EUNOIA JUNIOR COLLEGE, NAN YANG JUNIOR COLLEGE
JC2 Preliminary Examination 2024
General Certificate of Education Advanced Level
Higher 3

CANDIDATE NAME						
CIVICS GROUP	2	3	-			
REGISTRATION NUMBER						

PHYSICS 9814/01

Paper 1

19 September 2024

3 hours

Candidates answer on the Question Paper.

For Examiner's Use	
Section A	
1	**13**
2	**10**
3	**10**
4	**10**
5	**7**
6	**10**
Section B	
7	**20**
8	**20**
9	**20**
s.f.	
Total	**100**

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and registration number on
all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

The use of an approved scientific calculator is expected where appropriate.

Section A
Answer **all** questions.
You are advised to spend about 1 hour and 50 minutes on Section A.

Section B
Answer **two** questions only.
You are advised to spend about 35 minutes on each question in Section B.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question
or part question.

This document consists of **35** printed pages and **1** blank page.

Data

speed of light in free space,	c	=	3.00×10^{8} m s^{-1}
permeability of free space,	μ_o	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space,	ε_o	=	8.85×10^{-12} F m^{-1}
			$(1/(36\,\pi)) \times 10^{-9}$ F m^{-1}
elementary charge,	e	=	1.60×10^{-19} C
the Planck constant,	h	=	6.63×10^{-34} J s
unified atomic mass constant,	u	=	1.66×10^{-27} kg
rest mass of electron,	m_e	=	9.11×10^{-31} kg
rest mass of proton,	m_p	=	1.67×10^{-27} kg
molar gas constant,	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant,	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant,	k	=	1.38×10^{-23} J K^{-1}
gravitational constant,	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall,	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion,	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
moment of inertia of rod through one end	I	=	$\frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	I	=	$\frac{1}{2}M(r_1^2 + r_2^2)$
moment of inertia of solid sphere through centre	I	=	$\frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	I	=	$\frac{2}{3}MR^2$
work done on/by a gas,	W	=	$p\,\Delta V$
hydrostatic pressure,	p	=	ρgh
gravitational potential,	φ	=	$-\frac{Gm}{r}$
Kepler's third law of planetary motion	T^2	=	$\frac{4\pi^2 a^3}{GM}$
temperature,	T/K	=	T / °C + 273.15
pressure of an ideal gas,	p	=	$\frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$
mean translational kinetic energy of an ideal gas molecule	E	=	$\frac{3}{2}kT$
displacement of particle in s.h.m.	x	=	$x_o \sin \omega t$
velocity of particle in s.h.m.	v	=	$v_o \cos \omega t$
		=	$\pm \omega\sqrt{(x_o^2 - x^2)}$
electric current,	I	=	$Anvq$
resistors in series,	R	=	$R_1 + R_2 + \ldots$
resistors in parallel,	$1/R$	=	$1/R_1 + 1/R_2 + \ldots$

capacitors in series	$1/C = 1/C_1 + 1/C_2 + \ldots$
capacitors in parallel	$C = C_1 + C_2 + \ldots$
energy in a capacitor	$U = \frac{1}{2}CV^2$
electric potential,	$V = \frac{Q}{4\pi\varepsilon_o r}$
electric field strength due to a long straight wire	$E = \frac{\lambda}{2\pi\varepsilon_o r}$
electric field strength due to a large sheet	$E = \frac{\sigma}{2\varepsilon_o}$
alternating current/voltage,	$x = x_o \sin \omega t$
magnetic flux density due to a long straight wire	$B = \frac{\mu_o I}{2\pi d}$
magnetic flux density due to a flat circular coil	$B = \frac{\mu_o N I}{2r}$
magnetic flux density due to a long solenoid	$B = \mu_o n I$
energy in an inductor	$U = \frac{1}{2}LI^2$
RL series circuits	$\tau = \frac{L}{R}$
RLC series circuits (underdamped)	$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
radioactive decay,	$x = x_o \exp(-\lambda t)$
decay constant	$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$

Section A

Answer **all** questions in this section.

You are advised to spend about 1 hour 50 minutes on this section.

1 A frictionless hoop of mass M and radius R stands vertically on the ground. Two beads, both of mass m, are released at the same time from rest from the top of the hoop. They slide down the hoop, as shown in Fig. 1.1, where θ is the angular position of the left bead from the vertical direction.

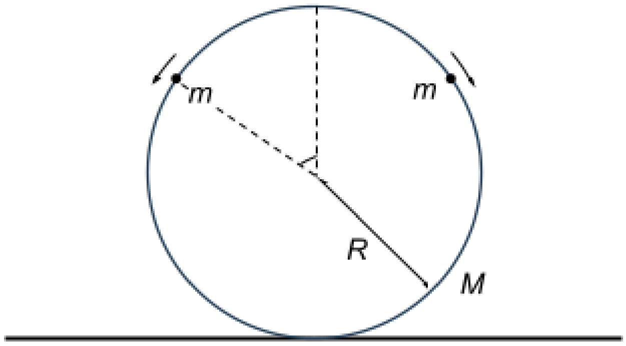


Fig. 1.1

(a) Derive an expression of the speed of the beads in terms of θ.

[1]

(b) Show that, for any angle θ with

$$\cos\theta < \frac{2}{3},$$

the normal force acting on the beads by the hoop is towards the centre of the loop.

[3]

(c) The hoop is not fixed to the ground but is free to move upwards if a large enough force acts on it.

Determine the maximum ratio between m and M, such that the hoop will not be lifted off the ground as the two beads slide down.

maximum $\dfrac{m}{M}$ = ... [6]

....

(d) The hoop is now fixed in position, and one of the beads is removed from the hoop. The remaining bead is displaced from the bottom of the hoop by a small angular displacement φ and then released, as shown in Fig. 1.2.

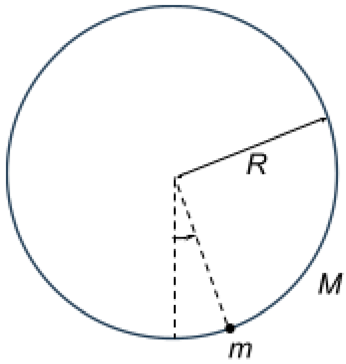


Fig. 1.2

Show that the subsequent motion of the bead is simple harmonic.

[3]

[Total: 13]

2 **(a)** Fig 2.1 shows a rigid uniform beam **AOB** of mass M. **O** is the mid-point of the beam. The beam rests on three vertical springs each having the same unstretched length, but with different spring constants k, $2k$ and $4k$ at **A**, **O** and **B** respectively, k being a constant. The bases of the springs are fixed to a horizontal platform.

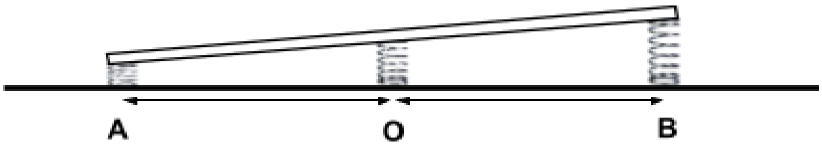


Fig. 2.1

Determine the compression of each spring in terms of M, k and g.

compression of A =…………….………….

compression of O = ….................………..

compression of B = ……………………. [6]

(b) A uniform cylinder of height 4.0 cm is held by a student and attached to an unstretched spring just above a beaker of fluid. The bottom end of the cylinder just touches the surface of the fluid, as shown in Fig. 2.2.

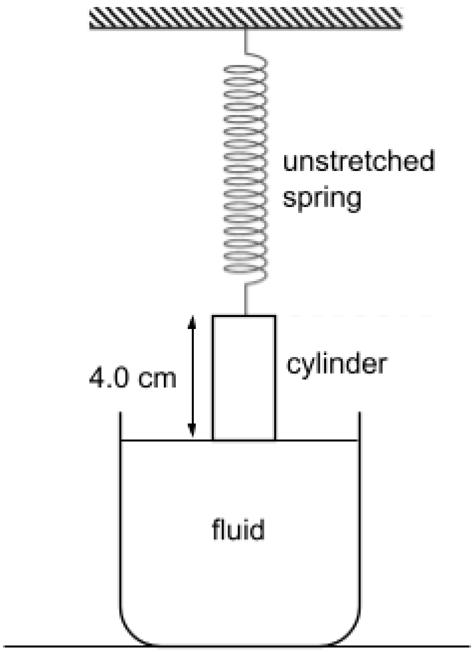


Fig. 2.2

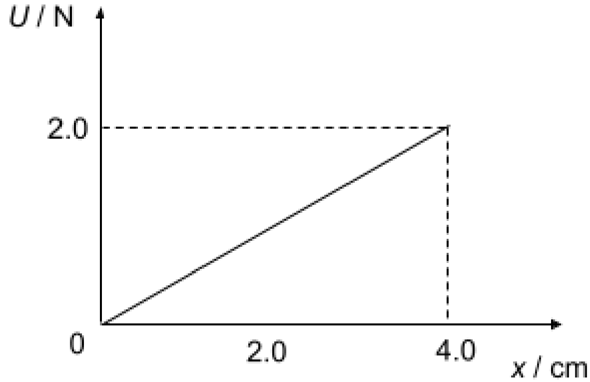


Fig. 2.3

The cylinder is then released. Fig. 2.3 shows the variation of the upthrust U acting on the cylinder with x, the extension of the spring, for $x \leq 4.0$ cm.

Given that the weight of the cylinder is 6.0 N and the force constant of the spring is 60 N m^{-1}, determine the speed of the cylinder when the extension on the spring is 6.0 cm. Assume that the drag forces are negligible.

speed = ……………….. m s^{-1} [4]

[Total: 10]

3 **(a)** A small beaker containing water can be fixed at various distances r from the axis of a horizontal turntable, which rotates with uniform angular velocity ω. The beaker is small enough for the surface of water in it to be taken as plane. It is observed that the plane of the water surface is inclined to the horizontal at an angle θ, which depends on r. These are shown in Fig 3.1.

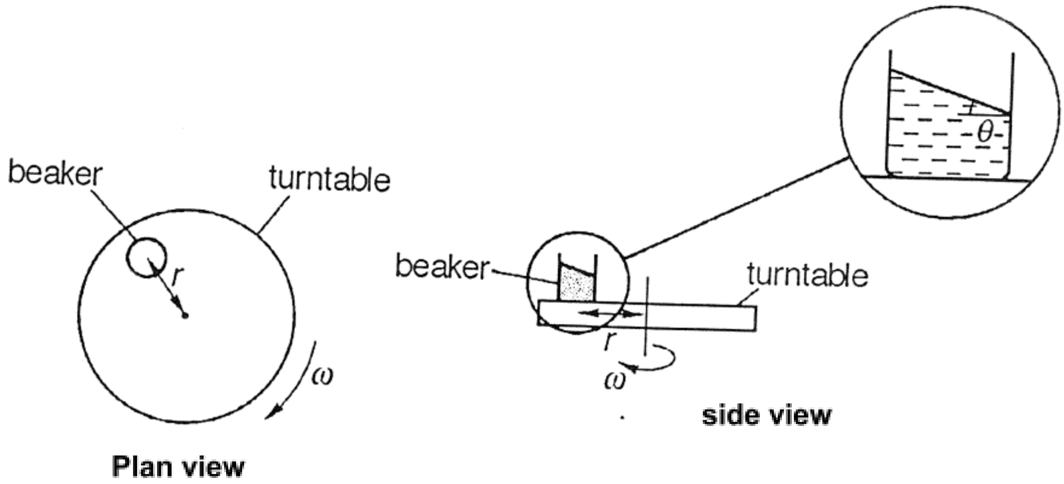


Fig. 3.1

(i) Explain why the water surface is not horizontal.

……………………………………………………………………………………………

……………………………………………………………………………………………... [1]

(ii) Derive an expression for θ in terms of r, ω and g, the acceleration of free fall.

θ = …………………… [4]

(b) A metal rod OA of length 0.90 m is hinged at O and is free to rotate in its plane about O. The angular displacement θ varies with time t according to

$$\theta = 0.15\,t^2,$$

where θ is in radians and t in seconds. Collar B slides along OA in such a way that its distance from O, r, is

$$r = 0.9 - 0.12t^2\,,$$

where r is in metres. This is shown in Fig. 3.3.

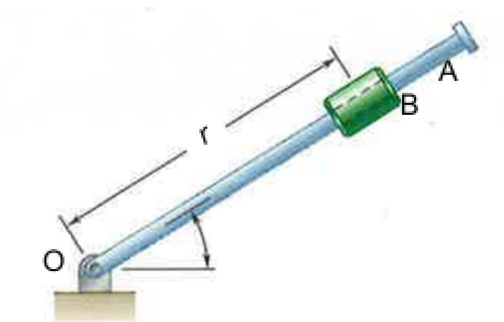


Fig. 3.3

When the angular displacement of OA is 30°, determine, for collar B,

(i) the tangential component of the velocity,

tangential velocity = …………………. m s^{-1} [2]

(ii) the radial component of the velocity,

radial velocity = …………………. m s^{-1} [2]

(iii) the speed.

speed = ………………. m s^{-1} [1]

[Total: 10]

4 A heat engine is a device that takes in thermal energy and, operating in a cyclic process, expels a fraction of that thermal energy and does work. The working substance of a heat engine is steam, which can be considered an ideal gas.

One way for a heat engine to achieve the maximum efficiency of η_{max} is by using the Carnot cycle as shown in Fig 4.1 below. The descriptions of the processes are shown in the table in Fig. 4.2.

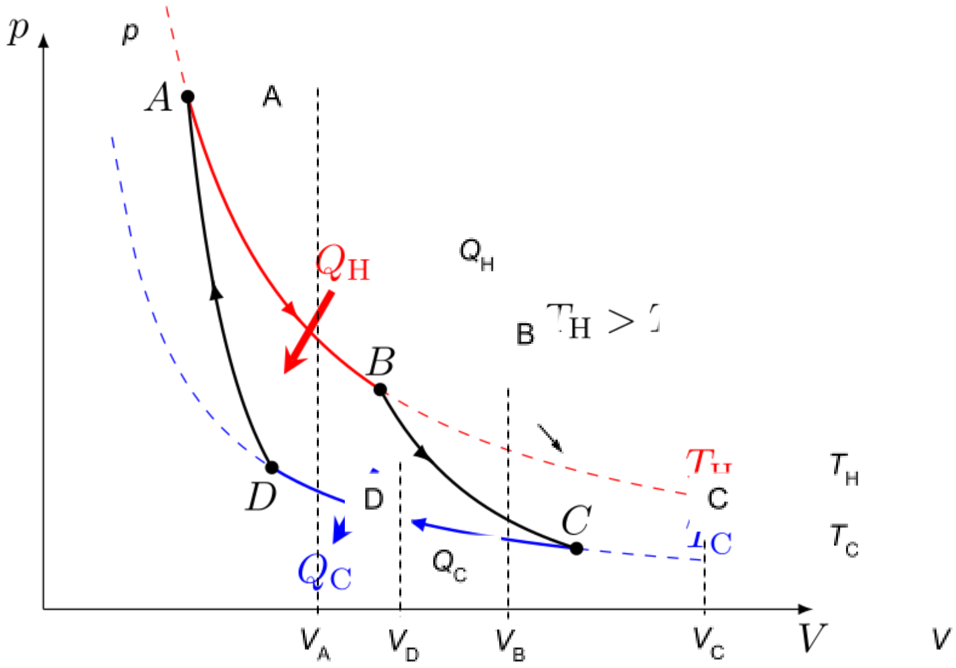


Fig. 4.1

Process	Type of process	Description of process
A to B	isothermal	Thermal energy Q_H is absorbed at temperature T_H and the steam expands. Work done on steam is W_{AB}.
B to C	adiabatic	The steam expands and its temperature falls from T_H to T_C. Work done on steam is W_{BC}
C to D	isothermal	Thermal energy Q_C is expelled at temperature T_C and the steam is compressed. Work done on steam is W_{CD}.
D to A	adiabatic	The steam is compressed, and its temperature increases from T_C to T_H. Work done on steam is W_{DA}.

Fig. 4.2

(a) State the First Law of Thermodynamics.

....

....

....

[1]

(b) Using the First Law of Thermodynamics, show that

(i) $W_{AB} = -Q_H$,

[1]

(ii) $W_{BC} = -W_{DA}$.

[2]

(c) **(i)** Show that the work done on the steam during the isothermal processes A to B and C to D are

$$W_{AB} = nRT_H \ln\left(\frac{V_A}{V_B}\right) \text{ and } W_{CD} = -nRT_C \ln\left(\frac{V_D}{V_C}\right)$$

respectively, where n is the number of moles of steam and R is the ideal gas constant.

[2]

(ii) For the adiabatic processes from B to C, the temperature T and the volume V are related by the equation,

$$TV^{x} = k_1 ,$$

where x and k_1 are constants. A similar equation holds for the adiabatic process from D to A,

$$TV^{x} = k_2 ,$$

where x is the same but k_2 is different constant from k_1.

Using the equations in **(b)** and the equation for efficiency

$$\eta = \frac{\text{net work output}}{\text{thermal energy input}} = \frac{W}{Q_\text{H}} ,$$

show that the thermal efficiency for the Carnot cycle is

$$\eta_\text{max} = 1 - \frac{T_\text{C}}{T_\text{H}} .$$

[4]

[Total: 10]

5 A photon of wavelength λ undergoes elastic head-on collision with a stationary object of mass m, which moves with speed v after the collision. The reflected photon is of wavelength λ', as shown in Fig. 5.1.

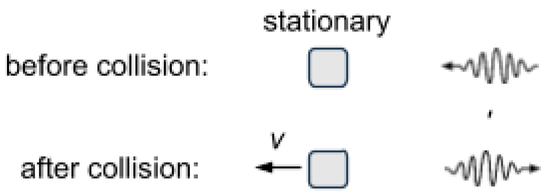


Fig. 5.1

There are no other forces acting on the system.
Ignore relativistic effects.

(a) Write down one equation for the conservation of linear momentum, and another for the conservation of energy, in terms of λ, λ', m and v.

[2]

(b) **(i)** Show that the final speed of the object is

$$v = \sqrt{c^2 + 4\frac{hc}{m\lambda}} - c$$

.

[2]

(ii) Determine an approximate expression for v, for the case of a macroscopic mass m and visible wavelength λ.

Explain your working clearly.

You may wish to use the approximation shown when x is small.

$$(1+x)^n \approx 1 + nx$$

[2]

(iii) Using conservation of energy and your answer in **(b)(ii)**, show that

$$\lambda' \approx \left(1+2\frac{h}{mc\lambda}\right)\lambda .$$

You may wish to use the approximation shown when x is small.

$$(1+x)^n \approx 1 + nx$$

[1]

[Total: 7]

6 **(a)** A yo-yo consists of two identical solid brass circular disks, of radius R, thickness t and density ρ, joined by a short axle of radius r. The total mass of the yo-yo is M.

Fig. 6.1 shows a falling yo-yo.

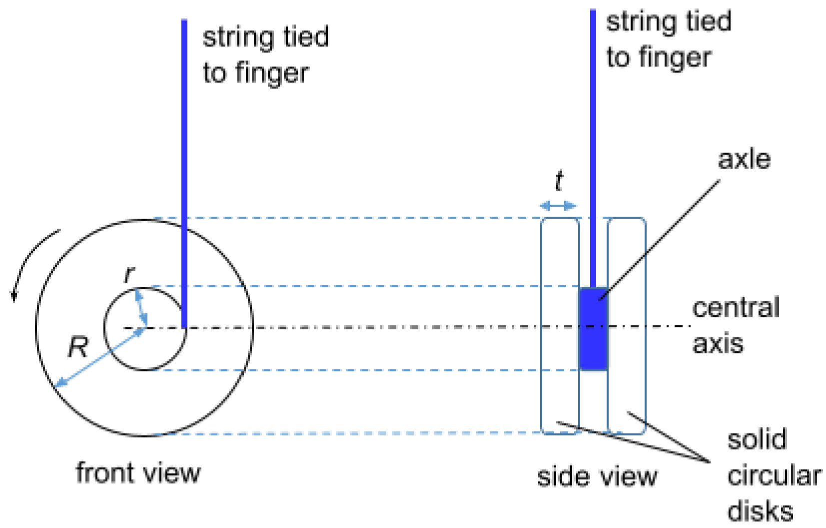


Fig. 6.1 (not to scale)

(i) Sketch and label the forces acting on the falling yo-yo in Fig. 6.2.

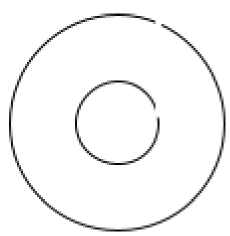


Fig. 6.2

[1]

(ii) Assuming that there is no slippage and the yo-yo starts rolling immediately after release, show that the yo-yo would roll down the string with constant acceleration of magnitude *a* given by

$$a = g\frac{1}{1+\frac{I}{Mr^2}}$$

where I is the moment of inertia of the yo-yo about its central axis.

[3]

(b) **(i)** The density of brass is 8400 kg m^{-3}. One particular yo-yo has brass disks of radii 3.6 cm and thickness 8.0 mm, joined by a short axle of radius 3.2 mm.

Assuming the moment of inertia of the axle is negligible, show that the minimum moment of inertia of the yo-yo is 3.55 × 10^{-4} kg m^2.

[2]

(ii) Using your answers to **(a)(ii)** and **(b)(i)**, show that the tension in the string of the yo-yo is 5.3 N.

[2]

(c) The yo-yo from **(b)** has a length of 1.2 m string wrapped around the shaft.

The yo-yo initially moves downwards with a linear velocity of 1.4 m s^{-1} in pure rotation.

Using your answer in **(b)(ii)**, or otherwise, calculate the final angular velocity of the yo-yo.

final angular velocity = ... rad s^{-1} [2]

[Total: 10]

Section B

Answer **two** questions from this section.

You are advised to spend about 35 minutes on each question.

7 **(a)** **(i)** Explain what is meant by an inertial reference frame.

....

....

.... [2]

(ii) Two objects P and Q are moving relative to each other.

Explain what is meant by the velocity of P relative to Q and how you can calculate it.

....

....

.... [2]

(b) Two cars P and Q travels towards the point O with a constant velocity of v_P and v_Q respectively as shown in Fig. 7.1.

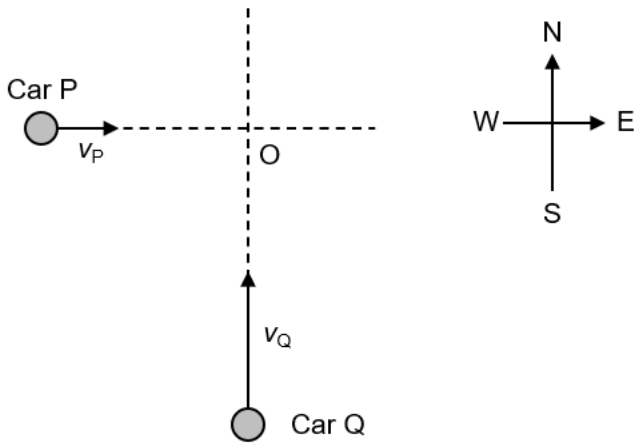


Fig. 7.1

At $t = 0$, car P and car Q are located at point P_0 and Q_0 respectively. At this point of time, car P is at a distance of D_P to the west of O while car Q is located at a distance of D_Q to the south of O as shown in Fig. 7.2. The angle that the line P_0Q_0 makes with the north is defined as θ.

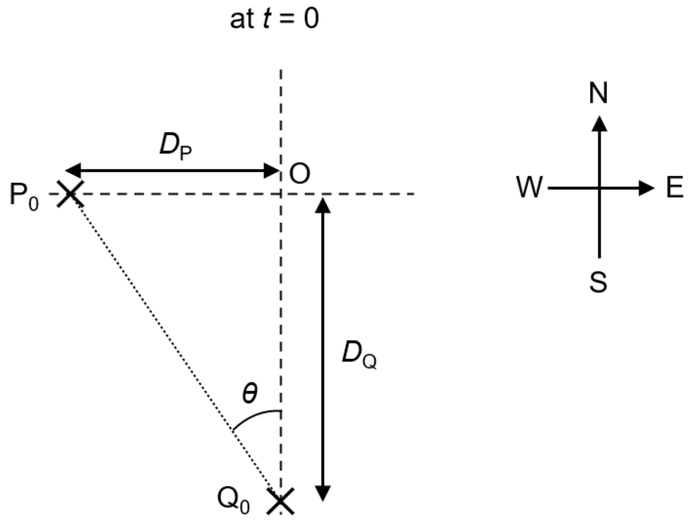


Fig. 7.2

(i) Given that the velocity of Q relative to P, v_{QP}, makes an angle ϕ with the north where $\phi < \theta$, draw in Fig. 7.3 a vector diagram that enables the vector v_{QP} to be determined. [1]

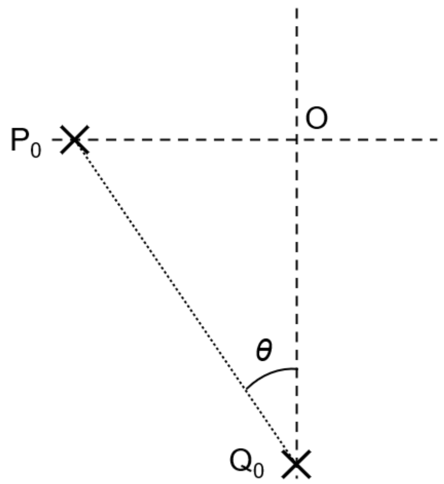


Fig. 7.3

(ii) Hence or otherwise, using the vector diagram drawn in **(b)(i)**, show that the shortest distance between car P and Q can be expressed as the following expression

$$d = \frac{D_P v_Q - D_Q v_P}{\sqrt{v_P^2 + v_Q^2}}.$$

You may want to consider using the following trigonometric relationship to aid your derivation

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B).$$

[4]

(iii) Show that the time t at which the distance between P and Q is the shortest can be given by

$$t = \frac{D_Q v_Q + D_P v_P}{v_P^2 + v_Q^2}$$

You may want to consider using the following trigonometric relationship to aid your derivation

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

[3]

(c) Two hockey pucks M and N has a mass of 0.600 kg and 0.400 kg respectively. M has an initial velocity of 5.0 m s^{-1} to the east. N is stationary initially. Upon an elastic collision between M and N, M moves with a final velocity of 3.0 m s^{-1} at an angle α to its initial direction of motion, as shown in Fig. 7.4.

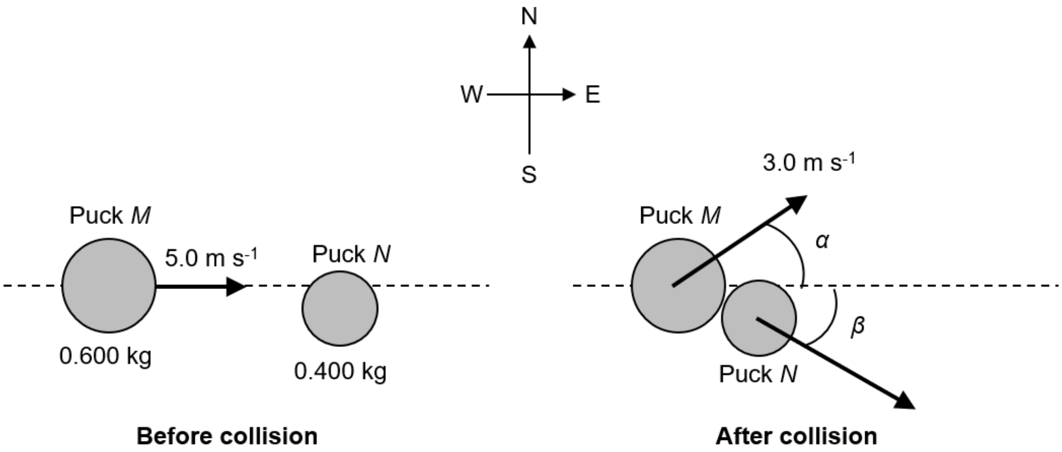


Fig. 7.4

(i) Explain what is meant by an *elastic collision*.

……………………………………………………………………………………………..

……………………………………………………………………………………………..

…………………………………………………………………………………………….. [2]

(ii) Using the centre of mass frame of reference, determine the angles α and β.

α = …..………………°

β = ……..……………° [4]

(iii) Describe an explain what will happen to α if the mass of puck *M* increases while mass of puck *N* is kept constant.

………………………………………………………………………………………..

………………………………………………………………………………………..

………………………………………………………………………………………..

………………………………………………………………………………………..

……………………………………………………………………………………….. [2]

[Total: 20]

8 **(a)** A long cylindrical conductor of radius *a* has two cylindrical cavities of diameter *a* through its entire length. A current I is directed out of the page and is uniform through a cross section of the conductor. P is a point a distance *r* from the centre of the conductor, as shown in Fig. 8.1.

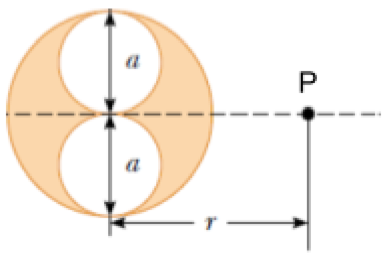


Fig. 8.1

Determine the magnitude and direction of the magnetic flux density at P in terms of μ_o, I, *r* and *a*.

magnitude = ……………………………

direction = …………………………..[6]

(b) A rectangular coil of sides 5.40 cm × 8.50 cm consists of 25 turns of wire and carries a current of 15.0 mA. A magnetic field of 0.350 T is applied parallel to the plane of the loop.

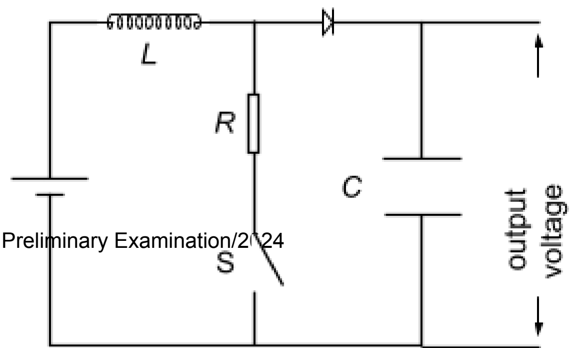
(i) Calculate the magnitude of its magnetic dipole moment.

magnitude of dipole moment = ………………… A m^2 [2]

(ii) Hence, determine the torque acting on the loop.

torque = …………………. N m [2]

(c) A DC voltage booster is used to output voltage that is higher than the e.m.f. of the DC source in the circuit. A simple DC voltage booster circuit is shown in Fig. 8.2. It consists of a source of e.m.f. ε, a resistor of resistance R, an inductor of inductance L, a capacitor of capacitance C, a switch S and a diode.



The voltage boosting is achieved by repeatedly closing and opening the switch S.

(i) After the circuit is built, S is closed for the first time and the steady state is reached.

State

1. the current in the inductor,

current = ...

2. the charge on the capacitor.

charge = ... [1
]

(ii) S is then opened at t = 0.

By the Kirchhoff's voltage law, the charge q on the capacitor and the current i in the circuit are such that

$$-L\frac{di}{dt}-\frac{q}{C}+\varepsilon=0$$
.

1. Derive the second order differential equation

$$\frac{d^2i}{dt^2}=-\frac{i}{LC}$$
.

[1]

2. Show that the solution to the second order differential equation is

$$i=i_0\sin(\omega t+\phi)$$
,

where i_0 and φ are constants, and

$$\omega=\frac{1}{\sqrt{LC}}$$
.

[1
]

3. By considering the above equations at t = 0, show that

a. $\phi = \dfrac{\pi}{2}$,

[2]

b. $i_0 = \dfrac{\varepsilon}{R}$.

[1
]

4. State the function of the diode.

....

.... [1
]

....

5. The switch remains open until a new equilibrium is reached.

a. Show that the total charge on the capacitor is

$$q_{tot} = C\varepsilon + \frac{\varepsilon}{R}\sqrt{LC}.$$

[2]

b. Determine the potential difference across the capacitor.

potential difference = ... [1]

[Total: 20]

9 **(a)** Planet P of mass m, is in elliptical orbit about a star, S of mass M where $M \gg m$. The distance of P from S at any instant is r and its corresponding velocity is v.

By considering components of v in the direction of r and perpendicular to r, show that the total energy, E_T of P is given by

$$E_T = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

where $\frac{dr}{dt}$ is the radial velocity of the planet, L is the planet's angular momentum and G is the universal gravitational constant.

[3]

(b) Fig. 9.1 shows the variation with distance r, of the effective radial potential U_{eff} of P.

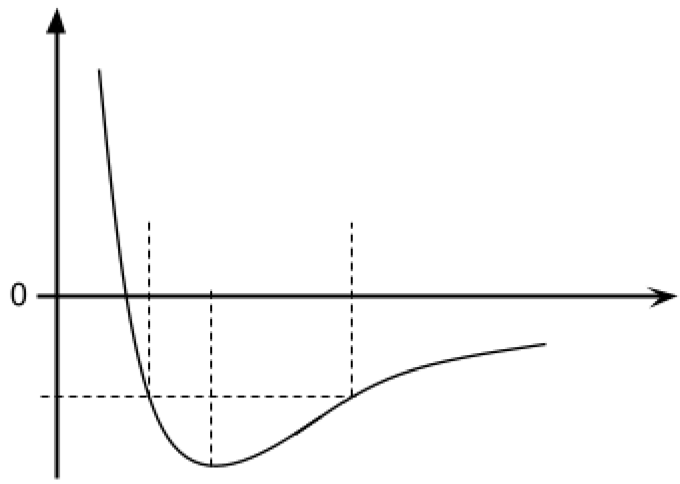


Fig. 9.1

The total energy of P is $-E$.

(i) Show that r_{min} and r_{max} are given by the following expressions:

$$r_{min} = \frac{GMm}{2E} - \sqrt{\left(\frac{GMm}{2E}\right)^2 - \left(\frac{L^2}{2mE}\right)} \quad \text{and} \quad r_{max} = \frac{GMm}{2E} + \sqrt{\left(\frac{GMm}{2E}\right)^2 - \left(\frac{L^2}{2mE}\right)}.$$

[3]

(ii) Hence or otherwise, derive an expression for the total energy E_T of P in terms of G, M, m and a where a represents the length of the semi-major axis of the elliptical orbit.

[3]

(iii) Show that the distance r_0 where the effective radial potential has a minimum value, is given by

$$r_0 = \frac{L^2}{GMm^2}$$

[2]

(c) The orbit of P relative to S is shown in Fig. 9.2. Distance *p* as indicated in Fig. 9.2 is called the semi latus rectum of the elliptical orbit, and *a* is the semi major axis. S and S' are the foci of the ellipse.

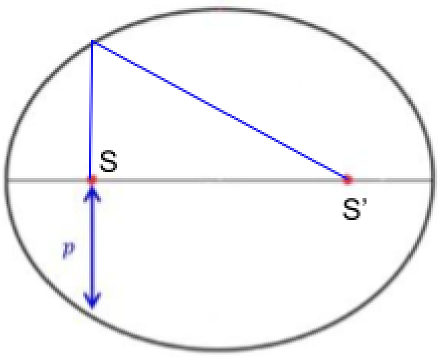


Fig. 9.2

Using your answers in part (b), show that $p = r_0$.

[5]

(d) Star S0-2 orbits the black hole at the centre of our galaxy. The orbital parameters are given in Fig 9.3, where 1 astronomical units (au) = 1.50×10^{11} m. Periapsis and Apoapsis are points on the orbit where the star is the closest and farthest, respectively, from the black hole.

star	period / years	periapsis / au	apoapsis / au
S0-2	15.2	119.5	1812

Fig. 9.3

Determine

(i) the mass of the black hole at the centre of our galaxy,

mass = ... kg [2]

(ii) the speed of S0-2 at the periapsis.

speed = ... $m\ s^{-1}$ [2]

[Total: 20]

[END OF PAPER]

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