RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 2A: Vectors I
Vector Algebra, Ratio Theorem, Scalar and Vector Products

1 Referred to an origin O, points A and B have position vectors given respectively by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} \text{ and } \overrightarrow{OB} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}.$$

The point P on AB is such that $AP : PB = \lambda : 1 - \lambda$. Show that

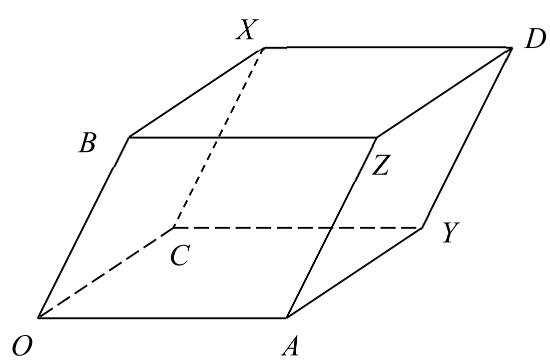
$$\overrightarrow{OP} = (1+\lambda)\mathbf{i} + (2-5\lambda)\mathbf{j} + (-2+8\lambda)\mathbf{k}.$$

(i) Find the value of λ for which OP is perpendicular to AB.

(ii) Find the value of λ for which angles AOP and POB are equal.

[(i) $\frac{5}{18}$ **(ii)** $\frac{3}{10}$**]**

2 Referring to an origin O, the position vectors of points A, B and C are given by $\overrightarrow{OA} = 7\mathbf{i}$, $\overrightarrow{OB} = \mathbf{i} + 4\mathbf{k}$ and $\overrightarrow{OC} = \mathbf{i} + 4\mathbf{j}$ respectively. A parallelepiped has OA, OB, OC as three edges, and the remaining vertices are X, Y, Z and D as shown in the diagram.



(i) Write down the position vector of Z in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

(ii) The point P divides CZ such that $\frac{CP}{PZ} = \lambda$. Given that $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{CZ}$, find the value of λ and evaluate $\left|\overrightarrow{OP}\right|$. [6]

[(i) $8\mathbf{i} + 4\mathbf{k}$ **(ii)** 4 **]**

3 The position vectors of A and B with respect to the origin O are $\mathbf{a}$ and $\mathbf{b}$ respectively. M is the mid-point of OA and C is the point on MB such that $\overrightarrow{CB} = 2\overrightarrow{MC}$.

Given that $|\mathbf{a}| = 6$, $|\mathbf{b}| = 5$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is 45°, evaluate the exact value of the scalar product of $-\mathbf{b}$ and $\mathbf{c}$.

[$-\frac{5}{3}\left(5+3\sqrt{2}\right)$]

4 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ such that

$$\mathbf{a} = \mathbf{i}+\mathbf{j}+\mathbf{k} \quad \text{and} \quad \mathbf{b} = \mathbf{i}+2\mathbf{j}+2\mathbf{k}.$$

(i) Find the size of angle OAB. [2]

The point C has position vector $\mathbf{c}$ given by $\mathbf{c} = \lambda\mathbf{a} + \mu\mathbf{b}$, where λ and μ are positive constants. Given that the area of triangle OAC is twice that of triangle OBC,

(ii) find μ in terms of λ, [3]

(iii) hence, if $OC = \sqrt{118}$, find the position vector $\mathbf{c}$. [4]

[**(i)** 144.7° **(ii)** $\mu = 2\lambda$ **(iii)** $\begin{pmatrix} 3\sqrt{2} \\ 5\sqrt{2} \\ 5\sqrt{2} \end{pmatrix}$]

5 With respect to the origin O, the position vectors of the points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Point C lies on AB such that $AC : CB = 1 : 2$. It is given that $\mathbf{a}$ is a unit vector and the length of OB is 2 units.

(i) Give a geometrical interpretation of $|\mathbf{a}\cdot\mathbf{c}|$. [1]

(ii) It is given that the angle AOB is 60°. By considering $(2\mathbf{a}-\mathbf{b})\cdot(2\mathbf{a}-\mathbf{b})$, find $|2\mathbf{a}-\mathbf{b}|$. [3]

(iii) Find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(iv) Hence by considering cosine of angle AOC and cosine of angle COB, determine if the line segment OC bisects the angle AOB. [3]

[**(ii)** 2 **(iii)** $\dfrac{\mathbf{b}+2\mathbf{a}}{3}$]

6 Relative to the origin O, the points A, B, and C, have non-zero position vectors $\mathbf{a}$, $\mathbf{b}$ and $3\mathbf{a}$ respectively. D lies on AB such that $AD = \lambda AB$, where $0 < \lambda < 1$.

(i) Write down the position vector of the point D. [1]

(ii) The point E is the midpoint of BC. Find the value of λ if E lies on the line OD, and write down the ratio $OE : ED$. [4]

[(i) $\overrightarrow{OD} = \lambda\mathbf{b} + (1-\lambda)\mathbf{a}$, $\lambda \in \mathbb{R}$ **(ii)** $2:1$ **]**

7 Referred to origin O, the points P, Q, R and S have position vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ and $\mathbf{s}$ respectively.

(i) Given that $\alpha\mathbf{p} + \beta\mathbf{q} + \mathbf{r} = \mathbf{0}$ and $\alpha + \beta + 1 = 0$, where α and β are non-zero constants. Show that P, Q and R are collinear. [3]

It is given that the point R lies on PQ produced. The point T lies on line RS produced such that $RT : ST = 3:2$.

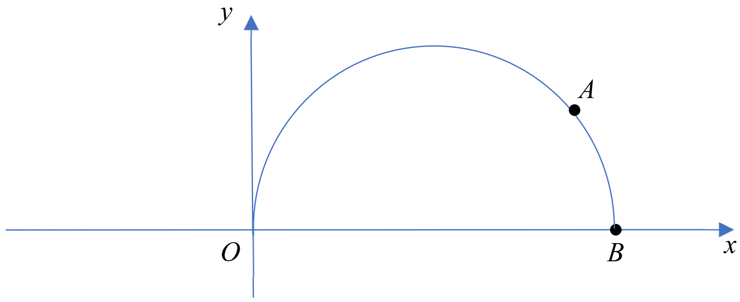
(ii) Find the position vector of the point T in terms of $\mathbf{r}$ and $\mathbf{s}$. [2]

(iii) Give a geometrical interpretation of $\dfrac{|(\mathbf{p}-\mathbf{r})\cdot(\mathbf{s}-\mathbf{r})|}{|\mathbf{s}-\mathbf{r}|}$. [1]

(iv) Find the area of triangle PRT in the form $\gamma|\mathbf{p}\times\mathbf{s} - \mathbf{p}\times\mathbf{r} - \mathbf{r}\times\mathbf{s}|$, where γ is a constant to be determined. [3]

[(ii) $\overrightarrow{OT} = 3\mathbf{s} - 2\mathbf{r}$ **(iv)** $\frac{3}{2}|\mathbf{p}\times\mathbf{s} - \mathbf{p}\times\mathbf{r} - \mathbf{r}\times\mathbf{s}|$ **]**

8 The diagram shows a semi-circle with diameter OB. Point A is on the circumference of the semi-circle and point C lies on chord AB such that $AC = 3CB$.



Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors.

(i) Show that $\mathbf{a}.\mathbf{b} = |\mathbf{a}|^2$. [2]

(ii) Find the position vector of C in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(iii) Hence find the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OB}$ if $|\mathbf{a}| = \sqrt{3}$ and $|\mathbf{b}| = 2$. [4]

[(ii) $\dfrac{\mathbf{a}+3\mathbf{b}}{4}$ **(iii)** $\dfrac{15}{8}$ **]**

9 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Point P is on the line AB such that $AP : PB = m : n$ where m and n are positive integers. Point C is on OP extended such that $OP : PC = 1 : 2$.

(i) Show that $\overrightarrow{AC} = \left(\dfrac{2n-m}{m+n}\right)\mathbf{a} + \left(\dfrac{3m}{m+n}\right)\mathbf{b}$. [3]

(ii) If $|\mathbf{a}| = |\mathbf{b}|$ and the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ is $\dfrac{\pi}{3}$, find the area of the triangle ABC in terms of $|\mathbf{a}|$. [4]

(iii) Find the ratio $AP : PB$ such that AC is parallel to OB. [2]

[(ii) $\dfrac{\sqrt{3}}{2}|\mathbf{a}|^2$ **(iv)** $2:1$ **]**

10 Referred to the origin O, three distinct and non-collinear points A, B and C have position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively. Point L is the mid-point of BC. The position vector of a point P is given by $(1-k)\mathbf{a}+\frac{k}{2}(\mathbf{b}+\mathbf{c})$, where k is a non-zero constant and $k \neq 1$.

(i) Show that A, L and P are collinear. [3]

(ii) Show that $\frac{1}{2}\left|\overrightarrow{CP}\times\overrightarrow{CB}\right| = \frac{|1-k|}{2}\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}-\mathbf{a}\times\mathbf{c}\right|$. [3]

For the rest of the question, let $k=\frac{1}{2}$.

Let point Q be a point on the line passing through A and L. P and Q are distinct points and the areas of triangle CPB and triangle CQB are equal.

(iii) By considering part **(ii)**, find the position vector of Q in terms of $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$. [4]

(iv) Given that $\left|\overrightarrow{BC}\right|=1$, interpret geometrically $\left|\overrightarrow{LP}\cdot\overrightarrow{BC}\right|$. [1]

[(iii) $-0.5\mathbf{a}+0.75(\mathbf{b}+\mathbf{c})$ **]**

11 With reference to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. A point P lies on AB between A and B such that $AP:PB=\lambda:1-\lambda$, $0<\lambda<1$.

(i) Show that $\cos(\angle AOP)=\frac{(1-\lambda)|\mathbf{a}|}{|(1-\lambda)\mathbf{a}+\lambda\mathbf{b}|}$. [4]

(ii) Prove that $\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]\cdot\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]=(1-\lambda)^2|\mathbf{a}|^2+\lambda^2|\mathbf{b}|^2$. Hence, given also that OP bisects $\angle AOB$, find the ratio of $\frac{|\mathbf{a}|}{|\mathbf{b}|}$, leaving your answer in terms of λ. [6]

[(ii) $\frac{\lambda}{1-\lambda}$ **]**

12 Relative to the origin O, the position vectors of the points P, Q and R are $\mathbf{i}$, $2\mathbf{j} - t\mathbf{k}$ and $t\mathbf{k}$ respectively, where t is a fixed constant. The points A and B divides both line segments PQ and QR respectively in the same ratio of $\mu : 1 - \mu$, where μ is a parameter such that $0 < \mu < 1$.

(i) Find the vector $\overrightarrow{AB}$ in terms of t and μ. [3]

(ii) Determine whether the points O, A, B are collinear. [1]

(iii) Find the values of μ such that the length of projection of $\overrightarrow{AB}$ onto $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ is $\frac{1}{5}$ unit. [2]

(iv) Given that angle AOB is a right angle, find the set of possible values of t, justifying your answer clearly. [3]

[(i) $\begin{pmatrix} \mu - 1 \\ 2 - 4\mu \\ -t + 3t\mu \end{pmatrix}$ **(iii)** $\frac{4}{13}$ or $\frac{6}{13}$ **(iv)** $t \in \mathbb{R} \setminus \{0\}$ **]**