

1 (a)

$$\frac{x+9}{8+2x-x^2} < 1$$

$$\frac{x+9}{8+2x-x^2} - 1 < 0$$

**Note:** Do not cross multiply.

$$\frac{x+9}{8+2x-x^2} - \frac{8+2x-x^2}{8+2x-x^2} < 0$$

$$\frac{x^2-x+1}{8+2x-x^2} < 0$$

$$\frac{x^2-x+1}{x^2-2x-8} > 0$$

$$\frac{x^2-x+1}{(x+2)(x-4)} > 0$$

**Alternatively,**

$$\frac{x^2-x+1}{8+2x-x^2} < 0$$

$$\frac{x^2-x+1}{(4-x)(2+x)} < 0$$

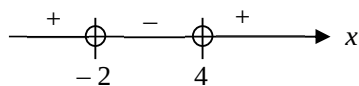
**Note:** Take note of coefficient of  $x^2$  and factorise correctly.

$$\text{Consider } x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\text{Since } \left(x - \frac{1}{2}\right)^2 \geq 0, \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0 \text{ for all real values of } x.$$

(OR coefficient of  $x^2 = 1 > 0$  and discriminant,  $b^2 - 4ac = (-1)^2 - 4(1)(1) = -3 < 0$  imply that  $x^2 - x + 1 > 0$  for all real values of  $x$ .)

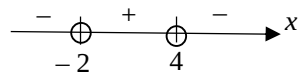
Therefore, we consider  $(x+2)(x-4) > 0$



$$\therefore x < -2 \text{ or } x > 4$$

**Alternatively,**

$$(4-x)(2+x) < 0$$



$$\therefore x < -2 \text{ or } x > 4$$

(b)

$$\frac{x^2-9}{x^4+2x^2-8} < 1$$

$$\frac{-x^2+9}{-x^4-2x^2+8} < 1$$

$$\frac{-x^2+9}{8+2(-x^2)-(-x^2)^2} < 1$$

By replacing  $x$  with  $-x^2$ , we have:

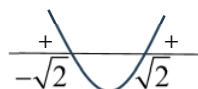
Using the (a) answer,  $-x^2 < -2$  or  $-x^2 > 4$  (No solution since  $-x^2 \leq 0$  for all real  $x$ )

$$x^2 > 2$$

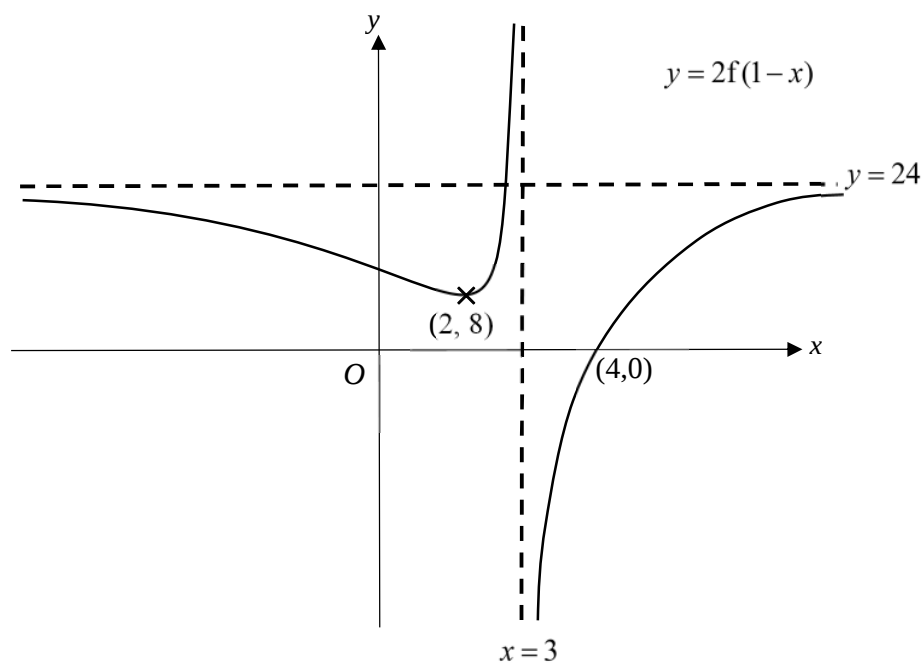
$$x^2 - 2 > 0$$

$$(x+\sqrt{2})(x-\sqrt{2}) > 0$$

$$x < -\sqrt{2} \text{ or } x > \sqrt{2}$$

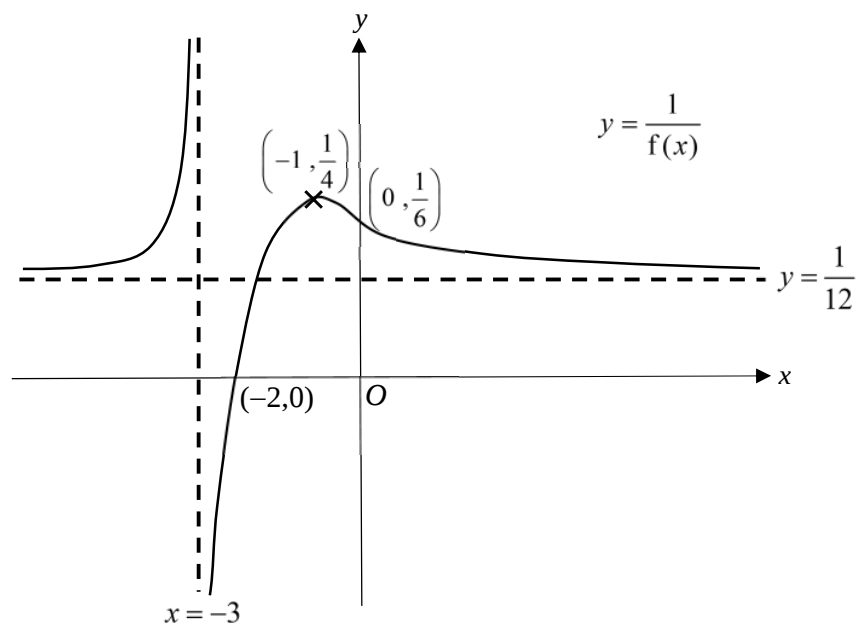


2 (a)



**Note:**  $(0,6)$  is transformed to  $(1,12)$  which is not required in this answer.

(b)



3 (a)  $\left(\frac{1}{2}a, b\right)$

(b)  $(-a, b), (a, b)$

(c)  $(a, -b)$

4

(a) Least value of  $k = 0$ .

(b) Let  $y = \frac{1}{x^2 - 4}$ ,  $x > 2$ .

$$x^2 y - 4y = 1$$

$$x^2 = \frac{1}{y} + 4$$

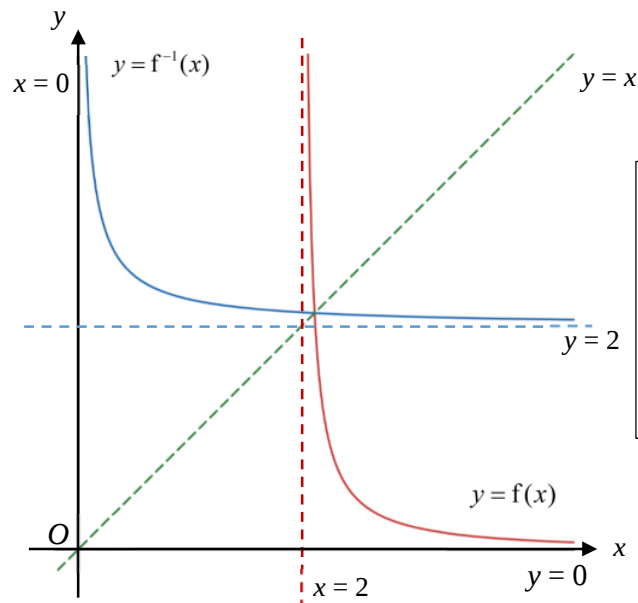
$$x = \pm \sqrt{\frac{1}{y} + 4}$$

$$\text{Since } x > 2, x = \sqrt{\frac{1}{y} + 4}$$

$$\text{Therefore, } f^{-1}(x) = \sqrt{\frac{1}{x} + 4}, x \in \mathbb{R}, x > 0$$

$$D_{f^{-1}} = (0, \infty)$$

(c)



**Note:**

1. 2 graphs must be symmetrical about the line  $y = x$
2. Line  $y = x$  must be drawn.

Relationship: Graphs of  $f$  and  $f^{-1}$  are reflections of each other about the line  $y = x$ .

**Note:** “symmetrical” is a property of the 2 graphs and not a relationship.

(d) Given  $f(x) = f^{-1}(x)$

Consider  $f(x) = x$

$$\frac{1}{x^2 - 4} = x$$

$$x^3 - 4x - 1 = 0, \text{ where } a = 1, b = -4, c = -1$$