
This Question Paper consists of **29** printed pages.

Data

speed of light in free space	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$ $(1/(36\pi)) \times 10^{-9} \text{ F m}^{-1}$
elementary charge	$e = 1.60 \times 10^{-19} \text{ C}$
the Planck constant	$h = 6.63 \times 10^{-34} \text{ J s}$
unified atomic mass constant	$u = 1.66 \times 10^{-27} \text{ kg}$
rest mass of electron	$m_e = 9.11 \times 10^{-31} \text{ kg}$
rest mass of proton	$m_p = 1.67 \times 10^{-27} \text{ kg}$
molar gas constant	$R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
the Avogadro constant	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
the Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
gravitational constant	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
acceleration of free fall	$g = 9.81 \text{ m s}^{-2}$

Formulae

uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$
moment of inertia of rod through one end	$I = \frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	$I = \frac{1}{2}M(r_1^2 + r_2^2)$
moment of inertia of solid sphere through centre	$I = \frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	$I = \frac{2}{3}MR^2$
work done on/by a gas	$W = p\Delta V$
hydrostatic pressure	$p = \rho gh$
gravitational potential	$\phi = -Gm/r$
Kepler's third law of planetary motion	$T^2 = \frac{4\pi^2 a^3}{GM}$
temperature	$T / \text{K} = T / ^\circ\text{C} + 273.15$
pressure of an ideal gas	$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle$

mean translational kinetic energy of an ideal gas molecule

$$E = \frac{3}{2}kT$$

displacement of particle in s.h.m.

$$x = x_0 \sin \omega t$$

velocity of particle in s.h.m.

$$v = v_0 \cos \omega t$$

$$= \pm \omega \sqrt{x_0^2 - x^2}$$

electric current

$$I = Anvq$$

resistors in series

$$R = R_1 + R_2 + \dots$$

resistors in parallel

$$1/R = 1/R_1 + 1/R_2 + \dots$$

capacitors in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

capacitors in parallel

$$C = C_1 + C_2 + \dots$$

energy in a capacitor

$$U = \frac{1}{2}CV^2$$

electric potential

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

electric field strength due to a long straight wire

$$E = \frac{\lambda}{4\pi\epsilon_0 r}$$

electric field strength due to a large sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

alternating current/voltage

$$x = x_0 \sin \omega t$$

magnetic flux density due to a long straight wire

$$B = \frac{\mu_0 I}{2\pi d}$$

magnetic flux density due to a flat circular coil

$$B = \frac{\mu_0 NI}{2r}$$

magnetic flux density due to a long solenoid

$$B = \mu_0 nI$$

energy in an inductor

$$U = \frac{1}{2}LI^2$$

RL series circuits

$$\tau = \frac{L}{R}$$

RLC series circuits (underdamped)

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

radioactive decay

$$x = x_0 \exp(-\lambda t)$$

decay constant

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$$

Section A

Answer **all** questions in this section.

- 1 Ball P strikes ball Q elastically in a head-on collision. The mass of ball Q is twice that of the ball P.
- (a) By using the centre of mass frame, determine the ratio of the final speed of the ball Q to the initial speed of the ball P.

ratio = [3]

- (b) Determine the percentage of initial kinetic energy transferred to the ball Q.

percentage = [2]

- (c) Determine the number of such collisions that is required to reduce the energy of the ball P from 10 kJ to 0.10 mJ.

number of collisions = [2]

- 2 (a) (i) A spherical asteroid of radius R is composed entirely of homogeneous titanium of density ρ .

A narrow shaft is bored from a point A on the surface of the asteroid to its centre B as shown in Fig. 2.1.

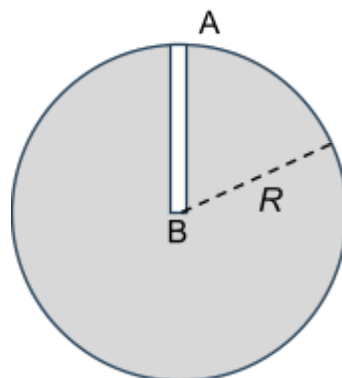


Fig. 2.1

Show that the magnitude of the gravitational force acting on a particle of mass m at a distance r from the centre of the asteroid (where $r < R$) is given by

$$F = \frac{4}{3}G\pi\rho mr$$

[2]

- (ii) A small particle is released from rest at point A and falls through the shaft to B. Determine the speed of the particle on arrival at B

[4]

- (b) Subsequently, mining removes material so that a spherical cavity of diameter $AB = R$ is formed within the asteroid, as shown in Fig. 2.2.

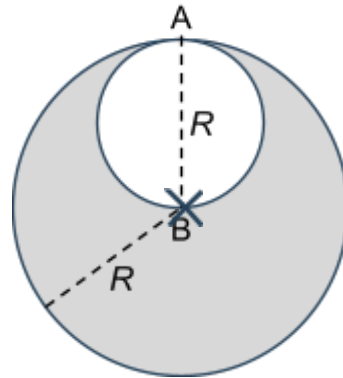


Fig. 2.2

- (i) By superposition, show that the gravitational field strength along AB is uniform and determine the magnitude of the field strength.

[2]

- (ii) A second particle is released from rest at A and falls to B. Determine the ratio of its impact speed at B to that found in (a)(ii).

[2]

- (c) A rocket ship with mass M_0 and loaded with fuel of mass m_0 takes off vertically in a uniform gravitational field. It ejects fuel with velocity U_0 with respect to the rocket ship. The ship is completely ejected during a time t_0 .

(i) Derive an equation of motion of the rocket in terms of $\frac{dM}{dt}$, U_0 , g and M , where M is the mass of the rocket at time t .

[2]

- (ii) Hence, determine the velocity of the vehicle at the instant t_0 when all the fuel has been ejected, in terms of M_0 , m_0 , g and t_0 .

[3]

- 3 (a) A parallel plate capacitor has capacitance C .

By considering the work dw needed to transport a small amount of charge dq from one plate to another, derive an expression for the electric potential energy U stored in the parallel plate capacitor in terms of the charge Q stored in the capacitor and its capacitance C . Show your working clearly.

[3]

- (b) In the circuit shown in Fig. 3.1, you may assume that the capacitors, battery and inductors have no resistance. The capacitors are initially uncharged, and the switch S has been set in position 1 until a steady-state is reached.

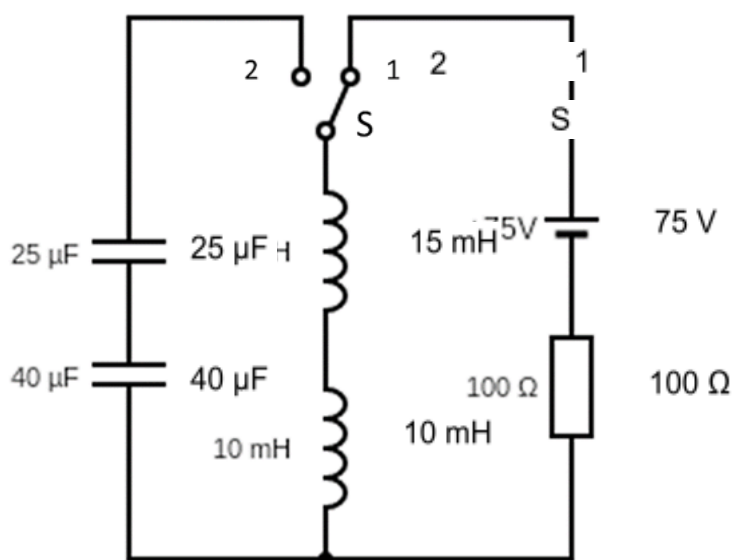


Fig. 3.1

- (i) State how energy is stored in an inductor.

.....
..... [1]

- (ii) Calculate the current in the circuit.

current = A [1]

- (iii) The switch is now flipped to position 2.

Calculate the maximum charge that each capacitor will receive.

maximum charge = C [3]

- 4 In 1784, George Atwood, a Cambridge mathematician, designed an apparatus to test Newton's laws of motion. Atwood's machine used two nearly equal masses connected by a light string over a pulley, with $m_2 > m_1$ so that the net accelerating force was small as shown in Fig. 4.1.

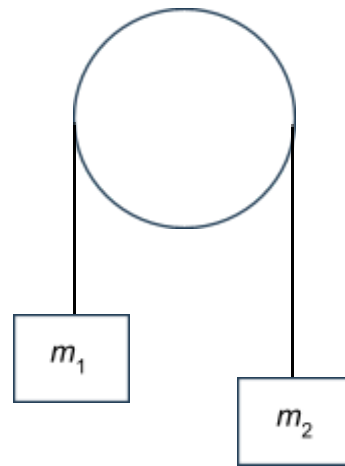


Fig. 4.1

This allowed motion that was slow enough to time accurately with the pendulum clocks of his era, providing the first quantitative laboratory verification of $F = ma$.

- (a) Assuming the pulley is fixed (i.e. not accelerating), derive expressions for:
- (i) the acceleration a of the masses, and

[3]

- (ii) the tension T in the string.

[2]

- (iii) In (a)(i), it was assumed the string is massless. Suppose instead that the string has a significant mass, and that the pulley is still massless and frictionless, and the string is inextensible.

State and explain how the acceleration in (a)(i) would be different if the string has mass.

.....
..... [1]

- (b) (i) Suppose now that the entire apparatus is inside an elevator which is descending with constant acceleration α .

Derive an expression for the tension in terms of m_1 , m_2 , g , and α .

[3]

- (ii) With reference to the expression in (b)(i), state what would happens to the tension as $\alpha \rightarrow g$.

.....
..... [1]

- (c) In a modified Atwood machine, two blocks of unequal masses are connected by a string over a smooth pulley as shown in Fig. 4.2.

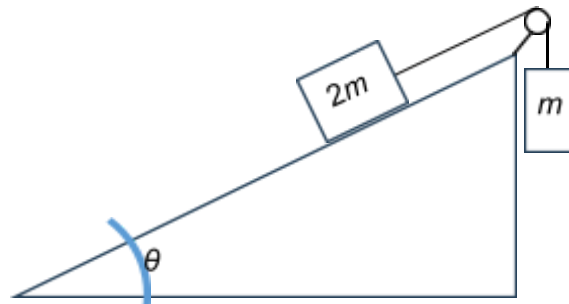


Fig. 4.2

If the coefficient of kinetic friction is μ_k , determine the angle θ of the incline that would allow the masses to move at a constant speed.

[5]

- 5 (a) (i)** When a sound wave passes along a straight pipe of constant cross-section, energy is lost from the wave.

The rate of loss of energy in each element Δx is proportional to the intensity of the wave at the beginning of that element. The constant of proportionality is k .

Derive an expression for the intensity I of the wave at a distance x along the pipe in terms of the intensity I at the source end where $x = 0$, k and x .

[3]

- (ii) When handling data with wide range of values of sound intensity, audio engineers sometimes use logarithms. Instead of just using the intensity I of a sound, it can be more convenient to use sound level β , defined as

$$\beta = (10 \text{ dB}) \log \log \frac{I}{I_0}$$

dB is the abbreviation for decibel, the unit for sound level, a name that was chosen to recognize the work of Alexander Graham Bell.

An audio engineer placed a listener and a speaker in a noisy room. The listener 1.20 m away from the speaker just barely discern a sound from the speaker. The speaker has fixed power and deliver sound uniformly in all directions.

The background noise is then increased by $\Delta\beta = 5 \text{ dB}$. The sound level at the listener must also increase in order to discern the same sound. To do that, the listener moves closer to the speaker.

Determine the new separation between the listener and the speaker so that the listener is able to discern the sound again.

new separation = cm [3]

- (b) In Fig. 5.1, sound with a 40.0 cm wavelength travels rightward from a source and through a tube that consists of a straight portion and a half-circle. Part of the sound wave travels through the half-circle and then joins the rest of the wave which goes directly through the straight portion.

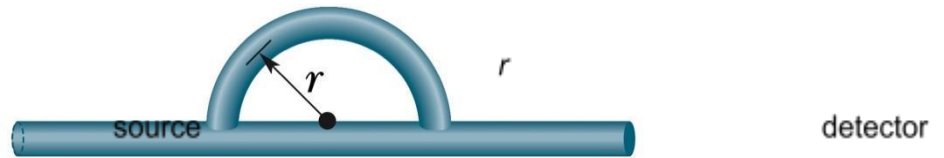


Fig. 5.1

Assume there is no loss in energy as the sound travels through the pipes.

Determine the smallest radius of the half-circle that results in an intensity minimum at the detector.

radius = cm [4]

- (c) Time-dependent variations of the displacements due to two sound waves of equal amplitude A can be written as

$$s_1 = A \cos \omega_1 t$$

$$s_2 = A \cos \omega_2 t$$

where ω_1 and ω_2 are the angular frequencies of the two sounds.

- (i) When the two waves meet in phase, the resultant amplitude s can be written as

$$s = 2A \cos \omega' t \cos \omega t$$

Determine ω' and ω in terms of ω_1 and ω_2 .

You may use the identity

$$\cos \alpha + \cos \beta = 2 \cos \left[\frac{1}{2}(\alpha + \beta) \right] \cos \left[\frac{1}{2}(\alpha - \beta) \right]$$

$$\omega' = \dots\dots\dots$$

$$\omega = \dots\dots\dots$$

[1]

- (ii) In a particular experiment, angular frequencies of the two sounds are set such that $\omega_1 \approx \omega_2$ but $\omega_1 \neq \omega_2$.

Using your answer in (c)(i), making reference to the frequency, state and explain the resultant sound you will hear. Explain your working clearly.

.....

.....

.....

..... [4]

Section B

Answer **all** questions in this section.

- 6 (a) A thin, uniform metal bar of length $L = 2.00$ m and mass $m_1 = 9.17$ kg is hanging vertically from the ceiling by a frictionless pivot as shown in Fig. 6.1.

It is struck at a distance $d = 1.50$ m below the ceiling by a small ball of $m_2 = 3.0$ kg, initially traveling horizontally at a speed $v_0 = 10 \text{ m s}^{-1}$. The ball rebounds in the opposite direction with a speed of $v = 6.0 \text{ m s}^{-1}$.

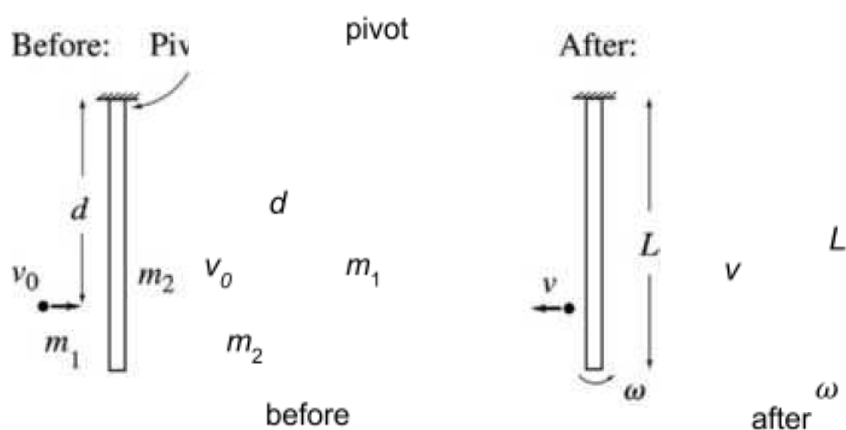


Fig. 6.1

- (i) Show from first principles, that the moment of inertia of a uniform rod of length L and mass M about an axis through one end of the bar is given by

$$I = \frac{1}{3} ML^2$$

[3]

- (ii) During the collision, explain whether the following quantities of the system are conserved.

1. linear momentum

.....
.....

2. angular momentum

.....
..... [2]

- (iii) Using the expression in (a)(i), determine the angular speed of the bar just after the collision.

angular speed = rad s^{-1} [2]

- (b) If a person of mass M_p simply moved forward with speed V , her kinetic energy would be $\frac{1}{2} M_p V^2$. However, in addition to possessing a forward motion, various parts of her body (such as the arms and legs) undergo rotation as shown in Fig. 6.2. Therefore, her total kinetic energy is the sum of the energy from her forward motion plus the rotational kinetic energy of her arms and legs. The purpose of this problem is to see how much this rotational motion contributes to the person's kinetic energy.

Biomedical measurements show that the arms and hands together typically make up 13% of a person's mass, while the legs and feet together account for 37%. For a rough calculation, we can model the arms and legs as thin uniform bars pivoting about the shoulder and hip, respectively.



Fig. 6.2

In a brisk walk, the arms and legs each move through an angle of about $\pm 30^\circ$ (a total of 60°) from the vertical in approximately 1.0 second. We shall assume that they are held straight, rather than being bent. Consider a 75-kg person walking at 5.0 km per hour, having arms 70 cm long and legs 90 cm long.

- (i) Determine the average angular velocity of person's arms and legs.

average angular velocity = rad s^{-1} [1]

- (ii) Using the expression in **(a)(i)** and your answer in **(b)(i)**, calculate the amount of rotational kinetic energy in this person's arms and legs as he walks.

rotational kinetic energy = J [2]

- (iii) Determine the percentage of his kinetic energy is due to the rotation of his legs and arms.

percentage = % [2]

- (c) A 16-kg roll of paper with radius $R = 18.0$ cm rests against the wall and is held in place by a bracket attached to a rod through the centre of the roll as shown in Fig. 6.3. The rod turns without friction in the bracket, and the moment of inertia of the paper and rod about the axis is 0.260 kg m². The other end of the bracket is attached by a frictionless hinge to the wall such that the bracket makes an angle of 30° with the wall.

The weight of the bracket is negligible. The coefficient of kinetic friction between the paper and the wall is $\mu = 0.25$. A constant vertical force $F = 60.0$ N is applied to the paper, and the paper unrolls.

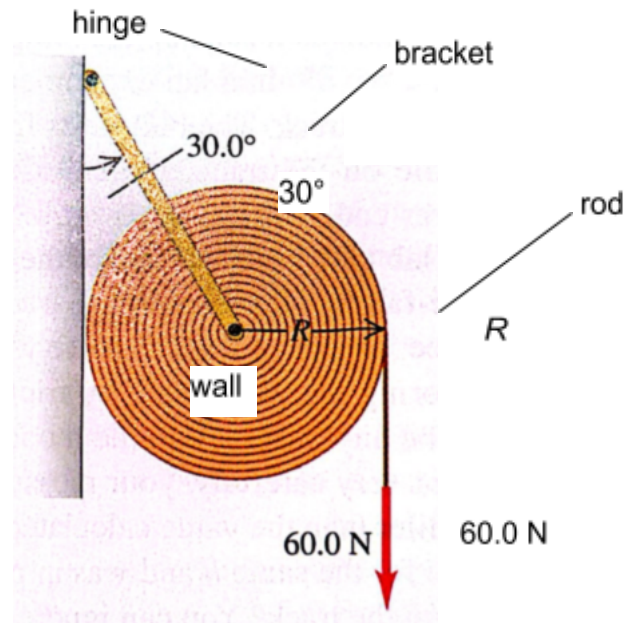


Fig. 6.3

- (i) Show that the magnitude of the force that the rod exerts on the paper as it unrolls is 293 N

[2]

- (ii) Determine the magnitude of the angular acceleration of the roll.

magnitude of the angular acceleration = rad s^{-2} [2]

- (iii) The vertical force F stops acting at the end of the roll of paper after 1.2 s. The paper continues to unroll but stops after a while.

Calculate the total length of paper that is unrolled when the setup comes to a rest. Assume the thickness of the paper is negligible compared to the radius of roll of paper.

total length of paper = m [4]

- 7 A circular parallel-plate capacitor of radius a and plate separation d is connected in series with a resistor R and a switch, initially open, to a constant voltage source V_0 as shown in Fig. 7.1.

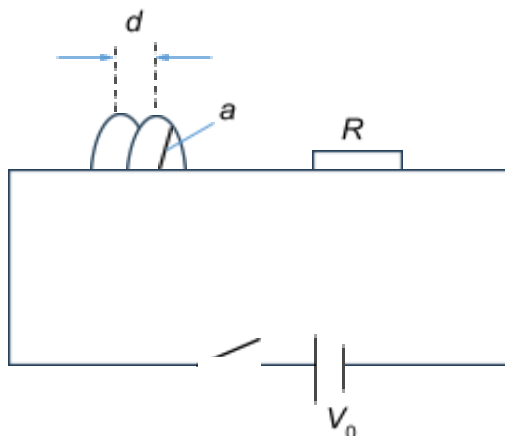


Fig. 7.1

The capacitance of the capacitor is C . Assume that $d \ll a$. The switch is closed at time $t = 0$.

- (a) By considering the potential difference in the circuit, show that the charge Q on the capacitor plates at time t is given by

$$Q = CV_0 \left(1 - e^{-\frac{1}{RC}t} \right)$$

[5]

(b) Charge density of the plate σ can be expressed as $\frac{\text{charge}}{\text{area}}$.

(i) State Gauss's law.

.....
.....
..... [2]

(ii) Use Gauss's law to show that the expression for charge density as a function of time can be written as

$$\frac{\text{charge}}{\text{area}} = \epsilon_0 E = E_0 \left(1 - e^{-\frac{1}{RC}t} \right)$$

Derive an expression for E_0 .

[4]

(c) Conduction currents produce an accompanying magnetic field in the vicinity of the current.

The British physicist James Clerk Maxwell in the 19th century predicted that a magnetic field also must be associated with a changing electric field even in the absence of a conduction current. He posited the idea of *displacement current* to explain magnetic fields that are produced by changing electric fields.

Take time derivative of the equation in **(b)** to derive an expression for the displacement current density j_d . This displacement current density is equal to the rate of change of $\epsilon_0 E$.

[2]

- (d)** Fig. 7.2 shows the electric field and an amperian loop of radius r between the two plates.

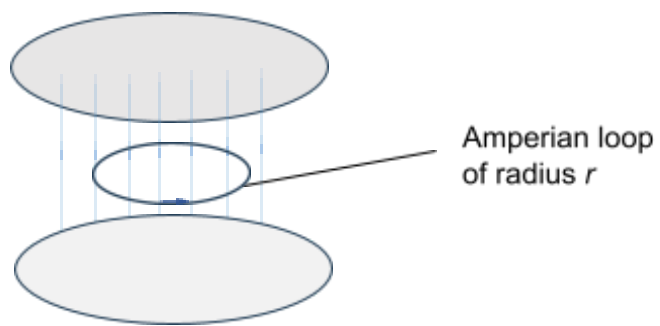


Fig. 7.2

The magnetic flux density as a function of time can be written as

$$B = B_0 e^{-\frac{1}{\tau}t}$$

Using Ampere's law to derive an expression for B_0 .

[4]

- (e) The circular plates have radius 4.00 cm and at a particular instant the conduction current in the wires is 0.280 A.

It can be assumed that conducting current equals to displacement current.

Calculate

- (i) the displacement current density in the air space between the plates, and

displacement current density = A m^{-2} [1]

- (ii) the induced magnetic flux density between the plates at a distance 2.00 cm from the axis.

induced magnetic flux density = T [2]

End of Paper