TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ______________________________

CIVICS GROUP: ______________________________

H2 FURTHER MATHEMATICS 9649/01

Paper 1

18 SEPTEMBER 2025

3 hours

Additional materials: Printed Answer Booklet

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer all the questions.

Write your answers in the spaces provided in the Printed Answer Booklet. Follow the instructions on the front cover of the Printed Answer Booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

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1 The terms in the sequence $\{u_n\}$ satisfy the recurrence relation

$$u_n = \tfrac{1}{2}u_{n-1} + 3$$

(a) Find an expression for u_n in terms of n in the case that $u_0 = 1$. [4]

(b) Describe the behaviour of the sequence as n tends to infinity. [2]

2 **(a)** Suppose S_1 and S_2 are the column spaces of matrix $\mathbf{A}_1$ and $\mathbf{A}_2$ where

$$\mathbf{A}_1 = \begin{pmatrix} 1 & -1 & a \\ 0 & 0 & b \\ 0 & 0 & c \end{pmatrix} \quad \text{and} \quad \mathbf{A}_2 = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 3 & 0 \\ 0 & 1 & c \end{pmatrix}.$$

Determine the possible values of a, b and c such that S_1 is a subspace of S_2. [4]

(b) Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for $\mathbb{R}^3$ and $\mathbf{N}$ be a non-singular 3×3 matrix with real entries. Determine whether the set $\{\mathbf{N}\mathbf{v}_1, \mathbf{N}\mathbf{v}_2, \mathbf{N}\mathbf{v}_3\}$ is a basis for $\mathbb{R}^3$. [3]

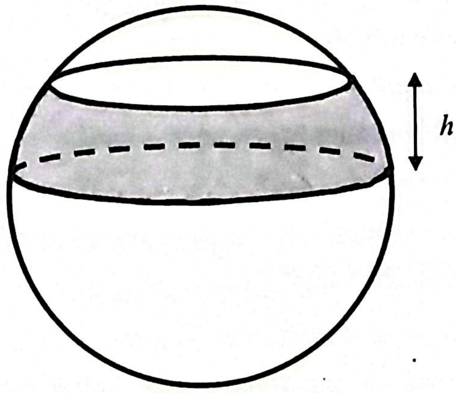
3 The complex number z satisfies $|\mathrm{i}z + 6 + \mathrm{i}| < |z - 5 - 6\mathrm{i}|$ and $\left|\frac{1}{2}z - 3 + 4\mathrm{i}\right| \le 5$.

(a) Sketch the locus of z. [3]

(b) Find the range of values of $\arg(z - 4\mathrm{i})$. [4]

4 The tangent plane to the point (a,b,c) on the surface with equation $(x-3)^2 + (y-5)^2 + z^2 = 4$ is a subspace of $\mathbb{R}^3$ where $a \ne 3, b \ne 5, c \ne 0$. Find the range of values of a. [6]

5 A spherical zone is the portion of the curved surface area of a sphere between two parallel planes h units apart as shown in the diagram below.



By using a suitable curve and rotation axis, explain if the location of the two planes will affect the curved surface area of the spherical zone. [8]

6 The region enclosed by the curve $y = x\ln x$, the $x-$axis and the vertical line $x = 2$ is rotated by 2π radians about the $y-$axis to form a solid.

(a) Show that the volume of the solid is $\int_1^2 2\pi g(x)\,dx$, where $g(x)$ is a function to be determined. Hence, find the exact volume of the solid. [4]

(b) Using Simpson's Rule with $n = 4$ strips, estimate the value of the integral $\int_1^2 2\pi g(x)\,dx$ up to 4 decimal places. You may assume that the intervals are equally spaced. [2]

(c) Calculate the percentage error in your answer in part **(b)**. [1]

(d) Without performing the calculations to find an approximation using the Trapezium Rule, explain with reasoning whether your answer will be an over-estimate or under-estimate. [2]

7 A differential equation is given by $\cos x\frac{dy}{dx} - y\sin x = 2\cos x$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) Show that the general solution of the differential equation is $y = 2\tan x + C\sec x$, where C is the arbitrary constant. [2]

(b) Explain why the solution curves have turning points when $C < -k$ or when $C > k$, where k is a constant to be determined. [4]

(c) On the same diagram, sketch the family of solution curves where $C \neq -k$ or k. [3]

8 **(a)** Let $z = \cos\theta + \mathrm{i}\sin\theta$. By considering z^5, show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta. \qquad [3]$$

(b) Using the result in part **(a)**,

(i) find the distinct real roots to the equation, giving your answer, where appropriate, in exact trigonometric form

$$16x^5 - 20x^3 + 5x + 1 = 0. \qquad [3]$$

(ii) find the exact value of $\tan^2\left(\frac{\pi}{5}\right)$. [4]

9 The polar curves C_1 and C_2 are given by the following equations in the respective intervals.

$$C_1: \; r = 2 + 2\sin\theta, \; 0 \le \theta < 2\pi$$
$$C_2: \; r = -2\sin\theta, \; \pi \le \theta < 2\pi$$

(a) On a single diagram, sketch C_1 and C_2. [4]

The region R is enclosed by the polar curves C_1 and C_2 in the 3rd and 4th quadrants.

(b) Show that the perimeter of the region R can be expressed as

$$4\int_p^q \sqrt{2 + 2\sin\theta}\; \mathrm{d}\theta + \frac{2\pi}{3}$$

where p and q are exact real numbers to be determined.

Evaluate this integral in exact form. [7]

[You may use the identity "$1 + \sin\theta = 2\sin^2\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$" without proof.]

10 In a population of plants, a particular gene has two versions (called alleles): A and a. A genotype is a combination of two alleles. Hence, for this particular gene, individual plants can have one of three genotypes: AA, Aa or aa. The order of the sequence of genotypes does not matter.

To breed a new plant, two plants, termed as its parents, are used. The new plant will inherit one gene from each of its parents' pairs of genes to form its own particular pair. For example, if one parent is of genotype Aa, it is equally likely that the new plant will inherit an A gene or an a gene from that parent, and if the other parent is of genotype aa, the new plant will definitely inherit an a gene from that parent. Thus, the new plant will either be of genotype Aa or aa.

Suppose a farmer has a large population of plants consisting of some distribution of all three possible genotypes AA, Aa and aa. He starts a breeding programme where each plant in the population is always bred with a plant with genotvpe Aa to produce the next generation of plants.

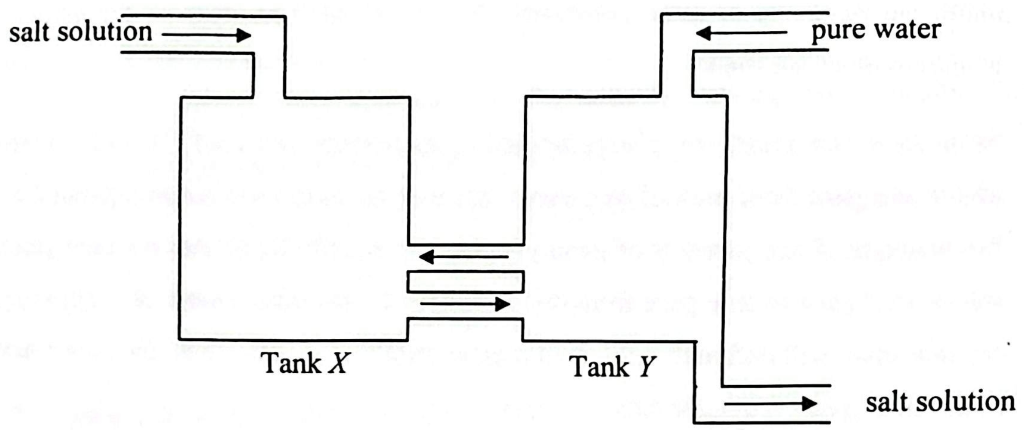
Let the vector $\begin{pmatrix} p_i \\ q_i \\ r_i \end{pmatrix}$ represent the proportion of plants with genotype AA, Aa and aa respectively in generation i, and T be the transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ that maps $\begin{pmatrix} p_i \\ q_i \\ r_i \end{pmatrix}$ to $\begin{pmatrix} p_{i+1} \\ q_{i+1} \\ r_{i+1} \end{pmatrix}$.

(a) Show that $T:\begin{pmatrix} p_i \\ q_i \\ r_i \end{pmatrix} \mapsto \begin{pmatrix} 2\beta & \beta & 0 \\ 2\beta & 2\beta & 2\beta \\ 0 & \beta & 2\beta \end{pmatrix}\begin{pmatrix} p_i \\ q_i \\ r_i \end{pmatrix}$, where β is a non-zero constant to be determined and find the nullity of T. [4]

(b) Find the eigenvalues of T and their corresponding eigenvectors of T. [7]

(c) By diagonalising the matrix representing T, find the proportion of the plants in the long run. [4]

11 Two tanks are filled with salt water and are interconnected by pipes as shown below.



Tank X and Tank Y initially contain 600 litres and 500 litres of salt solution respectively.

Salt water with a concentration of 1 gram per litre of salt enters Tank X at a rate of 2 litres per minute. Pure water enters Tank Y at a rate of 13 litres per minute. Through the connecting pipes, the salt solution in Tank X flows into Tank Y at 12 litres per minute. To keep the levels of the tanks the same, salt solution in Tank Y flows into Tank X at 10 litres per minute, and drains out at 15 litres per minute. The amount of salt in Tank X and Y are denoted as x grams and y grams respectively.

(a) Show that $\dfrac{dx}{dt} = 2 - \dfrac{1}{50}x + \dfrac{1}{50}y$ and $\dfrac{dy}{dt} = \dfrac{1}{50}x - \dfrac{1}{20}y$. [2]

(b) Find a second order differential equation involving y. [3]

(c) Find the general solution of the differential equation found in part **(b)**. [4]

(d) Find the ratio of the amount of salt in tank X to the amount of salt in tank Y in the long run. [3]

End of Paper