RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 2B: Vectors II – Equations of Straight Lines

Section A (Basic Questions)

1 For each of the following, write down a vector equation of the line l and convert it to Cartesian form.

(a) l passes through the point with position vector $-\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and is parallel to the vector $\mathbf{i}+\mathbf{j}$.

(b) l passes through the points $P(1,-1,3)$ and $Q(2,1,-2)$.

(c) l passes through the origin O and is parallel to the line $m:\mathbf{r}=\begin{pmatrix}1\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\3\end{pmatrix},\quad \lambda\in\mathbb{R}$.

(d) l passes through the point $C(1,0,1)$ and is parallel to the $y-$axis.

[(a) $x+1=y-2,\ z=1$ **(b)** $x-1=\dfrac{y+1}{2}=\dfrac{3-z}{5}$ **(c)** $x=\dfrac{y}{2}=\dfrac{z}{3}$ **(d)** $x=1,\ z=1$ **]**

2 For each of the following, find the acute angle between the lines l_1 and l_2.
Determine if l_1 and l_2 are parallel, intersecting or skew.
In the case of intersecting lines, find the position vector of the point of intersection.

(a) $l_1: x-1=-y=z-2$ and $l_2:\dfrac{x-2}{2}=-\dfrac{y+1}{2}=\dfrac{z-3}{2}$

(b) $l_1:\mathbf{r}=\begin{pmatrix}1\\0\\0\end{pmatrix}+\alpha\begin{pmatrix}4\\-2\\-3\end{pmatrix},\quad \alpha\in\mathbb{R}$ and $l_2:\mathbf{r}=\begin{pmatrix}0\\10\\1\end{pmatrix}+\beta\begin{pmatrix}3\\8\\1\end{pmatrix},\quad \beta\in\mathbb{R}$

(c) $l_1:\mathbf{r}=(\mathbf{i}-5\mathbf{k})+\lambda(\mathbf{i}-\mathbf{j}+\mathbf{k}),\quad \lambda\in\mathbb{R}$ and $l_2:\mathbf{r}=(\mathbf{i}-\mathbf{j}+\mathbf{k})+\mu(5\mathbf{i}-4\mathbf{j}-\mathbf{k}),\quad \mu\in\mathbb{R}$

[(a) $0°$, parallel **(b)** $81.3°$, skew **(c)** $44.5°$, intersecting, $\begin{pmatrix}6\\-5\\0\end{pmatrix}$ **]**

Section B (Discussion Questions)

1 The points P and Q have coordinates $(0,-1,-1)$ and $(3,0,1)$ respectively, and the equations of the lines l_1 and l_2 are given by

$$l_1:\mathbf{r}=\begin{pmatrix}0\\1\\-3\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\-1\end{pmatrix},\ \lambda\in\mathbb{R} \qquad \text{and} \qquad l_2:\mathbf{r}=\begin{pmatrix}-3\\3\\1\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\0\end{pmatrix},\ \mu\in\mathbb{R}.$$

(a) (i) Show that P lies on l_1 but does not lie on l_2.

(ii) Determine whether the lines l_1 and l_2 passes through Q.

(b) (i) Find the coordinates of the foot of perpendicular from P to l_2.

Hence, or otherwise, find the perpendicular distance from P to l_2.

(ii) Using **(b)(i)** or otherwise, find the length of projection of $\overrightarrow{PQ}$ onto l_2.

[(a)(ii) Q is on l_2 but not l_1 **(b)(i)** $(1,1,1)$, 3 units **(b)(ii)** $\sqrt{5}$ units**]**

2 The lines l and m are defined by the equations

$$l:\mathbf{r}=\mathbf{i}-\mathbf{k}+\lambda(2\mathbf{i}-6\mathbf{j}+3\mathbf{k}),$$
$$m:\frac{x-1}{4}=\frac{a-y}{a}=\frac{z+3}{4}.$$

(i) Given that the lines intersect, show that $a=6$. [2]

(ii) Find the position vector of N, the foot of perpendicular from the point $A(5,0,1)$ to the line l. [3]

(iii) Hence or otherwise, find the position vector of the two points on l that are 5 units from A. [3]

[(ii) $\frac{1}{7}\begin{pmatrix}11\\-12\\-1\end{pmatrix}$ **(iii)** $\frac{1}{7}\begin{pmatrix}17\\-30\\8\end{pmatrix}$, $\frac{1}{7}\begin{pmatrix}5\\6\\-10\end{pmatrix}$ **]**

3 The points A and B have position vectors $\mathbf{j}+2\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ respectively.

(i) Find a vector equation of the line l passing through the midpoint, M, of AB and the origin O.

(ii) Find the position vector of a point N where N is the foot of the perpendicular from A to l.

(iii) Find the position vector of a point C such that $AOCM$ is a parallelogram.

(iv) Show that the points A, N and C are collinear.

State, with a geometrical reason, the value of $\frac{CO}{CM}$.

[(i) $l:\mathbf{r}=t\begin{pmatrix}1\\1\\0\end{pmatrix},\ t\in\mathbb{R}$ **(ii)** $\overrightarrow{ON}=\frac{1}{2}\begin{pmatrix}1\\1\\0\end{pmatrix}$ **(iii)** $\overrightarrow{OC}=\begin{pmatrix}1\\0\\-2\end{pmatrix}$ **(iv)** 1**]**

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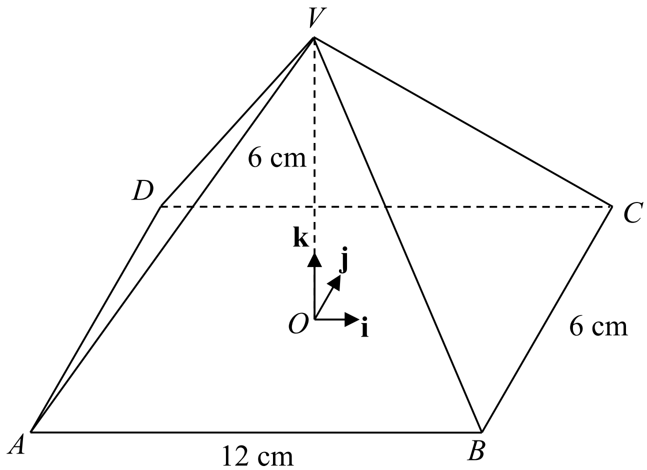


In the diagram, O is centre of the rectangular base $ABCD$ of a right pyramid with vertex V.
Perpendicular unit vectors **i**, **j**, **k** are parallel to AB, BC and OV respectively. The length of AB, BC and OV are 12 cm, 6 cm and 6 cm respectively.

A line l has cartesian equation $\dfrac{-x-4}{10} = y+2 = \dfrac{t-z}{-2}$.

(i) Find the vector equation of line AV. [2]

(ii) If the line l intersects line AV at M, find the position vector of M and the value of t. [5]

(iii) Find the acute angle between line AV and the line l. Hence find the perpendicular distance from A to the line l. [5]

[(i) $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$, $\lambda \in \mathbb{R}$ **(ii)** $\overrightarrow{OM} = \begin{pmatrix} -4 \\ -2 \\ 2 \end{pmatrix}$, $t = 2$ **(iii)** $60.8°$, 2.62**]**

5 Referred to the origin O, points A and B have position vectors **a** and **b** respectively. Point C lies on OA, between O and A, such that $OC : CA = 3 : 2$. Point D lies on OB, between O and B, such that $OD : DB = 5 : 6$.

(i) Find the position vectors $\overrightarrow{OC}$ and $\overrightarrow{OD}$, giving your answers in terms of **a** and **b**. [2]

(ii) Show that the vector equation of the line BC can be written as $\mathbf{r} = \frac{3}{5}\lambda\mathbf{a} + (1-\lambda)\mathbf{b}$, where λ is a parameter. Find in a similar form the vector equation of the line AD in terms of a parameter μ. [3]

(iii) Find, in terms of **a** and **b**, the position vector of the point E where the lines BC and AD meet and find the ratio $AE : ED$. [5]

[(i) $\frac{3}{5}\mathbf{a}, \frac{5}{11}\mathbf{b}$ **(ii)** $\mathbf{r} = \frac{5}{11}\mu\mathbf{b} + (1-\mu)\mathbf{a}$ **(iii)** $\frac{9}{20}\mathbf{a} + \frac{1}{4}\mathbf{b}$, $11 : 9$ **]**

6 When referred to the origin O, the points A and B have position vectors $5\mathbf{j}+15\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+3\mathbf{k}$ respectively.

The line l_1 has equation $\mathbf{r}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}$, $\lambda\in\mathbb{R}$.

(i) Find a vector equation of line l_2 passing through the points A and B. [2]

(ii) Find the coordinates of the point C, where l_1 and l_2 intersect. [3]

(iii) Find the position vector of the point F, the foot of the perpendicular from A to the line l_1. [4]

(iv) Find the vector equation of the line of reflection of l_2 in the line l_1. [3]

[(i) $\mathbf{r}=\begin{pmatrix}0\\5\\15\end{pmatrix}+\mu\begin{pmatrix}1\\-6\\-12\end{pmatrix}$, $\mu\in\mathbb{R}$ **(ii)** $(2,-7,-9)$ **(iii)** $\begin{pmatrix}-5\\7\\12\end{pmatrix}$**, (iv)** $\mathbf{r}=\begin{pmatrix}2\\-7\\-9\end{pmatrix}+\alpha\begin{pmatrix}-6\\8\\9\end{pmatrix}$, $\alpha\in\mathbb{R}$ **]**

7 One day, Eddie came home from a birthday party and brought back a helium filled balloon. After playing with it, he accidentally released the balloon at the point (1, 2, 3) and it floated vertically upwards (in a path parallel to the vector $\mathbf{k}$) at a speed of 1 unit per second. t seconds later, a sudden gust of wind caused the balloon to move in the direction of $\mathbf{i}+4\mathbf{j}+6\mathbf{k}$.

(i) Find the angle in which the balloon has changed in direction after the gust of wind blew it away. [3]

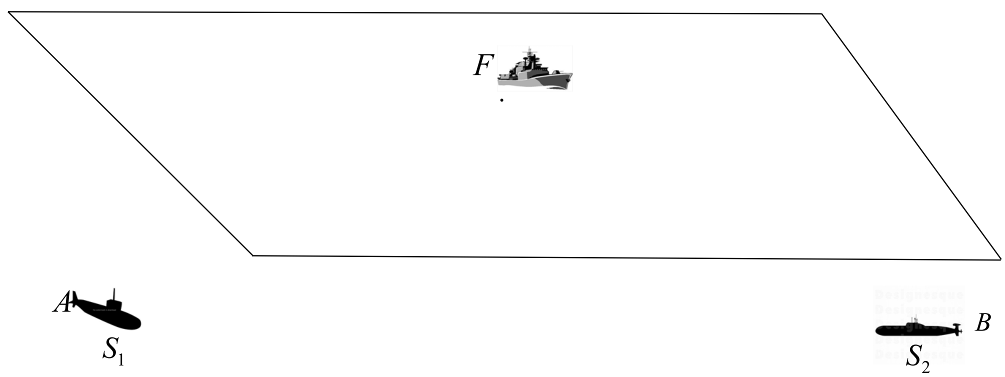
(ii) Given that the balloon eventually stayed at the point (2, 6, 12) on the ceiling, find the time t when the gust of wind blew the balloon away. [3]

Eddie decides to shoot the balloon down with his catapult.

(iii) Assume he was holding his catapult at (3, 2, 1) initially and he walked along the path parallel to $2\mathbf{i}+\mathbf{j}$. Find the position vector of the point where he should place his catapult so that the distance between his catapult and the balloon is at its minimum. Hence find this distance. [4]

[(i) 34.5^o **(ii)** $t=3$ **(iii)** $\frac{1}{5}\begin{pmatrix}19\\12\\5\end{pmatrix}$, 11.7]

8 A frigate is stationed at position $F(1,2,0)$. Two submarines S_1 and S_2 are under the sea surface. Submarine S_1 is at position $A(-2,-1,-1)$ and travelling in a path parallel to vector $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. An enemy submarine S_2 is detected at position $B(3,2,-2)$ travelling in a path parallel to vector $-2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.



(i) Determine if the paths of the submarines will intersect each other. [3]

(ii) The enemy submarine S_2 will launch a torpedo at the frigate when it is at a point P in its path that is closest to F. Find the co-ordinates of P. [4]

(iii) A depth charge is a countermeasure used against submarines. The frigate releases a depth charge which descends to the position $Q(1, 2, -k)$ to target S_2. The depth charge is detonated when the distance from Q to P is at a minimum. Find the value of k. [1]
When detonated the depth charge will cause damage within a 0.1 unit radius.
Explain whether S_2 will be damaged by the depth charge. [2]

[**(ii)** $P\left(\frac{15}{7},\frac{5}{7},-\frac{11}{7}\right)$ **(iii)** $k=\frac{11}{7}$]

THE END