2025 J2 H3 Preliminary Examinations
Mark Scheme

Section A

Question	Answer	Marks
1(a)	$(m + 2m)v_{CM} = mv$ $v_{CM} = \frac{v}{3}$ In centre of mass frame (before collision), P: $v_p = v - \frac{v}{3} = \frac{2v}{3}$ Q: $v_Q = 0 - \frac{v}{3} = -\frac{v}{3}$ In centre of mass frame (after collision), P: $v_p = -\frac{2v}{3}$ Q: $v_Q = \frac{v}{3}$ In lab frame (after collision), P: $v_p = -\frac{2v}{3} + \frac{v}{3} = -\frac{v}{3}$ Q: $v_Q = \frac{v}{3} + \frac{v}{3} = \frac{2v}{3}$ *Ratio* = $\frac{2v}{3}/v = \frac{2}{3}$	C1 M1 A1
(b)	$v_p = -\frac{KE_Q}{KE_0} = \frac{\frac{1}{2}(2m)\left(\frac{2v}{3}\right)^2}{\frac{1}{2}(m)(v)^2} = \frac{8}{9} = 89\ \%$	M1 A1
(c)	After n collisions, $\left(\frac{1}{9}\right)^n \times 10\times10^3 = 0.10\times10^{-3}$ $n = 8.38$ 9 collisions are needed	M1 A1

Question	Answer	Marks
2(a)(i)	$F = \frac{GMm}{r^2}$ $= \frac{G\rho \frac{4}{3}\pi r^3 m}{r^2}$ $= \frac{4}{3}G\rho\pi mr$	M1 M1 A0
(ii)	Acceleration of the particle = $= \frac{\frac{4}{3}G\rho\pi mr}{m} = \frac{4}{3}G\rho\pi r$. Since acceleration is always directed to B, its direction is opposite to that of the displacement. By comparison to $a = -\omega^2 x$, $\omega^2 = \frac{4}{3}G\rho\pi \Rightarrow \omega = \sqrt{\frac{4}{3}G\rho\pi}$ Since motion is SHM, speed of particle upon arrival at B = ωR $= R\sqrt{\frac{4}{3}G\rho\pi}$	M1 M1 M1 A1
(b)(i)	Gravitational field strength along AB can be taken to the superposition of field strength due to the original sphere as well as that of a "negative" mass at where the cavity is. $g_{net} = g_{originalsphere} - g_{cavitymass}$ At a point *r* away from centre of cavity $g_{net} = \frac{4}{3}G\rho\pi\left(\frac{R}{2} + r\right) - \frac{4}{3}G\rho\pi(r)$ $= \frac{4}{3}G\rho\pi\frac{R}{2}$ $= \frac{2}{3}G\rho\pi R$	 A1 M1

	Since $G\rho\pi R$ are all constants, the field strength is uniform.	
(ii)	Since the field strength is uniform, the particle will be accelerating at a constant rate. $v^2 = u^2 + 2as$ $v^2 = 2\left(\frac{2}{3}G\rho\pi R\right)R$ $v = \sqrt{\frac{4}{3}G\rho\pi R}$ Hence the ratio is 1:1	M1 A1
(c)(i)	To determine an expression for the motion, we consider a time interval Δt during which the rocket has mass M, travelling with velocity v, and subsequently ejects ΔM of gas to the surroundings to gain a velocity of Δv. The change in momentum during time interval Δt is therefore $(M - \Delta M)(v + \Delta v) - (u_0 + v)\Delta M - Mv = -Mg\Delta t$ As time interval becomes smaller, we get the following. $M\frac{dv}{dt} = -u_0\frac{dM}{dt} - Mg$	M1 A1
(ii)	By rearranging, we obtain the following expression $dv = -u_0\frac{dM}{M} - gdt$ $v = -u_0 \ln M - gt + K$ Since $M = M_0 + m_0$, and $v = 0$ at $t = 0$, $K = u_0 \ln(M_0 + m_0)$ $v = -u_0 \ln M_0 - gt_0 + u_0 \ln(M_0 + m_0)$ $= u_0 \ln\frac{(M_0 + m_0)}{M_0} - gt_0$	M1 M1 A1

Question	Answer	Marks
3(a)	$dw = Vdq.$ where V is the potential difference across the capacitor The total work done in charging up the capacitor is then given by: $W = \int_0^Q Vdq$ $W = \int_0^Q \frac{Q}{C}dq$ The electric potential energy stored is equal to the total work done in charging the capacitor, $U_i = \int_0^Q \frac{Q}{C}dq = \frac{Q^2}{2C}$	M1 M1 A1
(b)(i)	An inductor stores energy in the <u>magnetic field</u> generated by the current.	B1
(b)(ii)	At a steady-state, the inductors have no effect on the circuit as $\frac{dI}{dt} = 0$ Thus the current is that of a circuit with a battery and a resistor: $E = Ir$ $I = \frac{E}{r} = \frac{75}{100} = 0.75\ A$	 A1
(b)(iii)	There are now 2 capacitors in the circuit which are in series. We can find the effective capacitance: $\frac{1}{C_s} = \frac{1}{40x10^{-6}} + \frac{1}{25x10^{-6}}$ $\therefore C_s = 15.4$ μF For the 2 inductors in series: L_s = 15 +10 = 25 mH Using conservation of energy, the maximum energy that will be stored on the effective capacitance is equal to the maximum energy stored on the effective inductance at the maximum current. $\frac{q^2}{2C_s} = \frac{1}{2}L_s I^2$	 M1 M1

	$q = I\sqrt{L_s C_s}$ $=0.75\sqrt{25x10^{-3}x15.4x10^{-6}}$ $= 4.7 \times 10^{-4}$ C	A1

Question	Answer	Marks
4(a)(i)	$m_2 g - T = m_2 a$ $T - m_1 g = m_1 a$ $a = \frac{(m_2 - m_1)}{(m_2 + m_1)} g$	M1 M1 A1
(a)(ii)	$T - m_1 g - = m_1 \left(\frac{(m_2 - m_1)}{(m_2 + m_1)} g \right)$ $T = \frac{2m_1 m_2}{(m_2 + m_1)} g$	M1 A1
(a)(iii)	If the string has mass, <u>the total inertia of the system would be greater</u> and the net force would be exerted on the total mass of the system. The acceleration will therefore be $a = \frac{(m_2 - m_1)}{(m_2 + m_1 + m_{string})} g$ And hence acceleration will be <u>smaller.</u>	M0 A1
(b)(i)	$m_2 (g - \alpha) - T = m_2 a$ $T - m_1 (g - \alpha) = m_1 a$ $a = \frac{(m_2 - m_1)}{(m_2 + m_1)} (g - \alpha)$	M1 M1 A1

	$T = \frac{2m_1m_2}{(m_2+m_1)}(g-\alpha)$	
(b)(ii)	As $\alpha \rightarrow g$, the tension will tends to zero.	B1
(c)	Consider a point where the system is moving at constant speed For the hanging mass, $T = mg$ For mass on the slope, $N = 2mg\cos\theta$ $T - 2mg\sin\theta$ - $f = 0$ $mg - 2mg\sin\theta$ - $2\mu_k mg\cos\theta = 0$ $1 - 2\sin\theta$ - $2\mu_k\cos\theta = 0$ $\sin\theta = \frac{1 \pm \mu_k\sqrt{3+4\mu_k^2}}{2(1+\mu_k^2)}$	M1 M1 M1 M1 A1
Question	**Answer**	**Marks**
5(a)(i)	let change in intensity be ΔI, then we are given $\frac{\Delta I}{\Delta x} \propto -I$ or $\frac{\Delta I}{\Delta x} = -kI$; −ve sign is introduced because energy is lost	B1
	rearranging to form differential equation, $$\int_{I_0}^{I}\frac{1}{I}dI = -k\int_0^x dx$$ B1: forming d.e. B1: with correct limits	B2
	solving to get $I = I_0e^{-kx}$	A1
(ii)	$\Delta\beta = 5 = 10\log log \frac{I_f}{I_i}$, or $\log log \frac{I_f}{I_i} = \frac{1}{2}$ giving $\frac{I_f}{I_i} = \sqrt{10}$	M1
	Using $I \propto \frac{1}{r^2}$ or $\frac{I_f}{I_i} = \left(\frac{r_i}{r_f}\right)^2 = \sqrt{10}$	B1
	giving $r_f = \frac{1}{10}^{\frac{1}{4}} \times 1.2 = 0.67$ m	A1
5(b)	path difference = $\pi r - 2r$	B1
	We want smallest radius that produce minimum intensity, phase difference = π	B1
	$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi$ gives $\pi = \frac{\pi r - 2r}{\lambda} \times 2\pi$	M1

	$r = \frac{\lambda}{2(\pi-2)} = \frac{40.0}{2(\pi-2)} = 17.5$ cm	A1
5(c)(i)	$\omega_1 t + \omega_2 t = 2\left[\frac{1}{2}\left(\omega_1 - \omega_2\right)\right] \cos cos \left[\frac{1}{2}\left(\omega_1 + \omega_2\right)\right]$ $\omega' = \frac{1}{2}(\omega_1 - \omega_2)$ $\omega = \frac{1}{2}(\omega_1 + \omega_2)$	A1
(ii)	$\omega \gg \omega'$	M1
	since $\omega' = \frac{1}{2}(\omega_1 - \omega_2)$ $f' = \frac{1}{2}(f_1 - f_2)$ or $2f' = f_1 - f_2$	B1
	Hear a louder (4 times intensity) sound, since intensity ∝ amplitude^2	B1
	with frequency $= f_1 - f_2$	B1

Section B

Question	Answer	Marks
6(a)(i)	Consider a ring of mass *dm*, radius *r* and thickness *dr*. The mass of the ring is $dm = \lambda dx = \frac{M}{L}dx$, where λ is mass per unit length Since $r^2 = x^2$, $I_{rod\ at\ centre} = \int_{-L/2}^{L/2} r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{1}{12}ML^2$ Using parallel axis theorem, $I_{rod\ at\ end} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$	C1 M1 A1
(a)(ii)	There are no unbalanced torques about the pivot so angular momentum is conserved. But the pivot exerts an unbalanced horizontal external force on the system, so the linear momentum is not conserved.	B1 B1
(a)(iii)	$m_2 v_0 d = - m_2 vd + \frac{1}{3}m_1 L^2 \omega$ $(3.00)(10.0)(1.50) = - (3.00)(6.00)(1.50) + \frac{1}{3}(9.17)(2.00)^2\omega$ $\omega = 5.88$ rad/s	M1 A1
(b)(i)	$60° = \frac{\pi}{3}\ rad$ average angular velocity = $\frac{\frac{\pi}{3}\ rad}{1\ second}$ = 1.05 rad s^{-1}	A1
(b)(ii)	$I = \frac{1}{3}m_{arm}L_{arm}^{\ \ 2} + \frac{1}{3}m_{leg}L_{leg}^{\ \ 2}$ $I = \frac{1}{3}(0.13)(75)(0.70)^2 + \frac{1}{3}(0.37)(75)(0.90)^2 = 9.08\ kg\ m^2$ Rotational KE = $\frac{1}{2}I\omega^2 = \frac{1}{2}(9.08)(1.05)^2 = 5.0\,J$	M1 A1
(b)(iii)	Translation KE = $\frac{1}{2}mv^2 = \frac{1}{2}(75)(1.4)^2 = 72.34\,J$ Total KE = $72.35 + 5.0 = 77.34\,J$ percentage of his kinetic energy is due to the rotation = $\frac{5.0}{77.34} = 6.46\%$	M1 A1

(c)(i)	Balancing vertical forces, $F_{rod}\cos\theta = f + w + F$ $F_{rod}\sin\theta = n$ Since $f = \mu_k n$, $F_{rod}\cos\theta = \mu_k n + w + F$ $F_{rod}\cos\theta = \mu_k(F_{rod}\sin\theta) + w + F$ $F_{rod} = \frac{w+F}{\cos\theta - \mu_k \sin\theta} = \frac{(16.0)(9.8)+60.0}{\cos 30-(0.25)\sin 30} = 293\ N$	M1 A1
(c)(ii)	$f = \mu_k n$ $= \mu_k F_{rod}\sin\theta$ $= (0.25)(293)\sin 30$ $= 36.625\ N$ $\alpha = \frac{\tau}{I} = \frac{(60.0-36.625)(18\times10^{-2})}{0.260} = 16.2\ rad/s^2$	M1 A1
(c)(iii)	When the force is applied on the paper for 1.2 s, $\theta_1 = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(16.2)(1.2)^2 = 11.66\ rad$ $\omega_f = \omega_i + \alpha t = 0 + (16.2)(1.2) = 19.44\ rad\ s^{-1}$ When the force is removed, $\alpha_2 = \frac{\tau}{I} = \frac{(-36.625)(18\times10^{-2})}{0.260} = -25.36\ rad/s^2$ $0 = 19.44 + (-25.36)t$ $t = 0.7666\ s$ $\theta_2 = (19.44)(0.7666) + \frac{1}{2}(-25.36)(0.7666)^2 = 7.451\ rad$ Total length = $(11.66 + 7.451)(18\times10^{-2})$ = 3.44 m	M1 C1 M1 A1

Question	Answer	Marks
7(a)	$V_0 = \frac{Q}{C} + IR$	B1
	Write $I = \frac{dQ}{dt}$, arriving at differentiate equation $\frac{V_0}{R} = \frac{1}{RC} Q + \frac{dQ}{dt}$	B1
	Rearranging give $\int_0^t dt = \int_0^Q \frac{1}{\frac{V_0}{R} - \frac{1}{RC} Q} dQ$ Correct limits	B1
	Correct method of integration Example: $t - 0 = \ln(- RC) ln\left(\frac{\frac{V_0}{R} - \frac{1}{RC} Q}{\frac{V_0}{R}}\right)$ $\frac{V_0}{R} e^{-\frac{1}{RC}t} = \frac{V_0}{R} - \frac{1}{RC} Q$ Making Q the subject to get $Q = CV_0\left(1 - e^{-\frac{t}{RC}}\right)$ B1: correct integration B1: accounting the limits	B2
7(b)(i)	$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$	B1
	$\oint$: surface integral • : dot product Q : charge enclosed by surface	B1
(ii)	Gaussian surface: cylinder of radius a, with area normal to $\vec{E}$	C1
	LHS: $\oint E \cdot dA = E\pi a^2$ RHS: $\frac{Q}{\varepsilon_0} = \frac{CV_0\left(1 - e^{-\frac{t}{RC}}\right)}{\varepsilon_0}$	B1
	Equating LHS and RHS, and rearrange $\varepsilon_0 E = \frac{CV_0}{\pi a^2}\left(1 - e^{-\frac{t}{RC}}\right)$	M1
	$E_0 = \frac{CV_0}{\pi a^2}$	A1
(c)	Differentiating wrt time gives $\frac{CV_0}{\pi a^2}\left(-\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}\right)$	M1
	So $j_d = \frac{V_0}{\pi a^2 R} e^{-\frac{t}{RC}}$	A1

(d)	$\oint B \cdot dl = \mu_0 I$ gives $B 2\pi r = \mu_0 \left(j_d \pi r^2 \right)$	B1
	substituting j_d, $B 2\pi r = \mu_0 \left(\frac{V_0}{\pi a^2 R} e^{-\frac{t}{RC}} \right) \pi r^2$	M1
	rearranging to get $B = \frac{\mu_0 r}{2} \left(\frac{V_0}{\pi a^2 R} e^{-\frac{t}{RC}} \right)$	M1
	So $B = \frac{\mu_0 r}{2\pi a^2} \frac{V_0}{R}$	A1
(e)(i)	$j_d = \frac{0.280}{\pi 0.0400^2} = 55.7\, A\, m^{-2}$	B1
(ii)	$B 2\pi r = \mu_0 j_d \pi r^2$ gives $B = \frac{\mu_0 j_d r}{2}$	M1
	$B = \frac{4\pi \times 10^{-7} \times 55.7 \times 0.02}{2} = 7.00 \times 10^{-7}$ T	A1
Ref	adapted from Physics by Example Rees 29.42	