Amath practise prelim yay /70

1. *It is given that* $f(x)=4x^3+mx^2+nx-3$, *where* m *and* n *are constants. It is also given that* $(x-1)$ *is a factor of* $f(x)$ *and leaves a remainder of* 2 *when divided by* $(x+1)$. *Find the values of* m *and* n. [5]

2. *Evaluate* $\int_1^2 x^2+1-\frac{2}{x^2}\,\mathrm{d}x$. [3]

3. *Given that* $y = 3x^3 - 4x - 4$,

a) *Find the* expression *for* $\frac{dy}{dx}$ [1]

b) *Find the coordinates of the two stationary points* [3]

c) Determine the nature for these two stationary points. [3]

4. a) Prove the identity $(\mathrm{sinx} + \mathrm{cosx})^2 = 1 + \mathrm{sin2x}$ [2]

b) Hence, solve the equation $(\sin x + \cos x)^2 = \frac{3}{2}$, for $0^\circ \leq x \leq 360^\circ$ [3]

5. The line $x - y = 3$ intersects the curve $x^2 + y^2 - 3xy + 19 = 0$ at the points P and Q. Find the coordinates of the midpoint of PQ [6]

6. A stone is thrown vertically upwards such that its height, h metres from the ground at time t seconds after being thrown is given by the formula $h = -9t^2 + 12t + 1$.

a) Find the height of the stone when it is not thrown yet. [1]

b) Express h in the form $a(t+b)^2 + c$, where a, b and c are constants to be determined. [3]

c) Hence, find the maximum height of the stone when it is thrown and the time which it occurs. [2]

7. A right angled triangle has a base length of $(4-2\sqrt{2})$ cm and an area of $(16+6\sqrt{2})$ cm^2. Without using a calculator, find the perpendicular height of the triangle in the form of $(a+b\sqrt{c})$ cm. [4]

8. a) Find $\frac{d}{dx}[(3x+2)\sqrt{x+3}]$. [3]

b) Hence, evaluate $\int_1^3 \frac{9x+22}{2\sqrt{x+3}}\,dx$. [4]

9. $2y = x + 14$ is a tangent to the circle C. Given that the centre of C is at $(4, 4)$, find the equation of the circle C. [5]

10. It is given that $\frac{d^2y}{dx^2} = 12x^2 - 6$, and the gradient of the curve that passes through the point (1, 5) is 7. Find the equation of the curve. [5]

11. a) Factorise $8x^3 - 125$ completely. [2]

b) Hence explain why $8x^3 - 125 = 0$ has only one real solution. [2]

12. Express $\dfrac{8x^3 - x^2 - 12x + 12}{x^2 - 9}$ in partial fractions. [6]

13. A curve is defined as $y = 3\sin\frac{x}{2} + 1$.

a) State the amplitude and period of y. [2]

b) Sketch the graph of $y = 3sin\frac{x}{2} + 1$ for $0 \leq x \leq 4\pi$ [3]

c) By drawing a suitable line on your graph, determine the number of solutions of

$3sin\frac{x}{2} = -\frac{1}{2}x + 2$ [2]