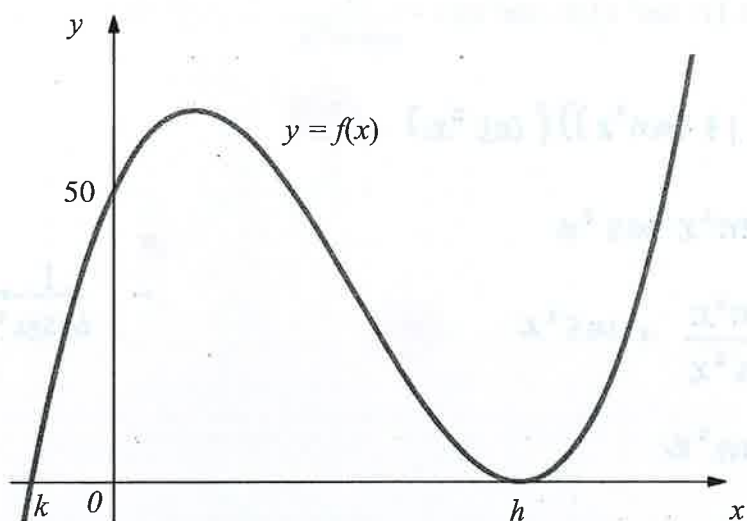
$$-\frac{4}{3} = -2 \sin^2\left(\frac{\theta}{2}\right)$$

$$\frac{2}{3} = \sin^2 \frac{\theta}{2} \quad (M1)$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{2}{3}} \text{ or } -\sqrt{\frac{2}{3}} \quad (M1)$$

(rej).

7.



The diagram represents a cubic function $f(x)$ which touches the x -axis at $x = h$ and intersects the x -axis at $x = k$. The coefficient of x^3 is 2 and the y -intercept is 50. $f(x)$ is divisible by $(x - 5)$.

(i) State the value of h .

[2]

$$\therefore f(x) = 2(x-h)^2(x-k) \quad (M1)$$

$$f(5) = 2(5-h)^2(5-k)$$

$$\therefore h = 5 \quad (A1)$$

(ii) Show that $k = -1$.

[2]

$$f(x) = 2(x-k)(x-5)^2$$

$$50 = 2(-k)(-5)^2 \quad (M1)$$

$$50 = 50(-k)$$

$$\therefore -k = 1$$

$$k = -1 \text{ (shown)} \quad (A1)$$

(iii) Find the remainder when $f(x)$ is divided by $(x - 1)$.

[2]

$$f(x) = 2(x+1)(x-5)^2$$

$$f(1) = 2(2)(-4)^2 \quad (M1)$$

$$= 64 \quad (A1)$$

8. (i) Prove that $(1 - \sec^2 x)(1 - \sin^2 x) = -\frac{1}{\operatorname{cosec}^2 x}$.

[3]

$$\text{LHS} = (1 - (1 + \tan^2 x))(\cos^2 x) \quad (\text{mi})$$

$$= -\tan^2 x \cos^2 x$$

$$= -\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x \quad (\text{mi})$$

$$= -\sin^2 x$$

$$= -\frac{1}{\operatorname{cosec}^2 x} = \text{RHS} \quad (\text{A1})$$

$$\begin{aligned} -\frac{1}{\operatorname{cosec}^2 x} &= -\frac{1}{\frac{1}{\sin^2 x}} \\ &= -\sin^2 x. \end{aligned}$$

- (ii) Hence, solve $(1 - \sec^2 2A)(1 - \sin^2 2A) = -\frac{1}{4}$, for $0 \leq x \leq \pi$.

Leave your answers in terms of π .

[4]

$$-\frac{1}{\operatorname{cosec}^2 2A} = -\frac{1}{4} \quad (\text{mi})$$

$$\operatorname{cosec}^2 2A = 4$$

$$\frac{1}{\sin^2 2A} = 4$$

$$\sin^2 2A = \frac{1}{4}$$

$$\sin 2A = \frac{1}{2} \quad \text{OR} \quad \sin 2A = -\frac{1}{2}$$

$$\begin{aligned} \alpha &= \sin^{-1}\left(\frac{1}{2}\right) \\ &= \frac{\pi}{6} \text{ rad} \end{aligned} \quad (\text{mi})$$

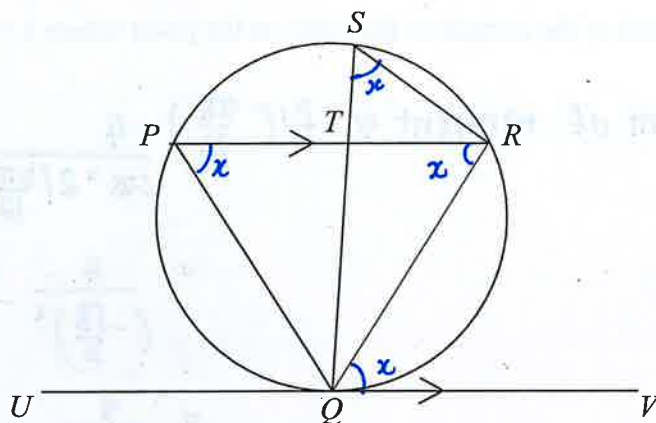
$$\therefore 2A = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$A = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

(A1)

(M)

9. The diagram shows triangles PQR and QRS whose vertices lie on the circumference of a circle. The chords QS and PR intersect at T and PR is parallel to UV . UV is a tangent to the circle at Q .



Show that

- (i) triangle QRS is similar to triangle QTR ,

[3]

Let $\angle RQV$ be x .

$$\angle QSR = \angle RQV = x \quad (\text{alt. seg. thm})$$

$$\angle TRQ = \angle VQR = x \quad (\text{alt. \(\angle\)'s, } PR \parallel QV)$$

$$\angle TQR = \angle RQS = \text{common.}$$

By AA similarity test, $\triangle QTR$ is similar to $\triangle QRS$.

- (ii) $QR^2 = QS \times QT$,

[2]

$$\frac{QT}{QR} = \frac{QR}{QS}$$

$$QT \times QS = QR^2$$

- (iii) triangle PQR is an isosceles triangle.

[2]

$$\angle QPR = \angle RQV = x \quad (\text{alt. seg. thm})$$

$$\text{Since } \angle QPR = \angle QRP = x$$

$$\therefore \triangle PQR \text{ is isos.}$$

10. The curve $y = f(x)$ is such that $f'(x) = 4\sec^2 2x - 3$.

- (i) Find the gradient of the normal to the curve at the point where $x = \frac{5\pi}{12}$.

[2]

$$\begin{aligned}
 \text{gradient of tangent} \Rightarrow f'\left(\frac{5\pi}{12}\right) &= \frac{4}{\cos^2 2\left(\frac{5\pi}{12}\right)} - 3 \\
 &= \frac{4}{\left(-\frac{\sqrt{3}}{2}\right)^2} - 3 \quad (\text{mi}) \\
 &= \frac{4}{\left(\frac{3}{4}\right)} - 3 \\
 &= \frac{7}{3}
 \end{aligned}$$

$$\therefore \text{grad of normal} = -\frac{3}{7} \quad (\text{A1})$$

- (ii) Explain why the curve $y = f(x)$ has no stationary point.

[2]

For stat pts, $f'(x) = 0$

$$4\sec^2 2x - 3 = 0$$

$$\sec^2 2x = \frac{3}{4}$$

$$\frac{1}{(\cos 2x)^2} = \frac{3}{4}$$

$$(\cos 2x)^2 = \frac{4}{3} \quad (\text{mi})$$

$\cos 2x = \text{no solution.}$

$\therefore$ there is no stationary point. A1

(iii) Given that the curve passes through the point (0, 5), find an expression for $f(x)$.

[3]

$$f(x) = \int 4 \sec^2 2x - 3 \, dx$$

$$= 4 \int \sec^2 2x \, dx - \int 3 \, dx$$

$$= 4 \left[\frac{\tan 2x}{2} \right] - 3x + c$$

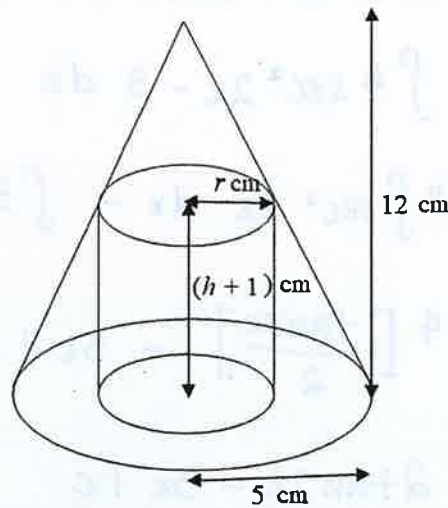
$$= 2 \tan 2x - 3x + c \quad (M1)$$

$$5 = 2 \tan(0) - 3(0) + c$$

$$c = 5 \quad (M1)$$

$$\therefore f(x) = 2 \tan 2x - 3x + 5 \quad (A1)$$

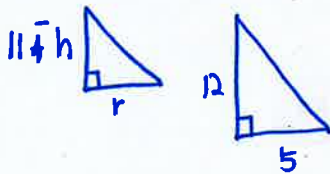
11.



The diagram shows a cylinder of radius r cm and height $(h+1)$ cm, inscribed in a right circular cone with a radius of 5 cm and height 12 cm.

- (i) Show that $h = 11 - \frac{12}{5}r$.

[2]



$$\frac{11-h}{12} = \frac{r}{5}$$

(M1)

$$55 - 5h = 12r$$

$$-5h = -55 + 12r$$

$$h = 11 - \frac{12}{5}r \text{ (shown). (A1)}$$

- (ii) Show that the volume of the cylinder, $V \text{ cm}^3$, is given by $V = \frac{1}{5}\pi r^2(60 - 12r)$.

[2]

$$V = \frac{1}{5}(\pi)(r^2)(h+1)$$

$$= \pi(r^2)\left(11 - \frac{12}{5}r + 1\right) \quad \text{(M1)}$$

$$= \pi(r^2)\left(12 - \frac{12}{5}r\right)$$

$$= \frac{1}{5}\pi r^2(60 - 12r) \text{ (shown). (A1)}$$

(iii) Given that r can vary, find the value of r which gives a stationary value of V . [3]

For stat value, $\frac{dV}{dr} = 0$

$$V = 12\pi r^2 - \frac{12}{5}\pi r^3$$

$$\frac{dV}{dr} = 24\pi r - \frac{36}{5}\pi r^2 \quad (M1)$$

$$24\pi r - \frac{36}{5}\pi r^2 = 0$$

$$r(24\pi - \frac{36}{5}\pi r) = 0 \quad (M1)$$

$$\therefore r = 0 \quad \text{or} \quad 24\pi - \frac{36}{5}\pi r = 0$$

(rej).

$$\frac{36}{5}\pi r = 24\pi$$

$$r = \frac{10}{3} \quad (A1)$$

(iv) Explain why this value of r gives the largest volume possible. [2]

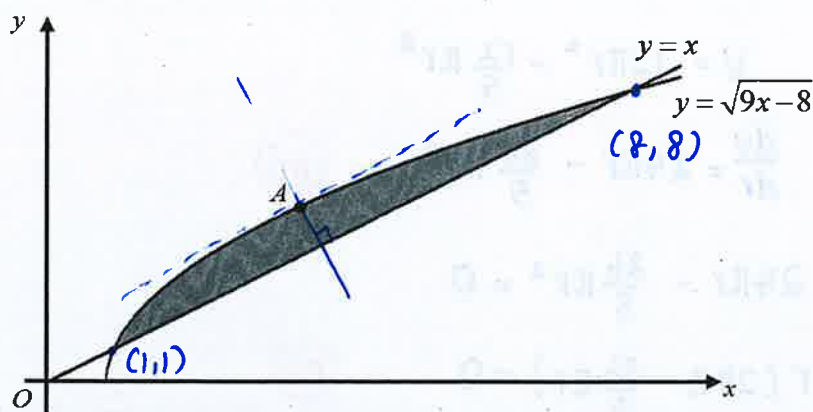
$$\frac{d^2V}{dr^2} = 24\pi - \frac{72}{5}\pi r \quad (M1)$$

$$\text{When } r = \frac{10}{3}, \quad \frac{d^2V}{dr^2} = -ve. \quad (A1)$$

thus it will give you the maximum value.

12. The diagram shows part of the curve $y = \sqrt{9x-8}$ intersecting the line $y = x$.

The point A lies on the curve and the normal at A is perpendicular to the line $y = x$.



(i) Find the x -coordinate of A .

[4]

$$y = (9x-8)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(9x-8)^{-\frac{1}{2}}(9)$$

$$= \frac{9}{2\sqrt{9x-8}}$$

(M1)

gradient of tangent = 1.

$$\frac{9}{2\sqrt{9x-8}} = 1$$

(M1)

$$9 = 2\sqrt{9x-8}$$

$$\sqrt{9x-8} = \frac{9}{2}$$

$$9x-8 = \frac{81}{4}$$

(M1)

$$9x = \frac{113}{4}$$

$$x = \frac{113}{36}$$

(M1)

(ii) Find the area of the shaded region.

[6]

$$x = \sqrt{9x-8}$$

$$x^2 = 9x-8$$

$$x^2 - 9x + 8 = 0$$

$$(x-8)(x-1) = 0$$

$$\therefore x = 8 \text{ or } x = 1$$

$$y = 8$$

$$y = 1$$

$$\text{Area} = \int_1^8 \sqrt{9x-8} - x \, dx$$

$$= \int_1^8 (9x-8)^{\frac{1}{2}} \, dx - \int_1^8 x \, dx$$

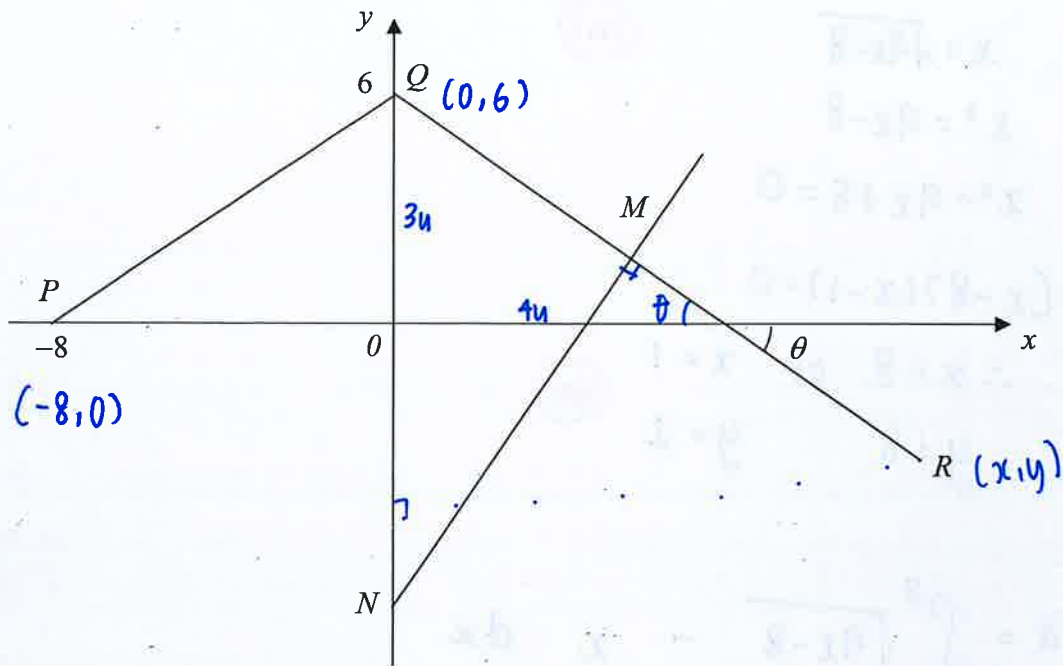
$$= \left[\frac{(9x-8)^{\frac{3}{2}}}{(\frac{3}{2})(9)} \right]_1^8 - \left[\frac{x^2}{2} \right]_1^8$$

$$= \left[\frac{2}{27} (9x-8)^{\frac{3}{2}} \right]_1^8 - \left[\frac{1}{2} x^2 \right]_1^8$$

$$= \frac{1024}{27} - \frac{2}{27} - \left[\frac{64}{2} - \frac{1}{2} \right]$$

$$= \frac{343}{54} \text{ unit}^2.$$

13. In the diagram, P and Q intersect the x and y axes at -8 and 6 respectively.



(i) Find the distance of PQ .

[1]

$$PQ = \sqrt{(-8)^2 + (6)^2}$$

$$= 10 \text{ units.}$$

(A1)

(ii) QR cuts the x -axis with angle θ such that $\tan \theta = \frac{3}{4}$. Find the equation of QR .

[2]

$$\text{gradient } QR = -\frac{3}{4}$$

(M1)

$$\text{Eqn } QR: y - 6 = -\frac{3}{4}(x - 0)$$

$$y = -\frac{3}{4}x + 6$$

(A1)

- (iii) Given that $\frac{QR}{PQ} = 1.5$, show that the coordinates of R is (12, -3).

[4]

$$\frac{QR}{10} = 1.5$$

$$QR = 15$$

Let Q be (x, y)

$$QR = \sqrt{(6-y)^2 + (0-x)^2}$$

$$15 = \sqrt{(6-y)^2 + x^2} \quad (M1)$$

$$225 = 36 - 12y + y^2 + x^2$$

$$225 = 36 - 12\left(-\frac{3}{4}x + 6\right) + \left(-\frac{3}{4}x + 6\right)^2 + x^2 \quad (M1)$$

$$225 = 36 + 9x - 72 + \frac{9}{16}x^2 - 9x + 36 + x^2$$

$$\frac{25}{16}x^2 = 225$$

$$x^2 = 144$$

$$x = 12 \text{ or } -12(\text{rej}) \quad (A1)$$

$$\therefore y = -\frac{3}{4}(12) + 6 = -3 \quad (A1)$$

$$\therefore R \text{ is } (12, -3).$$

- (iv) MN is the perpendicular bisector of QR. Find the y-intercept of line MN.

[2]

$$\text{grad}_{MN} = \frac{4}{3}$$

$$\text{midpt } QR = \left(\frac{0+12}{2}, \frac{6-3}{2}\right)$$

$$= (6, \frac{3}{2}) \quad (M1)$$

$$\text{Eqn MN: } y = \frac{4}{3}x + c$$

$$\frac{3}{2} = \frac{4}{3}(6) + c$$

$$\cancel{y-c} = \cancel{\frac{4}{3}(x-0)} \quad \frac{3}{2} = 8 + c$$

$$c = -\frac{13}{2} \quad (A1)$$

- (v) Find the area of parallelogram PQMN.

[2]

$$\text{Area} = \frac{1}{2} \begin{vmatrix} -8 & 0 & 6 & 0 & -8 \\ 0 & -\frac{13}{2} & \frac{3}{2} & 6 & 0 \end{vmatrix} \quad (M1)$$

$$= \frac{1}{2} [(52+36) - (-48-39)]$$

$$= \frac{1}{2} (175) = 87.5 \text{ unit}^2 \quad (A1)$$

End of Paper

NAME	CLASS	INDEX NO.
Solution		



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Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$.

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Identities

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$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle = \frac{1}{2}bc \sin A$$

1. Salmonella bacteria is commonly found in food that is not hygienically prepared. The number of bacteria, n , can be modelled by $n = Be^{kt}$ where n is the number of bacteria t minutes after the food is prepared, and B and k are constant.

- (i) State the initial number of bacteria. [1]

$$\text{when } t = 0, \quad n = Be^{k(0)}$$

$$n = B \#$$

- (ii) Show that $k = 0.0330$ if the number of bacteria doubles every 21 minutes. [2]

$$\text{when } t = 0, \quad n = B$$

$$2B = Be^{k(21)}$$

$$t = 21, \quad n = 2B$$

$$2B = e^{21k}$$

$$21k = \ln 2$$

$$k = \frac{\ln 2}{21}$$

$$k = 0.033007$$

$$= 0.0330 \text{ (shown).}$$

- (iii) It is found that food containing at least 10^6 bacteria will cause illness when consumed. To protect consumers, the Singapore Food Agency requires all food catering companies to indicate on the food packaging the "CONSUME BY" time that is not later than 4 hours from the time the food is cooked.

Find the maximum value of B , the initial number of bacteria, in order that the food is safe for consumption at $t = 240$ minutes. [3]

$$n = Be^{kt}$$

$$Be^{(0.033007)(240)} = 10^6$$

$$B = \frac{10^6}{e^{7.92168}}$$

$$B = 362.7923$$

$$\text{max value of } B = 363 \#$$

2. The equation of a circle is $x^2 + y^2 - 8x + 6y - 75 = 0$.

(i) Find the radius of the circle and the coordinates of its centre. [3]

$$2f = 6 \quad ; \quad 2g = -8$$

$$f = 3 \quad \quad g = -4$$

$$\therefore \text{centre} = (4, -3)$$

$$r = \sqrt{g^2 + f^2 - c}$$

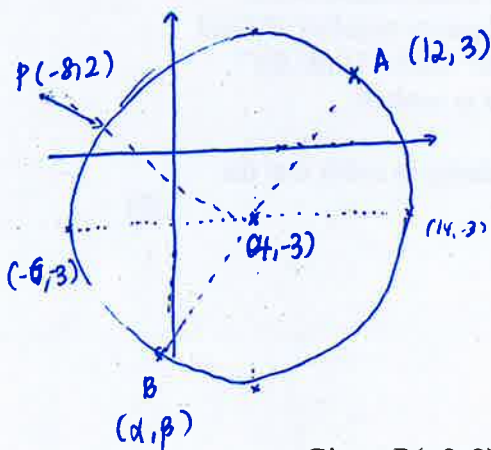
$$= \sqrt{(-4)^2 + (3)^2 + 75}$$

$$= \sqrt{100}$$

$$= 10 \text{ units.}$$

(ii) AB is a diameter of the circle. Find the coordinates of B given $A(12, 3)$. [2]

Sketch:



Let B be (α, β) .

$$\left(\frac{\alpha + 12}{2}, \frac{\beta + 3}{2} \right) = (4, -3)$$

$$\frac{\alpha + 12}{2} = 4$$

$$\alpha + 12 = 8$$

$$\alpha = -4$$

$$\frac{\beta + 3}{2} = -3$$

$$\beta = -6$$

$$\beta = -9$$

$$\therefore B \text{ is } (-4, -9)$$

(iii) Given $P(-8, 2)$, find the greatest distance from P to the circle. [2]

$$\text{Length} = \sqrt{(-8 - 4)^2 + (2 + 3)^2}$$

$$= 13 \text{ units.}$$

$$\text{Total distance} = 13 + 10$$

$$= 23 \text{ units.}$$

3. (i) In the expansion of $(2+3x)^n$,
the coefficient of x^3 is four times the coefficient of x^2 .
Show that $n = 10$. [4]

$$(2+3x)^n = 2^n + \binom{n}{1} 2^{n-1} (3x) + \binom{n}{2} 2^{n-2} (3x)^2 + \binom{n}{3} 2^{n-3} (3x)^3 + \dots$$

$$= 2^n + \binom{n}{1} 2^{n-1} (3x) + \binom{n}{2} 2^{n-2} (9x^2) + \binom{n}{3} 2^{n-3} (27x^3) + \dots \quad (m1)$$

$$\text{coeff of } x^3 : \binom{n}{3} 2^{n-3} (27)$$

$$= \frac{n!}{3!(n-3)!} (2^{n-3}) (27)$$

$$= \frac{n(n-1)(n-2)}{6} (2^{n-3}) (27) \quad (m1)$$

$$\text{coeff of } x^2 : \binom{n}{2} (2^{n-2}) (9)$$

$$= \frac{n!}{2!(n-2)!} (2^{n-2}) (9)$$

$$= \frac{n(n-1)}{2} (2^{n-2}) (9) \quad (m1)$$

$$\therefore \frac{\frac{n(n-1)(n-2)}{6} (2^{n-3}) (27)}{\frac{n(n-1)}{2} (2^{n-2}) (9)} = 4 \quad (m1)$$

$$\frac{n(n-1)(n-2)}{6} (2^{n-3}) (27) = 4 \left(\frac{n(n-1)}{2} (2^{n-2}) (9) \right)$$

$$(n-2) (2^{n-3-n+2}) = 4$$

$$(n-2) \left(\frac{1}{2}\right) = 4$$

$$n-2 = 8$$

$$n = 10 \quad \# \quad (Ai)$$

- (ii) Find the term independent of x in the expansion of $\left(4 - \frac{1}{128x^3}\right)(2+3x)^{10}$. [3]

$$\left(4 - \frac{1}{128x^3}\right)(2+3x)^{10}$$

$$= \left(4 - \frac{1}{128x^3}\right) \left[2^{10} + \dots + \frac{10(9)(8)}{6} (2^7) (27) + \dots\right] \quad (m1)$$

$$\text{Term independent} = (4)(2^{10}) + \left(-\frac{1}{128}\right)(414720) \quad \#$$

$$= 4096 - 3240 \quad (m1)$$

$$= 856 \quad \# \quad (Ai)$$

4. Given that $f(x) = 4\cos\left(\frac{x}{2}\right) + 2$,

- (i) state the amplitude and period of $f(x)$.

[2]

Amplitude = 4

(51)

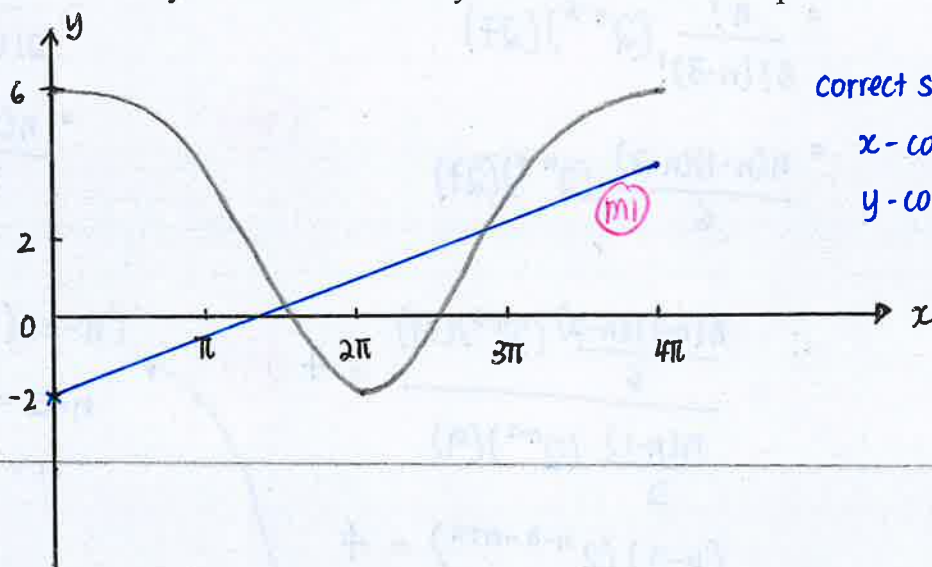
Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$ or 720° .

(B1)

- (ii) sketch the graph of $f(x)$ for $0 \leq x \leq 4\pi$.

Indicate clearly the coordinates of any maximum or minimum points.

[3]



correct shape [1]

x-coord [1]

y-coord [1]

- (iii) Hence, by using (b), sketch a suitable straight line, state the number of solutions to the equation $4\cos\frac{x}{2} - \frac{3x}{2\pi} = -4$ for $0 \leq x \leq 4\pi$.

[3]

$$4\cos\frac{x}{2} = \frac{3x}{2\pi} - 4$$

$$4\cos\frac{x}{2} + 2 = \frac{3x}{2\pi} - 2$$

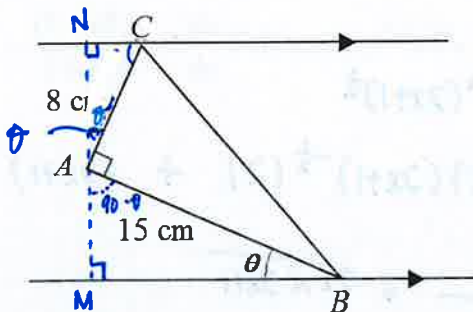
$$y = \frac{3x}{2\pi} - 2$$

(mi)

$\therefore$ there are 2 solutions.

(A1)

5.



The diagram shows two parallel lines and a right-angled triangle BAC with $AB = 15$ cm and $AC = 8$ cm. AB makes an acute angle θ with one of the lines as shown.

- (i) Show that the distance between the parallel lines, $D = 15 \sin \theta + 8 \cos \theta$ cm. [2]

$$\sin \theta = \frac{AM}{15}$$

$$\cos \theta = \frac{AN}{8}$$

$$AM = 15 \sin \theta$$

$$AN = 8 \cos \theta$$

$$\therefore D = 15 \sin \theta + 8 \cos \theta \text{ (shown).}$$

(M1)

(M1)

- (ii) Express D in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [2]

$$R = \sqrt{15^2 + 8^2}$$

$$\alpha = \tan^{-1}\left(\frac{8}{15}\right)$$

$$= 17$$

(M1)

$$= 0.48996$$

(M1)

$$\therefore D = 17 \sin(\theta + 0.490)$$

- (iii) Find the greatest possible value of D and the value of θ at which this occurs. [2]

$$\text{greatest value of } D = 17$$

(B1)

$$\sin(\theta + 0.48996) = 1$$

$$\theta + 0.48996 = \frac{\pi}{2}$$

$$\theta = 1.08083$$

$$= 1.08 \text{ (3sf)}$$

(M1)

6. (i) Given that $y = x^2\sqrt{2x+1}$, show that $\frac{dy}{dx} = \frac{x(5x+2)}{\sqrt{2x+1}}$.

[3]

$$\frac{dy}{dx} = x^2 \left(\frac{1}{2}\right) (2x+1)^{-\frac{1}{2}} (2) + (2x+1)^{\frac{1}{2}} (2x) \quad (M1)$$

$$= \frac{x^2}{\sqrt{2x+1}} + 2x\sqrt{2x+1}$$

$$= \frac{2x(2x+1) + x^2}{\sqrt{2x+1}} \quad (M1)$$

$$= \frac{5x^2 + 2x}{\sqrt{2x+1}}$$

$$= \frac{x(5x+2)}{\sqrt{2x+1}} \quad (\text{shown}). \quad (A1)$$

- (ii) Hence, evaluate $\int_1^5 \frac{5x^2 + 2x - 3}{\sqrt{2x+1}} dx$.

[4]

$$\int_1^5 \frac{5x^2 + 2x}{\sqrt{2x+1}} - \frac{3}{\sqrt{2x+1}} dx$$

$$= \int_1^5 \frac{5x^2 + 2x}{\sqrt{2x+1}} dx - \int_1^5 \frac{3}{\sqrt{2x+1}} dx$$

$$= \left[x^2\sqrt{2x+1} \right]_1^5 - 3 \left[\frac{(2x+1)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)(2)} \right]_1^5 \quad (M1)$$

$$= [25\sqrt{11} - 1\sqrt{3}] - 3[\sqrt{2x+1}]_1^5$$

$$= 25\sqrt{11} - \sqrt{3} - 3[\sqrt{11} - \sqrt{3}] \quad (M1)$$

$$= 22\sqrt{11} + 2\sqrt{3}$$

$$= 76.430$$

$$= 76.4 \text{ (3sf)}. \quad (A1)$$

7. The equation of a curve $y = 3xe^{-2x}$.

C1

(i) Find the value of x for which the curve has a stationary point.

[3] E

$$\begin{aligned}\frac{dy}{dx} &= 3xe^{-2x}(-2) + 3e^{-2x} \\ &= -6xe^{-2x} + 3e^{-2x} \quad [M1]\end{aligned}$$

$$\frac{dy}{dx} = 0, \quad e^{-2x}(-6x + 3) = 0 \quad [M1]$$

$$-6x + 3 = 0$$

$$x = \frac{1}{2} \quad [A1]$$

(ii) Express $\frac{d^2y}{dx^2}$ in the form $ae^{-2x}(b+x)$, where a and b are constants.

[3] E

$$\begin{aligned}\frac{d^2y}{dx^2} &= -6xe^{-2x}(-2) - 6e^{-2x} + 3e^{-2x}(-2) \quad [M1], [M1] \\ &= 12xe^{-2x} - 12e^{-2x} \\ &= 12e^{-2x}(-1 + x) \quad [A1]\end{aligned}$$

(iii) Hence, determine the nature of this stationary point.

[2] E

$$\begin{aligned}x = \frac{1}{2}, \quad \frac{d^2y}{dx^2} &= 12e^{-2(\frac{1}{2})}(-1 + \frac{1}{2}) \quad [M1] \\ &< 0.\end{aligned}$$

$\therefore$ the stationary point is a maximum point. [A1]

8. An object in motion with velocity v m/s is subjected to a resistive force of R Newton against its motion given by the equation

$$R = kv^\beta, \text{ where } k \text{ and } \beta \text{ are constants.}$$

The table shows experimental values of v and R .

v	10	20	30	40	50
R	96	290	530	840	1200

$$\begin{array}{l} \ln v \quad 2.30 \quad 3.00 \quad 3.40 \quad 3.69 \quad 3.91 \\ \ln R \quad 4.56 \quad 5.67 \quad 6.27 \quad 6.73 \quad 7.10 \end{array}$$

} [1]

- (a) Express $\ln R$ in terms of $\ln v$.

[1]

$$\begin{aligned} R &= kv^\beta \\ \ln R &= \ln k + \ln v^\beta \\ &= \beta \ln v + \ln k \end{aligned}$$

[A1]

- (b) Draw the graph of $\ln R$ plotted against $\ln v$, using a scale of 2 cm to 1 unit on the $\ln R$ axis and a scale of 4 cm to 1 unit on the $\ln v$ axis.

[4]

- (c) Use your graph to estimate the values of each of the constants k and β .

[4]

Accept

$$0.8 \leq \ln k \leq 1.2$$

[1]

$$2.23 \leq k \leq 3.33$$

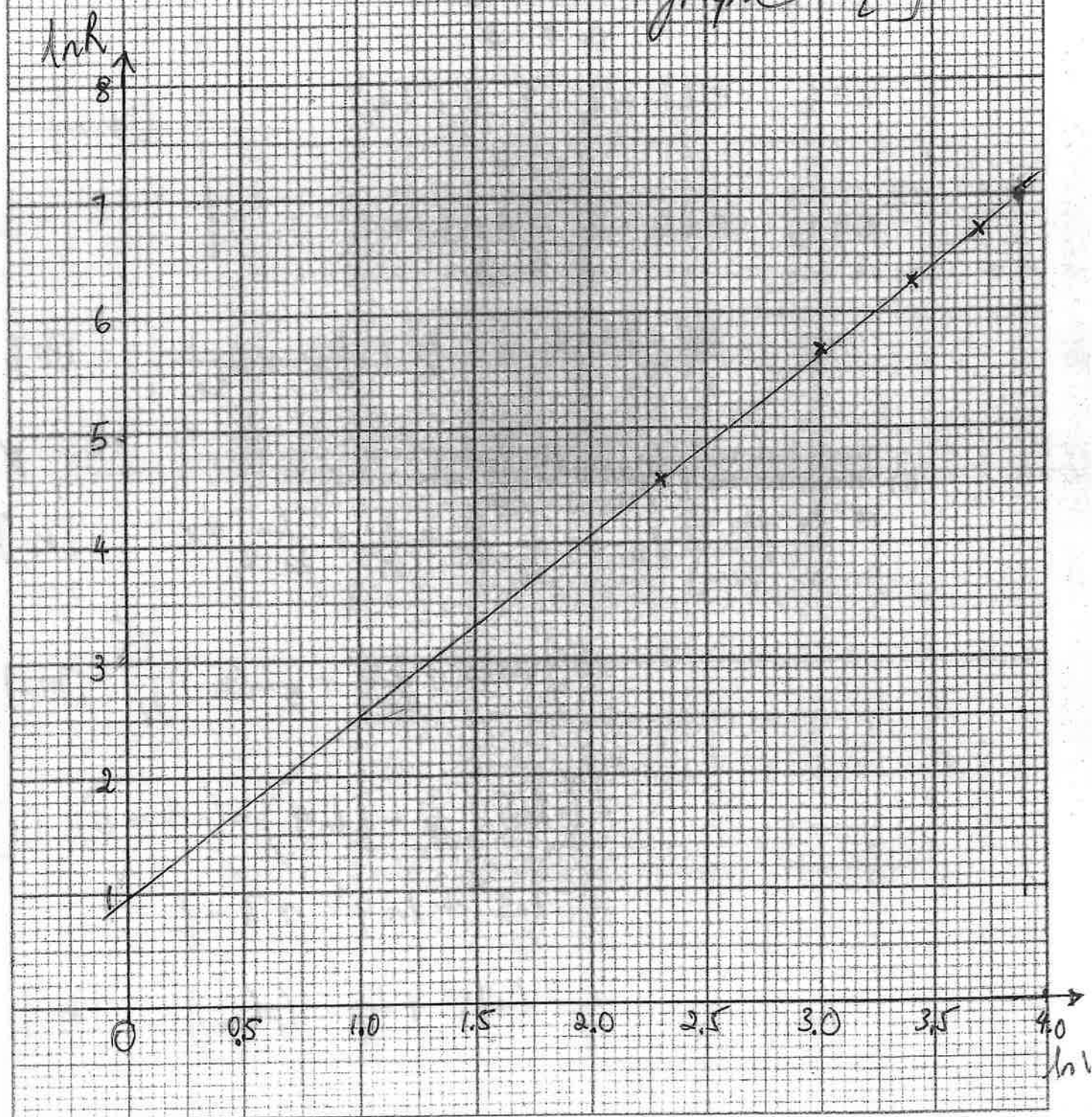
[1]

$$1.4 \leq \beta \leq 1.6$$

[1]

$$\begin{aligned} \beta &= \frac{7 - 2.5}{3.9 - 1.0} \\ &= 1.55 // \quad [1] \end{aligned}$$

Axes/Scale [1]
Points [1]
Graph [1]



9. (i) Express $\frac{5x^2+4x-3}{x^2(2x-1)}$ in partial fractions.

[4] E

M4

$$\text{Let } \frac{5x^2+4x-3}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$$

[1]

$$5x^2+4x-3 = Ax(2x-1) + B(2x-1) + Cx^2$$

$$x=0, \quad -3 = -B$$

$$B = 3$$

$$x=\frac{1}{2}, \quad 5\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3 = C\left(\frac{1}{2}\right)$$

$$C = 1$$

$$x=1, \quad 5+4-3 = A+3+1$$

$$A = 2$$

} [M2]

$$\therefore \frac{5x^2+4x-3}{x^2(2x-1)} = \frac{2}{x} + \frac{3}{x^2} + \frac{1}{2x-1} \quad [A1]$$

- (ii) Hence, show that $\int_1^5 \frac{5x^2+4x-3}{x^2(2x-1)} dx = \frac{12}{5} + \ln 75$.

[5] M

$$\int_1^5 \frac{5x^2+4x-3}{x^2(2x-1)} dx = \int_1^5 \frac{2}{x} + \frac{3}{x^2} + \frac{1}{2x-1} dx \quad [M1]$$

$$= \left[2\ln x + \frac{3x^{-1}}{(-1)} + \frac{1}{2}\ln(2x-1) \right]_1^5 \quad [M2]$$

$$= \left(2\ln 5 - \frac{3}{5} + \frac{1}{2}\ln 9 \right) - (-3 + 0) \quad [1]$$

$$= \ln 5^2 + \ln 9^{\frac{1}{2}} + 3 - \frac{3}{5}$$

$$= \ln 75 + \frac{12}{5} \quad (\text{shown}) \quad [A1]$$

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10. (i) A curve has equation $y = f(x)$, where $f(x) = \frac{5x-2}{x-3}$, $x > 3$.

C1

(a) Find expression for $f'(x)$.

[2] E

$$\begin{aligned} f'(x) &= \frac{5(x-3) - (5x-2)}{(x-3)^2} & [M1] \\ &= \frac{5x-15-5x+2}{(x-3)^2} \\ &= \frac{-13}{(x-3)^2} & [A1] \end{aligned}$$

- (b) Determine whether $y = f(x)$ is an increasing function or a decreasing function. Explain your answer.

[2] M

Since $(x-3)^2 > 0$, for $x > 3$,

$$\frac{-13}{(x-3)^2} < 0. \quad [M1]$$

$$\therefore f'(x) < 0 \quad [A1]$$

$f(x)$ is a decreasing function.

- (ii) Liquid gold is poured into a mold at a rate of $9 \text{ cm}^3/\text{s}$.
 The volume, $V \text{ cm}^3$ of liquid gold in the mold when the depth of liquid is $x \text{ cm}$,
 is given by $V = \frac{1}{30}(2x+5)^3 - \frac{2}{3}$.

Find the

- (a) rate of increase in the depth of liquid gold when $x = 5$,

[3] E

$$\begin{aligned}\frac{dV}{dx} &= 3\left(\frac{1}{30}\right)(2x+5)^2(2) \\ &= \frac{1}{5}(2x+5)^2 \quad [M1]\end{aligned}$$

$$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$$

$$9 = \frac{1}{5}(2x+5)^2 \cdot \frac{dx}{dt} \quad [M1]$$

$$x=5, \quad 9 = \frac{1}{5}(15)^2 \cdot \frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{1}{5} \text{ cm/s.} \quad [A1]$$

- (b) depth of liquid gold when the rate of increase of the depth is 1.2 cm/s .

[3] E

$$9 = \frac{1}{5}(2x+5)^2 \times 1.2 \quad [M1]$$

$$\frac{1}{5}(2x+5)^2 = \frac{9}{1.2}$$

$$(2x+5)^2 = \frac{9}{1.2} \times 5$$

$$2x+5 = \pm \sqrt{\frac{75}{2}}$$

[M1]

$$x = \frac{\sqrt{\frac{75}{2}} - 5}{2} \text{ or } \frac{-\sqrt{\frac{75}{2}} - 5}{2}$$

$$\approx 0.5618 \text{ or } -5.5618 \text{ (rej)}$$

$$= 0.562 \text{ cm}$$

[A1]

11. A particle moves in a straight line such that t seconds after leaving a fixed point O , its velocity, $v \text{ m s}^{-1}$, is given by $v = (0.5t - 3)^4 - 16$.

(i) Find the values of t when the particle is at instantaneous rest.

[3] E

C\

$$v = (0.5t - 3)^4 - 16$$

$$v = 0, \quad (0.5t - 3)^4 = 16 \quad \text{[M1]}$$

$$0.5t - 3 = 2 \quad \text{or} \quad 0.5t - 3 = -2 \quad \text{[M1]}$$

$$0.5t = 5 \quad \quad \quad 0.5t = 1$$

$$t = 10 \text{ s} \quad \quad \quad t = 2 \text{ s} \quad \text{[A1]}$$

(ii) Express the acceleration of the particle in terms of t .

[1] E

$$a = \frac{dv}{dt}$$

$$= 4(0.5t - 3)^3 (0.5)$$

$$= 2(0.5t - 3)^3 \text{ m/s}^2 \quad \text{[B1]}$$

(iii) Using your answer in (ii), explain why the velocity is decreasing, only when $0 \leq t < 6$.

[3] H

for velocity to be decreasing,

$$a < 0$$

$$2(0.5t - 3)^3 < 0 \quad \text{[M1]}$$

$$0.5t - 3 < 0$$

$$0.5t < 3$$

$$t < 6 \quad \text{[A1]}$$

$$\therefore 0 \leq t < 6 \quad \text{[A1]}$$

(iv) Express the displacement of the particle from O in terms of t .

[3] E

$$\begin{aligned}
 s &= \int v \, dt \\
 &= \int (0.5t - 3)^4 - 16 \, dt \\
 &= \frac{(0.5t - 3)^5}{(5)(0.5)} - 16t + C \quad [M1] \\
 &= \frac{2}{5}(0.5t - 3)^5 - 16t + C
 \end{aligned}$$

$$t=0, \quad s=0, \quad 0 = \frac{2}{5}(-3)^5 + C$$

$$C = \frac{486}{5} = 97.2 \quad [M1]$$

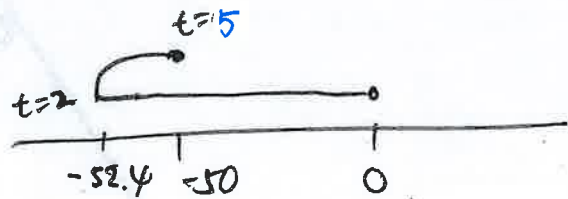
$$\therefore s = \frac{2}{5}(0.5t - 3)^5 - 16t + \frac{486}{5} \quad [A1]$$

(v) Calculate the distance travelled during the first 5 seconds.

[3] H

$$\begin{aligned}
 t=0, \quad s=0, \\
 t=2, \quad s=52.4
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} t=0, \quad s=0, \\ t=2, \quad s=52.4 \end{aligned}} \right\} [M1]$$

$$\begin{aligned}
 t=5, \quad s &= \frac{2}{5}(2.5 - 3)^5 - 80 + \frac{486}{5} \\
 &= -17.1875
 \end{aligned}$$



$$\begin{aligned}
 \text{Total Distance} &= 52.4 + (52.4 - 17.1875) \\
 &= 87.625 \text{ m} \\
 &\text{OR } 87 \frac{49}{80} \text{ m}
 \end{aligned}$$

End of Paper

87.6 m.

87 $\frac{49}{80}$

NAME

CLASS

INDEX NO.

MARKING SCHEME	4G3/5G2	
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ST. PATRICK'S SCHOOL

PRELIMINARY EXAMINATION 2023

SUBJECT : Additional Mathematics
Paper 1 (4049/01)

DATE : 17 August 2023

LEVEL : Secondary 4G3

DURATION : 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Score	/90

This question paper consists of 21 printed pages, including the cover page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$.

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}bc \sin A$$

- 1 If the roots of the equation $x^2 + 2x(1-k) + 3k - 5 = 0$ are real, find the range of values of k .

[4]

$$a=1, \quad b=2(1-k), \quad c=(3k-5)$$

For real roots,

$$\text{Discriminant} \geq 0$$

$$(b)^2 - 4ac \geq 0$$

$$(2-2k)^2 - 4(1)(3k-5) \geq 0$$

$$4 - 8k + 4k^2 - 12k + 20 \geq 0$$

$$4k^2 - 20k + 24 \geq 0$$

$$\div 4, \quad k^2 - 5k + 6 \geq 0$$

$$(k-3)(k-2) \geq 0$$

$$\therefore k \leq 2 \text{ or } k \geq 3$$



- 2 A curve has equation $y = \frac{2x+5}{x-1} - 12x$, where $x \neq 1$.

Explain, with working, whether y is an increasing or a decreasing function of x .

[4]

$$y = \frac{2x+5}{x-1} - 12x$$

$$\frac{dy}{dx} = \frac{(x-1) \frac{d}{dx}(2x+5) - (2x+5) \frac{d}{dx}(x-1)}{(x-1)^2} - 12$$

$$= \frac{2(x-1) - (2x+5)}{(x-1)^2} - 12$$

$$= \frac{2x-2-2x-5}{(x-1)^2} - 12$$

$$= \frac{-7}{(x-1)^2} - 12$$

Since $-7 < 0$, $(x-1)^2 > 0$ for all values of x , $x \neq 1$
& $-12 < 0$

$$\therefore \frac{dy}{dx} < 0$$

$\Rightarrow y$ is a decreasing function of x (explained)

- 3 (a) Express $y = 2x^2 - 10x + c$ in the form $a(x-h)^2 + k$ where a , c , h and k are constants. [2]

$$y = 2(x^2 - 5x) + c$$

$$= 2[(x - 2.5)^2 - (2.5)^2] + c$$

$$= 2(x - 2.5)^2 - 12.5 + c$$

- (b) If the minimum value of y is $-\frac{19}{2}$, find the value of c . [2]

$$\therefore -12.5 + c = -\frac{19}{2}$$

$$c = -\frac{19}{2} + 12.5$$

$$= 3$$

- 4 The speed, v m/s, of an aircraft, t s after passing a fixed point T on the runway, is given, for $t \geq 0$, by $v = 10 + Me^{kt}$, where M and k are constants. The table below shows corresponding values of t and v .

t	1	2	3	4
v	14.4	15.7	17.4	19.4

- (a) Draw the graph of $\ln(v - 10)$ plotted against t , using a scale of 2 cm for 1 unit on the t -axis and a scale of 5 cm for 1 unit on the $\ln(v - 10)$ -axis. [2]

t	1	2	3	4
$\ln(v-10)$	1.48	1.74	2.00	2.24

- (b) Use the graph to estimate
(i) the value of M ,

From the graph,

$$Y\text{-intercept} = 1.22$$

$$\ln M = 1.22$$

$$\log_e M = 1.22$$

$$M = e^{1.22}$$

$$= 3.387 \dots$$

$$= 3.39 \text{ (3 sf)}$$

- (ii) the time when the speed of the aircraft is 18 m/s. [2]

$$\text{When } v = 18$$

$$\ln(v-10) = \ln(18-10)$$

$$= \ln 8$$

$$= 2.07944 \dots$$

$$\approx 2.08$$

From the graph,

the time is 3.35 s

$$v = 10 + Me^{kt}$$

$$v-10 = Me^{kt}$$

$$\ln(v-10) = \ln Me^{kt}$$

$$= \ln M + \ln e^{kt}$$

$$\therefore \ln(v-10) = kt + \ln M$$

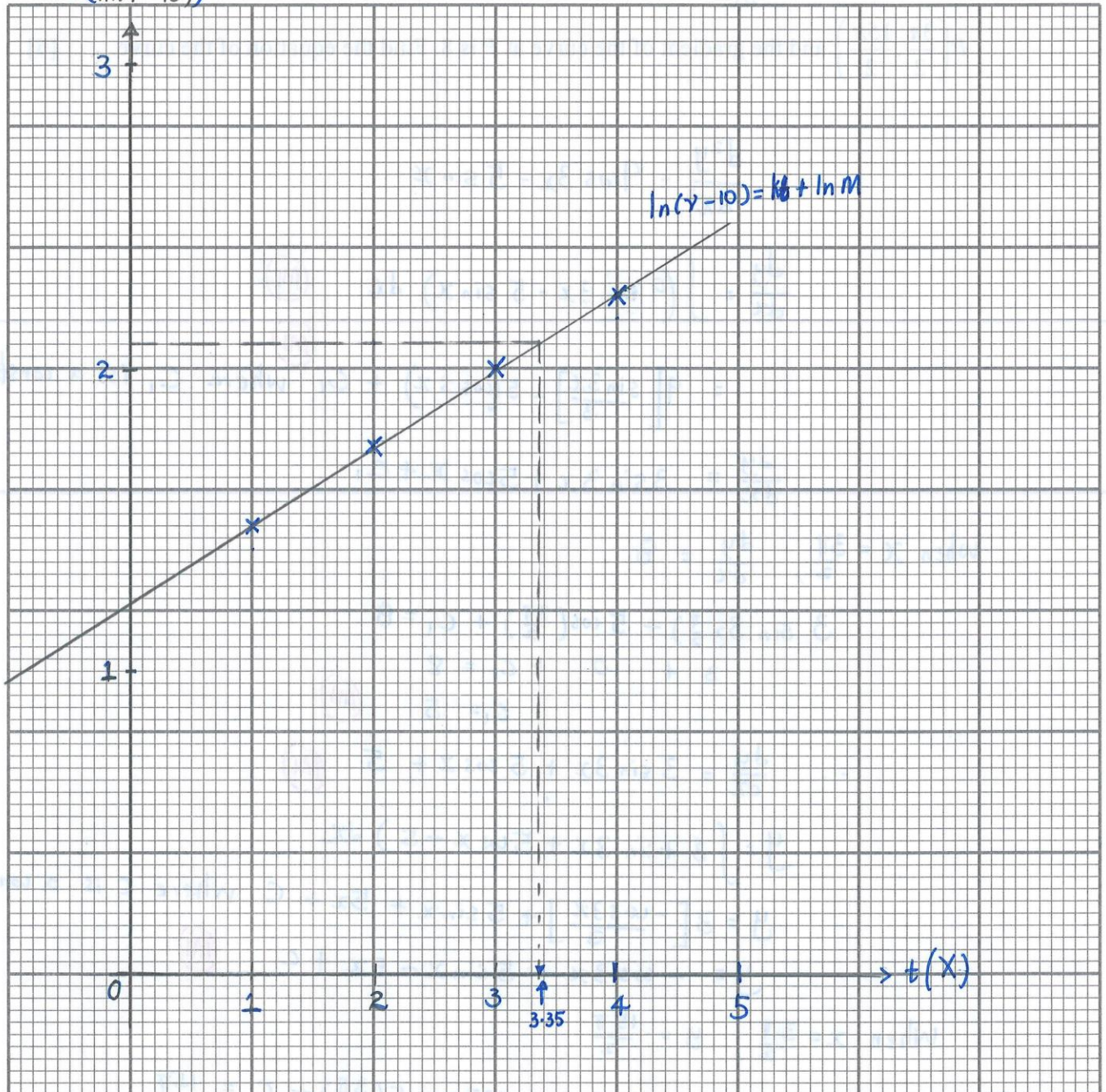
$$Y = mX + C$$

$$\text{where } Y = \ln(v-10), \quad m = k$$

$$X = t$$

$$C = \ln M$$

4

 $Y(\ln(Y-10))$ 

$$(iii) \quad C = 2$$

Eq of curve is

$$y = -\cos 3x + 2 \sin x + 2x + 2$$

- 5 A curve is such that $\frac{d^2y}{dx^2} = 9\cos 3x - 5\sin x$. The curve passes through the point $P\left(\frac{3\pi}{2}, \frac{15\pi}{2}\right)$ and the gradient of the curve at P is 8. Find the equation of the curve. [6]

$$\frac{d^2y}{dx^2} = 9\cos 3x - 5\sin x$$

$$\frac{dy}{dx} = \int (9\cos 3x - 5\sin x) dx$$

$$= 9\left[\frac{\sin 3x}{3}\right] - 5(-\cos x) + C_1 \text{ where } C_1 \text{ is a constant}$$

$$\frac{dy}{dx} = 3\sin 3x + 5\cos x + C_1$$

When $x = \frac{3\pi}{2}$, $\frac{dy}{dx} = 8$

$$3\sin 3\left(\frac{3\pi}{2}\right) + 5\cos\left(\frac{3\pi}{2}\right) + C_1 = 8$$

$$3 + 0 + C_1 = 8$$

$$C_1 = 5$$

$$\therefore \frac{dy}{dx} = 3\sin 3x + 5\cos x + 5$$

$$y = \int (3\sin 3x + 5\cos x + 5) dx$$

$$y = 3\left[-\frac{\cos 3x}{3}\right] + 5\sin x + 5x + C \text{ where } C \text{ is a constant}$$

$$y = -\cos 3x + 5\sin x + 5x + C$$

When $x = \frac{3\pi}{2}$, $y = \frac{15\pi}{2}$,

$$-\cos 3\left(\frac{3\pi}{2}\right) + 5\sin\left(\frac{3\pi}{2}\right) + 5\left(\frac{3\pi}{2}\right) + C = \frac{15\pi}{2}$$

$$0 + 5(-1) + \frac{15\pi}{2} + C = \frac{15\pi}{2}$$

$$C = 5$$

$\therefore$ Eqn of curve is

$$y = -\cos 3x + 5\sin x + 5x + 5$$

6 Express $\frac{x^3+1}{(x^2+1)(x-2)}$ in partial fractions.

$$(x^2+1)(x-2) = x^3 - 2x^2 + x - 2$$

[6]

$$\begin{array}{r} x^3 - 2x^2 + x - 2 \overline{) \begin{array}{r} x^3 + 0x^2 + 0x + 1 \\ (x^3 \oplus 2x^2 \oplus x \oplus 2) \\ \hline 2x^2 - x + 3 \end{array}} \end{array}$$

$$\therefore \frac{x^3+1}{(x^2+1)(x-2)} = 1 + \frac{2x^2-x+3}{(x^2+1)(x-2)}$$

$$\text{Let } \frac{2x^2-x+3}{(x^2+1)(x-2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-2}$$

$$2x^2 - x + 3 = (Ax+B)(x-2) + C(x^2+1)$$

By comparing coeff of

$$x^2: A + C = 2 \quad \text{--- (1)}$$

$$\text{constant: } -2B + C = 3 \quad \text{--- (2)}$$

when $x = 2$,

$$C(2^2+1) = 2(2)^2 - 2 + 3$$

$$5C = 9$$

$$C = \frac{9}{5}$$

$$\textcircled{1}: A = 2 - \frac{9}{5} = \frac{1}{5}$$

$$\textcircled{2}: -2B = 3 - \frac{9}{5}$$

$$B = -\frac{3}{5}$$

$$\therefore \frac{x^3+1}{(x^2+1)(x-2)} = 1 + \frac{x-3}{5(x^2+1)} + \frac{9}{5(x-2)}$$

when $x = 2$,

$$5C = 9$$

$$C = \frac{9}{5}$$

when $x = 0$,

$$B(-2) + \frac{9}{5}(1) = 3$$

$$-2B = 3 - \frac{9}{5}$$

$$B = -\frac{3}{5}$$

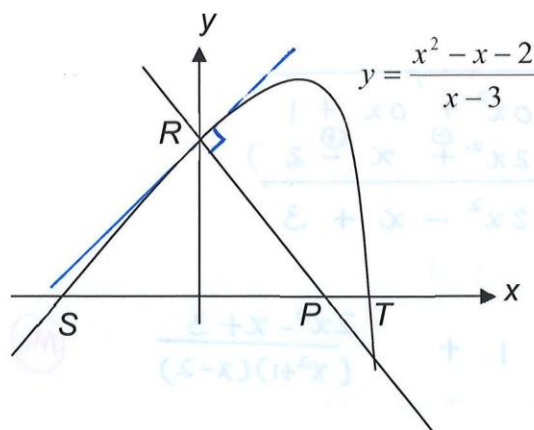
when $x = 1$,

$$(A - \frac{3}{5})(-1) + \frac{9}{5}(2) = 4$$

$$-A + \frac{3}{5} + \frac{18}{5} = 4$$

$$A = \frac{1}{5}$$

- 7 The diagram shows part of the graph of the curve $y = \frac{x^2 - x - 2}{x - 3}$.



- (a) Given that $\frac{dy}{dx} = 1 - \frac{k}{(x-3)^2}$. Find the value of k .

[3]

$$y = \frac{x^2 - x - 2}{x - 3}$$

$$\text{Let } u = x^2 - x - 2$$

$$v = x - 3$$

$$\frac{du}{dx} = 2x - 1$$

$$\frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = \frac{(2x-1)(x-3) - (x^2-x-2)}{(x-3)^2}$$

$$= \frac{2x^2 - 6x - x + 3 - x^2 + x + 2}{(x-3)^2}$$

$$= \frac{x^2 - 6x + 5}{(x-3)^2}$$

$$= \frac{(x-3)^2 - 9 + 5}{(x-3)^2}$$

$$= \frac{(x-3)^2 - 4}{(x-3)^2}$$

$$= 1 - \frac{4}{(x-3)^2}$$

$$= 1 - \frac{k}{(x-3)^2}$$

$$= 1 - \frac{k}{(x-3)^2}$$

$$\therefore k = 4$$

OR

$$\begin{array}{r} x+2 \\ x-3 \overline{) x^2-x-2} \\ \underline{-(x^2+3x)} \\ 2x-2 \\ \underline{-(2x+6)} \\ 4 \end{array}$$

$$y = x + 2 + \frac{4}{(x-3)}$$

$$y = x + 2 + 4(x-3)^{-1}$$

$$\frac{dy}{dx} = 1 - 4(x-3)^{-2}(1)$$

$$= 1 - \frac{4}{(x-3)^2}$$

$$= 1 - \frac{k}{(x-3)^2}$$

$$\therefore k = 4$$

- 7 (b) R , S and T are intersections of the graph with the axes. PR is the normal to the curve at R . Find the equation of PR . [3]

$R(0, ?)$

$$y = \frac{x^2 - x - 2}{x - 3}$$

when $x = 0$, $y = \frac{-2}{-3} = \frac{2}{3}$

$\therefore R(0, \frac{2}{3})$

$$\frac{dy}{dx} = 1 - \frac{4}{(x-3)^2}$$

When $x = 0$, $\frac{dy}{dx} = 1 - \frac{4}{(-3)^2}$

$$= 1 - \frac{4}{9}$$

$$= \frac{5}{9}$$

$\therefore \text{grad}_{\text{normal}} = -\frac{9}{5}$

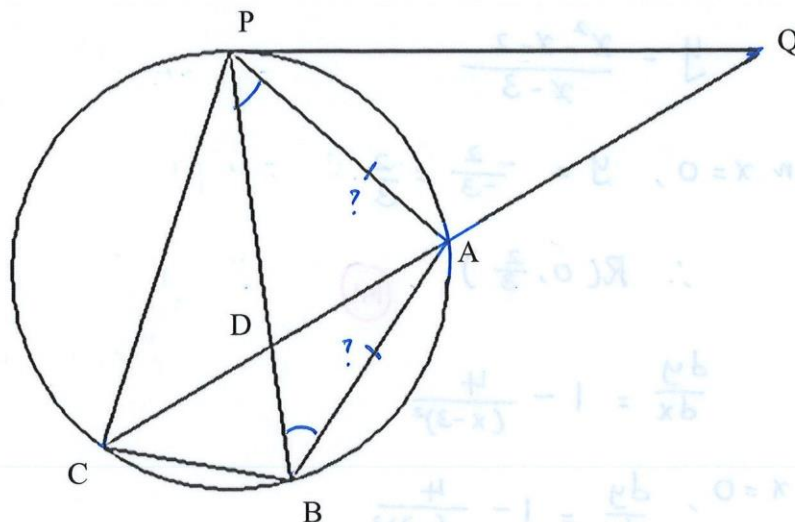
EQⁿ of PR :

$$y - y_1 = m(x - x_1)$$

$$y - \frac{2}{3} = -\frac{9}{5}(x - 0)$$

$$y = -\frac{9}{5}x + \frac{2}{3}$$

- 8 The diagram shows a point P on a circle and PQ is a tangent to the circle at P . Points A , B and C lie on the circle such that PA bisects angle QPB . QAC is a straight line. The lines QC and PB intersect at D .



- (a) Prove that $AP = AB$.

[3]

$$\angle APQ = \angle ABP \text{ (Alt. Seg Thm / Tangent-Chord Thm)}$$

Since PA bisects $\angle QPB$,

$$\angle APQ = \angle APB$$

$$\therefore \angle ABP = \angle APB$$

By base $\angle$ s of $\triangle$,
 $AP = AB$ (proven)

- (b) Prove that CD bisects angle PCB .

[3]

$$\angle ACB = \angle APB \text{ (}\angle\text{s in the same segment)}$$

$$\begin{aligned} \angle ACP &= \angle ABP \text{ (} \text{ " } \text{)} \\ &= \angle APB \text{ (from ca)} \end{aligned}$$

$$\therefore \angle ACB = \angle ACP$$

Hence CD bisects $\angle PCB$ (proven)

8 (c) Prove that triangles CDP and CBA are similar. [2]

$$\angle ACB = \angle PCD \text{ (from cb)})$$

$$\angle CPD = \angle CAB \text{ (}\angle\text{s in the same segment)}$$

Since there are 2 pairs of equal corresponding $\angle$ s,

$\therefore \triangle CDP$ and $\triangle CBA$ are similar (proven)

[AA Similarity Test]

- 9 (a) Show that the curve $y = 2e^{\frac{1}{2}x} + 5x$ has no stationary points. [3]

$$\frac{dy}{dx} = 2[e^{\frac{1}{2}x}][\frac{1}{2}] + 5$$

$$= e^{\frac{1}{2}x} + 5$$

Since $e^{\frac{1}{2}x} > 0$, $+5 > 0$

$$\Rightarrow \frac{dy}{dx} > 0 \neq 0$$

$\therefore$ the curve has no stationary points (shown).

- (b) Find the equation of the normal to the curve $y = \frac{2}{3\sqrt{2x-1}}$, $x \neq \frac{1}{2}$ at the point where the tangent is parallel to the line $3y + 2x = 1$. [5]

$$y = \frac{2}{3}(2x-1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{2}{3}[-\frac{1}{2}(2x-1)^{-\frac{3}{2}}(2)]$$

$$= -\frac{2}{3}(2x-1)^{-\frac{3}{2}}$$

Since tangent $\parallel 3y + 2x = 1$

$$\frac{dy}{dx} = -\frac{2}{3}$$

$$-\frac{2}{3}(2x-1)^{-\frac{3}{2}} = -\frac{2}{3}$$

$$\frac{1}{\sqrt{(2x-1)^3}} = 1$$

$$\sqrt{(2x-1)^3} = 1$$

$$(2x-1)^3 = 1$$

$$2x-1 = 1$$

$$2x = 2$$

$$x = 1$$

$$y = \frac{2}{3\sqrt{2-1}}$$

$$= \frac{2}{3}$$

point $(1, \frac{2}{3})$

$$3y = -2x + 1$$

$$y = -\frac{2}{3}x + \frac{1}{3}$$

$$\text{grad}_{\text{normal}} = \frac{3}{2}$$

$\therefore$ EQⁿ of normal:

$$y - \frac{2}{3} = \frac{3}{2}(x-1)$$

$$y = \frac{3}{2}x - \frac{3}{2} + \frac{2}{3}$$

$$y = \frac{3}{2}x - \frac{5}{6}$$

- 10 A boy cycling passes a point A at the bottom of the slope with a speed of 17 m/s. He then cycled up the slope and passes point B at the top of the slope, 5 seconds later, with a speed of 4.5 m/s. Between A to B , his velocity v m/s, is given by $v = 0.5t^2 - pt + q$, where t is the time in seconds since passing A , and p and q are constants.

(a) Show that $q = 17$ and find the value of p . [2]

$$v = 0.5t^2 - pt + q$$

$$\text{When } t = 0, v = 17 \text{ m/s}$$

$$0.5(0)^2 - p(0) + q = 17$$

$$\therefore q = 17 \text{ (shown)}$$

$$\text{When } t = 5, v = 4.5 \text{ m/s}$$

$$0.5(5)^2 - p(5) + 17 = 4.5$$

$$-5p = -25$$

$$p = 5$$

(b) Find the deceleration of the cyclist when his speed is 6.12 m/s. [4]

$$v = 0.5t^2 - 5t + 17$$

$$a = \frac{dv}{dt} = t - 5$$

$$\text{When } v = 6.12 \text{ m/s}$$

$$0.5t^2 - 5t + 17 = 6.12$$

$$0.5t^2 - 5t + 10.88 = 0$$

$$t^2 - 10t + \frac{544}{25} = 0$$

$$25t^2 - 250t + 544 = 0$$

$$(5t - 34)(5t - 16) = 0$$

$$t = \frac{34}{5} \quad \text{or} \quad t = \frac{16}{5}$$

$$= 6.8 \quad \quad \quad = 3.2$$

$$a = 6.8 - 5$$

$$= 1.8 \text{ (neg)}$$

$$a = 3.2 - 5$$

$$= -1.8$$

$$\therefore \text{deceleration} = 1.8 \text{ m/s}^2$$

10 (c) Find the distance AB in metres.

[3]

$$S = \int (0.5t^2 - 5t + 17) dt$$

$$= \frac{1}{2} \left[\frac{t^3}{3} \right] - 5 \left[\frac{t^2}{2} \right] + 17t + C \quad \text{where } C \text{ is a constant}$$

$$S = \frac{1}{6}t^3 - \frac{5}{2}t^2 + 17t + C$$

$$\text{When } t=0, S=0 \Rightarrow C=0$$

$$\therefore S = \frac{1}{6}t^3 - \frac{5}{2}t^2 + 17t$$

$$\text{When } t=5,$$

$$S = \frac{1}{6}(5)^3 - \frac{5}{2}(5)^2 + 17(5)$$

$$= 43\frac{1}{3}$$

$$\therefore AB = 43\frac{1}{3} \text{ m}$$

- 11 (a) The thirteenth term in the expansion of $\left(x^3 - \frac{4}{x}\right)^n$, is independent of x .

Find the value of n .

[3]

$$T_{r+1} = {}^nC_r (x)^{n-r} \left(-\frac{4}{x}\right)^r$$

$$T_{12+1} = {}^nC_{12} (x^3)^{n-12} \left(-\frac{4}{x}\right)^{12}$$

$$= {}^nC_{12} x^{3n-36} (-4)^r (x^{-1})^{12}$$

$$= {}^nC_{12} (-4)^r x^{3n-36-12}$$

$$= {}^nC_r (-4)^r x^{3n-48}$$

For term independent of x ,

$$3n - 48 = 0$$

$$3n = 48$$

$$n = 16$$

- (b) (i) Write down and simplify, in ascending powers of x , the first three terms in the expansion of $(1-3x)^5$.

[2]

$$= 1 + {}^5C_1 (-3x)^1 + {}^5C_2 (-3x)^2 + \dots$$

$$= 1 + 5(-3x) + 10(9x^2) + \dots$$

$$= 1 - 15x + 90x^2 + \dots$$

- (ii) Given that there is no term containing x in the expansion of $(x-a)^2(1-3x)^5$, find the value of a , where a is a non-zero constant.

[4]

$$(x-a)^2(1-15x+90x^2+\dots)$$

$$= (x^2 - 2ax + a^2)(1-15x+90x^2+\dots)$$

$$\text{coeff of } x = 0$$

$$-2a(1) + a^2(-15) = 0$$

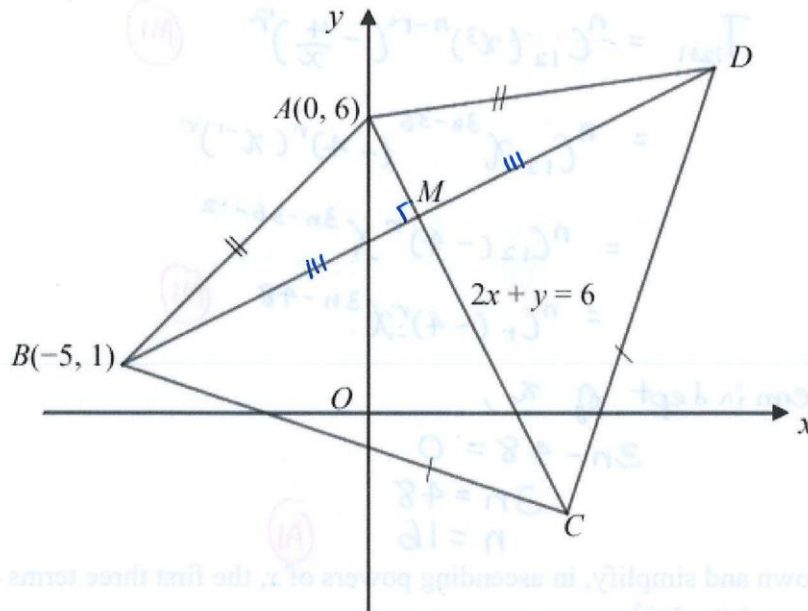
$$-2a - 15a^2 = 0$$

$$15a^2 + 2a = 0$$

$$a(15a+2) = 0$$

$$a = 0 \text{ (rej)} \text{ or } a = -\frac{2}{15}$$

- 12 The diagram shows a kite $ABCD$ with $AB = AD$ and $CB = CD$. The diagonals intersect at M . It is given that the coordinates of A and B are $(0,6)$ and $(-5,1)$ respectively and the equation of AC is $2x + y = 6$.



Find

- (a) the equation of BD .

[2]

$$\begin{aligned} AC : 2x + y &= 6 \\ y &= -2x + 6 \\ \text{grad}_{AC} &= -2 \end{aligned}$$

$ABCD$ is a kite: Since $AC \perp BD$,

$$\text{grad}_{BD} = \frac{1}{2}$$

$$\text{Eqn of } BD : (-5, 1)$$

$$y - 1 = \frac{1}{2}(x + 5)$$

$$y = \frac{1}{2}x + \frac{5}{2} + 1$$

$$y = \frac{1}{2}x + \frac{7}{2}$$

12 (b) the coordinates of M and of D .

[4]

$$AC: 2x + y = 6 \Rightarrow y = -2x + 6 \quad \text{--- ①}$$

$$BD: y = \frac{1}{2}x + \frac{7}{2} \quad \text{--- ②}$$

$$\text{①} = \text{②}, \quad -2x + 6 = \frac{1}{2}x + \frac{7}{2}$$

$$-\frac{5}{2}x = -\frac{5}{2}$$

$$x = 1$$

$$y = -2(1) + 6$$

$$= 4$$

$$\therefore M(1, 4)$$

(2) $ABCD$ is a kite :

Midpoint of $BD : M(1, 4)$

$B(-5, 1)$, $D(x, y)$

$$M\left(\frac{-5+x}{2}, \frac{1+y}{2}\right) = (1, 4)$$

$$\frac{-5+x}{2} = 1, \quad \frac{1+y}{2} = 4$$

$$-5+x = 2$$

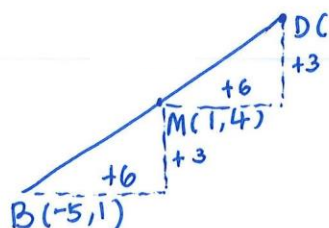
$$x = 7$$

$$1+y = 8$$

$$y = 7$$

$$\therefore D(7, 7)$$

OR



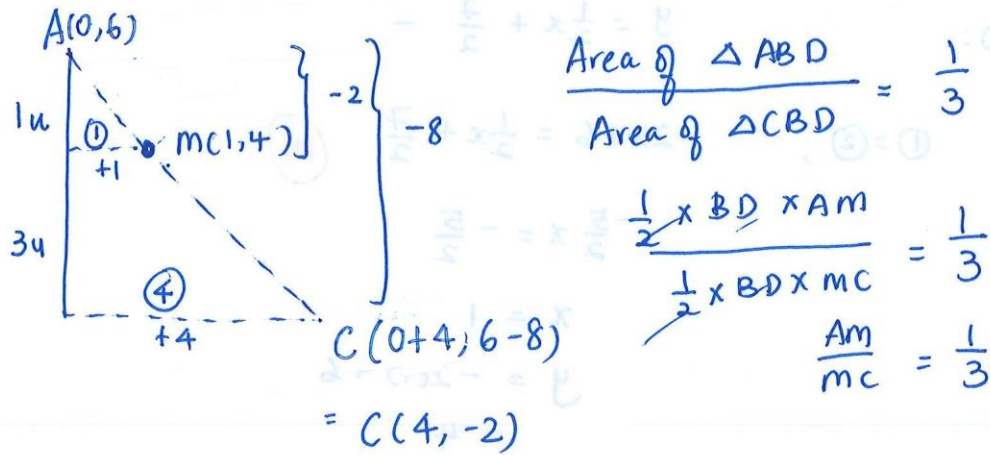
$$D(1+6, 4+3)$$

$$= D(7, 7)$$

- 12 Given further that the area of the triangle ABD is $\frac{1}{3}$ of the area of the triangle CBD ,

(c) find the coordinates of C ,

[2]



(d) find the area of the kite $ABCD$.

[2]

$$= \frac{1}{2} \begin{vmatrix} B & C & D & A & B \\ -5 & 4 & 7 & 0 & -5 \\ 1 & -2 & 7 & 6 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [(10 + 28 + 42 + 0) - (4 - 14 + 0 - 30)]$$

$$= \frac{1}{2} (120)$$

$$= 60 \text{ units}^2$$

- 13 (a) State the values between which the principal value of $\sin^{-1} x$ lies. [1]

$$-\frac{\pi}{2} \leq \text{principal value of } \sin^{-1} x \leq \frac{\pi}{2}$$

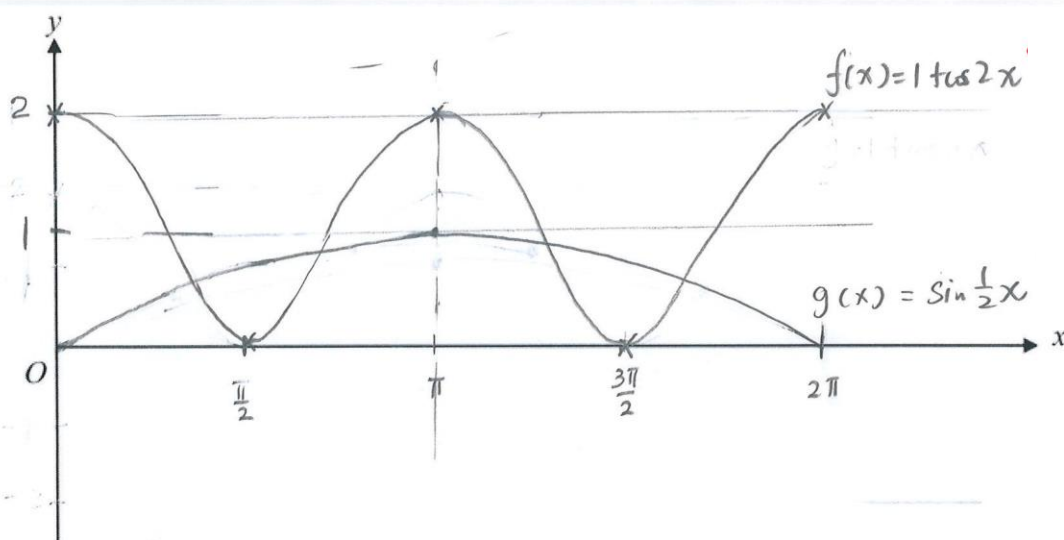
$-90^\circ \qquad \qquad \qquad 90^\circ$

- (a) Find the amplitude and period of

(i) $g(x) = \sin \frac{1}{2}x$ amplitude = 1, period = 4π [1]

(ii) $f(x) = 1 + \cos 2x$ amplitude = 1, period = π [2]

- (iii) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \leq x \leq 2\pi$. [4]



- (iv) Hence, state the number of solutions of the equation $\sin \frac{1}{2}x - 1 = \cos 2x$ for $0 \leq x \leq 2\pi$. [1]

$$\sin \frac{1}{2}x = 1 + \cos 2x$$

$$\text{No. of solns} = 4$$

- (v) The number of solutions of the equation $m \sin \frac{x}{2} = 1 + \cos 2x$, where m is a constant, for $0 \leq x \leq 2\pi$ is exactly 3. Find the value of m . [1]

$$m = 2$$

End of Paper

NAME	CLASS	INDEX NO.
MARKING SCHEME		



ST. PATRICK'S SCHOOL PRELIMINARY EXAMINATION 2023

**SUBJECT : Additional Mathematics
(4049/02)**

DATE : 22 AUGUST 2023

LEVEL : Secondary 4G3

DURATION : 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

<i>For Examiner's Use</i>	
<i>Score</i>	/90

This paper contains 23 printed pages including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1** A student learns a new topic from a teacher. At t days after learning the topic, the percentage, P %, of the topic that the student remembers can be modelled by the equation $P = 80e^{-kt} + 20$, where k is a constant.

- (a)** Explain why the student remembers 100% of what he learns from the teacher on the day that it is taught. [1]

When $t = 0$,

$$\begin{aligned} P &= 80e^{-k(0)} + 20 \\ &= 80 + 20 \\ &= 100 \end{aligned}$$

- (b)** A week later, the student remembers 40% of what he has learnt from the teacher. Find the value of k . [2]

$$\begin{aligned} 40 &= 80e^{-k(7)} + 20 \\ 20 &= 80e^{-k(7)} \\ 0.25 &= e^{-k(7)} \\ -7k &= \ln 0.25 \\ k &\approx 0.19804 \\ &= 0.198 \text{ (3sf)} \end{aligned}$$

- 1 (c) How many days will have elapsed when a student remembers at **most** half of what he has learnt from the teacher?

Give your answer to the nearest whole number.

[3]

$$80e^{-0.19804(t)} + 20 = 50$$

$$e^{-0.19804t} = 0.375$$

$$-0.19804t = \ln(0.375)$$

$$t = 4.95268$$

$$t = 5 \text{ days}$$

- (d) A parent claims that in the long run, the student remembers nothing about the Topic that he has learnt from the teacher. Do you agree with the parent?

Explain.

[2]

$$\text{When } t \rightarrow \infty$$

$$e^{-k(t)} \rightarrow 0$$

$$80e^{-k(t)} \rightarrow 0$$

$$80e^{-k(t)} + 20 \rightarrow 20$$

I do not agree as the student will at least remember 20% of what he has learnt.

- 2 Air is pumped into a spherical air balloon which changes the surface area at the rate of $400 \text{ cm}^2/\text{s}$.

Find the rate of increase of the radius when the volume is $288\pi \text{ cm}^3$, leaving your answer in terms of π . [4]

[Volume of sphere = $\frac{4}{3}\pi r^3$; Surface area of sphere = $4\pi r^2$]

$$V = \frac{4}{3}\pi r^3$$

$$288\pi = \frac{4}{3}\pi r^3$$

$$r^3 = 216$$

$$r = 6$$

$$A = 4\pi r^2$$

$$\frac{dA}{dr} = 8\pi r$$

$$\begin{aligned} \frac{dr}{dt} &= \frac{dr}{dA} \times \frac{dA}{dt} \\ &= \frac{1}{8\pi r} \times 400 \\ &= \frac{50}{\pi r} \end{aligned}$$

When $r = 6$,

$$\begin{aligned} \frac{dr}{dt} &= \frac{50}{\pi(6)} \\ &= \frac{25}{3\pi} \text{ cm/s} \end{aligned}$$

3 It is given that $f(x) = 4x^3 + 6x^2 - 9x + 2k$, where k is a constant.

(a) The expression $f(x)$ has a remainder of -125 when divided by $x + 2$.

Find the value of k .

[2]

By remainder theorem,

$$f(-2) = -125$$

$$4(-8) + 6(4) - 9(-2) + 2k = -125$$

$$10 + 2k = -125$$

$$2k = -135$$

$$k = -67.5$$

(b) Show that $(x - 3)$ is a factor of $f(x)$.

[2]

$$\begin{aligned} f(3) &= 4(27) + 6(9) - 9(3) + 2(-67.5) \\ &= 0 \end{aligned}$$

Thus, since $f(3) = 0$, by Factor theorem, $(x - 3)$ is a factor of $f(x)$.

- 3 (c) Explain why $f(x) = 0$ has only one real root. Show all your workings clearly. [3]

Thus $f(x) = 4x^3 + 6x^2 - 9x - 135$

By long division,

$$\begin{array}{r}
 4x^2 + 18x + 45 \\
 x - 3 \overline{) 4x^3 + 6x^2 - 9x - 135} \\
 \underline{-(4x^3 - 12x^2)} \\
 18x^2 - 9x \\
 \underline{-(18x^2 - 54x)} \\
 45x - 135 \\
 \underline{-(45x - 135)} \\
 0
 \end{array}$$

$$f(x) = (x - 3)(4x^2 + 18x + 45)$$

$$\begin{aligned}
 \text{Discriminant} &= b^2 - 4ac \\
 &= (18)^2 - 4(4)(45) \\
 &= -396
 \end{aligned}$$

Since discriminant < 0 , thus there are no real roots.

Therefore, $f(x) = 0$ has only one real root, which is $x = 3$.

- 4 (a) Prove the identity $\frac{2\sin x + \sin 2x}{1 - \cos 2x} = \frac{\sin x}{1 - \cos x}$. [4]

$$\begin{aligned}
 LHS &= \frac{2\sin x + \sin 2x}{1 - \cos 2x} \\
 &= \frac{2\sin x + 2\sin x \cos x}{1 - (1 - 2\sin^2 x)} \\
 &= \frac{2\sin x(1 + \cos x)}{2\sin^2 x} \\
 &= \frac{1 + \cos x}{\sin x} \times \frac{1 - \cos x}{1 - \cos x} \\
 &= \frac{1 - \cos^2 x}{\sin x(1 - \cos x)} \\
 &= \frac{\sin^2 x}{\sin x(1 - \cos x)} \\
 &= \frac{\sin x}{1 - \cos x} = RHS
 \end{aligned}$$

OR

$$\begin{aligned}
 LHS &= \frac{2\sin x + \sin 2x}{1 - \cos 2x} \\
 &= \frac{2\sin x + 2\sin x \cos x}{1 - (1 - 2\sin^2 x)} \\
 &= \frac{2\sin x(1 + \cos x)}{2\sin^2 x} \\
 &= \frac{2\sin x(1 + \cos x)}{2(1 - \cos^2 x)} \\
 &= \frac{2\sin x(1 + \cos x)}{2(1 - \cos x)(1 + \cos x)} \\
 &= \frac{\sin x}{1 - \cos x} = RHS
 \end{aligned}$$

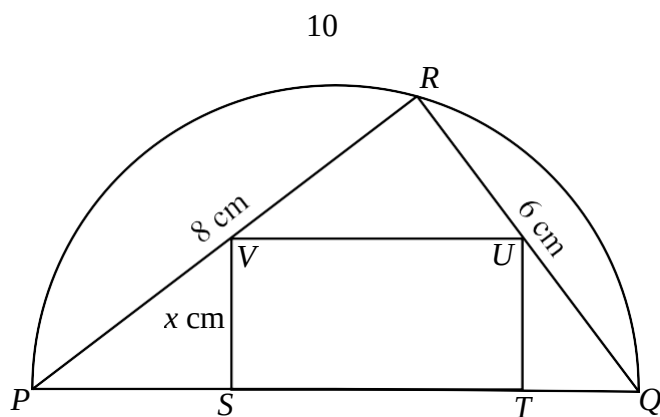
- 4 (b) **Hence**, solve the equation $\frac{2\sin 2x + \sin 4x}{1 - \cos 4x} = 3\cot 2x$ for $0 < x < \pi$.

Leave your answers in exact form.

[7]

$$\begin{aligned}\frac{2\sin 2x + \sin 4x}{1 - \cos 4x} &= 3\cot 2x \\ \frac{\sin 2x}{1 - \cos 2x} &= 3\cot 2x \\ \frac{\sin 2x}{1 - \cos 2x} &= \frac{3\cos 2x}{\sin 2x} \\ \sin^2 2x &= 3\cos 2x - 3\cos^2 2x \\ 1 - \cos^2 2x &= 3\cos 2x - 3\cos^2 2x \\ 2\cos^2 2x - 3\cos 2x + 1 &= 0 \\ (2\cos 2x - 1)(\cos 2x - 1) &= 0 \\ \cos 2x = 0.5 \quad \text{or} \quad \cos 2x &= 1 \\ \alpha = \cos^{-1}(0.5) \quad \text{or} \quad \alpha &= \cos^{-1}(1) \\ \alpha = \frac{\pi}{3} \quad \text{or} \quad \alpha &= 0 \\ 2x = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{or} \quad 2x &= 0, 2\pi \\ x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x &= 0(rej), \pi(rej)\end{aligned}$$

5



The diagram shows a semicircle with diameter PQ , with the point R on the circumference of the semicircle. A rectangle $STUV$ is drawn within the triangle PQR where ST lies on PQ . U and V are points on QR and PR respectively. It is given that $PR = 8$ cm and $QR = 6$ cm.

- (a) Find the perpendicular distance from R to PQ . [3]

Since angle $PRQ = 90^\circ$ (angle in semicircle), thus $PQ = \sqrt{8^2 + 6^2} = 10$ cm

Area of triangle $PRQ = 0.5 \times 6 \times 8 = 24$ cm².

$0.5 \times 10 \times \text{height} = 24$

Height = 4.8 cm

- (b) It is given that SV is x cm.
Show that the area of rectangle $STUV$, A cm², is given by $A = 10x - \frac{25}{12}x^2$. [2]

Triangle RPQ is similar to Triangle RVU .

$$\frac{VU}{10} = \frac{4.8 - x}{4.8}$$

$$VU = 10 - \frac{25}{12}x$$

$$\begin{aligned} A &= x \left(10 - \frac{25}{12}x \right) \\ &= 10x - \frac{25}{12}x^2 \text{ (shown)} \end{aligned}$$

- 5 (c) Calculate the value of x for which A has a stationary value. [3]

$$\frac{dA}{dx} = 10 - \frac{50}{12}x$$

For stat value, $\frac{dA}{dx} = 0$.

$$10 - \frac{50}{12}x = 0$$

$$x = 2.4$$

- (d) Determine whether this value of x gives a maximum or minimum value for the area of rectangle. [2]

$$\frac{d^2A}{dx^2} = -\frac{50}{12}$$

Since $\frac{d^2A}{dx^2} < 0$,

thus, it will give you a maximum value.

6 A calculator must not be used in this question.

- (a) Show that $\cos 105^\circ = \frac{\sqrt{2} - \sqrt{6}}{4}$. [4]

$$\begin{aligned}\cos 105^\circ &= \cos(60^\circ + 45^\circ) \\ &= \cos 60 \cos 45 - \sin 60 \sin 45 \\ &= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2} - \sqrt{6}}{4}\end{aligned}$$

- 6 (b) Use the result from **part (a)** to find an expression for $\sec^2 105^\circ$, in the form $a + b\sqrt{3}$, where a and b are integers. [3]

$$\begin{aligned}\sec^2(105^\circ) &= \frac{1}{(\cos 105^\circ)^2} \\&= \frac{1}{\left(\frac{\sqrt{2} - \sqrt{6}}{4}\right)^2} \\&= \frac{1}{\left(\frac{8 - 4\sqrt{3}}{16}\right)} \\&= \frac{4}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \\&= 8 + 4\sqrt{3}\end{aligned}$$

7 (a) Given that $3^{x+1} \times 2^{x+1} = 2^{x+2}$, evaluate 6^x .

[4]

$$3^{x+1} \times 2^{2x+1} = 2^{x+2}$$

$$3^{x+1} \times \frac{2^{2x+1}}{2^{x+2}} = 1$$

$$3^{x+1} \times 2^{2x+1-(x+2)} = 1$$

$$3^{x+1} \times 2^{x-1} = 1$$

$$(3^x)(3^1) \times (2^x)(2^{-1}) = 1$$

$$(6^x) \times \frac{3}{2} = 1$$

$$6^x = \frac{2}{3}$$

7 (b) Express y in terms of x if $\log_2 y = \log_8 x - \log_2 4$.

[4]

$$\log_2 y = \log_8 x - \log_2 4$$

$$\log_2 y = \frac{\log_2 x}{\log_2 8} - \log_2 4$$

$$\log_2 y = \frac{\log_2 x}{3} - \log_2 4$$

$$3 \log_2 y = \log_2 x - 3 \log_2 4$$

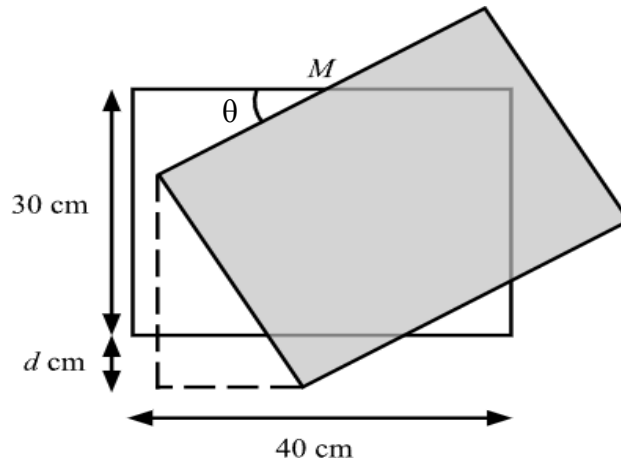
$$\log_2 y^3 = \log_2 x - \log_2 4^3$$

$$\log_2 y^3 = \log_2 \left(\frac{x}{64} \right)$$

$$y^3 = \left(\frac{x}{64} \right)$$

$$y = \frac{1}{4} \sqrt[3]{x}$$

8



The diagram shows a rectangular picture frame 40 cm by 30 cm hung on a wall.

The picture frame is rotated through an angle θ about the midpoint, M of the top edge.

- (a) Show that the vertical displacement, d cm, of the picture frame below its original bottom edge is given by $d = 20\sin\theta + 30\cos\theta - 30$. [2]

$$\begin{aligned}\sin\theta &= \frac{l_1}{20} & \cos\theta &= \frac{l_2}{30} \\ l_1 &= 20\sin\theta & l_2 &= 30\sin\theta \\ d + 30 &= 20\sin\theta + 30\sin\theta \\ d &= 20\sin\theta + 30\sin\theta - 30 \text{ (shown)}\end{aligned}$$

- (b) Express d in the form of $R\sin(\theta + \alpha) - 30$ where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$. [4]

$$\begin{aligned}R &= \sqrt{20^2 + 30^2} \\ &= \sqrt{1300} \\ \alpha &= \tan^{-1}\left(\frac{30}{20}\right) \\ &= 56.309^\circ \\ \therefore d &= 10\sqrt{13}\sin(\theta + 56.3^\circ) - 30\end{aligned}$$

- 8 (c) Find the value of d and the corresponding value of θ that will give the greatest vertical displacement of the picture frame below its original bottom edge. [3]

$$\begin{aligned}d &= 10\sqrt{13} - 30 \\&\approx 6.05555 \\&= 6.06\text{cm}(3sf)\end{aligned}$$

It occurs when $\sin(\theta + 56.309^\circ) = 1$

$$(\theta + 56.309^\circ) = 90^\circ$$

$$\begin{aligned}\theta &\approx 33.691^\circ \\&= 33.7^\circ(1dp)\end{aligned}$$

9 A curve has the equation $y = (x+5)\sqrt{7-x}$, where $x > 7$.

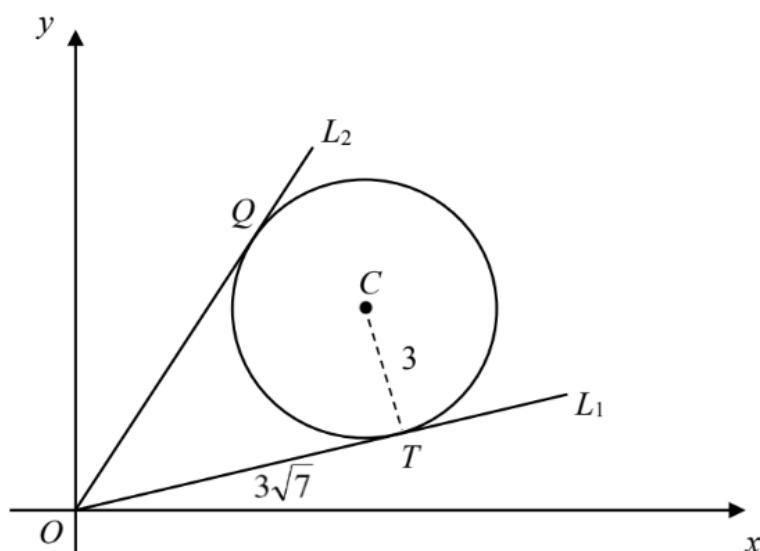
- (a) Show that $\frac{dy}{dx}$ can be expressed in the form $\frac{a+bx}{2\sqrt{7-x}}$. [4]

$$\begin{aligned}\frac{d}{dx}[(x+5)\sqrt{7-x}] &= (5+x)\left(\frac{1}{2}\right)(7-x)^{-\frac{1}{2}}(-1) + \sqrt{7-x}(1) \\ &= \frac{-5-x}{2\sqrt{7-x}} + \sqrt{7-x} \\ &= \frac{-5-x+2(7-x)}{2\sqrt{7-x}} \\ &= \frac{-5-x+14-2x}{2\sqrt{7-x}} \\ &= \frac{9-3x}{2\sqrt{7-x}} \quad (\text{shown})\end{aligned}$$

- 9 (b) Hence, evaluate $\int_3^6 \frac{3-x}{5\sqrt{7-x}} dx$. [4]

$$\begin{aligned}
 \int_3^6 \frac{3-x}{5\sqrt{7-x}} dx &= \frac{1}{5} \left(\frac{2}{3} \right) \int_3^6 \frac{3(3-x)}{2\sqrt{7-x}} dx \\
 &= \frac{1}{5} \left(\frac{2}{3} \right) \left[(x+5)\sqrt{7-x} \right]_3^6 \\
 &= \frac{1}{5} \left(\frac{2}{3} \right) \left[(6+5)\sqrt{7-6} \right] - \left[(3+5)\sqrt{7-3} \right] \\
 &= \frac{1}{5} \left(\frac{2}{3} \right) [11-16] \\
 &= -\frac{2}{3}
 \end{aligned}$$

10



The diagram shows a circle with centre C and radius 3 units. The tangent to the circle, L_1 , at the point T passes through the origin and $OT = 3\sqrt{7}$ units.

The x – coordinate of C is 6.

(a) Find the equation of the circle.

[4]

Let C be $(6, y)$.

$$(3)^2 + (3\sqrt{7})^2 = OC^2$$

$$OC^2 = 9 + 63$$

$$OC = \sqrt{72}$$

$$\sqrt{(6-0)^2 + (0-y)^2} = \sqrt{72}$$

$$36 + y^2 = 72$$

$$y^2 = 36$$

$$y = 6 \ (y > 0)$$

$$\text{Equation of circle} = (x-6)^2 + (y-6)^2 = 9$$

- 10 (b)** State the coordinates of the point on the circle which is farthest to the y - axis. [1]

The coordinates is (9,6)

A second tangent L_2 passes through the origin and touches point Q . Point R lies on the circle such that it is closer to the origin and is equidistant from the two tangents.

- (c)** Find the exact value of the y – coordinate of R . [4]

Equation of the line OC is $y = x$.

It is the intersection of the line OC and the equation of the circle.

Sub $y = x$ into equation,

$$(y-6)^2 + (y-6)^2 = 9$$

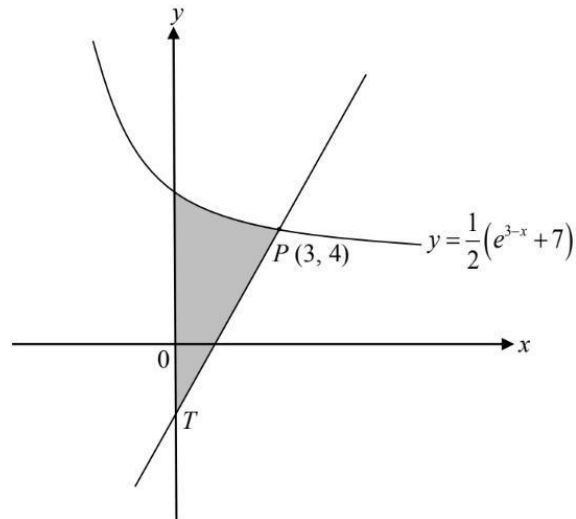
$$2(y-6)^2 = 9$$

$$(y-6)^2 = \frac{9}{2}$$

$$y-6 = \pm \frac{3}{\sqrt{2}}$$

$$y = 6 - \frac{3}{\sqrt{2}}$$

11



The diagram shows part of the curve $y = \frac{1}{2}(e^{3-x} + 7)$ and the normal to the curve at the point $P(3, 4)$ which meets the y -axis at T .

(a) Find the coordinates of the point T .

[4]

$$\begin{aligned}
 y &= \frac{1}{2}(e^{3-x} + 7) \\
 \frac{dy}{dx} &= \frac{1}{2}(-e^{3-x}) \\
 &= -\frac{1}{2}e^{3-x} \\
 \frac{dy}{dx} \Big|_{x=3} &= -\frac{1}{2}e^{3-3} \\
 &= -\frac{1}{2} \\
 \text{gradient of tangent} &= -\frac{1}{2} \\
 \text{gradient of normal} &= 2 \\
 y &= 2x + c \\
 (3, 4) \quad 4 &= 2(3) + c \\
 c &= -2 \\
 \text{Eqn of normal: } y &= 2x - 2 \\
 \text{Let } x &= 0, \\
 y &= -2 \\
 \therefore T(0, -2)
 \end{aligned}$$

- 11 (b) Find the area of the shaded region bounded by the curve, the normal and the y -axis, leaving your answer in the form $(A + Be^3)$ square units, where A and B are constants to be found. [5]

$$\text{Let } y=0, \quad 2x-2=0$$

$$2x=2$$

$$x=1$$

$$\therefore A(1, 0)$$

Area of region bounded

$$= \int_0^3 \frac{1}{2}(e^{3-x} + 7) dx - \frac{1}{2}(3-1)(4) + \frac{1}{2}(1)(2)$$

$$= \frac{1}{2} \left[\frac{e^{3-x}}{-1} + 7x \right]_0^3 - 4 + 1$$

$$= \frac{1}{2} [-e^{3-3} + 7(3) + e^{3-0} - 0] - 3$$

$$= \frac{1}{2} (-e^0 + 21 + e^3) - 3$$

$$= \frac{1}{2} (20 + e^3) - 3$$

$$= 10 + \frac{1}{2}e^3 - 3$$

$$= (7 + \frac{1}{2}e^3) \text{ units}^2, \text{ where } A=7, B=\frac{1}{2}$$

NAME

CLASS

INDEX No.

Mark Scheme		
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ST. PATRICK'S SCHOOL PRELIMINARY EXAMINATION 2024

**SUBJECT : Additional Mathematics
(4049/01)**

DATE : 13 AUGUST 2024

LEVEL : Secondary 4G3

DURATION : 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

<i>For Examiner's Use</i>	
<i>Score</i>	/90

This paper contains 19 printed pages including this cover page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 Find the values of p for which the line $y = x + 1$ is a tangent to the curve $y = x^2 + (2p + 3)x + p + 4$.

[4] E

$$y = x + 1 \quad \text{--- ①}$$

$$y = x^2 + (2p + 3)x + p + 4$$

Subst ② into ①

$$x^2 + (2p + 3)x + p + 4 = x + 1$$

[M1]

$$x^2 + (2p + 2)x + p + 3 = 0$$

tangent $\Rightarrow$ one solution

$\Rightarrow$ Discriminant = 0

$$(2p + 2)^2 - 4(1)(p + 3) = 0$$

[M1]

$$4p^2 + 8p + 4 - 4p - 12 = 0$$

$$4p^2 + 4p - 8 = 0$$

[M1]

$$p^2 + p - 8 = 0$$

$$(p + 2)(p - 1) = 0$$

$$p = -2 \text{ or } p = 1$$

[M1]

- 2 (a) Express $\frac{(x+2)^2}{x^2(x-2)}$ as a sum of 3 partial fractions.

[5] E

$$\text{Let } \frac{(x+2)^2}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} \quad \text{[M1]}$$

$$(x+2)^2 = Ax(x-2) + B(x-2) + Cx^2 \quad \text{[M1]}$$

$$\text{When } x=0, \quad 4 = B(-2)$$

$$B = -2$$

$$\text{When } x=2, \quad 16 = 4C$$

$$C = 4$$

$$\text{When } x=3, \quad 25 = A(3)(1) + (-2)(1) + 4(9)$$

$$25 = 3A - 2 + 36$$

$$A = -3 \quad \text{[A2]}$$

$$\therefore \text{Partial Fractions: } -\frac{3}{x} - \frac{2}{x^2} + \frac{4}{x-2} \quad \text{[A1]}$$

- (b) Hence, integrate $\frac{(x+2)^2}{x^2(x-2)}$ with respect to x . State the conditions, if any, that applies to x .

[3] E

$$\int \frac{(x+2)^2}{x^2(x-2)} dx = \int -\frac{3}{x} - \frac{2}{x^2} + \frac{4}{x-2} dx$$

X O

$$= -3 \ln x - \frac{2x^{-1}}{(-1)} + 4 \ln(x-2) + C \quad \text{[M1]}$$

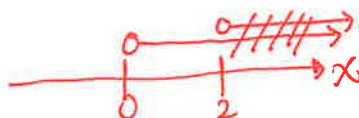
$$= -3 \ln x + \frac{2}{x} + 4 \ln(x-2) + C \quad \text{[A1]}$$

where C is a constant

Condition for x : $x > 2$ [A1]

$$x > 0 \quad \& \quad (x-2) > 0$$

$$x > 2$$



3 (a) Prove that $\frac{1}{1+\cos x} + \frac{1}{\cos x-1} = -2 \operatorname{cosec} x \cot x$.

[3] E

$$\text{LHS} = \frac{\cos x - 1 + 1 + \cos x}{(1 + \cos x)(\cos x - 1)} \quad \text{[M1]}$$

$$= \frac{2 \cos x}{\cos^2 x - 1}$$

$$= \frac{2 \cos x}{-\sin^2 x} \quad \text{[M1]}$$

$$= -2 \times \frac{\cos x}{\sin x} \times \frac{1}{\sin x}$$

$$= -2 \operatorname{cosec} x \cot x \quad \text{[A1]}$$

= RHS (shown)

(b) Given that $\cot \theta = -3$ and $\cos \theta > 0$, find the exact value of $\operatorname{cosec} 2\theta$.
(1st, 4th)

[3] E

$$\cot \theta = -3$$

$$\frac{1}{\tan \theta} = -3$$

$$\tan \theta = -\frac{1}{3} < 0 \quad (2^{\text{nd}}, 4^{\text{th}})$$

$\therefore \theta$ is in 4th Quadrant.

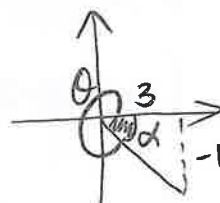
$$\operatorname{cosec} 2\theta = \frac{1}{\sin 2\theta}$$

$$= \frac{1}{2 \sin \theta \cos \theta} \quad \text{[M1]}$$

$$= \frac{1}{2 \left[-\frac{1}{\sqrt{10}} \left(\frac{3}{\sqrt{10}} \right) \right]}$$

$$= -\frac{10}{2(3)} \quad \text{[A1]}$$

$$= -\frac{5}{3}$$



$$\sin \theta = -\frac{1}{\sqrt{10}}$$

$$\cos \theta = \frac{3}{\sqrt{10}} \quad \text{[M1]}$$

4 (a) Show that $\frac{d}{dx} \left[x(3x-2)^{\frac{4}{3}} \right] = (7x-2)(3x-2)^{\frac{1}{3}}$.

[3] E

$$\frac{d}{dx} [x(3x-2)^{\frac{4}{3}}] = x \cdot \frac{4}{3}(3x-2)^{\frac{4}{3}-1} \cdot 3 + (3x-2)^{\frac{4}{3}} \cdot 1 \quad [M1]$$

$$= 4x(3x-2)^{\frac{1}{3}} + (3x-2)^{\frac{4}{3}}$$

$$= (3x-2)^{\frac{1}{3}} [4x + (3x-2)] \quad [M1]$$

$$= (3x-2)^{\frac{1}{3}} (7x-2)$$

$$= (7x-2)(3x-2)^{\frac{1}{3}} \quad [M1]$$

(b) Hence, integrate $\int x(3x-2)^{\frac{1}{3}} dx$.

[4] M

$$\int (7x-2)(3x-2)^{\frac{1}{3}} dx = x(3x-2)^{\frac{4}{3}} + C_1 \quad \text{where } C_1 \text{ is a constant} \quad [M1]$$

$$\int 7x(3x-2)^{\frac{1}{3}} dx - \int 2(3x-2)^{\frac{1}{3}} dx = x(3x-2)^{\frac{4}{3}} + C_1 \quad [M1]$$

$$\int 7x(3x-2)^{\frac{1}{3}} dx - \frac{2(3x-2)^{\frac{4}{3}}}{\frac{4}{3}(3)} = x(3x-2)^{\frac{4}{3}} + C_2 \quad [M1]$$

$$\int 7x(3x-2)^{\frac{1}{3}} dx = x(3x-2)^{\frac{4}{3}} + \frac{1}{2}(3x-2)^{\frac{4}{3}} + C_2$$

$$\int x(3x-2)^{\frac{1}{3}} dx = \frac{1}{7}x(3x-2)^{\frac{4}{3}} + \frac{1}{4}(3x-2)^{\frac{4}{3}} + C \quad [M1]$$

5 Solve the equation $3 \operatorname{cosec}^2 x + \frac{2}{\tan x} = 4$ for $-90^\circ \leq x \leq 90^\circ$.

[5] M

$$3(1 + \cot^2 x) + 2 \cot x = 4$$

[M1]

$$3 + 3 \cot^2 x + 2 \cot x = 4$$

$$3 \cot^2 x + 2 \cot x - 1 = 0$$

$$\text{let } m = \cot x$$

$$3m^2 + 2m - 1 = 0$$

$$(3m-1)(m+1) = 0$$

[M1]

$$m = \frac{1}{3} \quad \text{or} \quad m = -1$$

$$\cot x = \frac{1}{3}$$

$$\cot x = -1$$

$$\tan x = 3 \text{ (1st, 3rd)}$$

$$\tan x = -1 \text{ (2nd, 4th)} \quad [M1]$$

$$\text{ref } \angle = 71.565^\circ$$

$$\text{ref } \angle = 45^\circ$$

$\angle x$ is in 1st Q.

$\angle x$ is in 4th Q.

$$\therefore \angle x = 71.565$$

$$\angle x = -45^\circ$$

$$= 71.6^\circ \text{ (1dp)}$$

[A1]

[A1]

- 6 When the height of liquid in a container is h m, the volume of liquid is V m³ where $V = 0.06[(4h+1)^3 - 7]$ and $h \geq 0.25$.

(a) Find an expression for $\frac{dV}{dh}$.

[2] E

$$V = 0.06(4h+1)^3 - 0.42$$

$$\frac{dV}{dh} = \frac{3[0.06](4h+1)^2 \cdot 4}{1} = 0.72(4h+1)^2$$

[M1]

[A1]

$$0.72(16h^2 + 8h + 1)$$

The liquid leaks out of the container at a constant rate of 0.98 m³/s.

(b) Find the value of $\frac{dh}{dt}$ when $V = 1.2$.

[4] M

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$\frac{dV}{dt} = -0.98$$

$$-0.98 = 0.72(4h+1)^2 \times \frac{dh}{dt} \quad [M1]$$

when $V = 1.2$,

$$1.2 = 0.06[(4h+1)^3 - 7]$$

$$(4h+1)^3 = 27$$

$$4h+1 = 3$$

$$h = \frac{1}{2}$$

[A1]

$$\text{when } h = \frac{1}{2}, \quad -0.98 = 0.72[4(\frac{1}{2}) + 1]^2 \times \frac{dh}{dt} \quad [M1]$$

$$\frac{dh}{dt} = \frac{-0.98}{19.44} = -0.050411$$

$$\frac{dh}{dt} = -0.050411$$

$$= -0.0504 \text{ (3sf)}$$

$$= -\frac{49}{324} \text{ or } -0.151 \quad [A1]$$

(c) Briefly explain the significance of your answer for (b).

[1] M

$\frac{dh}{dt}$ is a negative value $\Rightarrow$ h is decreasing. [A1]

- 7 (a) Find the range of values of p for which the curve $y = 4x^2$ lies completely above the line $y = 4px - 1$. [4] M

$$y = 4x^2 \text{ --- (1)}$$

$$y = 4px - 1 \text{ --- (2)}$$

subst (1) into (2),

$$4x^2 = 4px - 1$$

$$4x^2 - 4px + 1 = 0$$

[M1]

No soln $\Rightarrow D < 0$

$$(-4p)^2 - 4(4)(1) < 0$$

[M1]

$$16p^2 - 16 < 0$$

$$p^2 - 1 < 0$$

$$(p-1)(p+1) < 0$$

$$-1 < p < 1$$

[M1]

[A1]



- (b) Hence, explain why $\frac{4x^2 - 4px + 1}{p+2}$ is always positive. [3] H

The curve above the line

$$\Rightarrow 4x^2 > 4px - 1$$

$$4x^2 - 4px + 1 > 0$$

[M1]

$$+1 +1 +1 \Rightarrow 0 < p+1 < 2$$

$$1 < p+2 < 3$$

$$\text{From (a), } -1 < p < 1 \Rightarrow p+1 > 0$$

[M1]

Since both $4x^2 - 4px + 1$ and $p+2 > 0$,

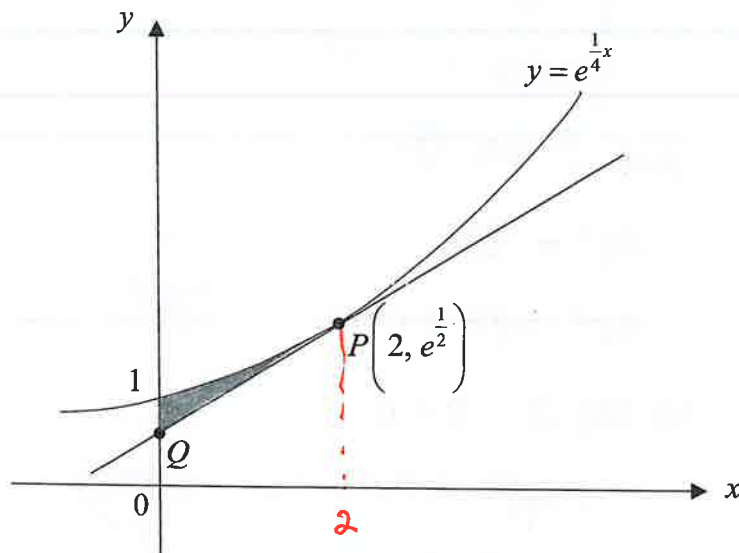
$$\frac{4x^2 - 4px + 1}{p+2} \text{ is always positive.}$$

[A1]

8. The diagram shows part of the curve $y = e^{\frac{1}{4}x}$ and the tangent to the curve at $P\left(2, e^{\frac{1}{2}}\right)$.

The tangent cuts the y -axis at Q . Find the area of the shaded region.

[5] M



To find equation of tangent:

$$\text{Grad} = \frac{dy}{dx} = e^{\frac{1}{4}x} \cdot \frac{1}{4} = \frac{1}{4}e^{\frac{1}{4}x}$$

when $x = 2$,

$$\text{Grad} = \frac{1}{4}e^{\frac{1}{2}} \quad 0.4218 \quad (M1)$$

$$\text{Eqn: } y = \frac{1}{4}e^{\frac{1}{2}}x + c$$

$$(2, e^{\frac{1}{2}})$$

$$e^{\frac{1}{2}} = \frac{1}{4}e^{\frac{1}{2}}(2) + c$$

$$c = \frac{1}{2}e^{\frac{1}{2}} \quad 0.82436 \dots$$

$$Q(0, \frac{1}{2}e^{\frac{1}{2}}) \quad (M1)$$

Area of shaded region:

$$= \int_0^2 e^{\frac{1}{4}x} dx - \frac{1}{2}(\frac{1}{2}e^{\frac{1}{2}} + e^{\frac{1}{2}}) \times 2$$

$2.59489 - 2.47308$

$$= \left[\frac{e^{\frac{1}{4}x}}{\frac{1}{4}} \right]_0^2 - \frac{3}{2}e^{\frac{1}{2}} \quad (M1), (M1)$$

$$= [4e^{\frac{1}{4}x}]_0^2 - \frac{3}{2}e^{\frac{1}{2}}$$

$$= 4e^{\frac{1}{2}} - 4 - \frac{3}{2}e^{\frac{1}{2}}$$

$$= \frac{5}{2}e^{\frac{1}{2}} - 4 \quad \text{units}^2 \quad 4.121803 \dots \quad (M1)$$

$$\text{or } 1.1218$$

$$= 1.12 \text{ units}^2 \text{ (3sf)}$$

$$= 0.121803 \dots$$

$$= 0.122$$

- 9 Given the function $y = \frac{ax^2 - 8}{x+3}$, $x > 0$.

(a) Find $\frac{dy}{dx}$.

[2] M

$$\frac{dy}{dx} = \frac{(x+3)(2ax) - (ax^2 - 8)(1)}{(x+3)^2} \quad [M1]$$

$$= \frac{2ax + 6ax - ax^2 + 8}{(x+3)^2}$$

$$= \frac{ax^2 + 6ax + 8}{(x+3)^2} \quad [M1]$$

- (b) Show that y is an increasing function for $0 < a < \frac{8}{9}$.

[3] M

Since $x > 0$

$$\Rightarrow (x+3)^2 > 0$$

For y to be an increasing function, $\frac{dy}{dx} > 0$ (M1)

$$\Rightarrow ax^2 + 6ax + 8 > 0, a > 0$$

$$\Rightarrow D < 0 \quad (M1)$$

$$(6a)^2 - 4(a)(8) < 0$$

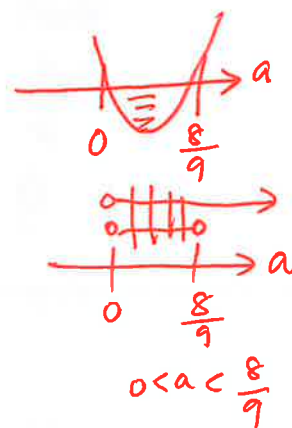
$$36a^2 - 32a < 0$$

$$9a^2 - 8a < 0$$

$$a(9a - 8) < 0 \quad (M1)$$

$$\underline{a > 0}$$

$$\therefore 0 < a < \frac{8}{9} \quad (\text{shown})$$



10 For a particular curve, $\frac{d^2y}{dx^2} = -2\cos x + 4\sin 2x$, where $0 \leq x \leq \frac{\pi}{2}$. 1st

The gradient of the curve at point $P\left(\frac{\pi}{3}, 1 - \frac{\sqrt{3}}{2}\right)$ is $(1 - \sqrt{3})$.

(a) Find the equation of the curve.

~~1st~~ [5] H

$$\text{grad } \frac{dy}{dx} = \int -2\cos x + 4\sin 2x \, dx$$

$$= -2(\sin x) + 4\left[-\frac{\cos 2x}{2}\right] + C_1$$

$$\frac{dy}{dx} = -2\sin x - 2\cos 2x + C_1$$

[M1]

$$\text{When } x = \frac{\pi}{3}, \frac{dy}{dx} = 1 - \sqrt{3}$$

$$1 - \sqrt{3} = -2\sin\left(\frac{\pi}{3}\right) - 2\cos\left(2\frac{\pi}{3}\right) + C_1$$

[M1]

$$1 - \sqrt{3} = -2\left(\frac{\sqrt{3}}{2}\right) - 2\left(-\frac{1}{2}\right) + C_1$$

$$C_1 = 1 - \sqrt{3} + \sqrt{3} - 1 = 0$$

[A1]

$$\frac{dy}{dx} = -2\sin x - 2\cos 2x$$

$$\begin{aligned} y &= \int -2\sin x - 2\cos 2x \, dx \\ &= -2(-\cos x) - 2\left(\frac{\sin 2x}{2}\right) + C_2 \\ &= 2\cos x - \sin 2x + C_2 \end{aligned}$$

[M1]

$$\text{When } x = \frac{\pi}{3}, y = 1 - \frac{\sqrt{3}}{2}$$

$$1 - \frac{\sqrt{3}}{2} = 2\cos\frac{\pi}{3} - \sin\frac{2\pi}{3} + C_2$$

$$1 - \frac{\sqrt{3}}{2} = 2\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2} + C_2$$

$$C_2 = 1 - \frac{\sqrt{3}}{2} - 1 + \frac{\sqrt{3}}{2} = 0$$

$$\therefore y = 2\cos x - \sin 2x$$

[A1]

- (b) Find the coordinates of the stationary point of the curve and determine the nature of this stationary point. [5] H

Stationary point $\Rightarrow \frac{dy}{dx} = 0$

$$-2\sin x - 2\cos 2x = 0$$

$$-2\sin x - 2(1 - 2\sin^2 x) = 0 \quad [M1]$$

$$-2\sin x - 2 + 4\sin^2 x = 0$$

$$4\sin^2 x - 2\sin x - 2 = 0$$

$$2\sin^2 x - \sin x - 1 = 0 \quad [M1]$$

$$(\sin x - 1)(2\sin x + 1) = 0$$

$$\sin x = 1 \quad \text{or} \quad \sin x = -\frac{1}{2} \quad \begin{matrix} [M1] \\ \text{No, 3rd} \end{matrix}$$

$$x = \frac{\pi}{2}$$

(N.A)

[A1]

x	$\frac{(1.5)\pi}{2}$	$\frac{\pi}{2}$	$\frac{(1.6)\pi}{2}$
$\frac{dy}{dx}$	-	0	-

$(\frac{\pi}{2}, 0)$ is a point of inflexion.

[A1]

When $x = \frac{\pi}{2}$

$$y = 2\cos\left(\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{2}\right)$$

$$= 2\cos\frac{\pi}{2} - \sin^2\pi$$

$$= 0 - 0$$

$$= 0$$

- 11 Points $A(8, 1)$ and $B(1, 2)$ lie on a circle whose centre is C . The line $4y = 3x - 20$ is tangent to the circle at A .

(a) Find the equation of the normal to the circle at A .

[2] E

$$4y = 3x - 20$$

$$y = \frac{3}{4}x - 5$$

grad of normal to the circle at $A = -\frac{4}{3}$

[M1]

Normal passes thru $A(8, 1)$

$$y = -\frac{4}{3}x + C$$

$$1 = -\frac{4}{3}(8) + C \Rightarrow C = \frac{35}{3}$$

$$\therefore y = -\frac{4}{3}x + \frac{35}{3}$$

[A1]

(b) Find the equation of the circle.

[6] M

To find perpendicular bisector of AB :

$$\text{midpt } AB = \left(\frac{9}{2}, \frac{3}{2}\right)$$

$$\text{Grad } AB = -\frac{1}{7}$$

Grad of perpendicular bisector = 7

[M1]

$$y = 7x + C$$

$$\frac{3}{2} = 7\left(\frac{9}{2}\right) + C \Rightarrow C = -30$$

Perpendicular Bisector of AB : $y = 7x - 30$

[A1]

$$y = 7x - 30 \quad \text{--- (1)}$$

$$y = -\frac{4}{3}x + \frac{35}{3} \quad \text{--- (2)}$$

Subst (1) into (2),

$$7x - 30 = -\frac{4}{3}x + \frac{35}{3}$$

[M1]

$$\Rightarrow x = 5$$

$$y = 5$$

Circle centre $(5, 5)$

[A1]

$$\begin{aligned}\text{radius} &= \sqrt{3^2 + 4^2} \\ &= 5 \text{ units}\end{aligned}$$

[A1]

$$\therefore \text{Eqn of circle : } (x-5)^2 + (y-5)^2 = 25 \quad \# \quad \text{[A1]}$$

(c) Explain why the coordinate axes are tangents to the circle.

[1] H

Since radius is 5 units and centre $(5,5)$,
the circle is touching the x and y -axis.

$\therefore$ coordinate axes are tangents to the curve.

[A1]

- 12 During an archery competition, an archer shoots an arrow towards a target board. Given that the arrow follows a parabolic path modelled by the function $y = -\frac{1}{16}x^2 + \frac{1}{8}x + \frac{7}{4}$, where x metres represent the horizontal distance the arrow travels and y metres represent the height of the arrow from the ground.

(a) State the initial height of the arrow.

[1] E

when $x=0$, $y = \frac{7}{4} = 1.75m$

Initial ht is $\frac{7}{4}m$.

[1/1]

(b) Express the function in the form $y = a(x-h)^2 + k$, where h and k are constants.

[3] M

$$y = -\frac{1}{16}x^2 + \frac{1}{8}x + \frac{7}{4}$$

$$= -\frac{1}{16}(x^2 - 2x - 28)$$

[M1]

$$= -\frac{1}{16}[x^2 - 2x + (-1)^2 - 1 - 28]$$

[M1]

$$= -\frac{1}{16}[(x-1)^2 - 29]$$

$$= -\frac{1}{16}(x-1)^2 + \frac{29}{16}$$

[M1]

- (c) Hence, state the maximum height attained by the arrow and the horizontal distance travelled when this occurs.

[2] E

$$\text{max height} = \frac{29}{16} \text{ m} = 1.8125$$

[A1]

it occurs when $x = 1 \text{ m}$.

[A1]

- (d) Explain if the arrow will hit the target board that is 1.4m in height and 3m away from the archer.

[2] H

When $x = 3 \text{ m}$,

$$y = -\frac{1}{16}(3)^2 + \frac{1}{8}(3) + \frac{7}{4}$$

[M1]

$$= 1.5625$$

When arrow travels a horizontal distance of 3m, the height of arrow is 1.5625 which is higher than the target board.

$\therefore$ It will not hit the target board.

[A1]

OR

Umayr

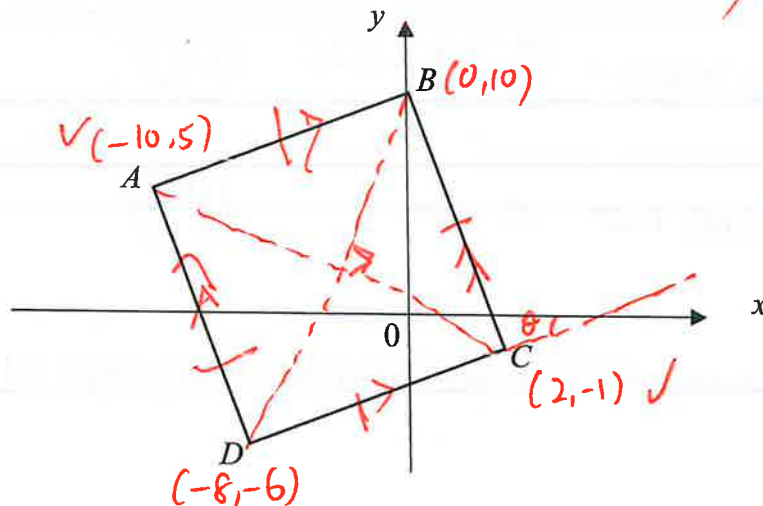
when $y = 1.4$

$\vdots$

$$x = 3.57 \text{ m or } x = -1.569046$$

No, - - -

- 13 The diagram shows a rhombus $ABCD$ in which the coordinates of the points A and C are $(-10, 5)$ and $(2, -1)$ respectively.



Given that the point B lies on the y -axis, find

- (a) the coordinates of B and D ,

[5] M

$$\text{Midpt } AC = (-4, 2)$$

$$\text{Grad } AC = \frac{5+1}{-10-2} = -\frac{1}{2}$$

$$\text{Grad } BD = 2$$

[M1]

Eqⁿ: $BD : y = 2x + c$

$$\text{Midpt } AC = \text{Midpt } BD$$

$$(-4, 2)$$

$$2 = 2(-4) + c$$

$$c = 10$$

[A1]

$$B(0, 10)$$

[A1]

$$\text{let } D(x, y)$$

$$B(0, 10)$$

$$\frac{x+0}{2} = -4$$

$$x = -8$$

[M1]

$$\frac{y+10}{2} = 2$$

$$y = -6$$

$$\therefore D(-8, -6)$$

[A1]

(b) the area of the rhombus,

[2] E

$$\text{Area} = \frac{1}{2} \begin{vmatrix} -10 & -8 & 2 & 0 & -10 \\ 5 & -6 & -1 & 10 & 5 \end{vmatrix} \quad \text{[M1]}$$

$$= \frac{1}{2} [60 + 8 + 20 + 0 - (-40 - 12 + 0 - 10)]$$

$$= 120 \text{ units}^2$$

[A1]

(c) the acute angle that the line DC produced makes with the x-axis.

[2] M

$$\tan \theta = \text{Gradient} = \frac{-6+1}{-8-2}$$

[M1]

$$\tan \theta = \frac{1}{2}$$

$$\theta = 26.6^\circ \quad (1 \text{ dp}) \quad \text{[A1]}$$

End of Paper

Mdm Soh.

NAME

CLASS

INDEX NO.

Marking Scheme



ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2024

SUBJECT : Additional Mathematics
(4049/02)

DATE : 15 AUGUST 2024

LEVEL : Secondary 4G3

DURATION : 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use		
Type of Penalties		Question Number
Units	-1	
	-2	
Accuracy	-1	
	-2	
Presentation	-1	
	-2	
		90

This paper contains 19 printed pages including this cover page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 (a) Given that $\log_3 a = x$ and $\log_a 5 = y$, express in terms of x and y ,

$$\log_a \frac{9\sqrt{a}}{125}.$$

[3]

$$\begin{aligned}
 &= \log_a 9 + \log_a a^{\frac{1}{2}} - \log_a 5^3 \\
 &= 2\log_a 3 + \frac{1}{2} - 3y \\
 &= 2\left(\frac{\log_3 3}{\log_3 a}\right) + \frac{1}{2} - 3y \\
 &= \frac{2}{x} + \frac{1}{2} - 3y =
 \end{aligned}$$

M2

A1

- (b) Solve $3^{2x+2} - 15 \times 3^x + 4 = 0$.

[5]

$$3^2(3^x)^2 - 15(3^x) + 4 = 0.$$

$$9y^2 - 15y + 4 = 0.$$

$$(3y - 1)(3y - 4) = 0$$

$$y = \frac{1}{3} \quad \text{or} \quad y = \frac{4}{3}$$

$$3^x = \frac{1}{3}$$

$$x \ln 3 = \ln \frac{1}{3}$$

$$x = \frac{\ln \frac{1}{3}}{\ln 3}$$

$$= -1$$

$$3^x = \frac{4}{3}$$

$$x \ln 3 = \ln \frac{4}{3}$$

$$x = \frac{\ln \frac{4}{3}}{\ln 3}$$

$$= 0.262 \text{ (3 sf)}$$

A2

- 2 (a) Show that the term x^{-1} does not exist in the expansion of $\left(\frac{x^2}{3} - \frac{2}{x}\right)^8$. [4]

$$\begin{aligned}
 T_{r+1} &= {}^8C_r \left(\frac{x^2}{3}\right)^{8-r} \left(-\frac{2}{x}\right)^r \\
 &= {}^8C_r \left(\frac{1}{3}\right)^{8-r} (-2)^r x^{2(8-r)} x^{-r} \\
 &= {}^8C_r \left(\frac{1}{3}\right)^{8-r} (-2)^r x^{16-3r}
 \end{aligned}$$

M1

M1

$$16 - 3r = -1$$

$$3r = 17$$

$$r = \frac{17}{3} \text{ (non integer)}$$

M1

$\therefore x^{-1}$ term does not exist.

- (b) Hence, find the term independent of x in the expansion of $\left(kx + \frac{4}{x}\right)\left(\frac{x^2}{3} - \frac{2}{x}\right)^8$. [3]

x x^1 term

$$\left(kx + \frac{4}{x}\right)\left(\frac{x^2}{3} - \frac{2}{x}\right)^8$$

$x(x^{-1}) = \text{does not exist}$

$$16 - 3r = 1$$

$$3r = 15$$

$$r = 5$$

M1

$\therefore$ the term = $4 \left[{}^8C_5 \left(\frac{1}{3}\right)^{8-5} (-2)^5 \right]$

M1

$$= -\frac{7168}{27}$$

A1 $-265\frac{13}{27}$

- 3 In an experiment, 150g of a particular radioactive substance is stored in a container. It is found that the substance is halved after 3.5 days. The amount of substance present after t days is given by $N = 150e^{-kt}$, where k is a constant.

Find

- (a) the value of k ,

[3]

$$\begin{aligned}
 75 &= 150 e^{-k(3.5)} & \text{M1} \\
 e^{-3.5k} &= \frac{1}{2} & \text{M1} \\
 -3.5k &= \ln \frac{1}{2} & \text{OR } e^{3.5k} = 2 \\
 k &= \frac{\ln \frac{1}{2}}{-3.5} & 3.5k = \ln 2 \\
 &\approx 0.19804 & k = \frac{\ln 2}{3.5} \\
 &= 0.198 \text{ (3 sf)} & \text{A1}
 \end{aligned}$$

- (b) the minimum number of days it takes before the amount is reduced to 10g,

[2] E

$$\begin{aligned}
 10 &= 150 e^{-0.19804 t} & \text{E} \\
 e^{0.19804 t} &= 15 & \text{M1} \\
 0.19804 t &= \ln 15 & \\
 t &= \frac{\ln 15}{0.19804} & \text{A1} \\
 &= 13.7 \text{ days (3 sf)} & \text{E} \\
 & & 14 \text{ days}
 \end{aligned}$$

- (c) the amount of substance left after 3 weeks.

[2] E

$$\begin{aligned}
 N &= 150 e^{-0.19804(3 \times 7)} & \text{M1} \\
 &\approx 2.3439 & \\
 &= 2.34 \text{ g (3 sf)} & \text{A1} \\
 & & 2\frac{11}{32} \text{ (exact!)}
 \end{aligned}$$

- 4 (a) Given $A \cos \theta + B \sin \theta = 4 \cos\left(\theta - \frac{\pi}{3}\right)$ and $A > 0, B > 0$,

[5]

find A and B .

$$\tan \frac{\pi}{3} = \sqrt{3}$$

(M1)

$$\frac{B}{A} = \sqrt{3}$$

(M1)

$$B = A\sqrt{3}$$

$$A^2 + B^2 = 4^2$$

$$A^2 + (A\sqrt{3})^2 = 16$$

(M1)

$$A^2 + 3A^2 = 16$$

$$4A^2 = 16$$

$$A^2 = 4$$

$$A = 2$$

$$B = 2\sqrt{3}$$

(A2)

use difference formula

$$A \cos \theta + B \sin \theta = 4 \cos\left(\theta - \frac{\pi}{3}\right)$$

$$= 4 \left[\cos \theta \cos \frac{\pi}{3} + \sin \theta \sin \frac{\pi}{3} \right] \quad (M1)$$

$$= (4 \cos \frac{\pi}{3}) \cos \theta + (4 \sin \frac{\pi}{3}) \sin \theta \quad (M2)$$

$$A \cos \theta + B \sin \theta$$

$$A = 4 \cos \frac{\pi}{3}$$

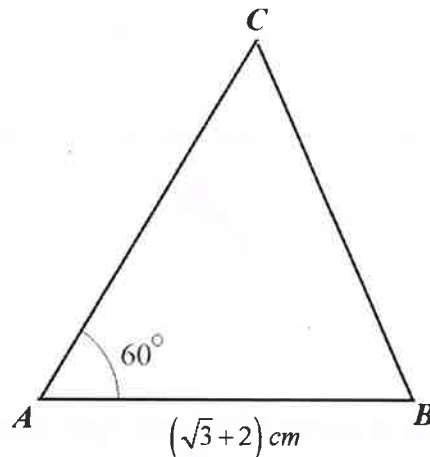
$$= 4\left(\frac{1}{2}\right) = 2 \quad (A)$$

$$B = 4 \sin \frac{\pi}{3}$$

$$= 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3} \quad (A)$$

- (b) The triangle ABC is such that its area is $\frac{1}{4}(12+9\sqrt{3}) \text{ cm}^2$.

The length of AB is $(\sqrt{3}+2) \text{ cm}$ and angle BAC is 60° .



Without using a calculator, find the length of AC in the form of $h+k\sqrt{3}$ where h and k are integers.

[5]

Area of $\triangle ABC$,

$$\frac{1}{2}(AC)(\sqrt{3}+2)\sin 60^\circ = \frac{1}{4}(12+9\sqrt{3})$$

$$AC\left(\frac{1}{2}\right)(\sqrt{3}+2)\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{4}(4+3\sqrt{3})$$

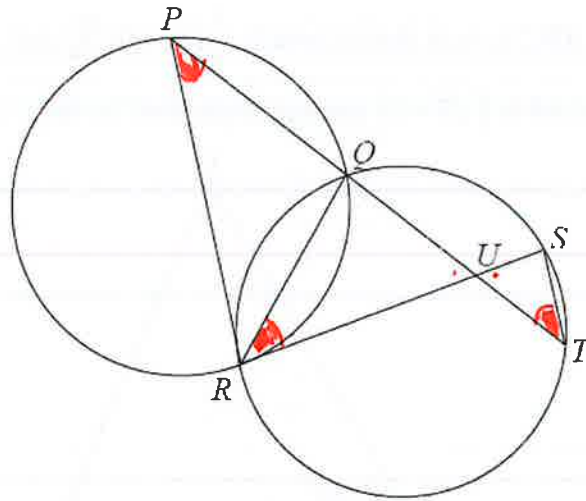
$$AC = \frac{3(4+3\sqrt{3})}{(3+2\sqrt{3})} \times \frac{(3-2\sqrt{3})}{(3-2\sqrt{3})}$$

$$= \frac{3(12+9\sqrt{3}-8\sqrt{3}-18)}{9-12}$$

$$= \frac{3(-6+\sqrt{3})}{-3}$$

$$= (6-\sqrt{3}) \text{ cm} //$$

5



PR and SR are tangents to circles $RQST$ and PQR respectively.
 PQT is a straight line and it intersects RS at U .

(a) Prove that PR is parallel to ST ,

[3]

$$\angle RPQ = \angle QRS \quad (\text{Alternate Segment}) \quad (M1)$$

$$\angle STQ = \angle QRS \quad (\angle s \text{ in the same segment}) \quad (M1)$$

$$\therefore \angle RPQ = \angle STQ$$

$$\therefore PR \parallel ST \quad (\text{Alternate } \angle s) \quad \text{By converse of alt. } \angle s \text{ are equal,} \quad (A1)$$

(b) Prove that triangle STU is similar to triangle RPU ,

[2]

$$\angle RUP = \angle TUS \quad (\text{vertically opposite}) \quad (M1)$$

$$\angle RPU = \angle STU \quad (\text{part (a)})$$

$$\therefore \triangle STU \text{ is similar to } \triangle RPU \quad (AA) \quad (A1)$$

(c) Name another triangle similar to triangle STU .

[1]

$\triangle STU$ is similar to $\triangle QRU$.

(d) Hence, prove that $(RU)^2 = PU \times QU$.

[2]

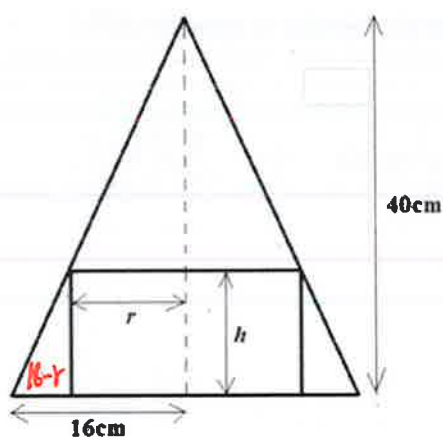
$\triangle RPU$ is similar to $\triangle QRU$.

$$\frac{RU}{QU} = \frac{PU}{RU}$$

$$(RU)^2 = PU \times QU \quad (\text{shown})$$

MI

AI



The diagram shows the cross-section of a solid cone of height 40 cm and the base radius is 16 cm. A cylinder of height h cm and radius r cm is to be extracted from the cone.

(a) Express h in terms of r .

[3]

By similar Δ s,

$$\frac{40-h}{40} = \frac{r}{16}$$

$$40-h = \frac{40r}{16}$$

$$h = 40 - \frac{5r}{2} \quad \text{or} \quad \frac{80-5r}{2}$$

M2

A1

(b) Show that the volume, $V \text{ cm}^3$, of the cylinder is

$$V = \frac{5\pi r^2}{2}(16-r).$$

[2]

$$V = \pi r^2 h$$

$$= \pi r^2 \frac{(80-5r)}{2}$$

$$= \frac{5\pi r^2(16-r)}{2} \quad (\text{shown})$$

M1

M1

- (c) Hence, given that r can vary, find the maximum volume of the cylinder and its radius. [5]

$$\begin{aligned}
 V &= \frac{5\pi r^2}{2} (16 - r) \\
 &= 40\pi r^2 - \frac{5\pi r^3}{2} \\
 \frac{dV}{dr} &= 80\pi - \frac{15\pi r^2}{2} \quad (M1)
 \end{aligned}$$

For stationary ~~points~~ ^{value},

$$\begin{aligned}
 80\pi - \frac{15\pi r^2}{2} &= 0 \quad (M1) \\
 \pi r \left(80 - \frac{15r}{2} \right) &= 0
 \end{aligned}$$

$$r = 0 \text{ (rej)}$$

$$80 - \frac{15r}{2} = 0$$

$$80 = \frac{15r}{2}$$

$$r = \frac{2 \times 80}{15}$$

$$= \frac{32}{3} \quad (M1)$$

$$\frac{d^2V}{dr^2} = 80\pi - 15\pi r$$

$$r = \frac{32}{3}$$

$$\begin{aligned}
 \frac{d^2V}{dr^2} &= 80\pi - 15\pi \left(\frac{32}{3} \right) \\
 &= -80\pi
 \end{aligned}$$

(max) (M1)

$$\therefore V_{\max} = \frac{5\pi}{2} \left(\frac{32}{3} \right)^2 \left(16 - \frac{32}{3} \right)$$

$$\approx 4765.9$$

$$= 4770 \text{ cm}^3$$

(A1)

- 7 (a) Show that $(x+1)$ is a factor of $f(x) = 2x^3 - 3x^2 + (k-5)x + k$, where k is a constant. [2]

$$x = -1, \quad f(-1) = 2(-1)^3 - 3(-1)^2 + (k-5)(-1) + k \quad (M1)$$

$$= -2 - 3 - k + 5 + k$$

$\stackrel{=0}{=} 0$
 $\therefore$ by Factor Theorem, $(x+1)$ is a factor of y (A1)

- (b) Express $f(x) = 2x^3 - 3x^2 + (k-5)x + k$ in the form of $f(x) = (x+1)(Ax^2 + Bx + k)$ where A and B are constants. [3]

$$2x^3 - 3x^2 + (k-5)x + k = (x+1)(Ax^2 + Bx + k)$$

x^3 term: By comparing coeff of x^3 :
 $2x^3 = Ax^3$ (M1)

$$A = 2$$

$$x = 1, \quad 2 - 3 + k - 5 + k = 2(2 + B + k)$$

$$2k - 6 = 4 + 2B + 2k$$

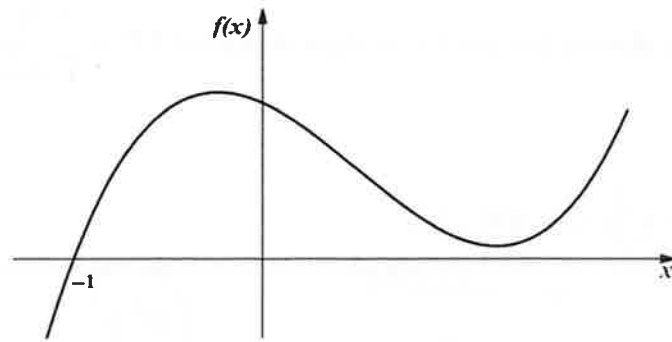
$$2B = -10$$

$$B = -5$$

$$y = (x+1)(2x^2 - 5x + k)$$

(M1)
(A1)

(c)



Given that $f(x) = 2x^3 - 3x^2 + (k-5)x + k$ can be represented by the graph above, find the range of values of k . Justify your answer.

[3]

$$f(x) = (x+1)(2x^2 - 5x + k)$$

Since there is only one real root,

$2x^2 - 5x + k$ has no real root.

$$\therefore (-5)^2 - 4(2)(k) < 0$$

$$25 - 8k < 0$$

$$k > \frac{25}{8}$$

M1
M1
A1

- 8 (a) Use the identity for $\cos 2A$ to show that $\cos 67.5^\circ = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$.

[4]

$$\begin{aligned}\cos(2 \times 67.5^\circ) &= \cos 135^\circ \\ &= -\cos 45^\circ \\ &= -\frac{1}{\sqrt{2}}\end{aligned}$$

(M1)

$$\begin{aligned}\cos 2A &= 2\cos^2 A - 1 \\ \cos 135^\circ &= 2\cos^2 67.5^\circ - 1 \\ -\cos 45^\circ &= 2\cos^2 67.5^\circ \\ 1 - \frac{1}{\sqrt{2}} &= 2\cos^2 67.5^\circ \\ \cos^2 67.5^\circ &= \frac{1}{2} \left[\frac{\sqrt{2}-1}{\sqrt{2}} \right]\end{aligned}$$

(M1)

$$\cos 67.5^\circ = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} \quad \sim \quad -\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} \text{ rej. 1st quadrant.}$$

(A2)

$$\frac{\text{max} + \text{min}}{2} = A, \quad \frac{\text{max} - \text{min}}{2} = 3 \Rightarrow \frac{\text{max} - (-10)}{2} = 3$$

$$\frac{\text{max} - 10}{2} = A \Rightarrow 2A = \text{max} + 10 = 6 \Rightarrow \text{max} = 6 - 10 = -4$$

- (b) The temperature ($^{\circ}\text{C}$) in a chemical storage chamber is given by the function $H(t) = A + 3 \sin Bt$, where A and B are constants and t represents the time in hours.

$a - c : \text{min}$

$a + c : \text{max}$

- (i) State the value of A if the lowest temperature in the chamber is set to -10°C . Hence, find the maximum temperature in the chamber. [2]

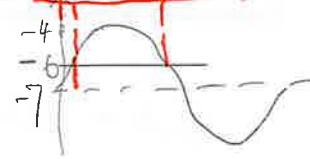
$$A + 3(-1) = -10$$

$$A = -7$$

$$\text{Max temperature} = -7 + 3 = -4^{\circ}\text{C}$$

$$-3 \leq 3 \sin Bt \leq 3$$

$$A - 3 \leq A + 3 \sin Bt \leq A + 3$$



$$A - 3 = -10$$

$$A = -7$$

- (ii) Given that one cycle of $H(t)$ is 6 hours, show that $B = \frac{\pi}{3}$. [2]

$$B(T) = 2\pi$$

$$6B = 2\pi$$

$$B = \frac{\pi}{3}$$

$$\text{period} = \frac{2\pi}{B}$$

$$6 = \frac{2\pi}{B}$$

$$6B = 2\pi$$

$$B = \frac{\pi}{3} \text{ (shown)}$$

- (iii) Find the duration per cycle, to the nearest minute, when the temperature is above -6°C . [5]

$$y = 3 \sin \frac{\pi}{3}t - 7$$

$$-7 + 3 \sin \frac{\pi}{3}t = -6$$

$$\sin \frac{\pi}{3}t = \frac{1}{3} \text{ (1st, 2nd)}$$

$$-7 \leq y \leq -4$$

$$\text{Reference } \theta = \sin^{-1}\left(\frac{1}{3}\right) \approx 0.33984$$

$$\frac{\pi}{3}t = 0.33984, \quad (\pi - 0.33984)$$

$$t = \frac{3}{\pi}(0.33984), \quad \frac{3}{\pi}(\pi - 0.33984)$$

$$\text{Duration} = \frac{3}{\pi}(\pi - 0.33984) - \frac{3}{\pi}(0.33984)$$

$$= 3 - 2 \times \frac{3}{\pi}(0.33984)$$

$$\approx 2.35095$$

$$= 2 \text{ h } 0.35095 \times 60 \text{ min}$$

$$= 2 \text{ h } 21 \text{ min}$$

- 9 An object in motion with velocity v m/s is subjected to a resistive force of R N (Newton) against its motion. The table shows experimental values of the variables v and R .

v	5	10	15	20	25
R	32	96	180	290	410

R and v are known to be related by the equation $R = kv^\beta$, where k and β are constants.

- (a) Express $\ln R$ in terms of $\ln v$.

[1]

$$\ln R = \ln k + \beta \ln v \quad (A1)$$

$$Y = mX + C \Rightarrow Y = \ln R, \quad m = \beta$$

$$X = \ln v, \quad C = \ln k$$

- (b) Complete the following table and plot the graph of $\ln R$ against $\ln v$ in the grid provided.

[3]

$\ln v$	1.6	2.3	2.7	3.0	3.2
$\ln R$	3.5	4.6	5.2	5.7	6.0

(M1)

- (c) Use your graph to estimate the values of k and β .

[4]

$$\ln R = \beta \ln v + \ln k$$

From graph,
 $\ln k = 1$

$$k = e$$

(M1)

(A1)

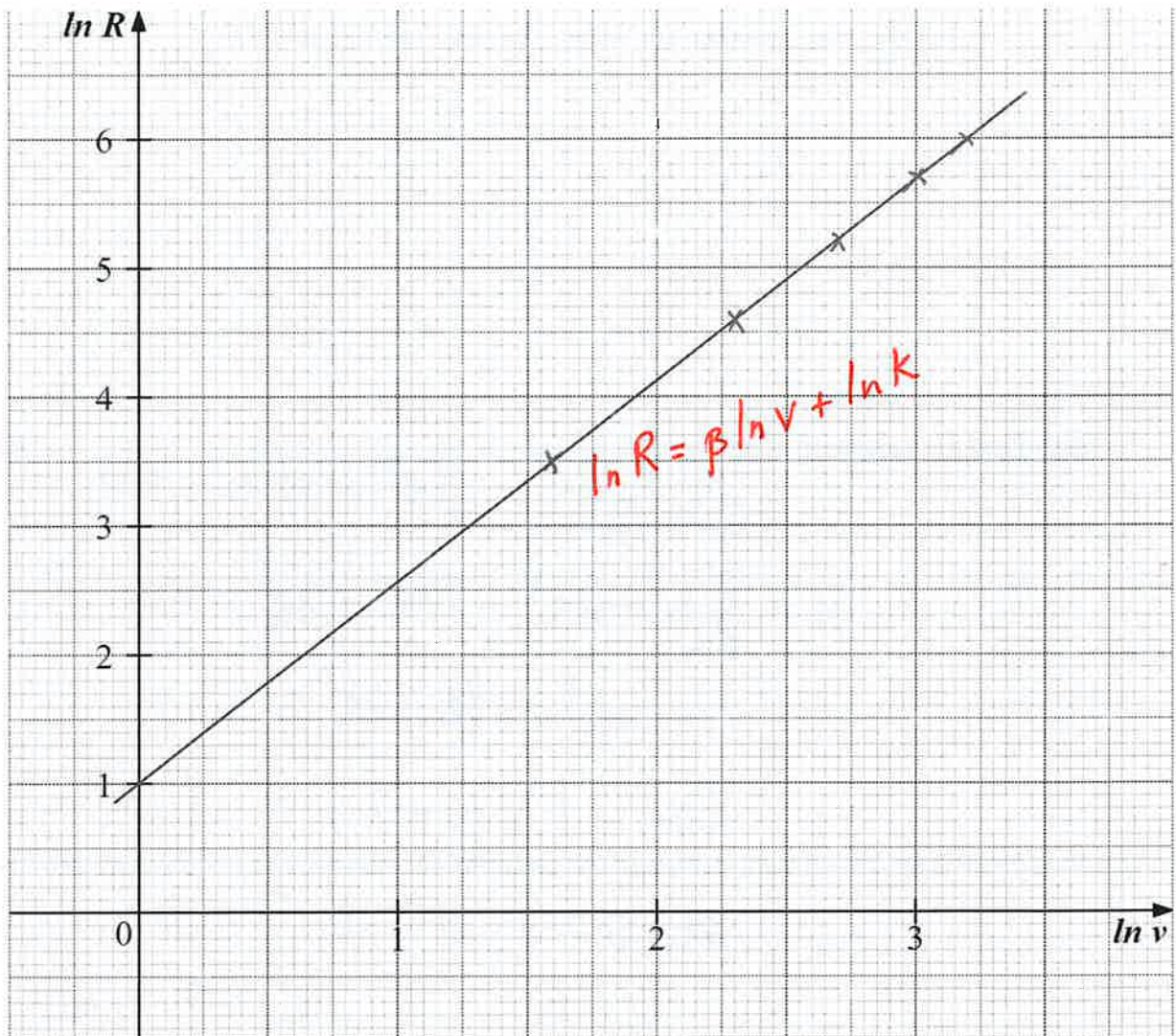
$$\text{Gradient } \beta = \frac{6 - 3.5}{3.2 - 1.6}$$

$$= 1.56$$

(M1)

(A1)

Use table value.



correct points
graph

(M1)

(M1)

- 10 A particle moves in a horizontal straight line so that its displacement, s cm, from a fixed point O , after t seconds, is given by

$$s = \frac{2}{3}t^3 - \frac{19}{2}t^2 + 35t - 40.$$

The particle reverses its direction of motion at time t_1 and t_2 , where $t_2 > t_1$.

All positions on the right-hand side of the fixed point O are considered positive displacements.

- (a) State the distance of the particle and its direction with respect to the fixed point O initially.

[1]

Initially the particle is 40cm to the left of point O . (B1)

- (b) Find the values of t_1 and t_2 .

[4]

$$s = \frac{2}{3}t^3 - \frac{19}{2}t^2 + 35t - 40$$

$$v = \frac{d}{dt}s = 2t^2 - 19t + 35 \quad (M1)$$

$$2t^2 - 19t + 35 = 0.$$

$$(2t - 5)(t - 7) = 0 \quad (M1)$$

$$2t - 5 = 0 \text{ or } t - 7 = 0$$

$$t_1 = \frac{5}{2}$$

$$t_2 = 7 \quad (A2)$$

(c) Find the total distance travelled by the particle in the first 10 seconds.

[4]

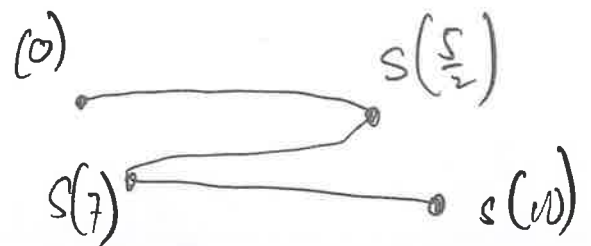
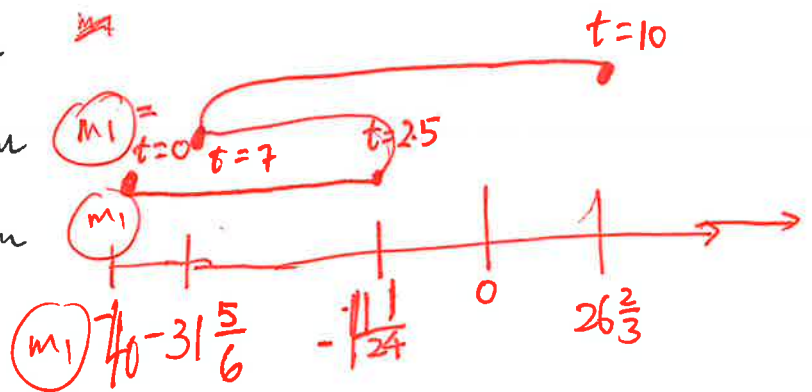
$$s = \frac{2}{3}t^3 - \frac{19}{2}t^2 + 35t - 40$$

$$s(0) = -40 \text{ cm}$$

$$s\left(\frac{5}{2}\right) = -\frac{35}{24} \text{ cm}$$

$$s(7) = -\frac{191}{6} \text{ cm}$$

$$s(10) = \frac{80}{3}$$



$$\begin{aligned} \text{Total Distance} &= s\left(\frac{5}{2}\right) - s(0) \\ &+ s\left(\frac{5}{2}\right) - s(7) \\ &+ s(10) - s(7) \end{aligned}$$

$$= \frac{925}{24} + \frac{243}{8} + \frac{117}{2}$$

$$= \frac{1529}{12} \text{ cm} \text{ or } 127\frac{5}{12} \text{ cm}$$

(AI)

(d) Find the acceleration of the particle when $t = t_2$.

[2]

$$v = 2t^2 - 19t + 35$$

$$a = \frac{dv}{dt} = 4t - 19$$

$$t=7, \quad a = 4(7) - 19$$

$$= 9 \text{ cm/s}^2$$

(m1)

(AI)

NAME	FORM CLASS	ACAD CLASS	INDEX NO.
MARKING SCHEME			



ST. PATRICK'S SCHOOL PRELIMINARY EXAMINATION 2025

SUBJECT : Additional Mathematics
(4049/01)

DATE : 18 August 2025

LEVEL : Secondary 4 Express

DURATION : 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Form Class, Acad Class and Index No. in the spaces at the top of the page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value of 3.142, unless the equation requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use			
Type of Penalties		Question Number	90
Units	−1		
	−2		
Presentation	−1		
	−2		

This paper consists of 20 printed pages and 1 blank page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

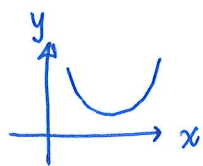
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 The equation of a curve is $y = kx^2 + kx + p$, where p and k are constants.

(a) Show that $p > \frac{k}{4}$ for which the curve lies completely above the x -axis [3]



$k > 0$ &
no real roots

$\Rightarrow$ Discriminant < 0

$$b^2 - 4ac < 0$$

$$(k)^2 - 4(k)(p) < 0 \quad (M1)$$

$$k^2 - 4kp < 0$$

$$k(k - 4p) < 0$$

Since $k > 0$

$$\therefore k - 4p < 0 \quad (M1)$$

$$k < 4p$$

$$4p > k$$

$$p > \frac{k}{4} \quad (\text{shown}) \quad (A1)$$

- (b) In the case where $k = 2$ and $p = 4$, find the values of m for which the line $y = mx - 4$ is a tangent to the curve. [4]

$$y = 2x^2 + 2x + 4 \quad \text{---} \quad (1)$$

$$y = mx - 4 \quad \text{---} \quad (2)$$

$$(1) = (2),$$

$$2x^2 + 2x + 4 = mx - 4$$

$$2x^2 + (2 - m)x + 8 = 0 \quad (M1)$$

For line is a tangent to the curve,

discriminant, $D = 0$

$$\Rightarrow b^2 - 4ac = 0 \quad (M1)$$

$$(2 - m)^2 - 4(2)(8) = 0$$

$$4 - 4m + m^2 - 64 = 0$$

$$m^2 - 4m - 60 = 0$$

$$(m - 10)(m + 6) = 0$$

$$m = 10 \quad \text{or} \quad m = -6 \quad (A2)$$

Minus 1
mark if
this step
not shown

- 2 (a) Express $\frac{2x^3 + 7x^2 + 5x + 6}{x(x^2 + 2)}$ as the sum of a polynomial and a proper fraction. [1]

$$\begin{array}{r} 2 \\ x^3 + 2x \overline{) 2x^3 + 7x^2 + 5x + 6} \\ \underline{-(2x^3 + 4x)} \\ 7x^2 + x + 6 \end{array}$$

$$\therefore \frac{2x^3 + 7x^2 + 5x + 6}{x(x^2 + 2)} = 2 + \frac{7x^2 + x + 6}{x(x^2 + 2)} \quad (AI)$$

- (b) Hence, express $\frac{2x^3 + 7x^2 + 5x + 6}{x(x^2 + 2)}$ in partial fractions. [5]

$$\text{Let } \frac{7x^2 + x + 6}{x(x^2 + 2)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 2} \quad (MI)$$

$$7x^2 + x + 6 = A(x^2 + 2) + x(Bx + C)$$

When $x = 0$,

$$A(0 + 2) = 6$$

$$2A = 6$$

$$A = 3 \quad (MI)$$

By comparing coeff of

$$x^2: A + B = 7$$

$$3 + B = 7$$

$$B = 4 \quad (MI)$$

$$x: C = 1 \quad (MI)$$

$$\therefore \frac{2x^3 + 7x^2 + 5x + 6}{x(x^2 + 2)} = 2 + \frac{3}{x} + \frac{4x + 1}{x^2 + 2} \quad (AI)$$

- 3 Prove the identity $\frac{\cos x}{\operatorname{cosec} x - 1} + \frac{\cos x}{\operatorname{cosec} x + 1} = 2 \tan x$. [5]

$$\text{LHS} = \frac{\cos x}{\operatorname{cosec} x - 1} + \frac{\cos x}{\operatorname{cosec} x + 1}$$

$$= \frac{\cos x (\operatorname{cosec} x + 1) + \cos x (\operatorname{cosec} x - 1)}{(\operatorname{cosec} x)^2 - (1)^2} \quad (M1)$$

$$= \frac{\cos x \left[\frac{1}{\sin x} + 1 \right] + \cos x \left[\frac{1}{\sin x} - 1 \right]}{\operatorname{cosec}^2 x - 1} \quad (M1)$$

$$| 1 + \cot^2 x = \operatorname{cosec}^2 x |$$

$$= \frac{\frac{\cos x}{\sin x} + \cancel{\cos x} + \frac{\cos x}{\sin x} - \cancel{\cos x}}{\cot^2 x}$$

$$\frac{\cos x}{\sin x} = \cot x$$

$$= \frac{2 \cot x}{\cot^2 x} \quad (M1)$$

$$= \frac{2}{\cot x} \quad (M1)$$

$$= 2 \tan x$$

$$= \text{RHS (shown)} \quad (A1)$$

- 4 (a) Given that $y = (x-1)\sqrt{4x+1}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{px+q}{\sqrt{4x+1}}$ where p and q are constants. [4]

$$\text{Let } u = x-1$$

$$\frac{du}{dx} = 1$$

$$v = \sqrt{4x+1}$$

$$v = (4x+1)^{\frac{1}{2}}$$

$$\frac{dv}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}}(4) = \frac{2}{\sqrt{4x+1}}$$

$$\therefore \frac{dy}{dx} = \sqrt{4x+1} + \frac{2(x-1)}{\sqrt{4x+1}}$$

$$= \frac{4x+1 + 2x-2}{\sqrt{4x+1}}$$

$$= \frac{6x-1}{\sqrt{4x+1}} \quad (\text{shown})$$

- (b) Hence evaluate $\int_2^6 \frac{x}{\sqrt{4x+1}} dx$ [5]

$$\int_2^6 \frac{6x-1}{\sqrt{4x+1}} dx = \left[(x-1)\sqrt{4x+1} \right]_2^6$$

$$\int_2^6 \frac{6x}{\sqrt{4x+1}} dx - \int_2^6 \frac{1}{\sqrt{4x+1}} dx = \left[(6-1)\sqrt{24+1} \right] - \left[(2-1)\sqrt{8+1} \right]$$

$$6 \int_2^6 \frac{x}{\sqrt{4x+1}} dx = \int_2^6 (4x+1)^{-\frac{1}{2}} dx + 5(5) - (1)(3)$$

$$= \left[\frac{[4x+1]^{-\frac{1}{2}+1}}{\frac{1}{2}(4)^{\frac{1}{2}}} \right]_2^6 + 25 - 3$$

$$= \frac{1}{2} \left[\sqrt{4x+1} \right]_2^6 + 22$$

$$= \frac{1}{2} [\sqrt{25} - \sqrt{9}] + 22$$

$$= \frac{1}{2} [5-3] + 22$$

$$= 23$$

$$\therefore \int_2^6 \frac{x}{\sqrt{4x+1}} dx = \frac{23}{6}$$

5 Solve the equation $3 \cos A - \sec A = 5 \tan A$ for $0 \leq A \leq 2\pi$.

[5]

$$3 \cos A - \frac{1}{\cos A} = \frac{5 \sin A}{\cos A} \quad (M1)$$

$$\frac{3 \cos^2 A - 1}{\cos A} = \frac{5 \sin A}{\cos A}$$

$$\frac{3 [1 - \sin^2 A] - 1}{\cos A} = \frac{5 \sin A}{\cos A} \quad (M1)$$

$$\frac{3 - 3 \sin^2 A - 1}{\cos A} = \frac{5 \sin A}{\cos A}$$

$$\frac{2 - 3 \sin^2 A}{\cos A} = \frac{5 \sin A}{\cos A}$$

$$\cos A [2 - 3 \sin^2 A - 5 \sin A] = 0$$

$$\div (-1), \quad \cos A [3 \sin^2 A + 5 \sin A - 2] = 0$$

$$\cos A (3 \sin A - 1)(\sin A + 1) = 0$$

$$\cos A = 0 \quad \text{or} \quad (3 \sin A - 1)(\sin A + 2) = 0 \quad (M1)$$

(rejected $\therefore$

$\frac{1}{\cos A}$ cannot be defined)

(A1)

$$\sin A = \frac{1}{3} > 0 \quad \text{or} \quad \sin A = -2 \quad (\text{rejected})$$

$$\text{ref } A = 0.33983 \dots$$

$$\therefore A = 0.33983, 2.801755 \dots$$

$$= 0.340, 2.80 \quad (3 \text{ sf})$$

(A1)

- 6 It is given that $y = x^3 + px^2 + qx + 10$ where p and q are integers. The only values of x for which y is a decreasing function of x are those values for which $3 < x < 7$. Find the value of p and of q . [4]

For decreasing function,

$$\frac{dy}{dx} < 0$$

$$3x^2 + 2px + q < 0 \quad (M1)$$

Also $\Rightarrow 3 < x < 7$ (given)

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= 3(x-3)(x-7) \\ &= 3[x^2 - 7x - 3x + 21] \\ &= 3[x^2 - 10x + 21] \\ &= 3x^2 - 30x + 63 \quad (M1) \end{aligned}$$

By comparing,

$$\begin{aligned} 2p &= -30 \\ p &= -15 \quad (A1) \end{aligned}$$

$$\& \quad q = 63 \quad (A1)$$

7

The line $2x + 3y = 12$ intersects the curve $y^2 = 4x - 8$ at points A and B .

Find the value of p and q for which the length of AB can be expressed as $p\sqrt{q}$.

[6]

From ①, $2x = 12 - 3y$ ——— ③

Subst. ③ into ②,

$$y^2 = 2(12 - 3y) - 8 \quad (M1)$$

$$y^2 = 24 - 6y - 8$$

$$y^2 + 6y - 16 = 0$$

$$(y + 8)(y - 2) = 0 \quad (M1)$$

$$y = -8 \quad \text{or} \quad y = 2$$

$$2x = 12 - 3(-8)$$

$$2x = 36$$

$$\therefore x = 18$$

$$\therefore A(18, -8) \quad (A1)$$

$$2x = 12 - 3(2)$$

$$2x = 6$$

$$x = 3$$

$$B(3, 2) \quad (A1)$$

$$\text{Length of } AB = \sqrt{(18-3)^2 + (-8-2)^2}$$

$$= \sqrt{225 + 100}$$

$$= \sqrt{325}$$

$$= \sqrt{25 \times 13}$$

$$= 5\sqrt{13} \text{ units}$$

$$= p\sqrt{q}$$

By comparison, $p = 5$, $q = 13$. $(A2)$

8 The equation of a curve is $y = 5e^{-\frac{1}{2}x} + e^{\frac{1}{2}x}$.

- (a) Find the coordinates of the stationary point, in logarithmic and/or surd form where appropriate. [5]

$$y = 5e^{-\frac{1}{2}x} + e^{\frac{1}{2}x}$$

$$\frac{dy}{dx} = 5[e^{-\frac{1}{2}x}] \left[-\frac{1}{2}\right] + e^{\frac{1}{2}x} \left(\frac{1}{2}\right)$$

$$= -\frac{5}{2}e^{-\frac{1}{2}x} + \frac{1}{2}e^{\frac{1}{2}x} \quad (M1)$$

for stationary pts,

$$\frac{dy}{dx} = 0$$

$$\frac{5}{2}e^{-\frac{1}{2}x} = \frac{1}{2}e^{\frac{1}{2}x} \quad (M1)$$

$$x e^{\frac{1}{2}x}, \quad \frac{5}{2}e^0 = \frac{1}{2}e^x$$

$$e^x = 5$$

$$\ln e^x = \ln 5$$

$$x = \ln 5 \quad (M1)$$

$$y = 5e^{-\frac{1}{2}\ln 5} + e^{\frac{1}{2}\ln 5}$$

$$= 5e^{\ln 5^{-1/2}} + e^{\ln 5^{1/2}}$$

$$= 5[5^{-1/2}] + 5^{1/2}$$

$$= 5^{1/2} + 5^{1/2}$$

$$= 2\sqrt{5} \quad (M1)$$

$\therefore$ stationary pt is $(\ln 5, 2\sqrt{5})$ (A1)

$$a^{\log_a b}$$

$$= b$$

$$x = a^{\log_a b}$$

$$\log_a x = \log_a a^{\log_a b}$$

$$\log_a x = \log_a b$$

$$\therefore x = b$$

- (b) Determine the nature of the stationary point of the curve.

[2]

$$\frac{dy}{dx} = -\frac{5}{2} e^{-\frac{1}{2}x} + \frac{1}{2} e^{\frac{1}{2}x}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= -\frac{5}{2} \left[e^{-\frac{1}{2}x} \right] \left[-\frac{1}{2} \right] + \frac{1}{2} \left[e^{\frac{1}{2}x} \right] \left[\frac{1}{2} \right] \\ &= +\frac{5}{4} e^{-\frac{1}{2}x} + \frac{1}{4} e^{\frac{1}{2}x} \quad (\text{M1}) \end{aligned}$$

When $x = \ln 5$,

$$\frac{d^2y}{dx^2} > 0$$

$\Rightarrow (\ln 5, 2\sqrt{5})$ is a minimum point. (A1)

- (c) Explain why the $\frac{dy}{dx}$ is an increasing function for all real values of x .

[2]

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2y}{dx^2} = +\frac{5}{4} e^{-\frac{1}{2}x} + \frac{1}{4} e^{\frac{1}{2}x}$$

$$\begin{aligned} \text{Since } e^{-\frac{1}{2}x} > 0, \quad e^{\frac{1}{2}x} > 0 \quad (\text{M1}) \\ \Rightarrow +\frac{5}{4} e^{-\frac{1}{2}x} > 0, \quad \frac{1}{4} e^{\frac{1}{2}x} > 0 \end{aligned}$$

$$\therefore \frac{d^2y}{dx^2} > 0$$

$\Rightarrow$ gradient is an increasing function for all real values of x . (explained) (A1)

- 9 A particle moves along the curve $y = \frac{3(x+6)}{x+4}$, where $x \neq -4$, in such a way that the y -coordinate of the particle is increasing at a constant rate of $\frac{4}{27}$ units per second. $\frac{dy}{dt} = \frac{4}{27} \text{ units/s}$

Find the x -coordinates of the particle at that instant that the x -coordinates of the particle is decreasing at a rate of 2 units per second. $\frac{dx}{dt} = -2 \text{ units/s}$

[5]

$$y = \frac{3x+18}{x+4}$$

Let $u = 3x+18$, $v = x+4$
 $\frac{du}{dx} = 3$ $\frac{dv}{dx} = 1$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{3(x+4) - (3x+18)}{(x+4)^2} \\ &= \frac{3x+12-3x-18}{(x+4)^2} \\ &= \frac{-6}{(x+4)^2} \end{aligned}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{4}{27} = \frac{-6}{(x+4)^2} \times (-2)$$

$$\frac{12}{(x+4)^2} = \frac{4}{27}$$

$$4(x+4)^2 = 324$$

$$(x+4)^2 = 81$$

$$\begin{aligned} x+4 &= 9 & \text{or} & \quad x+4 = -9 \\ x &= 5 & & \quad x = -13 \end{aligned}$$

- 10 Jade is a parkour enthusiast who enjoys jumping between built structures or obstacles. One day, Jade decides to jump from one 2-metre-high structure to another of the same height. The path of Jade's jump can be modeled by the quadratic equation:

$$h = -\frac{5}{2}t^2 + 5t + 1 \text{ where } h \text{ is the height in metres above the ground after } t \text{ seconds.}$$

- (a) Rewrite h in the form $b - k(t - a)^2$. [3]

$$\begin{aligned} h &= -\frac{5}{2}t^2 + 5t + 1 \\ &= -\frac{5}{2}[t^2 - 2t] + 1 \quad (M1) \\ &= -\frac{5}{2}[(t-1)^2 - (1)^2] + 1 \quad (M) \\ &= -\frac{5}{2}(t-1)^2 + \frac{5}{2} + 1 \\ &= -\frac{5}{2}(t-1)^2 + 3\frac{1}{2} \\ &= 3\frac{1}{2} - \frac{5}{2}(t-1)^2 \quad (A1) \end{aligned}$$

- (b) Hence determine
(i) the maximum height that Jade reaches during the jump, [1]

$$= 3\frac{1}{2} \text{ m} \quad (B1)$$

- (ii) the time at which this maximum height occurs. [1]

$$= 1 \text{ s} \quad (B1)$$

- (c) Find the total time Jade spends in the air before landing on the other building. [3]

$$\text{When } h = 2 \text{ m}$$

$$-\frac{5}{2}(t-1)^2 + 3\frac{1}{2} = 2$$

$$-\frac{5}{2}(t-1)^2 = -\frac{3}{2}$$

$$5(t-1)^2 = 3 \quad (M1)$$

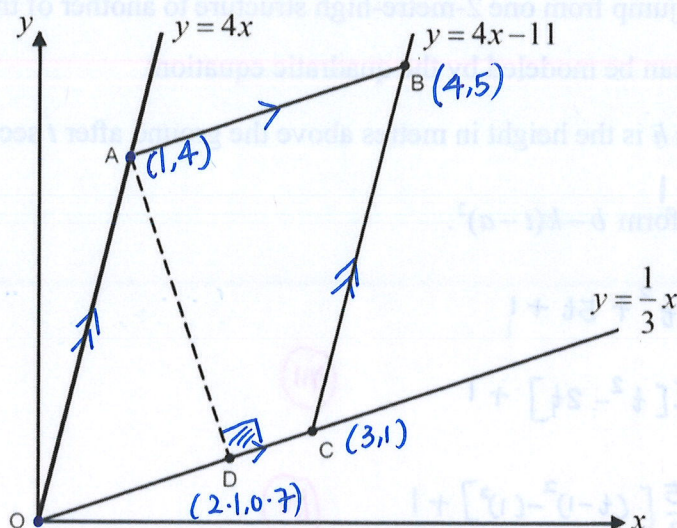
$$(t-1)^2 = \frac{3}{5}$$

$$t-1 = \sqrt{\frac{3}{5}} \quad \text{or} \quad t-1 = -\sqrt{\frac{3}{5}} \quad (M1)$$

$$t = 1.774 \dots \quad t = \cancel{1.5485} 0.2254 \dots$$

$$\begin{aligned} \therefore \text{total distance} &= 1.774 - \cancel{1.5485} 0.2254 \\ &= 1.5485 \dots \\ &= 1.55 \text{ s (3 sf)} \quad (A1) \end{aligned}$$

11



The diagram shows a parallelogram $OABC$ where O is the origin and A is the point $(1,4)$.

The equations OA , OC and CB are $y = 4x$, $y = \frac{1}{3}x$ and $y = 4x - 11$ respectively.

The perpendicular from A to OC meets OC at the point D .

Find

- (a) the coordinates of C , of B and of D .

[6]

$$\begin{aligned} C: \quad y &= 4x - 11 & \text{--- ①} & \text{EQ of CB} \\ y &= \frac{1}{3}x & \text{--- ②} & \text{EQ of OC} \end{aligned}$$

$$\begin{aligned} \text{①} = \text{②}, \quad 4x - 11 &= \frac{1}{3}x & \text{(m)} \\ \frac{11}{3}x &= 11 \end{aligned}$$

$$x = 3$$

$$y = \frac{1}{3}(3) = 1$$

$$\therefore C(3, 1) \quad \text{(A)}$$

$$D: \quad \text{Let } D(m, \frac{1}{3}m)$$

$$\text{grad}_{AD} = -3 \quad (\because AD \perp OC)$$

$$\frac{\frac{1}{3}m - 4}{m - 1} = -3 \quad \text{(m)}$$

$$\frac{1}{3}m - 4 = -3(m - 1)$$

$$\frac{1}{3}m - 4 = -3m + 3$$

$$\frac{10}{3}m = 7$$

$$\therefore m = 2.1, \quad \frac{1}{3}m = 0.7$$

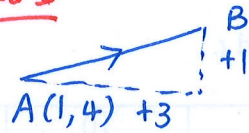
$$\therefore D(2.1, 0.7) \quad \text{(A)}$$

[Turn over]

(a) continuation . . .

(b) area of parallelogram OABC.

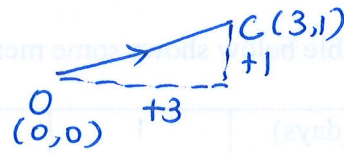
[2]

Method 1

$$\therefore B(4, 5)$$

(A1)

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$



$$O \rightarrow C$$

(M1)

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Let $B(h, k)$

Method 2 Since OABC is a //logram
OR Midpoint $A C = \text{Midpoint } O B$

$$\left(\frac{1+3}{2}, \frac{4+1}{2} \right) = \left(\frac{0+h}{2}, \frac{0+k}{2} \right)$$

$$\left(2, \frac{5}{2} \right) = \left(\frac{h}{2}, \frac{k}{2} \right)$$

$$\therefore h = 4$$

$$k = 5$$

$$\therefore B(4, 5)$$

(M1)

(b) Area of //logram OABC

$$= \frac{1}{2} \begin{vmatrix} 0 & C & B & A & O \\ 0 & 3 & 4 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2} \left[(0+15+16+0) - (0+4+5+0) \right]$$

$$= \frac{1}{2} [31 - 9]$$

$$= \frac{1}{2} [22]$$

$$= 11 \text{ units}^2$$

(A1)

Method (3)

$$\text{Eqn } AB: y = \frac{1}{3}x + \frac{11}{3}$$

$$\text{Eqn } BC: y = 4x - 11$$

$$\vdots$$

$$B(4, 5)$$

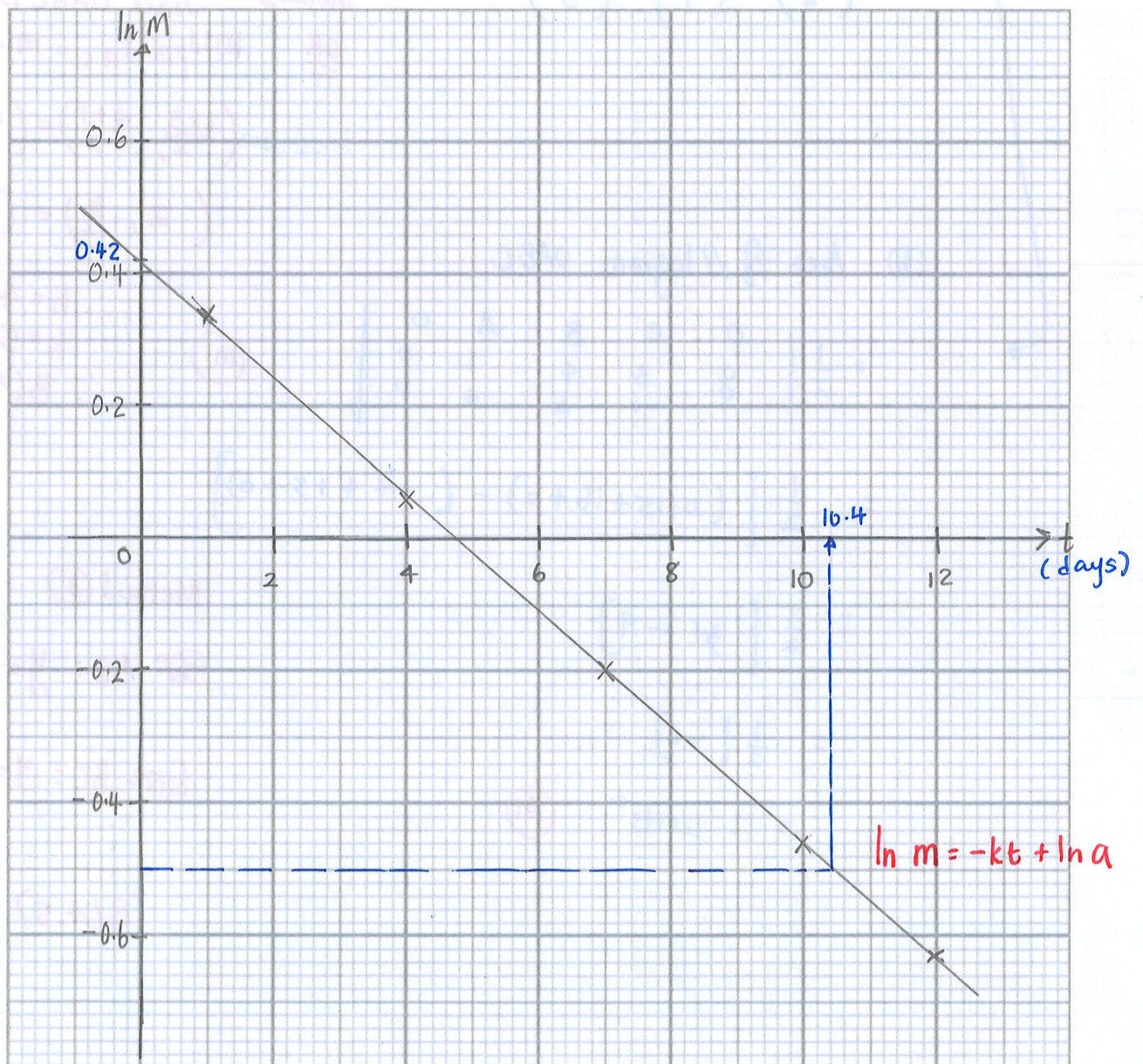
- 12 A radioactive substance undergoes decay. Its mass, M grams, after t days is defined by $M = ae^{-kt}$, where a and k are constants.

The table below shows some measurements of M and t .

t (days)	1	4	7	10	12
M (grams)	1.40	1.06	0.82	0.63	0.53
$\ln M$	0.34	0.06	-0.20	-0.46	-0.63

- (a) Plot $\ln M$ against t and draw a straight line graph to illustrate the information.

[2]



$$M = ae^{-kt}$$

$$\ln m = \ln ae^{-kt}$$

$$= \ln a + \ln e^{-kt}$$

$$\ln m = -kt + \ln a$$

$$Y = MX + C$$

where $Y = \ln m$, $m = -k$
 $X = t$, $C = \ln a$

- (M1) • CORRECT POINTS/LINE ^{straight}
- (m1) • Label/correct scale

[Turn over]

- (b) Use your graph to estimate the value of a and of k .

[4]

From the graph,

$$\ln a = 0.42 \quad (M1)$$

$$\log_e a = 0.42$$

$$a = e^{0.42}$$

$$= 1.52196$$

$$= 1.52 \text{ (3 s.f.)} \quad (A1)$$

$$\text{gradient} = \frac{-0.63 + 0.2}{12 - 7} \quad (M1)$$

$$= -0.086$$

$$-k = -0.086$$

$$\therefore k = 0.086 \quad (A1)$$

- (c) Use your graph to estimate the time taken, to the nearest day, when at least 60% of the radioactive substance has decayed.

[2]

60% decayed — 40% present

$$40\% M = \frac{40}{100} \times 1.52196$$

$$= 0.6087$$

$$\ln 0.6087 = -0.49629$$

$$\approx -0.50 \quad (M1)$$

From the graph

$$t = 10.4 \text{ days}$$

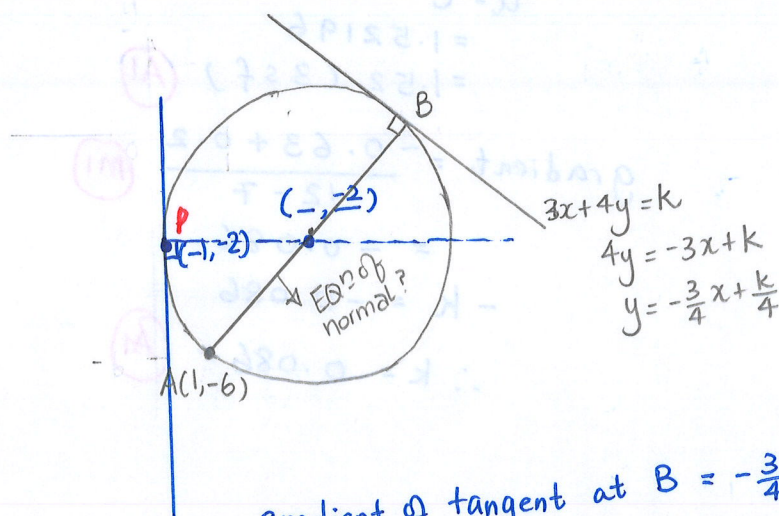
$$\therefore \text{time taken} = 11 \text{ days} \quad (A1)$$

(nearest day)

- 13 A circle has a diameter AB . The point A has coordinates $(1, -6)$ and the equation of the tangent to the circle at B is $3x + 4y = k$.

$$4y = -3x + k$$

- (a) Show that the equation of the normal to the circle at the point A is $4x - 3y = 22$. [3]



gradient of tangent at $B = -\frac{3}{4}$

gradient of normal at $B = \frac{4}{3} = \text{grad normal at A}$

Eqn of normal at $A: A(1, -6)$

$$y - y_1 = m(x - x_1)$$

$$y + 6 = \frac{4}{3}(x - 1)$$

$$y = \frac{4}{3}x - \frac{4}{3} - 6$$

$$y = \frac{4}{3}x - \frac{22}{3}$$

$$\frac{4}{3}x - y = \frac{22}{3}$$

$$\times 3, \quad 4x - 3y = 22 \text{ (shown)}$$

- (b) It is also given that line $x = -1$ touches the circle at the point $(-1, -2)$.
Find the coordinates of the centre and the radius of the circle.
Hence, state the equation of the circle.

[5]

Since $x = -1$ touches the circle at pt $(-1, -2)$,

So equation of normal at $(-1, -2)$ is

$y = -2$ & this passes thro' the centre of circle

$$y = -2 \quad \text{--- (1)}$$

$$4x - 3y = 22 \quad \text{--- (2)}$$

subst. (1) into (2),

$$4x - 3(-2) = 22$$

$$4x = 22 - 6$$

$$4x = 16$$

$$x = 4$$

$\therefore$ centre of circle is $(4, -2)$

C $(4, -2)$, A $(1, -6)$

$$\text{Radius} = \sqrt{(4-1)^2 + (-2+6)^2}$$

$$= \sqrt{9 + 16}$$

$$= 5 \text{ units}$$

$\therefore$ EQⁿ of circle:

$$(x-4)^2 + (y+2)^2 = 25$$

(c) Find the value of k .

[2]

$$A(1, -6), C(4, -2), B(m, n)$$

$$\text{Mid-point of } AB, M\left(\frac{1+m}{2}, \frac{-6+n}{2}\right) = (4, -2)$$

$$\frac{1+m}{2} = 4$$

$$1+m=8$$

$$m=7$$

$$\frac{-6+n}{2} = -2$$

$$-6+n=-4$$

$$n=2$$

$$\therefore B(7, 2) \quad \text{(mi)}$$

$$3x + 4y = k$$

$$\therefore k = 3(7) + 4(2)$$

$$= 29$$

$$\text{(AI)}$$

- END OF PAPER -

NAME	FORM CLASS	ACAD CLASS	INDEX NO.
Solutions			



ST. PATRICK'S SCHOOL PRELIMINARY EXAMINATION 2025

SUBJECT : Additional Mathematics **DATE : 21 August 2025**
(4049/02)

LEVEL : Secondary 4 Express **DURATION : 2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Form Class, Acad Class and Index No. in the spaces at the top of the page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value of 3.142, unless the equation requires the answer in terms of π

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90 .

For Examiner's Use			
Type of Penalties		Question Number	
Units	–1		
	–2		
Presentation	–1		
	–2		

This paper consists of 19 printed pages including this cover page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

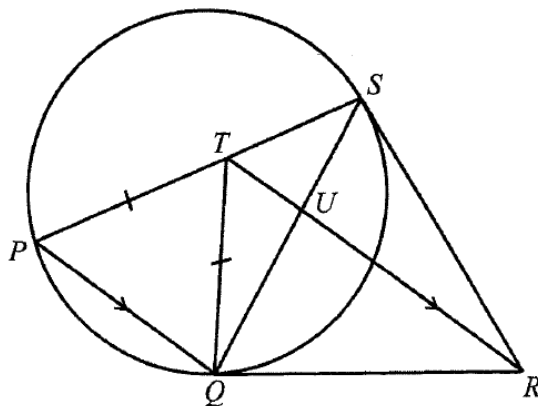
Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1



In the diagram, P , Q and S lie on a circle. The tangents to the circle at Q and S meet at R , and PQ is parallel to TR . SQ and TR intersect at U and $PT = QT$.

(a) Prove that $\triangle TQU$ is similar to $\triangle SRU$.

[5]

$$\begin{aligned}
 \text{Let } \angle TPQ &= \angle TQP = x, & (TP = TQ) \\
 \angle STR &= \angle TQ R = x & (\text{Cor. } \angle s, TR \parallel PQ) \quad m1 \\
 \angle QTU &= \angle TQP = x & (\text{alt } \angle s, TR \parallel PQ) \quad m1 \\
 \angle QSR &= \angle SPQ = x & (\text{alternate segment}) \quad m1 \\
 \angle QUT &= \angle RUS & (\text{vertically opposite}) \quad m1 \\
 \therefore \triangle TQU &\text{ is similar to } \triangle SRU \quad (AA) \quad m1
 \end{aligned}$$

(b) Hence, show that a circle can be drawn passing through Q , R , S and T .

[1]

$$\text{Since } \angle QTA = \angle QSA,$$

QR is a chord of circle $QRST$ ($\angle s$ in the same segment).

(c) What conclusion can you make of angle QTS and QRS ?

[1]

$$\angle QTS + \angle QRS = 180^\circ \quad (\text{Sum of } \angle s \text{ in opposite segment}).$$

- 2 (a) Solve the equation $\log_2 x^2 + \log_{16} x = -2.5$.

[3]

$$2 \log_2 x + \frac{\log_2 x}{\log_2 2^4} = -2.5 \quad m1$$

$$2 \log_2 x + \frac{\log_2 x}{4} = -2.5$$

$$8 \log_2 x + \log_2 x = -10$$

$$\log_2 x = -\frac{10}{9}$$

$$x = 2^{-\frac{10}{9}} \quad m1$$

$$= 0.463 \text{ (3 sf)} \quad \text{AI}$$

- (b) A cylinder of volume $(3\sqrt{7}-6)\pi \text{ cm}^3$ has a height of $(2+\sqrt{7})$ cm and a radius of r cm. Without using a calculator, obtain expression for r^2 in the form $(h+k\sqrt{7})$, where h and k are integers.

[4]

$$\pi r^2 (2+\sqrt{7}) = (3\sqrt{7}-6)\pi \quad m1$$

$$r^2 = \frac{3\sqrt{7}-6}{(2+\sqrt{7})} \times \frac{(2-\sqrt{7})}{(2-\sqrt{7})} \quad m1$$

$$= \frac{6\sqrt{7}-12-3(7)+6\sqrt{7}}{2^2-7} \quad m1$$

$$= \frac{-33+12\sqrt{7}}{-3}$$

$$= (11-4\sqrt{7}) \quad m1$$

- 3 A man suffering from insomnia took a sleeping pill at 9:30 p.m. The initial drug level in his blood is 6 mg per litre. After T hours, the amount of drug in his blood is N , where $N = 6(0.7)^T$ mg per litre.

Solutions

- (a) Calculate the amount of drug at midnight in per litre of blood.

[2]

$$\begin{aligned}
 N &= 6(0.7)^T \\
 T &= 2.5 & N &= 6(0.7)^{2.5} & \text{M1} \\
 & & &\approx 2.45978 \\
 & & &= 2.46 \text{ mg/l.} & \text{A1}
 \end{aligned}$$

- (b) It is found that when the amount of drug has fallen to below $\frac{3}{4}$ of the original level, the man would fall asleep.

Estimate the time, to the nearest minutes, that the man is expected to fall asleep. [3]

$$\begin{aligned}
 N &= 6(0.7)^T \\
 \frac{N}{6} &= \frac{3}{4} = 0.7^T \\
 \ln\left(\frac{3}{4}\right) &= T \ln 0.7 & \text{M1} \\
 T &= \frac{\ln\left(\frac{3}{4}\right)}{\ln 0.7} \\
 &\approx 0.80656 \text{ h } (\times 60) \\
 &\approx 48.394 & \text{M1} \\
 \text{Time} &= 9:30 + 49 \text{ min.} \\
 &= 10:19 \text{ pm} // & \text{A1}
 \end{aligned}$$

- 4 (a) Given that $(1+kx)^n = 1-12x+63x^2+\dots$, find the value of k and of n . [4]

$$1 + {}^nC_1(kx)^1 + {}^nC_2(kx)^2 = 1 - 12x + 63x^2$$

$$nk = -12$$

m1

$$k = -\frac{12}{n} \quad \text{--- ①}$$

$$\frac{n!}{2!(n-2)!} k^2 = 63$$

m1

$$n(n-1)k^2 = 126 \quad \text{--- ②}$$

Put ① into ②,

$$n(n-1)\left(-\frac{12}{n}\right)^2 = 126$$

$$\frac{144}{n}(n-1) = 126$$

$$144n - 144 = 126n$$

$$18n = 144$$

$$n = \frac{144}{18}$$

$$= 8$$

$$k = -\frac{12}{8}$$

$$= -1.5$$

$$n=8, \quad k=-1.5 \quad \underline{\underline{A2}}$$

(b) In the expansion of $(1-2x^3)\left(3x^2 + \frac{1}{2x}\right)^9$, find the term independent of x .

[5]
[4]

$$(3x^2 + \frac{1}{2x})^9$$

$$\begin{aligned} T_{r+1} &= {}^9C_r (3x^2)^{9-r} \left(\frac{1}{2x}\right)^r & m1 \\ &= {}^9C_r (3)^{9-r} x^{2(9-r)} \left(\frac{1}{2}\right)^r x^{-r} \\ &= {}^9C_r (3)^{9-r} \left(\frac{1}{2}\right)^r x^{18-2r-r} \end{aligned}$$

$$\begin{aligned} \text{For } x^0, \quad 18-3r &= 0 \\ r &= 6 & m1 \end{aligned}$$

$$\begin{aligned} \text{Independent term} &= {}^9C_6 (3)^{9-6} \left(\frac{1}{2}\right)^6 \\ &= \frac{567}{16} \end{aligned}$$

$$\begin{aligned} \text{For } x^{-3}, \quad 18-3r &= -3 \\ 2r &= 21 \\ r &= 7 & m1 \end{aligned}$$

$$\begin{aligned} T_{r+1} &= {}^9C_7 (3)^{9-7} \left(\frac{1}{2}\right)^7 x^{-3} \\ &= \frac{81}{32} x^{-3} \end{aligned}$$

$$\begin{aligned} \therefore \text{term independent of } x &= \frac{567}{16} + (-2) \frac{81}{32} & m1 \\ &= \frac{8991}{16} & A1 \end{aligned}$$

- 5 (a) A , B , and C are the angles of a triangle.

Show that $\cos C = -\cos(A+B)$

[2]

$$C = 180^\circ - (A+B)$$

$$\tan C = -\tan(A+B) \text{ (shown)}$$

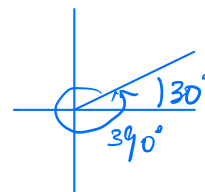


- (b) (i) State the exact value of $\cos(+390^\circ)$

[1]

$$\cos(390^\circ) = \cos(30^\circ)$$

$$= \frac{\sqrt{3}}{2}$$



- (ii) Hence, by using double angle formula, find the exact value of $\sin(+195^\circ)$.

[4]

$$\text{Let } 2x = 390^\circ$$

$$x = 195^\circ$$

$$\cos 2x = 1 - 2\sin^2 x \quad M1$$

$$2\sin^2 x = 1 - \cos 2x$$

$$\sin^2(195^\circ) = \frac{1 - \cos(390^\circ)}{2} \quad M1$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{2}$$

$$= \frac{2 - \sqrt{3}}{4}$$

$$\sin(195^\circ) = \frac{\sqrt{2 - \sqrt{3}}}{2} \approx -\frac{\sqrt{2 - \sqrt{3}}}{2}$$

(rej)



A2

- 6 (a) The graph of $y = a \sin bx + h$ has a minimum point at $\left(\frac{3\pi}{4}, -2\right)$ followed by a maximum point at $\left(\frac{5\pi}{4}, 8\right)$.

Find the values of the constants a , b and h .

[3]

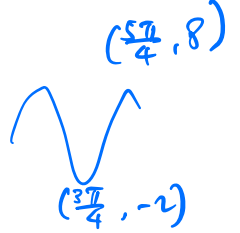
$$\text{Amplitude, } a = \frac{8 - (-2)}{2} = 5 \quad m1$$

$$\begin{aligned} \frac{1}{2} \text{ Period} &= \frac{5\pi}{4} - \frac{3\pi}{4} \\ &= \frac{\pi}{2} \end{aligned}$$

$$1 \text{ Period} = \pi$$

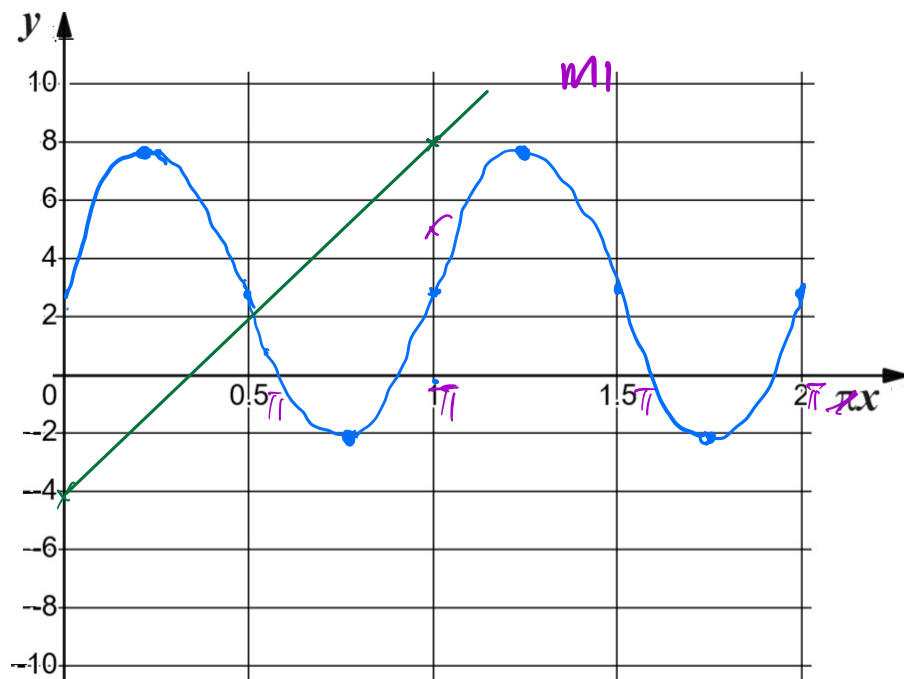
$$b = 2 \quad m1$$

$$\begin{aligned} h &= -2 + 5 \\ &= 3 \quad m1 \end{aligned}$$



- (b) Hence,
(i) sketch the graph of $y = a \sin bx + h$.

[2]



- (ii) By drawing a suitable straight line on the axes above, find the number of solutions for $a \sin bx + h = \pi x + 4$.

[2]

1 solution. A1

- 7 (a) Show that $(x+2)$ is a factor of $4x^3 - 3x^2 - 16x + 12$ and hence factorise $4x^3 - 3x^2 - 16x + 12$ completely. [5]

$$\text{let } f(x) = 4x^3 - 3x^2 - 16x + 12$$

$$f(-2) = 4(-2)^3 - 3(-2)^2 - 16(-2) + 12$$

$$= -32 - 12 + 32 + 12$$

$$= 0$$

By Factor theorem, $(x+2)$ is a factor of $f(x)$. m1

$$\begin{array}{r} 4x^2 - 11x + 6 \\ x+2 \overline{) 4x^3 - 3x^2 - 16x + 12} \\ \underline{4x^3 + 8x^2} \end{array}$$

$$-11x^2 - 16x$$

$$\underline{-11x^2 - 22x}$$

$$6x + 12$$

$$\underline{6x + 12}$$

$$f(x) = (x+2)(4x^2 - 11x + 6)$$

$$= (x+2)(x-2)(4x-3)$$

(b) Hence, solve the equation $4(9^y) + 12(3^{-y}) = 3^{y+1} + 16$.

[5]

$$4(3^{2y}) + \frac{12}{3^y} = 3(3^y) + 16.$$

M1

$$4(3^y)^3 + 12 = 3(3^y)^2 + 16(3^y)$$

$$4(3^y)^3 - 3(3^y)^2 - 16(3^y) + 12 = 0$$

M1

$$(3^y + 2)(3^y - 2)(4(3^y) - 3) = 0.$$

M1

$$3^y = -2 \text{ (rej)}$$

$$3^y = 2$$

$$y = \frac{\ln 2}{\ln 3}$$

$$= 0.631$$

$$3^y = \frac{3}{4}$$

$$y = \frac{\ln(\frac{3}{4})}{\ln 3}$$

$$= -0.262$$

(3 sf)

A2

- 8 A particle, travelling in a straight line, has velocity, v m/s, given by $v = 4\cos 3t + 2$, where t is the time, in seconds, after passing through O.

(a) Find the first two instances when the particle changes direction of motion. [3]

$$v = 4\cos 3t + 2 = 0.$$

M1

$$\cos 3t = -\frac{2}{4} = -\frac{1}{2}$$

$$\text{Reference } \angle = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3}$$

$$3t = \pi - \frac{\pi}{3}, \quad \pi + \frac{\pi}{3}.$$

$$t = \frac{2\pi}{9}, \quad \frac{4\pi}{9}$$

A2

- (b) Find the total distance travelled by the particle from $t = 0$ s to $t = \cancel{\pi/2}$ s. $\frac{\pi}{2}$ s. [6]

$$s = \int 4\cos 3t + 2 \, dt$$

$$= \frac{4}{3}\sin 3t + 2t + C$$

$$s(0) = 0 + 0 + C = 0$$

$$\therefore s = \frac{4}{3}\sin 3t + 2t \quad \text{M1}$$

$$s\left(\frac{2\pi}{9}\right) = \frac{4}{3}\sin\frac{2\pi}{3} + \frac{4\pi}{9}$$

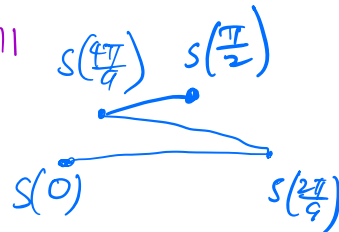
$$\approx 2.55096 \, \text{m} \quad \text{M1}$$

$$s\left(\frac{4\pi}{9}\right) = \frac{4}{3}\sin\left(\frac{4\pi}{3}\right) + \frac{8\pi}{9}$$

$$\approx 1.63782 \quad \text{M1}$$

$$s\left(\frac{\pi}{2}\right) = \frac{4}{3}\sin\frac{3\pi}{2} + 2\left(\frac{\pi}{2}\right)$$

$$= -\frac{4}{3} + \pi \quad \text{M1}$$

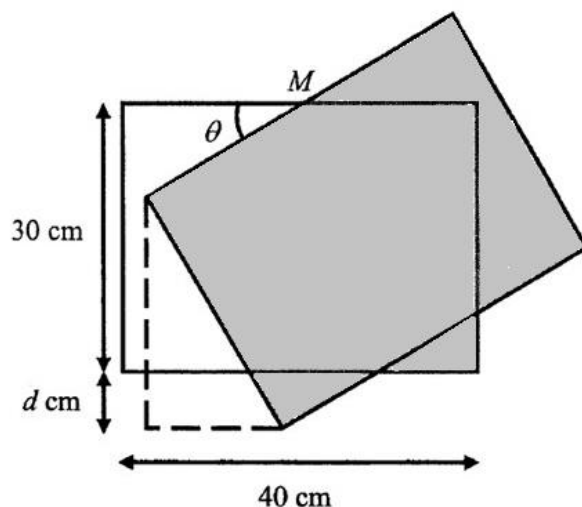


$$\text{Total distance} = s\left(\frac{2\pi}{9}\right) - s(0) + s\left(\frac{2\pi}{9}\right) - s\left(\frac{4\pi}{9}\right) + s\left(\frac{\pi}{2}\right) - s\left(\frac{4\pi}{9}\right)$$

$$= 2s\left(\frac{2\pi}{9}\right) + s\left(\frac{\pi}{2}\right) - 2s\left(\frac{4\pi}{9}\right)$$

$$= 3.63 \quad \text{A1}$$

9



The diagram shows a rectangular picture frame 40 cm by 30 cm hung on the wall. The picture frame is rotated through an angle θ about the midpoint, M of the top edge of the frame.

- (a) Show that the vertical displacement, d cm, of the picture frame below its original bottom edge is given by $d = 20 \sin \theta + 30 \cos \theta - 30$. [3]

$$\begin{aligned} \triangle ABM \quad \sin \theta &= \frac{AB}{20} & AB &= 20 \sin \theta \\ \triangle BCD \quad \cos \theta &= \frac{BC}{30} \quad m1 & BC &= 30 \cos \theta \quad m1 \\ d &= AB + BC - 30 \\ &= 20 \sin \theta + 30 \cos \theta - 30 \quad A1 \end{aligned}$$

- (b) Express d in the form of $R \sin(\theta + \alpha) - 30$, where $R > 0$ and $0 < \alpha < 90^\circ$. [3]

$$\begin{aligned} R &= \sqrt{20^2 + 30^2} \\ &= \sqrt{1300} \quad m1 \\ \alpha &= \tan^{-1}\left(\frac{30}{20}\right) \\ &= 56.3099^\circ \quad m1 \\ d &= \sqrt{1300} \sin(\theta + 56.3^\circ) - 30 \quad A1 \end{aligned}$$

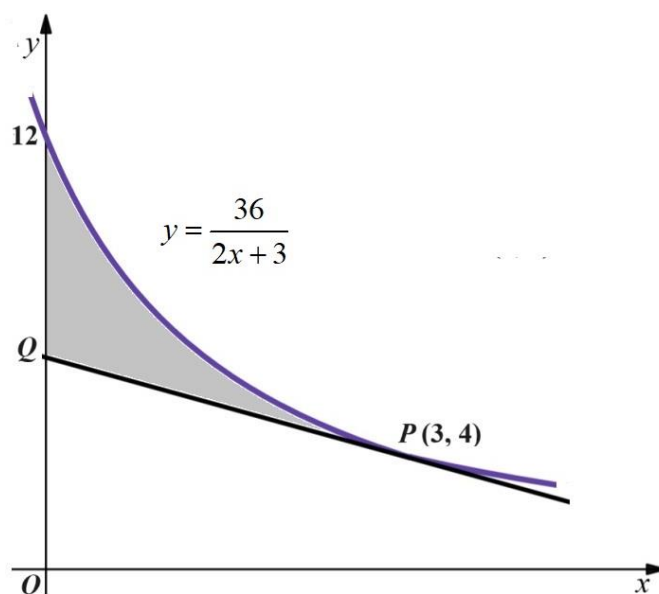
- (c) Find the maximum value of d and the corresponding value of θ .

[2]

$$\begin{aligned} \text{Max } d &= \sqrt{1300} - 30 \\ &= 6.06 \text{ m (3 sf)} \end{aligned} \quad A1$$

$$\begin{aligned} \theta + 56.3099^\circ &= 90^\circ \\ \theta &= 33.7^\circ \text{ (1 dp)} \end{aligned} \quad A1$$

10



The diagram shows part of the curve $y = \frac{36}{2x+3}$ and its y -intercept is 12.

The tangent to the curve at the point $P(3, 4)$ intersects the y -axis at Q .
Find the exact area of the shaded region.

[10]

Ans : $18\ln 3 - 6$

$$y = 36(2x+3)^{-1}$$

$$\frac{dy}{dx} = -36(2x+3)^{-2}(2)$$

$$= -\frac{72}{(2x+3)^2}$$

m1

$P(3, 4)$, $\frac{dy}{dx} = -\frac{72}{(6+3)^2}$

$$= -\frac{72}{81} = -\frac{8}{9}$$

m1

Equation of PQ: $y = -\frac{8}{9}x + c$

$$4 = -\frac{8}{9}(3) + c$$

$$c = 4 + \frac{8}{3}$$

$$= \frac{20}{3}$$

$$y = -\frac{8}{9}x + \frac{20}{3}$$

$Q(0, \frac{20}{3})$ m1

$$\text{Shaded Area} = \int_0^3 \frac{36}{(2x+3)} - \left[-\frac{8}{9}x + \frac{20}{3} \right] dx \quad M2$$

$$= \int_0^3 18 \left(\frac{2}{2x+3} \right) + \frac{8}{9}x - \frac{20}{3} dx \quad M1$$

$$= \left[18 \ln(2x+3) + \frac{4x^2}{9} - \frac{20}{3}x \right]_0^3 \quad M1$$

$$= [18 \ln 9 + 4 - 20] - [18 \ln 3 + 0] \quad M1$$

$$= 18(\ln 9 - \ln 3) - 16 \quad M1$$

$$= 18 \ln 3 - 16 \quad A1$$

OR

$$\text{Area under curve} = \int_0^3 \frac{36}{2x+3} dx$$

$$= 18 \int_0^3 \frac{2}{2x+3} dx$$

$$= 18 [\ln(2x+3)]_0^3$$

$$= 18 [\ln 9 - \ln 3]$$

$$= 18 \ln 3 \text{ unit}^2$$

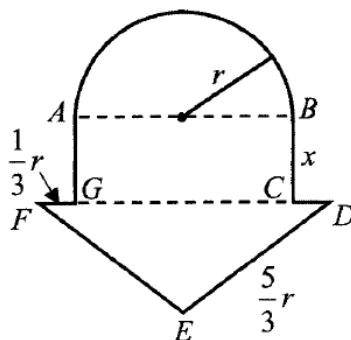
$$\text{Area of trapezium, OQPR} = \frac{1}{2} \left(\frac{20}{3} + 4 \right) (3)$$

$$= \frac{1}{2} \left(\frac{32}{3} \right) (3)$$

$$= 16 \text{ unit}^2$$

$$\therefore \text{Shaded area} = (18 \ln 3 - 16) \text{ unit}^2$$

11



A baker uses 131 cm of wire to enclose a cake mould of the shape shown in the diagram. The shape consists of a semicircle with diameter AB , a rectangle $ABCG$ and an isosceles triangle FED such that $FE = ED$.

It is given that $AB = 2r$ cm, $BC = x$ cm, $ED = \frac{5}{3}r$ cm and $FG = CD = \frac{1}{3}r$ cm.

(a) Express x in terms of r and π .

[3]

$$\begin{aligned}\pi r + 2x + \frac{2}{3}r + \frac{10}{3}r &= 131 \\ 2x &= 131 - 4r - \pi r \\ x &= \frac{131 - (4 + \pi)r}{2}\end{aligned}$$

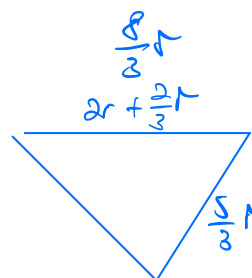
(b) Show that the area of the mould, A cm², is given by $A = 131r - \frac{\pi}{2}r^2 - \frac{8}{3}r^2$. [4]

$$\text{Area} = \frac{\pi r^2}{2} + 2rx + \frac{1}{2}\left(\frac{8}{3}r\right)r \quad M1$$

$$= \frac{\pi r^2}{2} + r[131 - (4 + \pi)r] + \frac{4}{3}r^2$$

$$= \frac{\pi r^2}{2} + 131r - 4r^2 - \pi r^2 + \frac{4}{3}r^2 \quad M2$$

$$= 131r - \frac{\pi r^2}{2} - \frac{8}{3}r^2 \quad (\text{shown}) \quad A1$$



$$\frac{28}{9} - \frac{16}{9} = \frac{12}{9}$$

(c) Given that r can vary, find the stationary of A and determine its nature.

[5]

$$P = 131r - \frac{\pi r^2}{2} - \frac{8}{3}r^2$$

$$\frac{dP}{dr} = 131 - \pi r - \frac{16}{3}r$$

M1

for stationary value, $\frac{dP}{dr} = 0$

$$r\left(\pi + \frac{16}{3}\right) = 131$$

M1

$$r = \frac{131}{\pi + \frac{16}{3}}$$

$$= 15.457$$

$$= 15.5 \text{ cm (3 sf)}$$

M1

$$\frac{d^2P}{dr^2} = -\pi$$

M1

$$\therefore \max A = 131r - \frac{\pi r^2}{2} - \frac{8}{3}r^2$$

$$\approx 3037.37$$

$$= 3040 \text{ cm}^2 \text{ (3 sf)} \quad A1$$