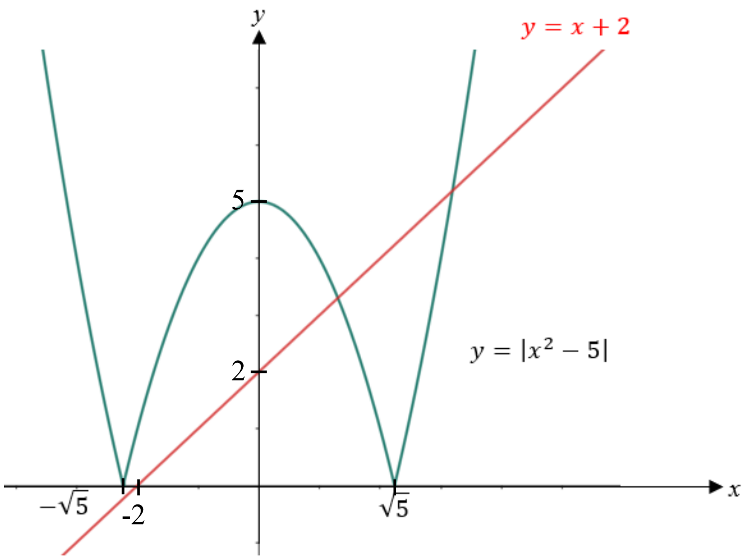
2025 EJC H2 Maths Promo Paper **Duration: 3 hrs** **Marks: 100**
Attempt all questions.

1 Four housewives bought three different kinds of organic vegetables at a Goodprice supermarket. They cannot remember the individual prices per kilogram, but three of them can remember the total amount that they each paid. The weights of vegetables purchased and the total amounts paid are shown in the following table.

	Amy	Betty	Carol	Denise
Pumpkins (kg)	1.20	1.50	1.30	2.30
Carrots (kg)	2.50	2.70	2.60	3.10
Broccoli (kg)	1.25	1.35	1.10	2.10
Total amount paid ($)	21.86	25.20	21.90	?

Given that, for each variety of vegetable, the price per kilogram paid by each of the housewives is the same, find the amount that Denise paid for her purchase. [4]

2



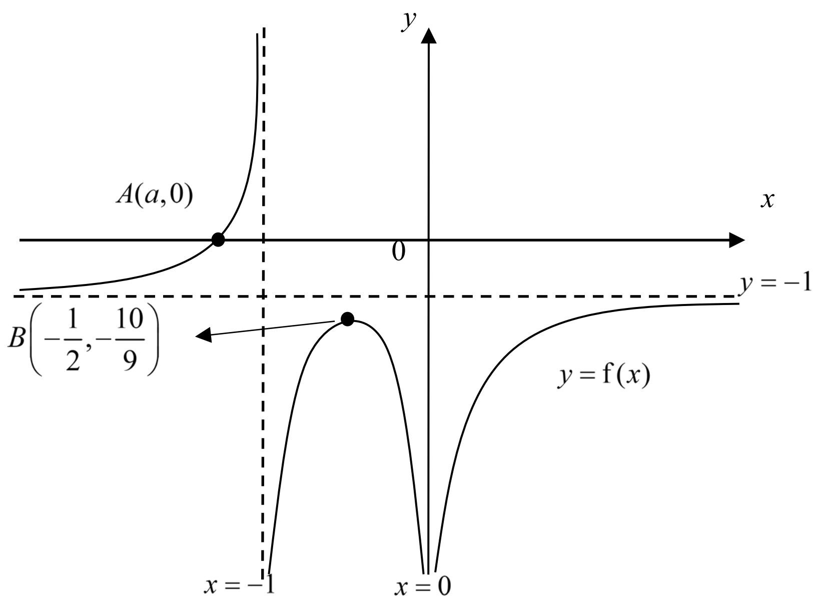
The diagram shows the sketch of the curves $y=\left|x^2-5\right|$ and $y=x+2$. Solve the inequality $\left|x^2-5\right|>x+2$ exactly. [4]

3 **(a)** Given that $\mathbf{a}\times\mathbf{b}=\mathbf{0}$, what can be deduced about the vectors **a** and **b**? [2]

(b) Find a unit vector **n** such that $\mathbf{n}\times\left(2\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)=\mathbf{0}$. [1]

(c) Find the cosine of the acute angle between $2\mathbf{i}-\mathbf{j}+3\mathbf{k}$ and the y-axis. [2]

4 The diagram below shows the curve $y = \mathrm{f}(x)$. The curve crosses the x-axis at point $A(a, 0)$, where $a < -1$ and has a maximum point $B\left(-\frac{1}{2}, -\frac{10}{9}\right)$. The curve also has a horizontal asymptote $y = -1$ and vertical asymptotes $x = -1$ and $x = 0$.



On separate diagrams, sketch the graphs of

(a) $y = 3\mathrm{f}(x-1)$, [2]

(b) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

showing clearly in each case, the equations of any asymptotes, the coordinates of any stationary points and axial intercepts in terms of *a*.

The curve $y = \mathrm{f}(x)$ undergoes the following transformations in succession:

P: Reflection in the y-axis

Q: Scaling parallel to the x-axis by a scale factor of k

(c) Determine the coordinates of the x-intercept of the transformed curve. [2]

5 The sequence $x_1, x_2, x_3, \ldots$ satisfies the recurrence relation

$$x_{n+1} = x_n^2\left(2 - x_n\right) \text{ for } n \geq 1$$

(a) Prove algebraically that, if the sequence converges, then it converges to either 1 or 0. [2]

(b) Determine the behaviour of the sequence for each of the following cases.

(i) $x_1 = 0.5$ [1]

(ii) $x_1 = 1.5$ [1]

(iii) $x_1 = -1$ [1]

(c) By considering the graph of $y = 2x^2 - x^3 - x$, show that if $0 < x_n < 1$, then $x_{n+1} - x_n < 0$. State briefly what this result tells us about the sequence in part **(b)(i)**. [2]

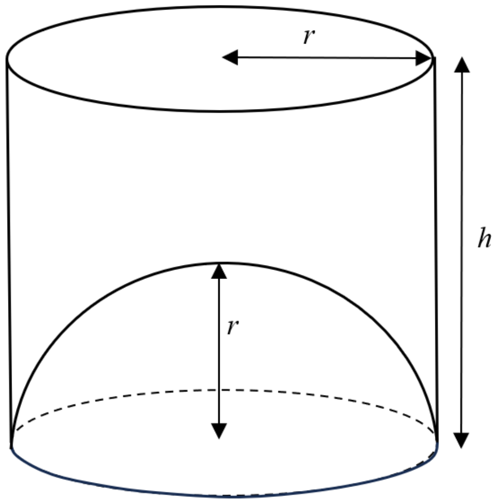
6 A curve C is defined parametrically by $x = 6t,\ y = t^2$.

(a) Show that the equation of the normal at the point $P\left(6p, p^2\right)$ is $p^3 + \left(18 - y\right)p - 3x = 0$. [3]

(b) The normal to the curve at the point P meets the curve again at the point $Q\left(-54, 81\right)$. Find the possible values of p. [2]

(c) Sketch the graph of C, indicating clearly the coordinates of Q and the possible points of P. Sketch also the normals at each of these points P. [3]

7 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$.]



A model of a toy is made up of three parts.

- The top of the toy is modelled by a circular disc of radius r cm.
- The sides of the toy are modelled by the curved surface of a cylinder of radius r cm and height h cm.
- The base of the toy is an empty space which is modelled by the curved surface of a hemisphere of radius r cm.

The three parts are joined together as shown in the diagram. The model is made of material of negligible thickness.

It is given that the volume of the toy is a fixed value k cm^3, and the external surface area is a minimum. Use differentiation to find the values of r and h in terms of k. Simplify your answers. [7]

8 The function f is defined by

$$\mathrm{f} : x \mapsto \frac{1}{1-x},\ x \in \mathbb{R},\ x \neq 1.$$

(a) Explain why f^2 does not exist. [2]

The function g is defined by

$$\mathrm{g} : x \mapsto \frac{1}{1-x},\ x \in \mathbb{R},\ x \neq 0, 1.$$

(b) Given that g^2 exists, find $\mathrm{g}^2(x)$. State its domain and range. [3]

(c) Show that $\mathrm{g}^3(x) = x$ and hence solve the equation $\mathrm{g}^{2025}(x) + \mathrm{g}(x) = 0$. [3]

9 The curve C_1 has equation

$$(x+3)^2 - 4(y+1)^2 = 16.$$

(a) Sketch the curve C_1, stating the equations of any asymptote(s) and other key features clearly. [3]

Another curve C_2 has parametric equations $x = t - \dfrac{1}{t-2}$ and $y = -t+5$ where $-2 \le t < 2$.

(b) Find the coordinates of any point(s) of intersection between C_1 and C_2. [2]

(c) Sketch the curve C_2 on the same diagram as part **(a)**, showing clearly any asymptote(s) and the coordinates of any axial intercept(s). [4]

10 It is given that $y = \tan\left[\ln(1+2x)\right]$, for $x > -0.5$.

(a) Show that $(1+2x)\dfrac{dy}{dx} = 2(1+y^2)$. [2]

(b) Hence find the first three non-zero terms of the Maclaurin expansion of $\tan\left[\ln(1+2x)\right]$. [4]

(c) It is given that the three terms found in part **(b)** are equal to the first three terms in the series expansion of $2x(1+ax)^b$. Find the exact values of the constants a and b. [3]

11 **(a)** Find $\displaystyle\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}}\,dx$. [2]

(b) Find $\displaystyle\int \frac{1}{4x^2+4x+17}\,dx$. [3]

(c) Use the substitution $u = \sqrt{2x+1}$ to find $\int x\sqrt{2x+1}\,dx$. [4]

12 Lines l_1 and l_2 have equations

$$l_1: \ \mathbf{r} = \begin{pmatrix} a \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} b \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad l_2: \ \frac{x-7}{3} = y - 2 = \frac{z+2}{-5}$$

respectively, where λ is a parameter and a, b are constants.

(a) It is given that lines l_1 and l_2 intersect each other at a right angle at point A. Find the values of a and b, and show that point A has coordinates $(4, 1, 3)$. [5]

Plane p_1 has equation $2x + z = -4$.

(b) Find the shortest distance from point A to plane p_1. [2]

Plane p_2 contains both lines l_1 and l_2, and has equation $-11x + 8y - 5z = -51$.

(c) Find the acute angle between planes p_1 and p_2. [2]

(d) Hence, or otherwise, find the shortest distance of point A to line l_3, the line of intersection between p_1 and p_2. (Give your answer to three significant figures.) [2]

13 The *effective range* of an electric car refers to the distance that an electric car can travel on a single full charge. Electrica, an electric car manufacturer, releases the Model LX with an initial effective range of 460 km to celebrate its 10^{th} anniversary.

The effective range decreases over time due to battery degradation. In the first year of ownership, Model LX's effective range decreases by 5.6 km. In each subsequent year, the decrease in effective range is 0.75 km more than the decrease in the previous year. For instance, the effective range decreases further by 6.35 km in the 2^{nd} year.

(a) Show that the decrease in effective range in the 7^{th} year is 10.1 km. [2]

(b) A battery replacement is recommended once the effective range falls below 75% of the initial range. Determine the year in ownership when a battery replacement is first recommended. [4]

On 1 January 2025, Ben purchased a brand-new Model LX and took up a car loan of \$150 000 from a bank. The loan is such that an interest of 0.2% is added to the outstanding loan amount at the start of every month, with the first interest amount added on 1 January 2025. Ben makes a monthly repayment of \$$x$ on the 12^{th} of every month, with the first repayment on 12 January 2025.

(c) Show that the outstanding loan amount, in dollars, at the end of n months is

$$150\ 000\left(1.002^n\right) - 500x\left(1.002^n - 1\right).$$ [3]

(d) **(i)** Suppose Ben made the last monthly repayment of \$$x$ on 12 Dec 2029 to fully pay off the loan. Find the value of \$$x$ to the nearest cent. [2]

(ii) Use your answer in part **d(i)** to calculate the total interest paid on the loan. [1]