Chapter 1(c) : Planes

1. Equation of a Plane

A plane is commonly defined using

(i) two vectors parallel to the plane and a point on the plane or

(ii) three non-collinear points or

(iii) the direction of the normal to the plane and one point on the plane.

In this section, we will learn the three different forms of equation of a plane, namely

1.1 Vector equation (in parametric form): $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where $\lambda, \mu \in \mathbb{R}$;

1.2 Vector equation (in scalar product form): $\mathbf{r} \bullet \mathbf{n} = d$, where $d \in \mathbb{R}$; and

1.3 Cartesian equation: $ax + by + cz = d$, where $a,\ b,\ c,\ d \in \mathbb{R}$.

Recall: An equation of a line is of the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, $\lambda \in \mathbb{R}$

where **r** represents position vector of a point on the line

a represents position vector of a given (known) point on the line

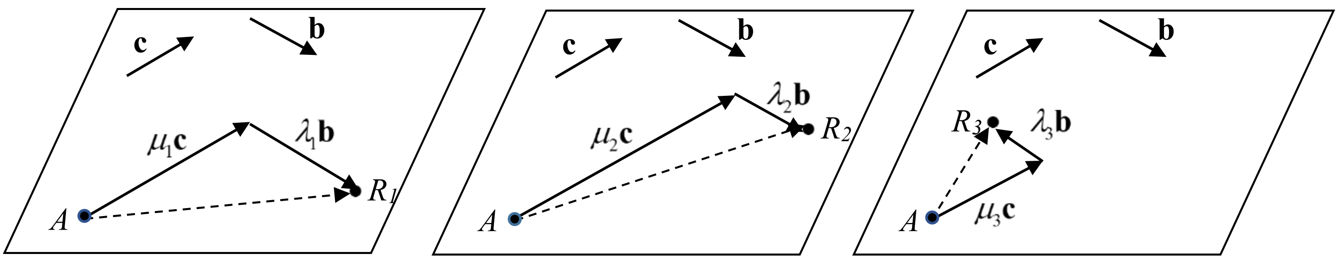
b represents a vector parallel to the line

1.1 Vector Equation of a Plane in Parametric Form

Refer to the diagrams below. A is a fixed point on a plane and R_1, R_2, R_3, ……etc are different points on the same plane. **b** and **c** are 2 vectors parallel to the plane.
Then, by vector addition,

$$\overrightarrow{AR_1} = \lambda_1\mathbf{b} + \mu_1\mathbf{c}$$
$$\overrightarrow{AR_2} = \lambda_2\mathbf{b} + \mu_2\mathbf{c}$$
$$\overrightarrow{AR_3} = \lambda_3\mathbf{b} + \mu_3\mathbf{c}$$

Generalising, for any point R lying on the plane, $\overrightarrow{AR} = \lambda\mathbf{b} + \mu\mathbf{c}$, where $\lambda, \mu \in \mathbb{R}$.
In fact, any vectors on the plane or parallel to the plane can also be expressed as $\lambda\mathbf{b} + \mu\mathbf{c}$.



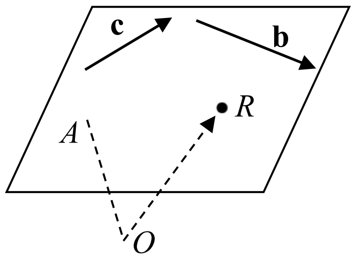
We can obtain the vector equation of a plane in parametric form for the following cases:

➢ **Given a point on the plane and two vectors parallel to the plane**

The equation of the plane **<u>through any given point</u>** A, with position vector $\mathbf{a}$, and parallel to vectors $\mathbf{b}$ and $\mathbf{c}$ can be found as follows.

$\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR}$ where R is any variable point on the plane.

$\Rightarrow \mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, $\lambda, \mu \in \mathbb{R}$



Vector equation of the plane in parametric form: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, $\lambda, \mu \in \mathbb{R}$

<u>Example 1</u>

Write down the vector equation of the plane in parametric form

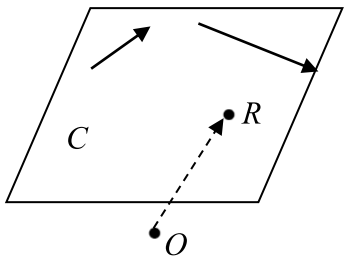
(a) through point $C(4, -1, 2)$ and parallel to the vectors $\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $-\mathbf{i} + 4\mathbf{j} - \mathbf{k}$,

(b) through the points $A(3, -2, 0)$, $B(2, 0, 3)$ and $C(1, -1, 1)$

(c) that contains the lines $\mathbf{r} = \mathbf{j} - \mathbf{k} + t(\mathbf{i} + \mathbf{k})$ and $\mathbf{r} = \mathbf{j} - \mathbf{k} + s(2\mathbf{i} - \mathbf{j} - \mathbf{k})$

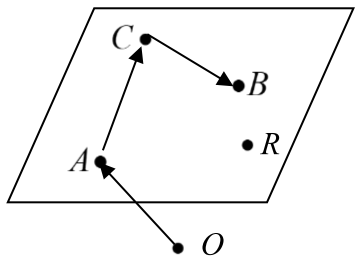
Solution

(a) , $\lambda, \mu \in \mathbb{R}$

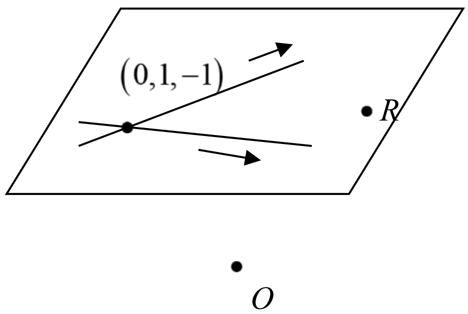


(b)

, $\lambda, \mu \in \mathbb{R}$



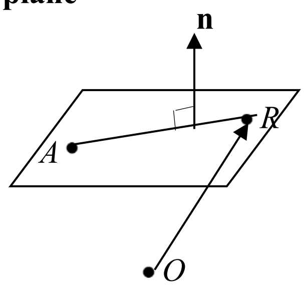
(c) , $\lambda, \mu \in \mathbb{R}$



1.2 Vector Equation of Plane in Scalar Product Form

We can obtain the equation of a plane in scalar product form for the following cases:

- **Given the vector normal to the plane and one point on the plane**



The equation of a plane can also be given / found in a form involving the scalar product of vectors. If **n** is a vector **normal** to the plane, then we can find a simple expression for the equation of the plane.

Let R be any point and A is a given point on the plane.

Then $\overrightarrow{AR} \perp \mathbf{n}$.

$\overrightarrow{AR} \bullet \mathbf{n} = 0$

$\left(\overrightarrow{OR} - \overrightarrow{OA}\right) \bullet \mathbf{n} = 0$

$(\mathbf{r} - \mathbf{a}) \bullet \mathbf{n} = 0$

$\mathbf{r} \bullet \mathbf{n} - \mathbf{a} \bullet \mathbf{n} = 0$

$\mathbf{r} \bullet \mathbf{n} = \mathbf{a} \bullet \mathbf{n} \; (= d)$

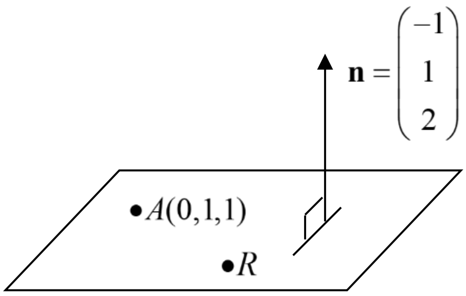
Vector equation of the plane in scalar product form:
$\mathbf{r} \bullet \mathbf{n} = d$.

Example 2

Find the equation of the plane passing through the point (0, 1, 1) and perpendicular to the vector $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

Solution

$\mathbf{r} \bullet \mathbf{n} = \mathbf{a} \bullet \mathbf{n}$



Hence, the equation of the plane is

THINK: What does it imply if the RHS of the scalar equation of plane is **zero**?

Example 3

Find the equation of the following planes in scalar product form

(a) the plane passing through the points $A(2, -1, 4)$, $B(3, 2, -6)$ and $C(4, 1, 5)$;

(b) the plane containing the lines $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j})$ and $\mathbf{r} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{k})$, where λ and μ are parameters.

Solution

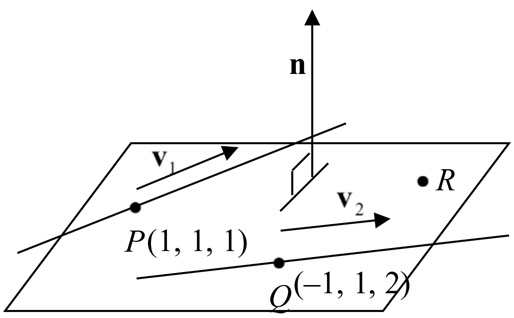
(a) Find **n** using vector product of $\overrightarrow{AB}$ & $\overrightarrow{AC}$ or other choices:

Let $\mathbf{n} = \begin{pmatrix} 23 \\ -21 \\ -4 \end{pmatrix}$ $\left\{ \textit{or take } \mathbf{n} = \begin{pmatrix} -23 \\ 21 \\ 4 \end{pmatrix} \right\}$

Equation of plane in scalar product form:

i.e.

(b) Identify vectors // to plane:



Find **n** using vector product:

Let $\mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

Equation of plane in scalar product form:

Think: How would you check your answer in (a) & (b)?

1.3 Equation of Plane in Cartesian Form (an equation in x, y , z only)

This is obtained from expanding the scalar product form of the plane's equation by writing the position vector of R as $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$.

If the equation of a plane is given as $\mathbf{r}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}=2$,

$$\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}=2$$

$$\Rightarrow x-2y+3z=2$$

Thus the Cartesian equation of the plane is $x-2y+3z=2$.

Example 3 (continued)

Write down the equation of the planes in Cartesian form for (a) and (b) in the above example.

Solution

(a) Equation of plane in Cartesian form:

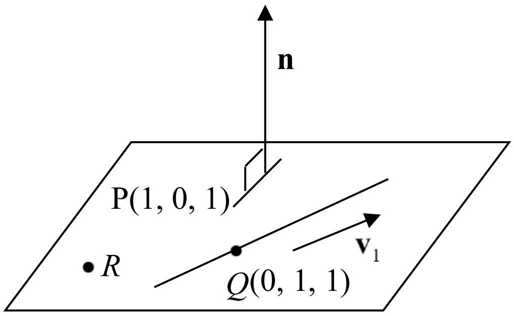
(b) Equation of plane in Cartesian form:

Example 4

Find the equation of the plane passing through a point (1, 0, 1) and containing the line

$$\mathbf{r} = \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{k}),\ \lambda \in \mathbb{R}$$

(i) in scalar product form, and (ii) in Cartesian form.



Solution

(i)

Equation of plane in scalar product form:

(ii) Equation of plane in Cartesian form:

Think: What is the normal vector of a plane that is parallel to the plane given by $\mathbf{r} \bullet \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = 3$?

Special Planes

You may come across terms such as x-y plane, x-z plane and y-z plane.

The x-y plane is a plane which contains **both the x and y– axis**.

Think about the following for x-y plane:

- What is the axis that is perpendicular to the x-y plane?
- What is the normal of the x-y plane?
- What is a known point lying on the x-y plane?
- What is the equation of the x-y plane in scalar product form and Cartesian form?

Apply the same questioning to obtain the equations of the x-z plane and y-z plane in scalar product form and Cartesian form.

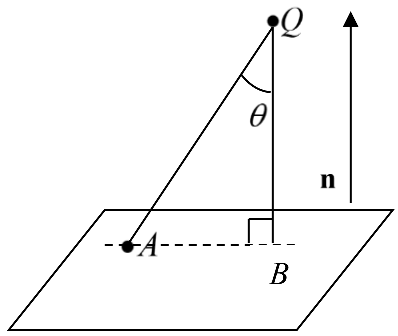
2. Perpendicular Distance (Shortest Distance) from a Point (or Origin) to a Plane

Distance from point Q to the plane

= QB

= length of projection of $\overrightarrow{AQ}$ onto $\mathbf{n}$

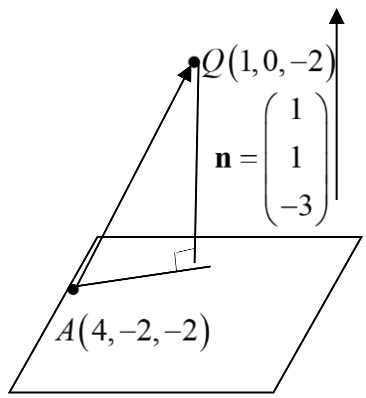
= $\left|\overrightarrow{AQ} \bullet \hat{\mathbf{n}}\right|$, where A is any point on the plane



Example 5

Find, in scalar product form, the equation of the plane p through the point $A(4,-2,-2)$ and perpendicular to $\mathbf{i}+\mathbf{j}-3\mathbf{k}$. Show that point $Q(1, 0, –2)$ does not lie on the plane p and find the shortest distance from Q to p,

Solution



Equation of plane:

$$\mathbf{r} \bullet \mathbf{n} = \mathbf{a} \bullet \mathbf{n}$$

i.e.

Now,

$\therefore$ Point Q does not lie on the plane.

$$\overrightarrow{AQ} = \begin{pmatrix}1\\0\\-2\end{pmatrix} - \begin{pmatrix}4\\-2\\-2\end{pmatrix} = \begin{pmatrix}-3\\2\\0\end{pmatrix}$$

Shortest distance from Q to plane

$=$

$$= \frac{1}{\sqrt{11}}$$

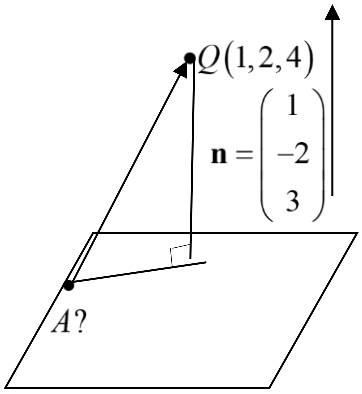
<u>Example 6</u>

Find the perpendicular distance from the point $Q(1,\ 2,\ 4)$ to the plane p with equation $\mathbf{r} \bullet \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = -4$.

[Comparing with Example 5, what is the missing information?]

[How do you obtain the missing information? Trial and Error? Inspection? Solving?]

Solution



By inspection,

is a point on the plane.

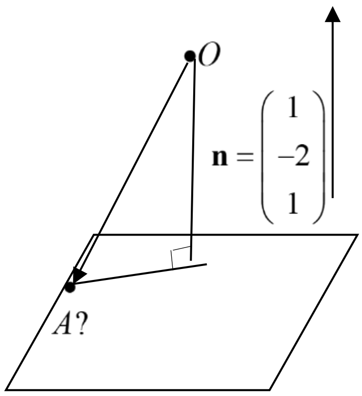
$\overrightarrow{AQ} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$

Shortest distance from Q

<u>Example 7</u>

Find the perpendicular distance from the origin to the plane p with $\mathbf{r} \bullet \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = -5$.

Solution



By inspection,

is a point on the plane.

Shortest distance from O

Observe the answer $\dfrac{5}{\sqrt{6}}$.

How is the denominator $\sqrt{6}$ obtained?

How is the numerator 5 related to the equation of the plane $\mathbf{r} \bullet \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = -5$?

<u>Distance from the Origin to a Plane</u>

Given a plane with equation $\mathbf{r} \bullet \mathbf{n} = d$,

the shortest distance from the origin to the plane =

<u>Note</u>: If the equation of the plane is given as $\mathbf{r} \bullet \hat{\mathbf{n}} = c$,

then the shortest distance from the origin to the plane = $|c|$

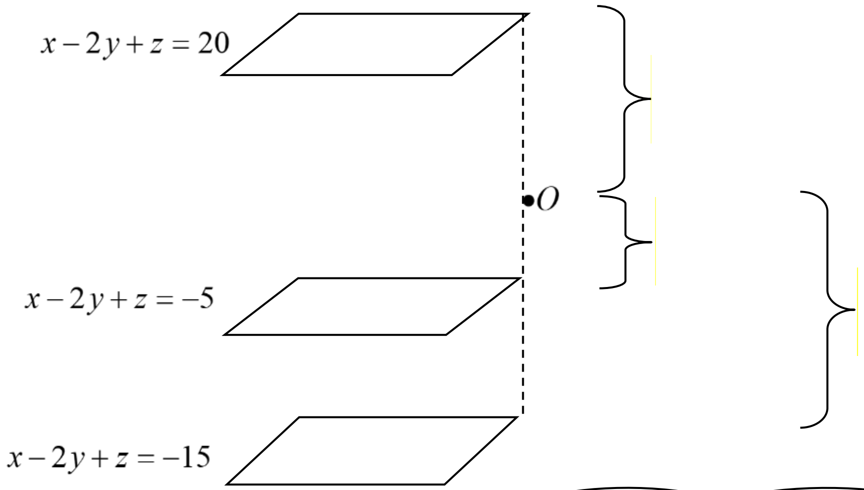
<u>Think</u>:

Consider 3 planes with equations $x-2y+z=-5$, $x-2y+z=-15$ and $x-2y+z=20$

(i) Observe the normal of the planes. What can you say about the planes?
(ii) What is the distance of each plane from the origin?
Using the origin as a reference point,

(iii) What is the distance between the plane $x-2y+z=-5$ and $x-2y+z=-15$? Ans :

(iv) What is the distance between the plane $x-2y+z=-5$ and $x-2y+z=20$? Ans :

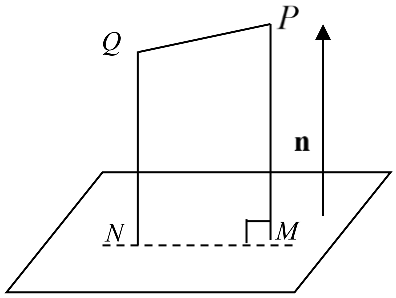


What is the significance of the **sign** of the number on the right hand side of the equation of a plane?

3. Length of Projection of a Vector onto a Plane

Given a vector $\overrightarrow{PQ}$ and a plane with equation $\mathbf{r} \bullet \mathbf{n} = d$, then the length of projection of the vector $\overrightarrow{PQ}$ onto the plane is given by $MN = \left|\overrightarrow{PQ} \times \hat{\mathbf{n}}\right|$.

(Think: why cross product?)



Example 8

With respect to the origin, the position vectors of P and Q are $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ respectively. Find the length of projection of the vector $\overrightarrow{PQ}$ onto the plane $\mathbf{r} \bullet \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix} = 3$.

Solution

$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

The length of projection required =

4. Points and Lines on a Plane

There are three cases for the relation between a line and a given plane:

(1) A line intersects the plane at one point

(2) A line intersects the plane at an infinite number of points

(3) A line does not intersect the plane

4.1 Finding the Point of Intersection between a Line and a Plane

(line intersects the plane at one point)

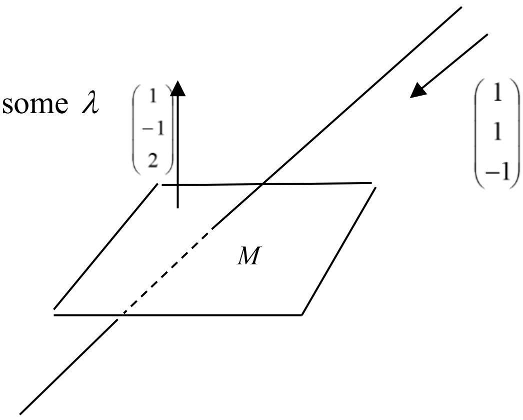
Example 9

Find the position vector of the point of intersection of the line $\mathbf{r}=\begin{pmatrix}-1\\3\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$, $\lambda\in\mathbb{R}$**,** and the plane $\mathbf{r}\bullet\left(\mathbf{i}-\mathbf{j}+2\mathbf{k}\right)=4$.

Solution

Let M be the point of intersection.

Then M lies on the line, for some λ



and M lies on the plane,

Substituting $\overrightarrow{OM}$,

$\therefore$ position vector of M, $\overrightarrow{OM}=\begin{pmatrix}-1\\3\\1\end{pmatrix}-3\begin{pmatrix}1\\1\\-1\end{pmatrix}=\begin{pmatrix}-4\\0\\4\end{pmatrix}$

4.2 Finding the Foot of Perpendicular from a Point to a Plane

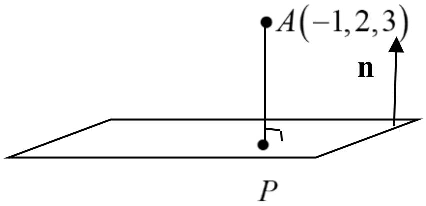
(line intersects the plane at one point)

Think: Find a vector equation of the line which passes through the point (0, 1, 1) and is perpendicular to the plane $\mathbf{r}\bullet\begin{pmatrix}1\\-2\\1\end{pmatrix}=4$.

Example 10

Find the coordinates of P, the foot of perpendicular from the point $A(-1,\ 2,\ 3)$ to plane $2x-3y+4z+8=0$.

Solution



Equation of plane: -----(1)

Since the line AP is normal to the plane, direction vector of line AP =

Equation of line AP: , $\lambda\in\mathbb{R}$ **-----**(2)

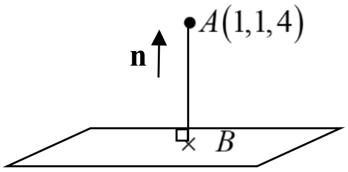
At point of intersection P, $\overrightarrow{OP}$ must satisfy equations (1) and (2):

Point of intersection:

<u>Example 11</u>

A plane p has equation $\mathbf{r}\cdot(2\mathbf{i}+3\mathbf{j})=-8$. Find, in vector form, an equation for the line passing through the point A with position vector $\mathbf{i}+\mathbf{j}+4\mathbf{k}$ and normal to the plane p. Hence find the position vector of point B, the foot of the perpendicular from A to the plane p. Deduce the shortest distance from the point A to the plane p.

Solution



Since the line is normal to the plane, direction vector of line = .

Equation of line: **,** $\lambda\in\mathbb{R}$ **-----**(1)

Equation of plane: $\mathbf{r}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=-8$ -----(2)

At point of intersection B, $\overrightarrow{OB}$ must satisfy equations (1) and (2):

$\overrightarrow{OB}=\begin{pmatrix}1\\1\\4\end{pmatrix}-\begin{pmatrix}2\\3\\0\end{pmatrix}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}$ and $\overrightarrow{AB}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}=\begin{pmatrix}-2\\-3\\0\end{pmatrix}$

Shortest distance =

4.3 Line Parallel to a Plane

When a line is parallel to the plane, there are 2 cases to consider:

(1) The line does not intersect the plane; or

(2) The line lies on the plane and have an infinite number of solutions.

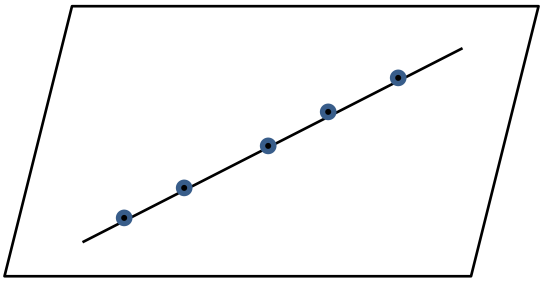
Example 12

Determine whether the line $\mathbf{r} = \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{k})$, $\lambda \in \mathbb{R}$ lies on the plane $\mathbf{r} \bullet \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = -1$

Solution

Method 1

Line: $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$



Since

The line lies on the plane.

Note: If LHS $\neq$ RHS, the line does not lie on the plane.

Method 2

Check whether the line is parallel to the plane. i.e. check if

Since

Line is ______________________________, hence line is parallel to the plane.

Check if a known point on the line lies on the plane. Check if

Since

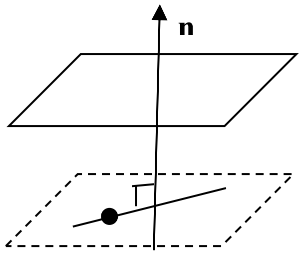
Since a point on the line is on the plane, the line lies on the plane.

<u>Note</u>:

If LHS $\neq$ RHS, the point on the line does not lie on the plane.

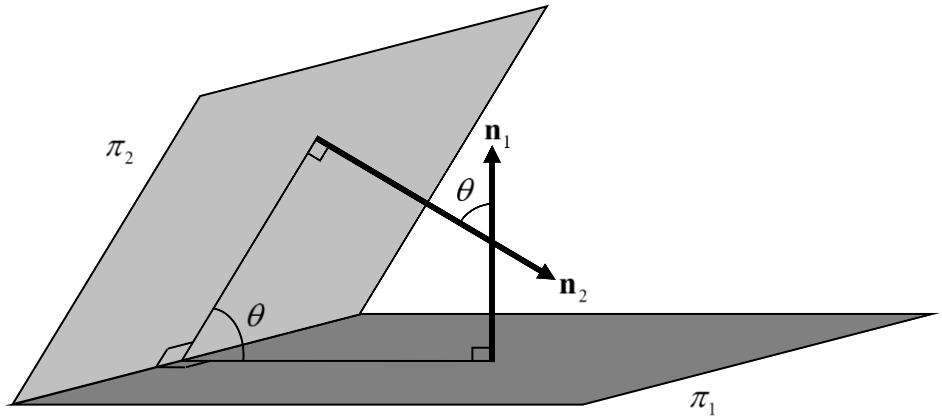
Therefore, the line also does not lie on the plane.

The line does not intersect the plane. (see diagram on the right.)



5. <u>Acute Angle Between 2 Planes</u>

Let π_1 and π_2 be two planes with vector equations $\mathbf{r}\bullet\mathbf{n}_1 = d_1$ and $\mathbf{r}\bullet\mathbf{n}_2 = d_2$ respectively.



Acute angle between π_1 and π_2 = Acute angle between $\mathbf{n}_1$ and $\mathbf{n}_2 = \theta$

$$\boxed{\cos\theta = \frac{|\mathbf{n}_1\bullet\mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|}}$$

<u>Note</u>: By convention, we normally find the acute angle between 2 planes.

<u>Example 13</u>

The planes Π_1 and Π_2 have the equations $2x-3y+6z=0$ and $x+2y+3z=12$ respectively.

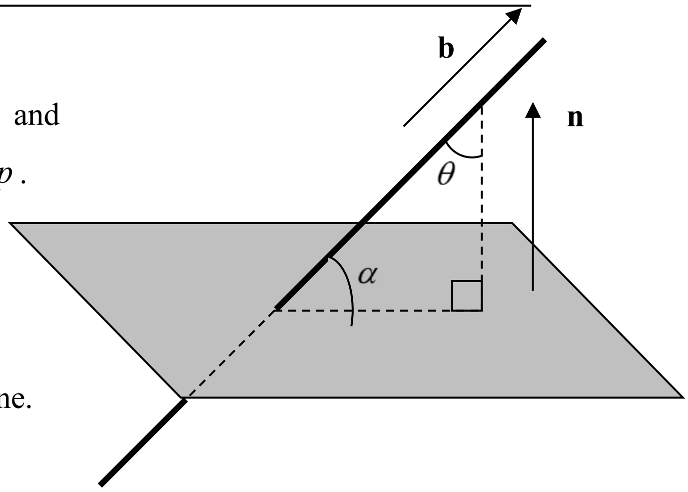
Find the acute angle between the planes Π_1 and Π_2, giving your answer to the nearest 0.1°.

Solution

Equation of $\Pi_1 : \mathbf{r}\bullet\begin{pmatrix}2\\-3\\6\end{pmatrix}=0$ Equation of $\Pi_2 : \mathbf{r}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}=12$

$\theta = 57.7°$

6. Acute Angle between Line and Plane



Let θ be the acute angle between the line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and the **normal** to the plane with equation $\mathbf{r} \bullet \mathbf{n} = p$.

Then $\cos\theta = \dfrac{|\mathbf{b} \bullet \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$

From the diagram,

α is the acute angle between the line and plane.

Hence, $\alpha = 90° - \theta = 90° - \cos^{-1}\dfrac{|\mathbf{b} \bullet \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$.

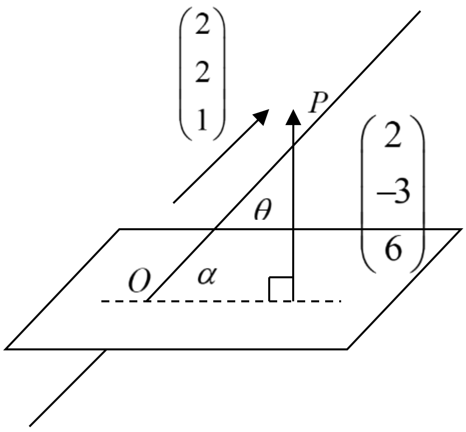
$$\boxed{\alpha = 90° - \cos^{-1}\frac{|\mathbf{b} \bullet \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}}$$

Note: By convention, we normally find the acute angle between a line and a plane.

Example 14

The plane Π has equation $2x - 3y + 6z = 0$. The point P has coordinates (2, 2, 1), and O is the origin. Find the acute angle between the line OP and the plane Π, giving your answer correct to 0.1°.

Solution



Equation of $\Pi : \mathbf{r} \bullet \begin{pmatrix}2\\-3\\6\end{pmatrix} = 0$

Equation of line OP:

acute angle between OP and Π,

$\alpha =$

Example 15

Two planes p_1 and p_2 have equations $\mathbf{r} \bullet \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = 4$ and $\mathbf{r} \bullet \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 2$ respectively.

(i) Find the acute angle between the planes p_1 and p_2, giving your answer to the nearest $0.1°$.

(ii) Find the acute angle between the line *AB* and the plane p_2, where $A(1, 0, 1)$ and $B(0, 1, -1)$.

(iii) Find the position vector of the foot of the perpendicular from $A(1, 0, 1)$ to p_1.

Solution

(i) Acute angle between 2 planes:

$$\cos\theta = \frac{\left|\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}\right|}{\sqrt{11}\sqrt{6}} = \frac{|-2|}{\sqrt{66}} = \frac{2}{\sqrt{66}} \qquad \Rightarrow \quad \therefore\ \theta = 75.7°$$

(ii) $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$

Acute angle between line and plane:

$\alpha =$

(iii) Let foot of the perpendicular be *Q*.

Since $\overrightarrow{AQ} \,//\, \mathbf{n}_1$,

Vertical equation of AQ: where λ is a parameter.

Since Q lies on the line, $\overrightarrow{OQ} =$ for some λ.

Since Q lies on plane,

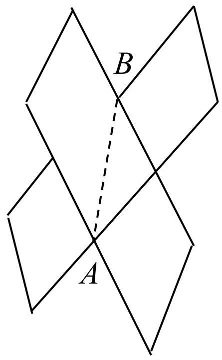
$$11\lambda = 4 \quad \Rightarrow \quad \lambda = \frac{4}{11}$$

Position vector of Q, $\overrightarrow{OQ} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \frac{4}{11}\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 15 \\ 12 \\ 7 \end{pmatrix}$

7. **<u>Intersection between 2 planes</u>**

Two non-parallel planes will intersect at a line, AB, and AB is perpendicular to both $\mathbf{n}_1$ and $\mathbf{n}_2$. Hence, direction vector of the line of intersection is parallel to $\mathbf{n}_1 \times \mathbf{n}_2$.

To find the equation of the line of intersection,



<u>Method 1 (Using Cartesian Equation)</u>:

Use GC APPS to solve the simultaneous equations of the Cartesian equations of the 2 planes.

Consider 2 planes π_1 and π_2 with Cartesian equations:

π_1: $a_1x + b_1y + c_1z = p_1$ and π_2: $a_2x + b_2y + c_2z = p_2$.

We can use GC to find a vector equation of the line of intersection, if any.

<u>Method 2 (Using Vector Equation)</u>:

(i) Find direction vector of the line of intersection by evaluating $\mathbf{n}_1 \times \mathbf{n}_2$;

(ii) Find a point lying on both planes;

(iii) The vector equation of the line will be obtained.

Example 16

Find a vector equation of the line of intersection of the two planes

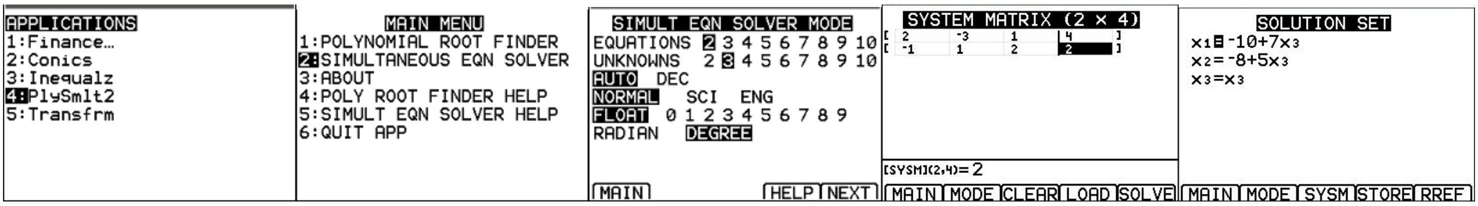
$$2x-3y+z=4, \qquad \text{and} \qquad -x+y+2z=2.$$

Solution

Note that $x_1 = x,\quad x_2 = y,\quad x_3 = z$.

Let $z=\lambda$. Then

$\therefore$ The 2 planes intersect along a line with vector equation , $\lambda \in \mathbb{R}$.



Example 17

Two non-parallel planes Π_1 and Π_2 have equations $\mathbf{r}\bullet\begin{pmatrix}1\\-3\\2\end{pmatrix}=-4$ and $\mathbf{r}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}=1$ respectively.

(i) Find the acute angle between the planes, giving your answer to the nearest 0.1°.

(ii) Find the equation of line of intersection between the planes.

Solution

(i) Acute angle between 2 planes:

$$\cos\theta = \frac{\left|\begin{pmatrix}1\\-3\\2\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}\right|}{\sqrt{14}\sqrt{3}} = \frac{|-4|}{\sqrt{42}} = \frac{4}{\sqrt{42}}$$

$\theta = 51.9°$

(ii) $\Pi_1 : x-3y+2z=-4 \qquad (1)$

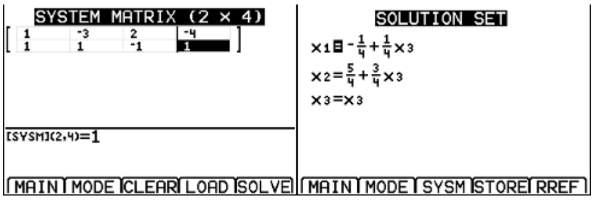
$\Pi_2 : x+y-z=1 \qquad (2)$

Method 1 is used here.

Using GC,

Let $z=\lambda$, Then

Equation of line of intersection:



Example 18

Show that the point (–1, 0, –1) lies in the planes $\mathbf{r}\bullet\begin{pmatrix}2\\1\\2\end{pmatrix}=-4$ and $\mathbf{r}\bullet\begin{pmatrix}\alpha\\1\\-\alpha\end{pmatrix}=0$. Hence, find an equation of the line of intersection between the two planes.

Solution

Method 2 is used here.

Sub (–1, 0, –1) into equation of each plane

$$\begin{pmatrix}-1\\0\\-1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\2\end{pmatrix}=-2-2=-4$$

$$\begin{pmatrix}-1\\0\\-1\end{pmatrix}\bullet\begin{pmatrix}\alpha\\1\\-\alpha\end{pmatrix}=-\alpha+\alpha=0$$

Hence the point lies on both planes.

Direction vector of line of intersection =

Equation of the line of intersection: $\quad , \lambda\in\mathbb{R}$.