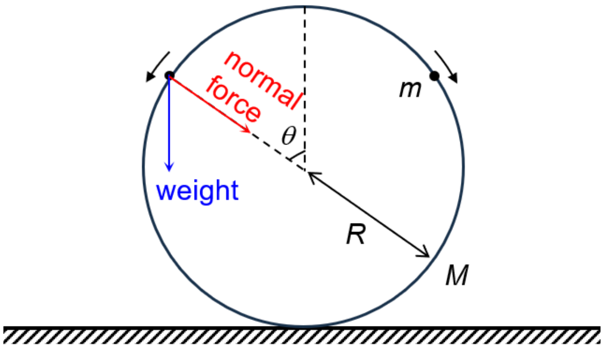
EUNOIA JUNIOR COLLEGE, NAN YANG JUNIOR COLLEGE
JC2 Preliminary Examination 2024
General Certificate of Education Advanced Level
Higher 3

H3 PHYSICS (9814)

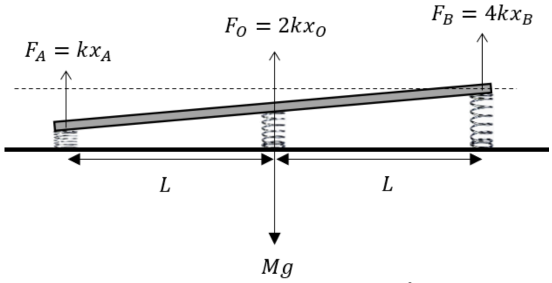
Paper 1

SUGGESTED SOLUTIONS

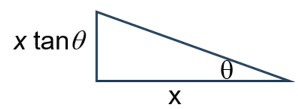
Question	Suggested Solution	Marks
1(a)	Loss in GPE = Gain in KE $mgR(1-\cos\theta)=\frac{1}{2}mv^2 \Rightarrow v=\sqrt{2gR(1-\cos\theta)}$	
1(b)	 Using expressions in **(a)**, $m\frac{v^2}{R}=2mg(1-\cos\theta)$ In order for the normal force on the beads to be radially inward, the centripetal force required (shown above) must exceed the radial component of weight. Thus, $2mg(1-\cos\theta)>mg\cos\theta$ $2>3\cos\theta$ $\cos\theta<\frac{2}{3}$	

Question	Suggested Solution	Marks
1(c)	Let the normal force be N. By Newton's 2nd Law, $F_{\text{centripetal}} = N + mg\cos\theta = m\dfrac{v^2}{R}$ $\Rightarrow\ N = m\dfrac{v^2}{R} - mg\cos\theta = 2mg(1-\cos\theta) - mg\cos\theta = mg(2-3\cos\theta)$ This normal force (hoop on beads) is radially inwards (when the beads are moving fast). By Newton's 3rd Law, <u>the beads exert on the hoop a normal force radially outwards of the same magnitude</u>. The sum of the upward components of N from the two beads is $N_{\text{vert}} = 2N\cos\theta = mg(4\cos\theta - 6\cos^2\theta)$. If this vertical force on the hoop is greater than the weight of the hoop, the hoop will be lifted off the ground. Thus, we require $N_{\text{vert}} = 2N\cos\theta = mg(4\cos\theta - 6\cos^2\theta) \le Mg$ $\Rightarrow \dfrac{m}{M} \le \dfrac{1}{4\cos\theta - 6\cos^2\theta}$ To find the maximum value of the denominator, $\dfrac{d}{d\theta}(4\cos\theta - 6\cos^2\theta) = -4\sin\theta + 12\cos\theta\sin\theta = 0 \Rightarrow \cos\theta = \dfrac{1}{3}$ Hence, at its maximum, $4\cos\theta - 6\cos^2\theta = \dfrac{4}{3} - \dfrac{6}{9} = \dfrac{2}{3}$, which implies $\dfrac{m}{M} \le \dfrac{3}{2}$. The maximum value of the ratio is then $\dfrac{m}{M} = \dfrac{3}{2}$. **Note:** Initially, the beads move slowly, the required centripetal force ($\propto v^2$) is small. The radial component of weight (= $mg\cos\theta$) is more than enough to supply the centripetal force. Hence, the normal force is radially outwards, and $mg\cos\theta - N$ is the centripetal force. As the beads speed up, the centripetal force required exceeds $mg\cos\theta$ (which decreases with θ). The normal force will then be radially inwards, such that $mg\cos\theta + N$ is the centripetal force. Since the hoop exerts on the beads an inward normal force, by Newton's 3rd Law, the beads exert on the hoop and outward normal force, which has an upward component. If this upward force component is greater than the weight of the hoop, the hoop will be lifted off the ground.	

Question	Suggested Solution	Marks

2(a)	Let the compression of the spring at A, O and B be x_A, x_O and x_B respectively. $\sum$Moments about O: $(kx_A)L = (4kx_B)L$ $x_A = 4x_B \quad ---(1)$  $\sum$Moments about A: $MgL = (2kx_O)L + (4kx_B)(2L)$ $2kx_O = Mg - 8kx_B \quad ---(2)$ Since the beam is rigid, from similar triangles: $\frac{x_O - x_B}{L} = \frac{x_A - x_B}{2L}$ $2x_O = x_A + x_B$ From (1): $2x_O = 4x_B + x_B \quad \Rightarrow \quad x_O = \frac{5}{2}x_B$ From (2): $2k\left(\frac{5}{2}x_B\right) = Mg - 8kx_B$ Solving: $x_B = \frac{Mg}{13k}$; $x_A = \frac{4Mg}{13k}$; $x_O = \frac{5Mg}{26k}$	

Question	Suggested Solution	Marks
2(b)	As uthrust is constant when immersed in the fluid (from 4.0 to 6.0 cm) $WD_{\text{against upthrust}} = \text{Area under graph} + (2.0\times10^{-2})(\text{max upthrust})$ $= \frac{1}{2}(2.0)(4.0\times10^{-2}) + (2.0\times10^{-2})(2.0)$ $= 0.080\text{J}$ energy stored in spring $= \frac{1}{2}kx^2 = \frac{1}{2}(60)(0.06)^2 = 0.108\text{J}$ Loss in GPE = Gain in KE + energy stored in spring + $WD_{\text{against upthrust}}$ $(6.0)(0.06) = \frac{1}{2}\left(\frac{6.0}{9.81}\right)v^2 + 0.108 + 0.080$ $v = 0.75\text{ m s}^{-1}$	

3(a)(i)	So that there is a net pressure and <u>thus a net force pointing to the right</u> to <u>provide the centripetal force</u>.	
3(a)(ii)	Let the depth be z and the density be ρ. Area $= (x \tan\theta) z$ Mass of water $= \frac{1}{2}\rho(x^2 \tan\theta)z$  Average force $=$ average pressure $\times$ area $= \frac{1}{2}\rho g x^2 z \tan^2\theta$ Since average force provides for centripetal force, $\frac{1}{2}\rho g x^2 z \tan^2\theta = mr\omega^2$ $\frac{1}{2}\rho g x^2 z \tan^2\theta = \left(\frac{1}{2}\rho(x^2\tan\theta)z\right)r\omega^2$ $\tan\theta = \frac{\omega^2}{g}$ $\therefore \theta = \tan^{-1}\left(\frac{\omega^2}{g}\right)$	

Question	Suggested Solution	Marks
3(b)(i)	At 30° $\theta = 30° = \left(\dfrac{30}{360}\right)2\pi = 0.524\text{ rad}$ Using $\theta = 0.15t^2$, $t = \sqrt{\dfrac{\theta}{0.15}} = \sqrt{\dfrac{0.524}{0.15}} = 1.869\text{ s}$ Note : $\omega = \dfrac{d\theta}{dt} = 0.30t$ $v_{\text{tangential}} = r\omega = (0.9 - 0.12t^2)(0.30t)$ $= [0.9 - 0.12(1.869)^2][0.30(1.869)]$ $= 0.270\text{ m s}^{-1}$	
3(b)(ii)	$v_{\text{radial}} = \dfrac{dr}{dt} = -0.24t$ $= -0.24(1.869)$ $= -0.449\text{ m s}^{-1}$	
3(b)(iii)	$v = \sqrt{v_{\text{radial}}^2 + v_{\text{tangential}}^2}$ $= \sqrt{(-0.449)^2 + (0.270)^2}$ $= 0.524\text{ m s}^{-1}$	

4(a)	The first law of thermodynamics states that the increase in internal energy of a system is equal to the sum of the heat supplied to the system and the work done on the system.	
4(b)(i)	From the First Law of Thermodynamics, $\Delta U = Q + W$ For the isothermal process A to B, $\Delta U = 0$ $\therefore W_{AB} = -Q_H$	

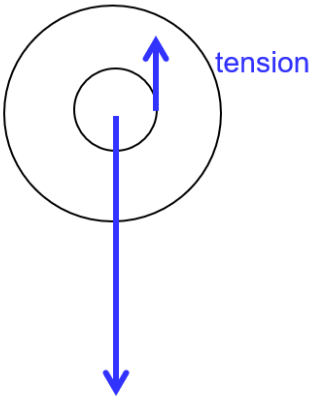
Question	Suggested Solution	Marks
4(b)(ii)	From the First Law of Thermodynamics, $\Delta U = Q + W$ For the adiabatic process B to C, $Q = 0$ $\therefore W_{\text{BC}} = \Delta U_{\text{BC}}$, similarly, $W_{\text{DA}} = \Delta U_{\text{DA}}$ But since ΔT is the same, $\Delta U_{\text{BC}} = -\Delta U_{\text{DA}}$ $\therefore W_{\text{BC}} = -W_{\text{DA}}$	
4(c)(i)	Work done on the steam from A to B is = $W_{\text{AB}} = -\int_{V_\text{A}}^{V_\text{B}} p \, \text{d}V$ $= -nRT_{\text{H}} \int_{V_\text{A}}^{V_\text{B}} \frac{1}{V} \, \text{d}V \quad \left(\text{using the ideal gas equation } pV = nRT\right)$ $= nRT_{\text{H}} \ln\left(\frac{V_A}{V_\text{B}}\right)$ Similarly, the work done on the steam from C to D is = $W_{\text{CD}} = -nRT_{\text{C}} \ln\left(\frac{V_\text{D}}{V_\text{C}}\right)$	

4(c)(ii)	$\eta = \dfrac{\text{work output}}{\text{thermal energy input}} = \dfrac{W}{Q_H}$ For one complete Carnot cycle, the net work done on the steam is $W = W_{ABCD}$ where $W_{ABCD} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$ $= W_{AB} + W_{CD}$ $= -Q_H + Q_C$ ∴ the net work done by the steam is = $Q_H - Q_C$ $\therefore \eta_{max} = \dfrac{W}{Q_H} = \dfrac{Q_H - Q_C}{Q_H}$ $= 1 - \dfrac{Q_C}{Q_H}$ $= 1 - \dfrac{W_{CD}}{W_{AB}}$ $= 1 - \dfrac{nRT_C \ln\left(\dfrac{V_C}{V_D}\right)}{nRT_H \ln\left(\dfrac{V_B}{V_A}\right)}$ $= 1 - \dfrac{T_C \ln\left(\dfrac{V_C}{V_D}\right)}{T_H \ln\left(\dfrac{V_B}{V_A}\right)}$ For the adiabatic processes B to C and D to A, $T_B V_B^{\,x} = T_C V_C^{\,x} = k_1$ $T_A V_A^{\,x} = T_D V_D^{\,x} = k_2$ $\therefore \dfrac{T_B V_B^{\,x}}{T_A V_A^{\,x}} = \dfrac{T_C V_C^{\,x}}{T_D V_D^{\,x}}$ But, $T_A = T_B$, and $T_C = T_D$, $\therefore \dfrac{V_B^{\,x}}{V_A^{\,x}} = \dfrac{V_C^{\,x}}{V_D^{\,x}} \Rightarrow \dfrac{V_B}{V_A} = \dfrac{V_C}{V_D}$	

Question	Suggested Solution	Marks
	$$\eta_{\max} = 1 - \frac{T_C \ln\left(\frac{V_C}{V_D}\right)}{T_H \ln\left(\frac{V_B}{V_A}\right)} = 1 - \frac{T_C}{T_H}$$ Hence,	

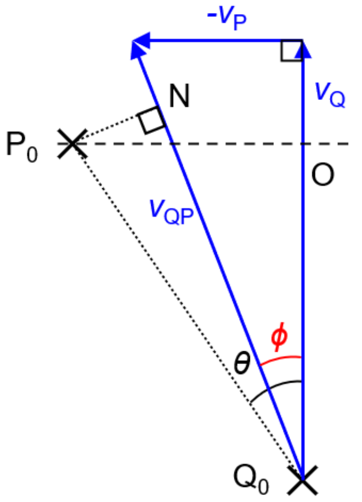
5(a)	Conservation of momentum: $\frac{h}{\lambda} = mv - \frac{h}{\lambda'}$ Conservation of energy: $\frac{hc}{\lambda} = \frac{1}{2}mv^2 + \frac{hc}{\lambda'}$	
5(b)(i)	Rearranging the above equations: $\frac{h}{\lambda} + \frac{h}{\lambda'} = mv$ ...... (1) $\frac{h}{\lambda} - \frac{h}{\lambda'} = \frac{mv^2}{2c}$ ...... (2) (1)+(2) to eliminate λ′, and rearranging: $mv^2 + 2mcv - 4\frac{hc}{\lambda} = 0$ Solving the quadratic equation for v: $v = \frac{-2mc + \sqrt{4m^2c^2 + 16\frac{hmc}{\lambda}}}{2m}$ or $\frac{-2mc - \sqrt{4m^2c^2 + 16\frac{hmc}{\lambda}}}{2m}$ (rejected since $v > 0$) $= \sqrt{c^2 + 4\frac{hc}{m\lambda}} - c$	
5(b)(ii)	$v = \sqrt{c^2 + 4\frac{hc}{m\lambda}} - c = c\sqrt{1 + 4\frac{h}{m\lambda c}} - c$ Since $h \approx 10^{-34}, m \approx 1,\ c \approx 10^8$ and $\lambda \approx 10^{-7}, \frac{h}{mc\lambda} \approx \frac{10^{-34}}{10^{8-7}} = 10^{-35} << 1$ $v = c\sqrt{1 + 4\frac{h}{m\lambda c}} - c \approx c\left(1 + 2\frac{h}{m\lambda c}\right) - c = 2\frac{h}{m\lambda}$	

Question	Suggested Solution	Marks
5(b)(iii)	Conservation of energy: $\frac{hc}{\lambda}=\frac{1}{2}mv^2+\frac{hc}{\lambda'}$ Rearranging, $\frac{1}{\lambda'}=\frac{1}{\lambda}-\frac{mv^2}{2hc}=\frac{1}{\lambda}-\frac{m}{2hc}\left(2\frac{h}{m\lambda}\right)^2=\frac{1}{\lambda}\left(1-2\frac{h}{mc\lambda}\right)$ $\Rightarrow \lambda'=\frac{1}{1-2\frac{h}{mc\lambda}}\lambda$ $\approx\left(1+2\frac{h}{mc\lambda}\right)\lambda$	

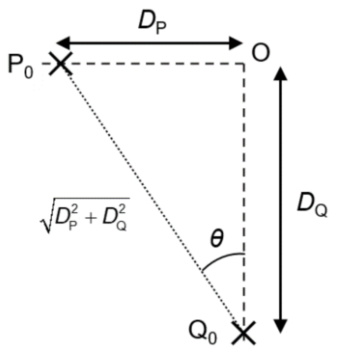
6(a)(i)	weight must be larger than tension  	

Question	Suggested Solution	Marks
6(a)(ii)	Let T be the tension in string. Apply Newton's Second Law in both its translational and rotational forms : $\sum F = Mg - T = Ma$ --- (1) $\sum \tau = Tr = I\alpha$ --- (2) Sub (2) into (1) $Mg - \frac{I\alpha}{r} = Ma$ --- (3) Using $a = r\alpha$ into (3), $Mg - \frac{I}{r}\left(\frac{a}{r}\right) = Ma$ $Mgr^2 - Ia = Mar^2$ $a = g\frac{Mr^2}{Mr^2 + I}$	
6(b)(i)	$M = \rho V = (8400)[2\pi(0.036)^2(8.0\times10^{-3})] = 0.547210$ $I = \frac{1}{2}MR^2 = \frac{1}{2}(0.547210)(0.036)^2 = 3.55\times10^{-4}$ kg m^2	
6(b)(ii)	$a = g\frac{Mr^2}{Mr^2+I} = (9.81)\frac{0.547210\left(3.2\times10^{-3}\right)^2}{0.547210\left(3.2\times10^{-3}\right)^2 + 3.55\times10^{-4}} = 0.1526$ $T = Mg - Ma$ $= 0.547210(9.81 - 0.1526)$ $= 5.3$ N	
6(c)	$\text{WD}_{\text{by tension}} = \text{gain in rotational KE}$ $TL = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$ $5.285\times1.2 = \frac{1}{2}(3.55\times10^{-4})\left[\omega_f^2 - \left(\frac{1.4}{0.0032}\right)^2\right]$ $\omega_f = 480$ rad s^{-1}	

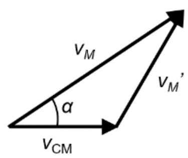
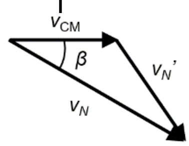
Question	Suggested Solution	Marks
7(a)(i)	An inertial frame of reference refers to one in which Newton's first law of motion is valid. The frame is either at rest or moving at a constant velocity.	
7(a)(ii)	The velocity of P as observed by Q and is given by the vector equation $\vec{V}_{PQ} = \vec{V}_P - \vec{V}_Q$	

<table>
<tr><th>Question</th><th>Suggested Solution</th><th>Marks</th></tr>
<tr><td>7(b)(i)</td><td>


Correct direction of vector diagram and $\phi < \theta$ correctly labelled.</td><td></td></tr>
<tr><td>7(b)(ii)</td><td>
From vector diagram in (b)(i),
$\vec{v}_{QP} = \vec{v}_Q - \vec{v}_P$ and $v_{QP}^2 = v_P^2 + v_Q^2$



Using Pythagoras' Theorem on diagram above,
$P_0Q_0 = \sqrt{D_P^2 + D_Q^2}$
Let N be the foot of perpendicular of point P_0 on $\vec{v}_{QP}$,
P_0N will be the shortest distance between car P and Q = $P_0Q_0 \sin(\theta - \phi$
Using $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$,
$P_0N = \sqrt{D_P^2 + D_Q^2}\left[\sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi)\right]$
$= \sqrt{D_P^2 + D_Q^2}\left[\left(\frac{D_P}{\sqrt{D_P^2 + D_Q^2}}\right)\left(\frac{v_Q}{v_{QP}}\right) - \left(\frac{D_Q}{\sqrt{D_P^2 + D_Q^2}}\right)\left(\frac{v_P}{v_{QP}}\right)\right]$
$= \frac{D_P v_Q - D_Q v_P}{v_{QP}}$
$= \frac{D_P v_Q - D_Q v_P}{\sqrt{v_P^2 + v_Q^2}}$ (shown)
</td><td></td></tr>
</table>

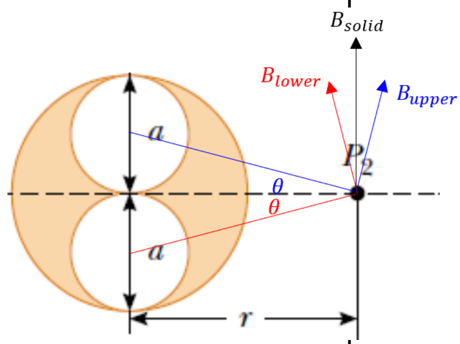
Question	Suggested Solution	Marks
7(b)(iii)	Let t be the time at which the distance between the cars is the shortest. $t = \dfrac{\text{Q}_0\text{N}}{v_{\text{QP}}}$ $= \dfrac{\text{P}_0\text{Q}_0 \cos\left(\theta - \phi\right)}{v_{\text{QP}}}$ $= \dfrac{\sqrt{D_{\text{P}}^2 + D_{\text{Q}}^2}}{v_{\text{QP}}}\left[\cos\left(\theta\right)\cos\left(\phi\right) + \sin\left(\theta\right)\sin\left(\phi\right)\right]$ $= \dfrac{\sqrt{D_{\text{P}}^2 + D_{\text{Q}}^2}}{v_{\text{QP}}}\left[\left(\dfrac{D_{\text{Q}}}{\sqrt{D_{\text{P}}^2 + D_{\text{Q}}^2}}\right)\left(\dfrac{v_{\text{Q}}}{v_{\text{QP}}}\right) + \left(\dfrac{D_{\text{P}}}{\sqrt{D_{\text{P}}^2 + D_{\text{Q}}^2}}\right)\left(\dfrac{v_{\text{P}}}{v_{\text{QP}}}\right)\right]$ $= \dfrac{D_{\text{Q}}v_{\text{Q}} + D_{\text{P}}v_{\text{P}}}{v_{\text{QP}}^2}$ $= \dfrac{D_{\text{Q}}v_{\text{Q}} + D_{\text{P}}v_{\text{P}}}{v_{\text{P}}^2 + v_{\text{Q}}^2}$	
7(c)(i)	An elastic collision is one in which <u>the total kinetic energy of the system of colliding bodies</u> is conserved <u>before and after</u> the collision.	
7(c)(ii)	$v_{\text{CM}} = \dfrac{(0.600)(5.0) + (0.400)(0.0)}{0.600 + 0.400} = 3.0\ \text{m s}^{-1}$ Using CM frame of reference, $u'_M = u_M - v_{\text{CM}} = 5.0 - 3.0 = 2.0\ \text{m s}^{-1}$ $u'_N = u_N - v_{\text{CM}} = 0.00 - 3.0 = -3.0\ \text{m s}^{-1}$ Since 2D collision, to conserve momentum and KE, $\lvert v'_{\text{M}}\rvert = \lvert u'_{\text{M}}\rvert = 2.0\ \text{m s}^{-1}$ and $\lvert v'_{\text{N}}\rvert = \lvert u'_{\text{N}}\rvert = 3.0\ \text{m s}^{-1}$ To conserve KE : $\sum \text{KE}_{\text{i}} = \sum \text{KE}_{\text{f}}$ $\frac{1}{2}(0.600)(5.0)^2 = \frac{1}{2}(0.600)(3.0)^2 + \frac{1}{2}(0.400)(v_{\text{N}})^2$ $v_{\text{N}} = 4.90\ \text{m s}^{-1}$ Using vector equations : $\vec{v}_{\text{M}} = \vec{v}'_{\text{M}} + \vec{v}_{\text{CM}}$ to obtain the vector triangle and using cosine rule, $(2.0)^2 = (3.0)^2 + (3.0)^2 - 2(3.0)(3.0)\cos(\alpha)$ $\alpha = 38.9°$ Using vector equations : $\vec{v}_{\text{N}} = \vec{v}'_{\text{N}} + \vec{v}_{\text{CM}}$ to obtain the vector triangle and using cosine rule, $(3.0)^2 = (4.90)^2 + (3.0)^2 - 2(4.90)(3.0)\cos(\beta)$ $\beta = 35.2°$  	
7(c)(iii)	α will take on a smaller value. $v_{\text{M}}{}'$ will take on a smaller value because v_{CM} will have a magnitude closer to v_{M}. Hence, using cosine rule, α will take on a smaller value.	

Question	Suggested Solution	Marks

Question	Suggested Solution	Marks
8(a)	At P :	

Flux density due to conductor with full solid circular cross-section:

$$B_{solid}(2\pi r) = \mu_0 J\left(\pi a^2\right)$$



$$B_{solid} = \frac{\mu_0 J\left(a^2\right)}{2r}$$

The horizontal components of B_{lower} and B_{upper} cancels off and hence we only need to consider vertical components.

Flux density at P due to the conductor with odd shaped cross-section:

$$B_{odd} = B_{solid} - B_{upper} - B_{lower}$$

$$B_{odd} = \frac{\mu_0 J\left(a^2\right)}{2r} - 2\left(\frac{\mu_0 J\left(\frac{\pi a^2}{4}\right)}{2\pi\sqrt{\left(r^2+\frac{a^2}{4}\right)}}\right)\cos\cos\theta$$ where

$$\cos\cos\theta = \frac{r}{\sqrt{\left(r^2+\frac{a^2}{4}\right)}}$$

$$B_{odd} = \frac{\mu_0 J\left(a^2\right)}{2r} - 2\left(\frac{\mu_0 J\left(\frac{\pi a^2}{4}\right)r}{2\pi\left(r^2+\frac{a^2}{4}\right)}\right) = \frac{\mu_0 J\left(a^2\right)}{2r} - 2\left(\frac{\mu_0 J a^2 r}{8\left(r^2+\frac{a^2}{4}\right)}\right) = \frac{\mu_0 J\left(a^2\right)}{2r} - \left(\frac{\mu_0 J}{(4r^2}\right.$$

$$B_{odd} = \mu_0 J a^2\left(\frac{1}{2r} - \left(\frac{r}{\left(4r^2+a^2\right)}\right)\right) = \mu_0 J a^2\left(\frac{4r^2+a^2-2r^2}{2r\left(4r^2+a^2\right)}\right) = \mu_0 J a^2\left(\frac{2r^2+a^2}{2r\left(4r^2+a^2\right)}\right)$$

Current through the odd shaped cross-section, $I = J\left(\frac{\pi a^2}{2}\right)$

$$\Rightarrow \quad J = \frac{2I}{\pi a^2}$$

Hence, $B_{odd} = \mu_0\left(\frac{2I}{\pi a^2}\right)a^2\left(\frac{2r^2+a^2}{2r\left(4r^2+a^2\right)}\right) = \left(\frac{\mu_0 I}{\pi r}\right)\left(\frac{2r^2+a^2}{4r^2+a^2}\right)$

upwards.

Question	Suggested Solution	Marks
8(b)(i)	μ = *NIA* = 25 x 15 x 10^{-3} x 0.054 x 0.085 = 1.72 x 10^{-3} A m^2	
8(b)(ii)	τ = μB = 1.72 x 10^{-3} x 0.35 = 6.02 x 10^{-4} N m	
8(c)(i)1	$\frac{\varepsilon}{R}$	
8(c)(i)2	$C\varepsilon$	
8(c)(ii)1	$-L\frac{di}{dt}-\frac{q}{C}+\varepsilon=0$ Differentiating both sides with respect to *t*, $-L\frac{d^2i}{dt^2}-\frac{i}{C}=0 \Rightarrow \frac{d^2i}{dt^2}=-\frac{i}{LC}$ where $i=\frac{dq}{dt}$ and $\frac{d\varepsilon}{dt}=0$.	
8(c)(ii)2	Differentiating $i=i_0\sin(\omega t+\phi)$ with respect to *t* twice, $\frac{d^2i}{dt^2}=-\omega^2 i_0\sin(\omega t+\phi)=-\frac{i_0\sin(\omega t+\phi)}{LC}=-\frac{i}{LC}$. Hence, $i=i_0\sin(\omega t+\phi)$ satisfies the differential equation.	
8(c)(ii)3a	At *t* = 0, the charge on the inductor is $q(t=0)=C\varepsilon$. Using the Kirchhoff's voltage law equation, $-L\frac{di}{dt}-\frac{q}{C}+\varepsilon=0 \Rightarrow \frac{di}{dt}\Big\vert_{t=0}=\frac{1}{L}\left(\varepsilon-\frac{q(t=0)}{C}\right)=\frac{1}{L}\left(\varepsilon-\frac{C\varepsilon}{C}\right)=0$ Thus, $\frac{di}{dt}\Big\vert_{t=0}=\frac{d}{dt}i_0\sin(\omega t+\phi)=\omega i_0\cos(\omega t+\phi)\vert_{t=0}=\omega i_0\cos\phi=0$ $\Rightarrow \cos\phi=0$ Hence, $\phi=\frac{\pi}{2}$.	
8(c)(ii)3b	At *t* = 0, the current in the inductor is $i(t=0)=\frac{\varepsilon}{R}$. Thus, $i(t=0)=\frac{\varepsilon}{R}=i_0\sin(\phi)=i_0\sin(\frac{\pi}{2})=i_0$.	

Question	Suggested Solution	Marks
8(c)(ii)4	To prevent the current from flowing in the opposite direction (hence the discharging of the capacitor).	
8(c)(ii)5a	Since the diode prevents the current flowing in the opposite direction (which will lead to the discharging of the capacitor), only positive current needs to be considered. Since $i = i_0 \sin(\omega t + \frac{\pi}{2}) = i_0 \cos(\omega t)$, we consider $0 < \omega t < \frac{\pi}{2}$. Thus, $$\Delta q = \frac{1}{\omega}\int_0^{\frac{\pi}{2}} i_0 \cos(\omega t) d(\omega t) = \frac{i_0}{\omega}\sin(\omega t)\Big\|_0^{\frac{\pi}{2}} = \frac{i_0}{\omega} = \frac{\varepsilon}{R}\sqrt{LC},$$ and $$q_{\text{tot}} = q(t=0) + \Delta q = C\varepsilon + \frac{\varepsilon}{R}\sqrt{LC}.$$	
8(c)(ii)5b	$$\text{potential difference} = \frac{q_{\text{tot}}}{C} = \frac{C\varepsilon + \frac{\varepsilon}{R}\sqrt{LC}}{C} = \varepsilon\left(1 + \frac{1}{R}\sqrt{\frac{L}{C}}\right)$$	
9(a)	$v^2 = (v_r\ r\omega) \bullet (v_r\ r\omega) = v_r^2 + r^2\omega^2$ where $v_r = \frac{dr}{dt}$ $E_T = k.e. + g.p.e.$ $= \frac{1}{2}m\left(v_r^2 + r^2\omega^2\right) - \frac{GMm}{r}$ $= \frac{1}{2}mv_r^2 + \frac{1}{2}mr^2\omega^2 - \frac{GMm}{r}$ Since angular momentum, $L = mr^2\omega$, $\omega = \frac{L}{mr^2}$ $E_T = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$ (shown)	

Question	Suggested Solution	Marks
9(b)(i)	From $E_T = \frac{1}{2}mv_r^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r} = -E$ $\frac{1}{2}mv_r^2 = -E - \frac{L^2}{2mr^2} + \frac{GMm}{r} = 0$ Multiplying throughout by − r^2 and rearranging terms gives $Er^2 - GMmr + \frac{L^2}{2m} = 0$ Solving for *r* gives $r = \frac{GMm \pm \sqrt{(GMm)^2 - 4E\left(\frac{L^2}{2m}\right)}}{2E} = \frac{GMm}{2E} \pm \sqrt{\left(\frac{GMm}{2E}\right)^2 - \left(\frac{L^2}{2mE}\right)}$ $r_{min} = \frac{GMm}{2E} - \sqrt{\left(\frac{GMm}{2E}\right)^2 - \left(\frac{L^2}{2mE}\right)}$ $r_{max} = \frac{GMm}{2E} + \sqrt{\left(\frac{GMm}{2E}\right)^2 - \left(\frac{L^2}{2mE}\right)}$	
9(b)(ii)	$2a = r_{min} + r_{max}$ $= \frac{GMm}{E} \quad \Rightarrow \quad E = \frac{GMm}{2a}$ The total energy of P, $E_T = -E$ $E_T = -\frac{GMm}{2a}$	
9(b)(iii)	From $U_{eff} = \frac{L^2}{2mr^2} - \frac{GMm}{r}$ $\frac{dU_{eff}}{dr} = -\frac{L^2}{mr^3} + \frac{GMm}{r^2}$ $\frac{dU_{eff}}{dr} = 0$ $\frac{L^2}{mr^3} = \frac{GMm}{r^2}$ $r = \frac{L^2}{GMm^2} = r_0$ (shown)	

Question	Suggested Solution	Marks
9(c)	From Pythagoras theorem: $(2a - p)^2 = p^2 + (SS')^2$ Where $SS' = r_{max} - r_{min} = 2\sqrt{\left(\frac{GMm}{2E}\right)^2 - \left(\frac{L^2}{2mE}\right)}$ $(SS')^2 = \left(\frac{GMm}{E}\right)^2 - \left(\frac{2L^2}{mE}\right)$ $4a^2 - 4ap + p^2 = p^2 + \left(\frac{GMm}{E}\right)^2 - \left(\frac{2L^2}{mE}\right)$ $4a^2 - \left(\frac{GMm}{E}\right)^2 + \left(\frac{2L^2}{mE}\right) = 4ap$ $p = \frac{4a^2 - \left(\frac{GMm}{E}\right)^2 + \left(\frac{2L^2}{mE}\right)}{4a}$ $= \frac{\left(\frac{GMm}{E}\right)^2 - \left(\frac{GMm}{E}\right)^2 + \left(\frac{2L^2}{mE}\right)}{\frac{2GMm}{E}}$ $= \left(\frac{2L^2}{mE}\right) \times \left(\frac{E}{2GMm}\right) = \frac{L^2}{GMm^2}$ Since $r_0 = \frac{L^2}{GMm^2}$ Hence $p = r_0$	
9(d)(i)	From Kepler's 3rd law, $T^2 = \frac{4\pi^2 a^3}{GM}$ $M = \frac{4\pi^2 a^3}{GT^2}$ $a = \frac{119.5+1812}{2} = 965.75$ au = 1.448 625 x 10^{14} m $M = \frac{4\pi^2 (1.448\,625 \times 10^{14})^3}{6.67\times10^{-11}(15.2\times365\times24\times3600)^2}$ $M = 7.83\times10^{36}$ kg	
9(d)(ii)	From energy conservation, $\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$ $v^2 = \frac{2GM}{r} - \frac{GM}{a}$ $= \left(6.67\times10^{-11}\times7.83\times10^{36}\right)\left(\frac{2}{119.5\times1.50\times10^{11}} - \frac{1}{1.448\,625 \times 10^{14}}\right)$ $v^2 = 5.47\times10^{13}$ $v = 7.39\times10^{6}$ m s^{-1}.	